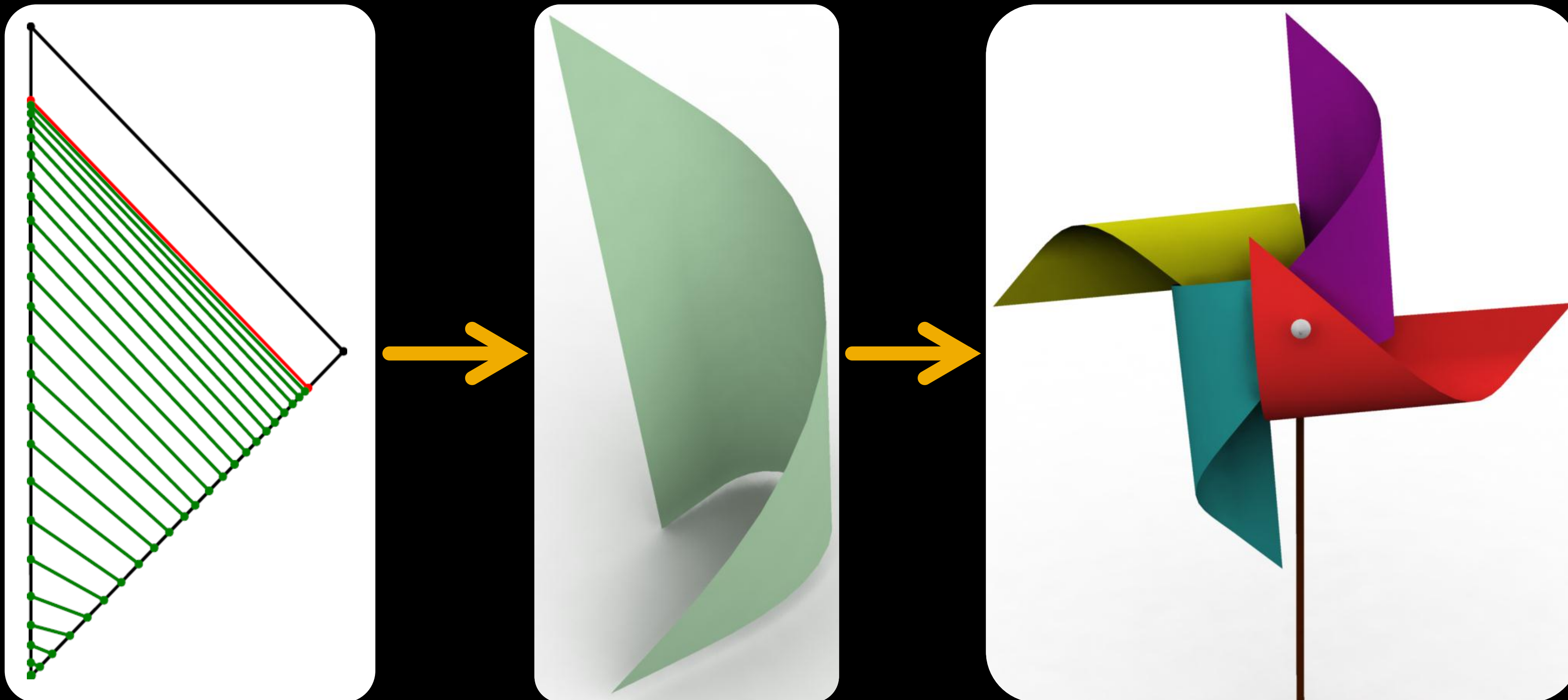


Flexible Developable Surfaces

Justin Solomon
Stanford University

Etienne Vouga
Columbia University

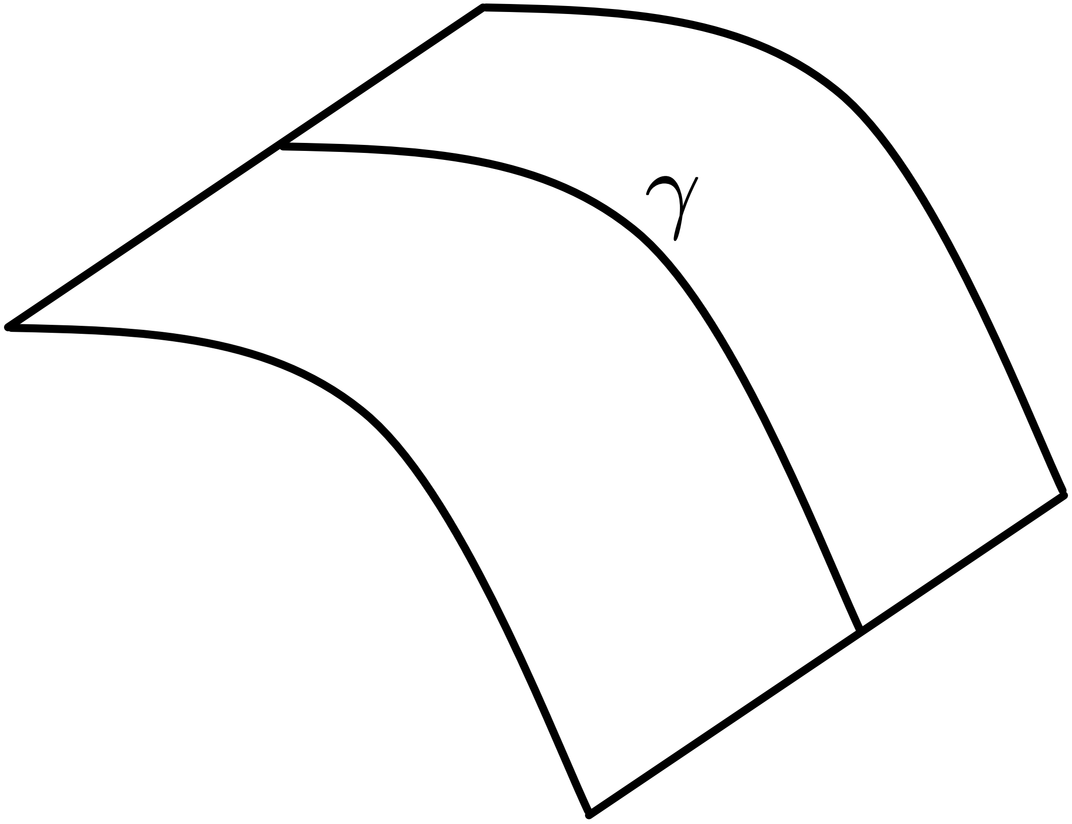
Eitan Grinspun
Columbia University

Max Wardetzky
Univ. of Göttingen

Isometric Deformations

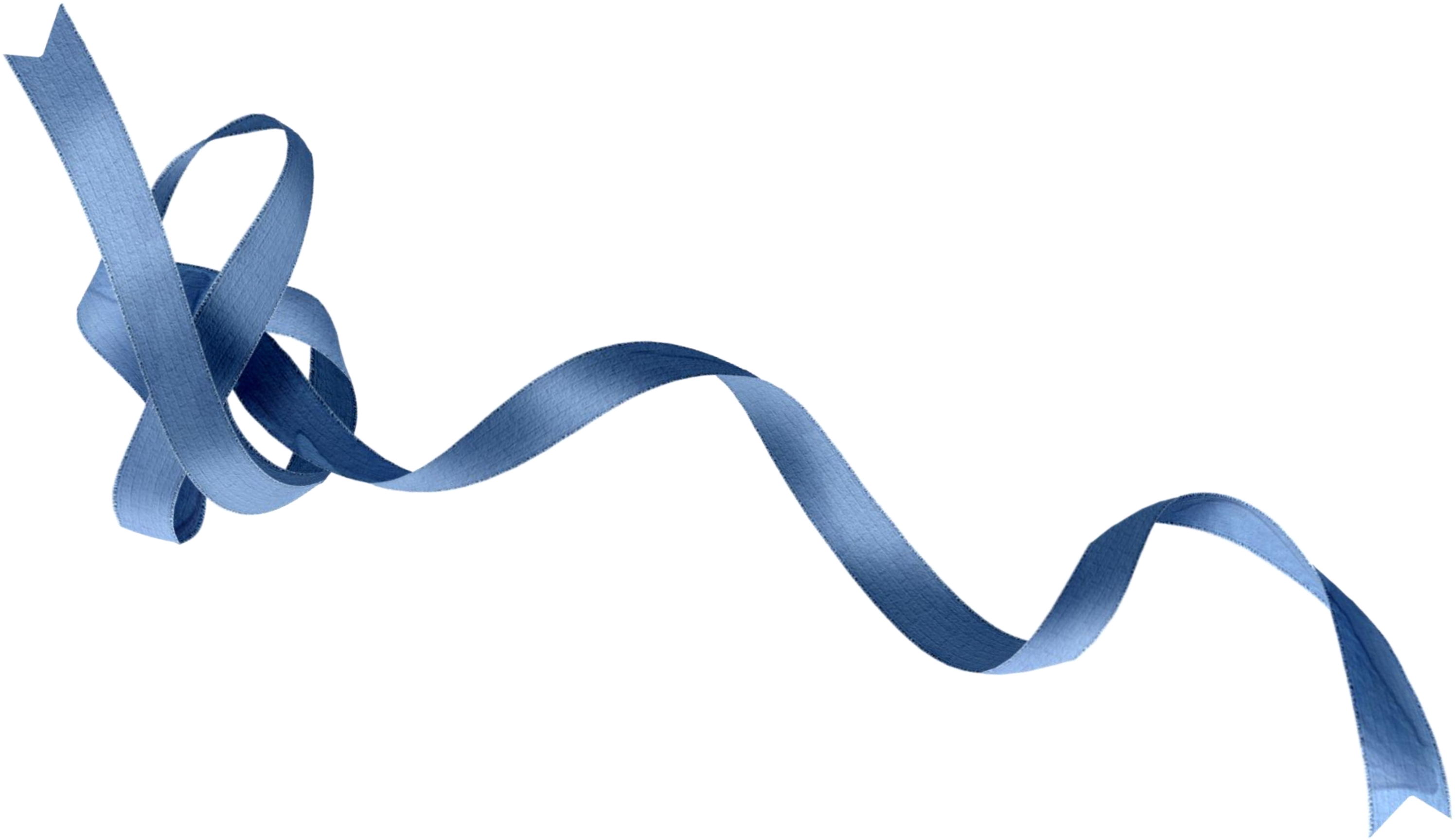
Bending without stretching

Isometric Deformations



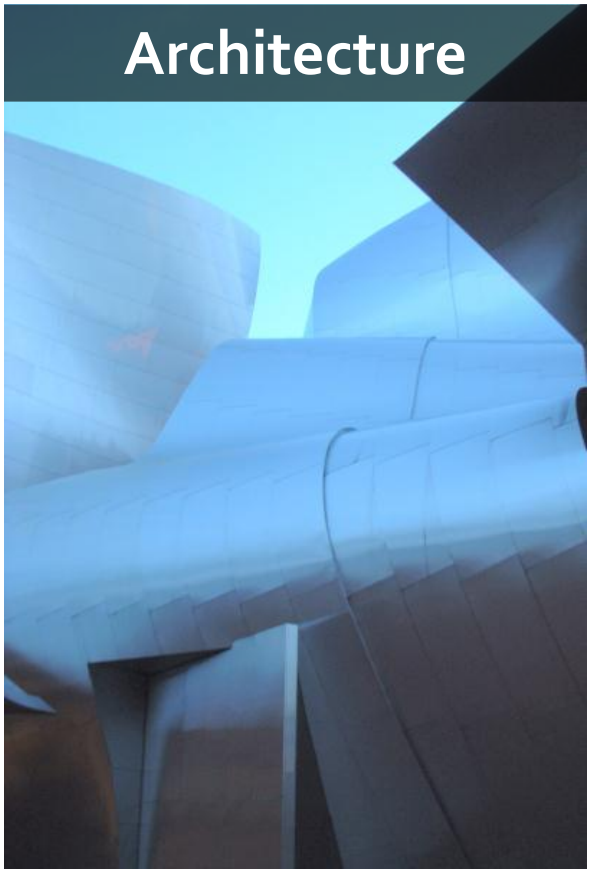
Preserve lengths of curves

Developable Surfaces



Can flatten onto the plane

Developable Surfaces



http://www.trekearth.com/gallery/Europe/Germany/West/Nordrhein-Westfalen/Herford/photo310089.htm

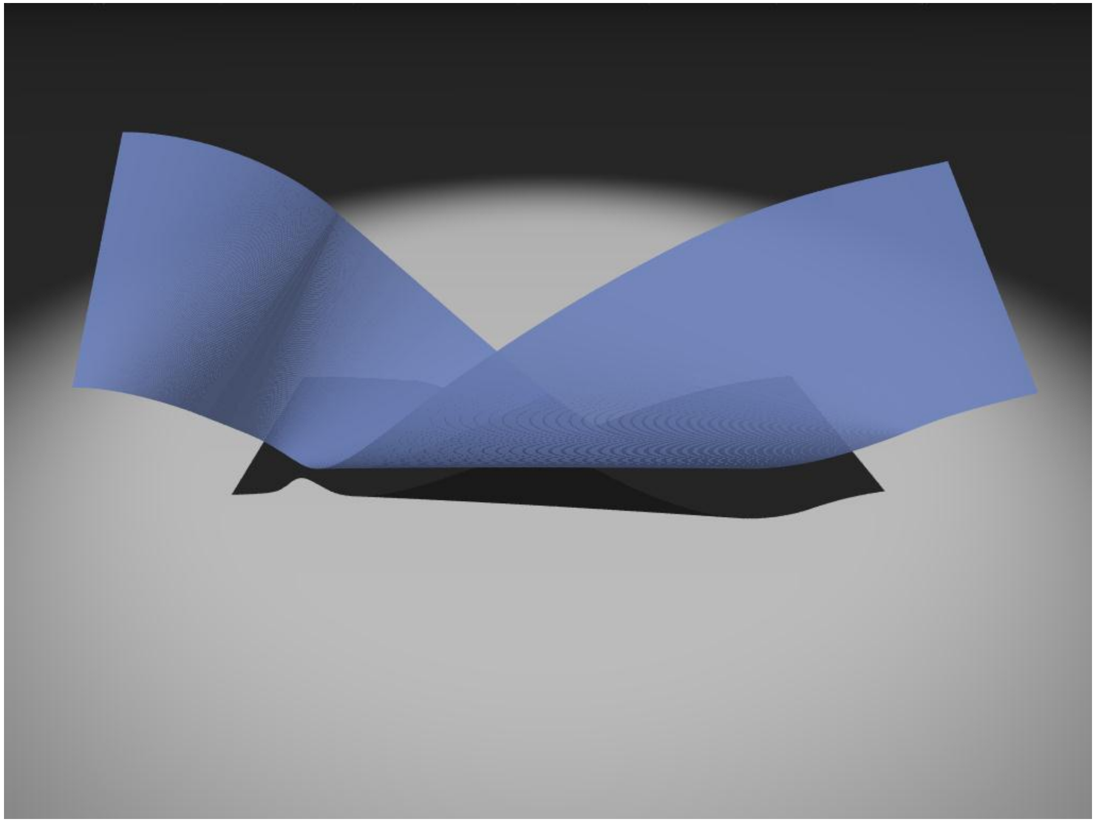
http://www.otherlab.com/news/2012/01/23/developable-surface-bike/

http://www.matterdesignstudio.com/2009/07/drawn-dress-fashioning-digital-fabrication/

Can flatten onto the plane

Developable Surfaces

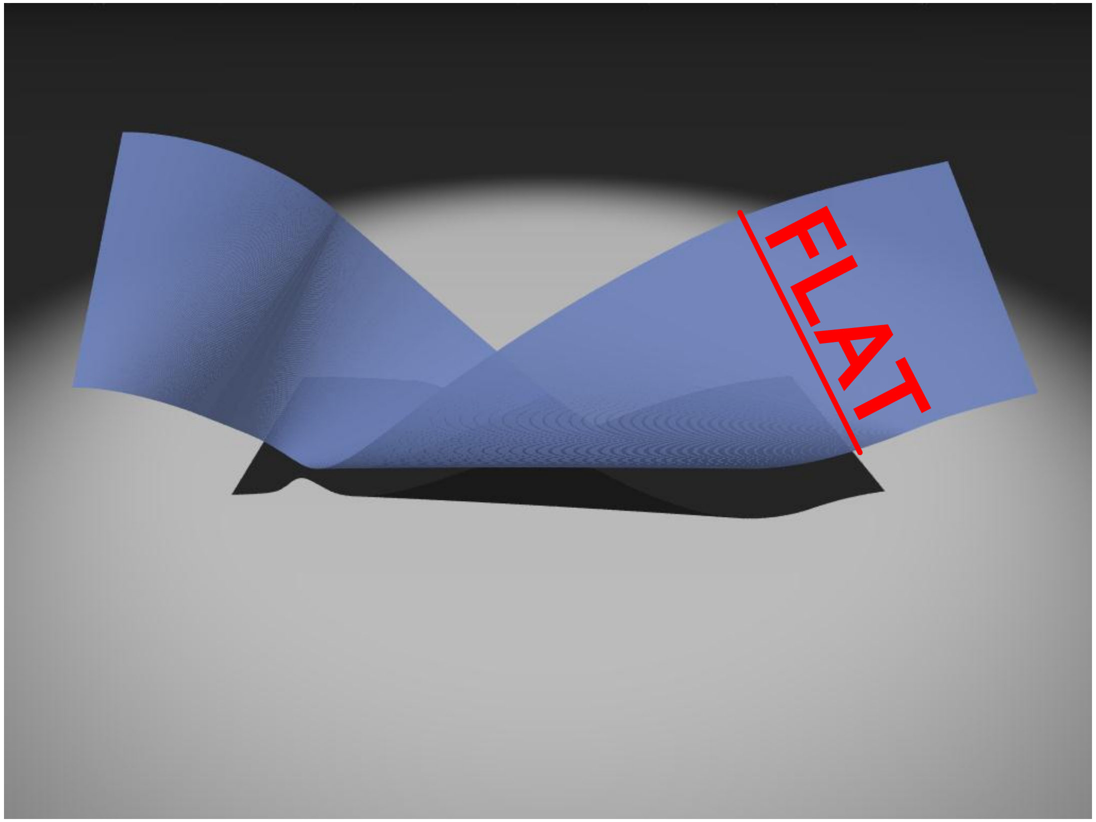
$$\boxed{K = \kappa_1 \kappa_2 \equiv 0}$$



http://graphics.stanford.edu/~niloy/research/folding/paper_docs/folding_sig_08.pdf

Developable Surfaces

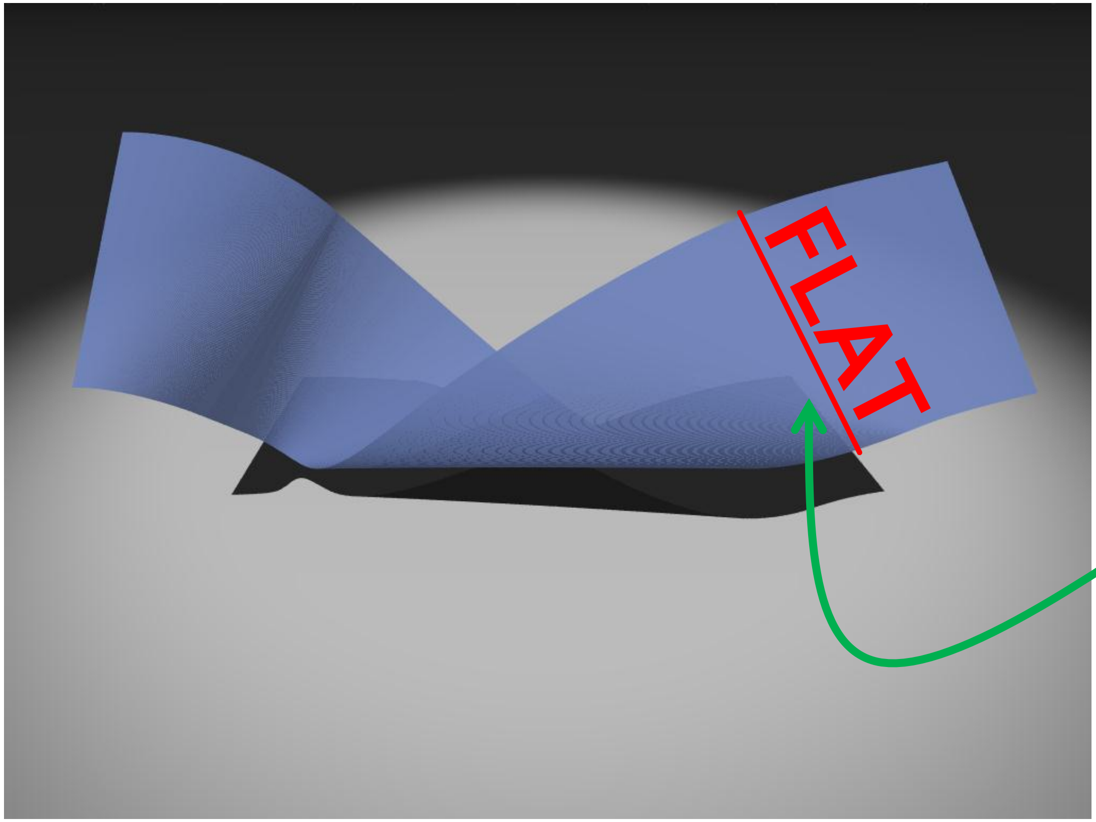
$$\boxed{\kappa_1 \equiv 0 \text{ or } \kappa_2 \equiv 0}$$



http://graphics.stanford.edu/~niloy/research/folding/paper_docs/folding_sig_08.pdf

Ruled Developable Surfaces

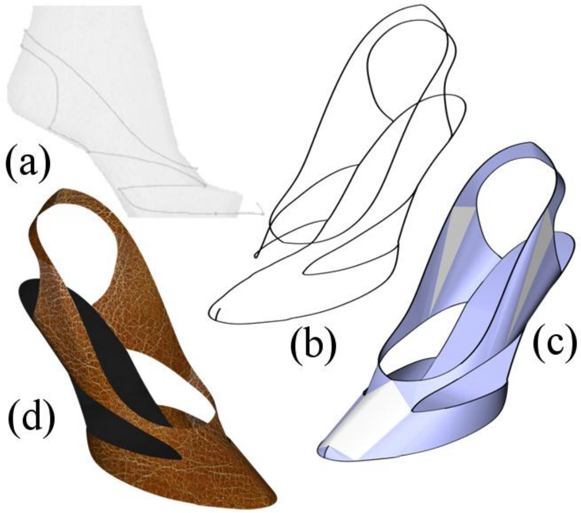
$$\boxed{\kappa_1 \equiv 0 \text{ or } \kappa_2 \equiv 0}$$



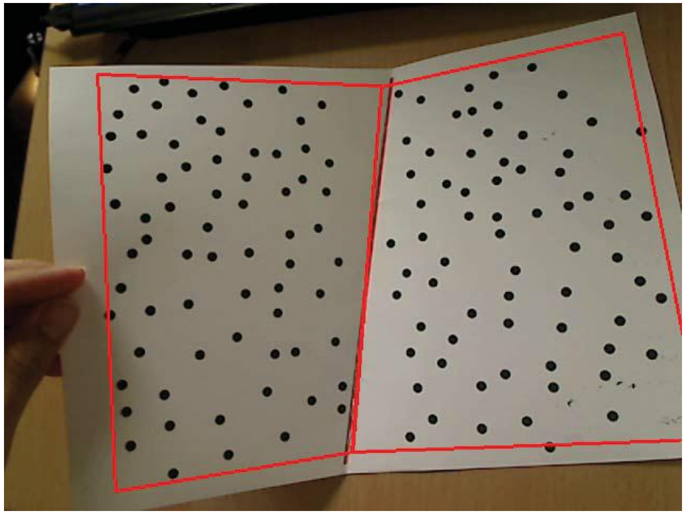
Ruling direction

http://graphics.stanford.edu/~niloy/research/folding/paper_docs/folding_sig_08.pdf

Previous Work



Synthesis



Approximation

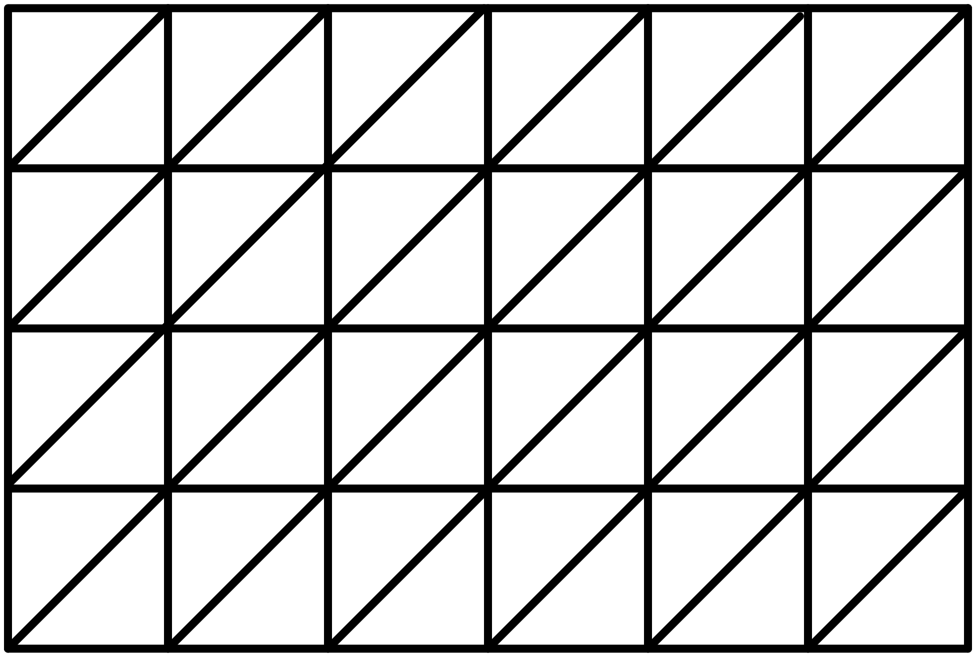
Deformation

http://ljk.imag.fr/membres/Boris.Thibert/publications/SGP07.pdf
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5960933
http://www.springerlink.com/content/j31444p7v7611256/fulltext.pdf

Locking

Failure of a discretization to navigate the full space of isometric deformations of a developable surface smoothly

Locking

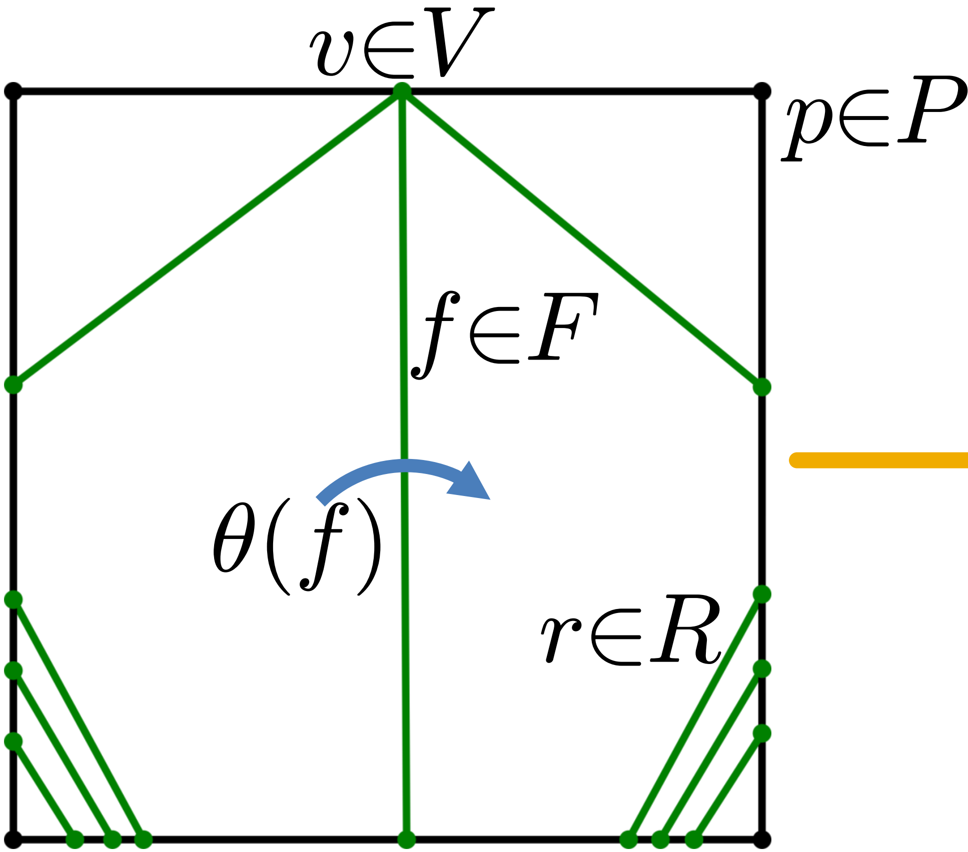


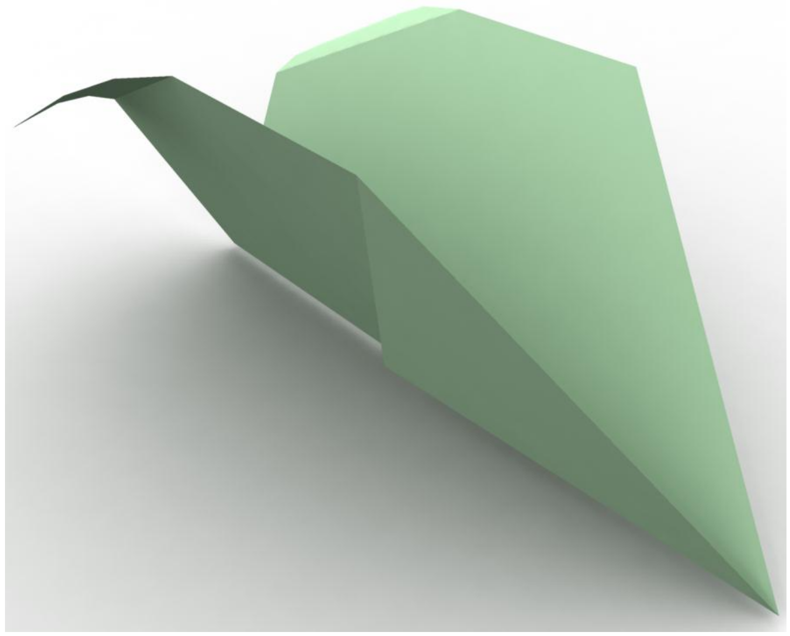
Fixing edge lengths maintains flexibility along edges but makes surface stiff in other directions.

Goal

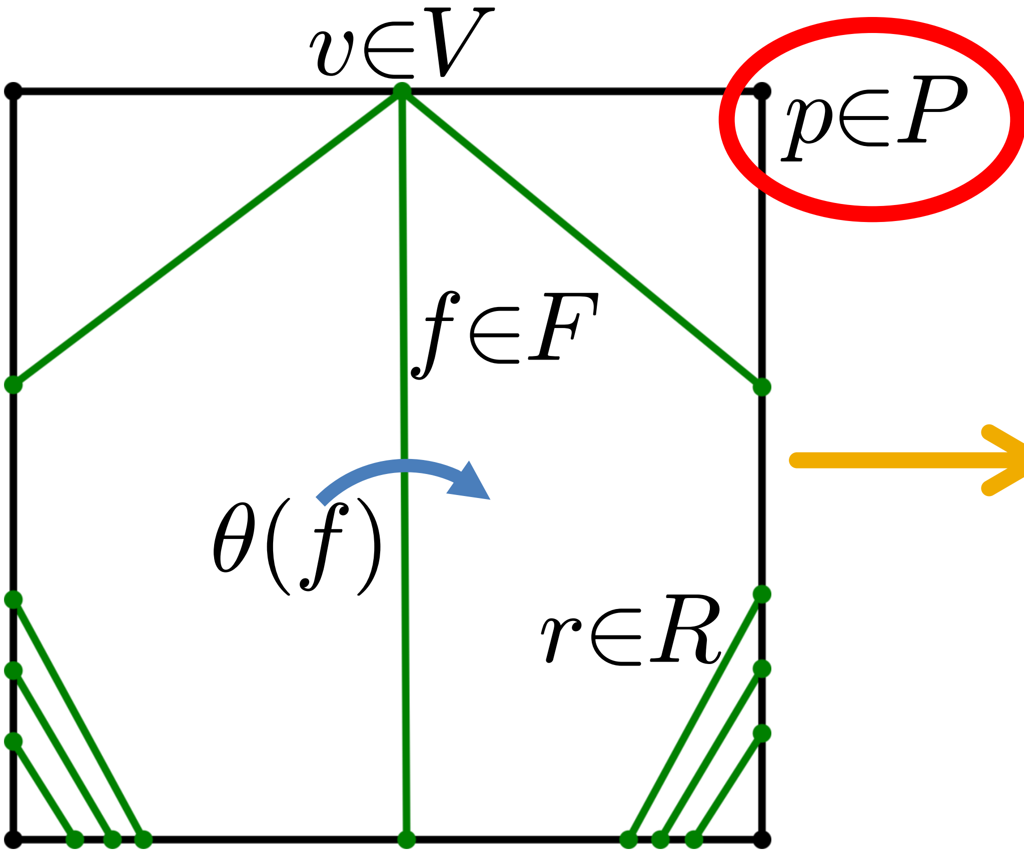
Flexible developable surfaces that can be navigated interactively, subdivided, and smoothed.

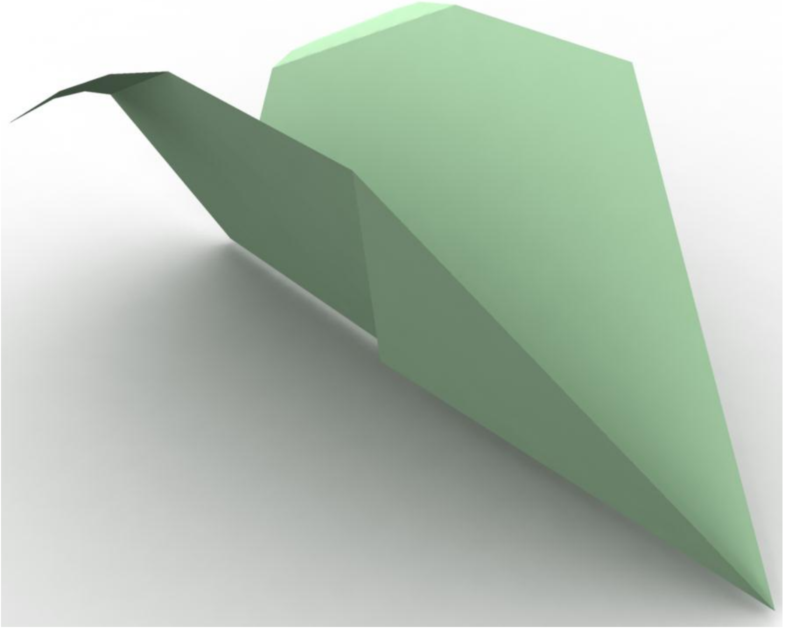
Discrete Developable Surfaces





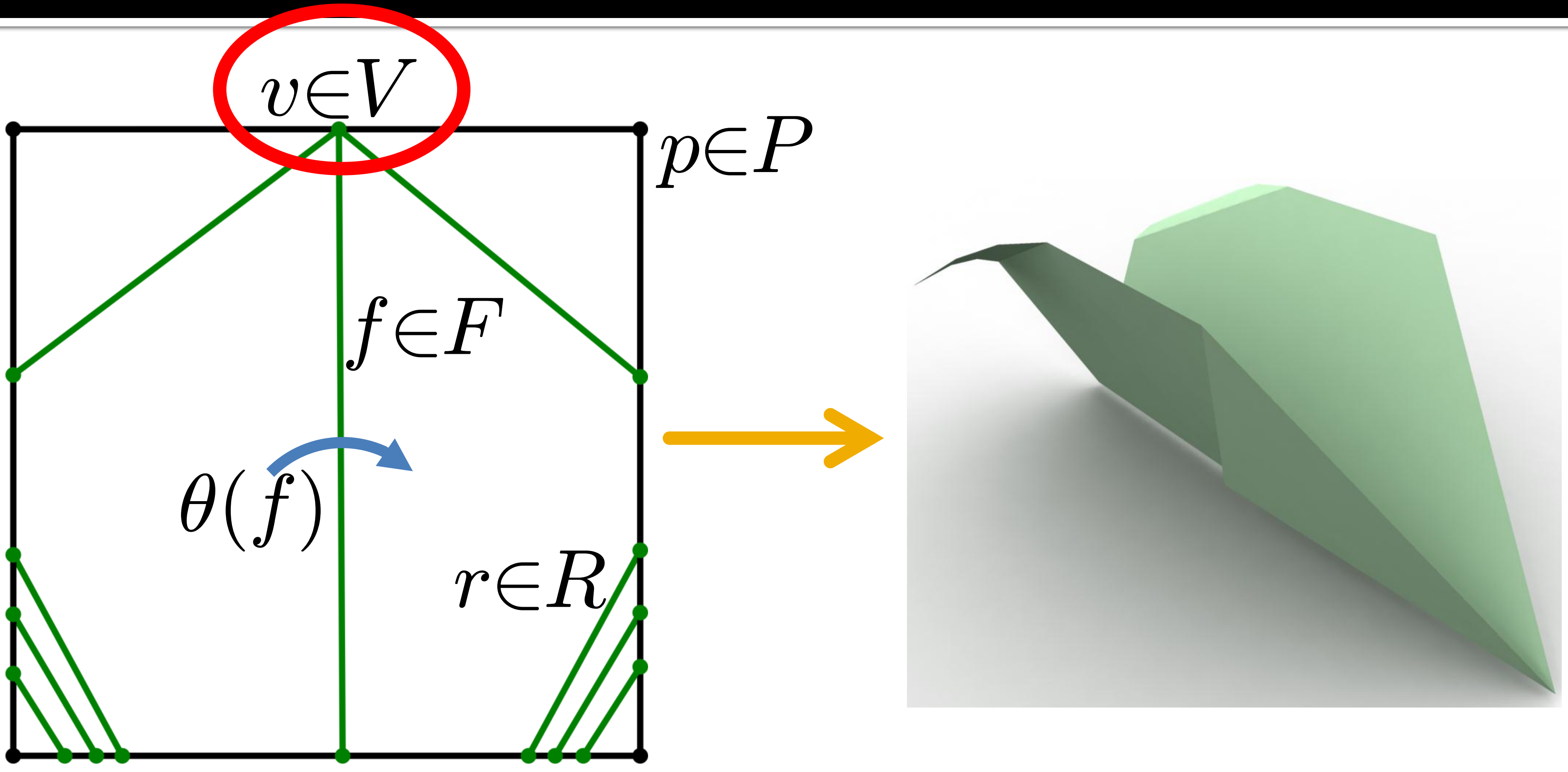
Discrete Developable Surfaces





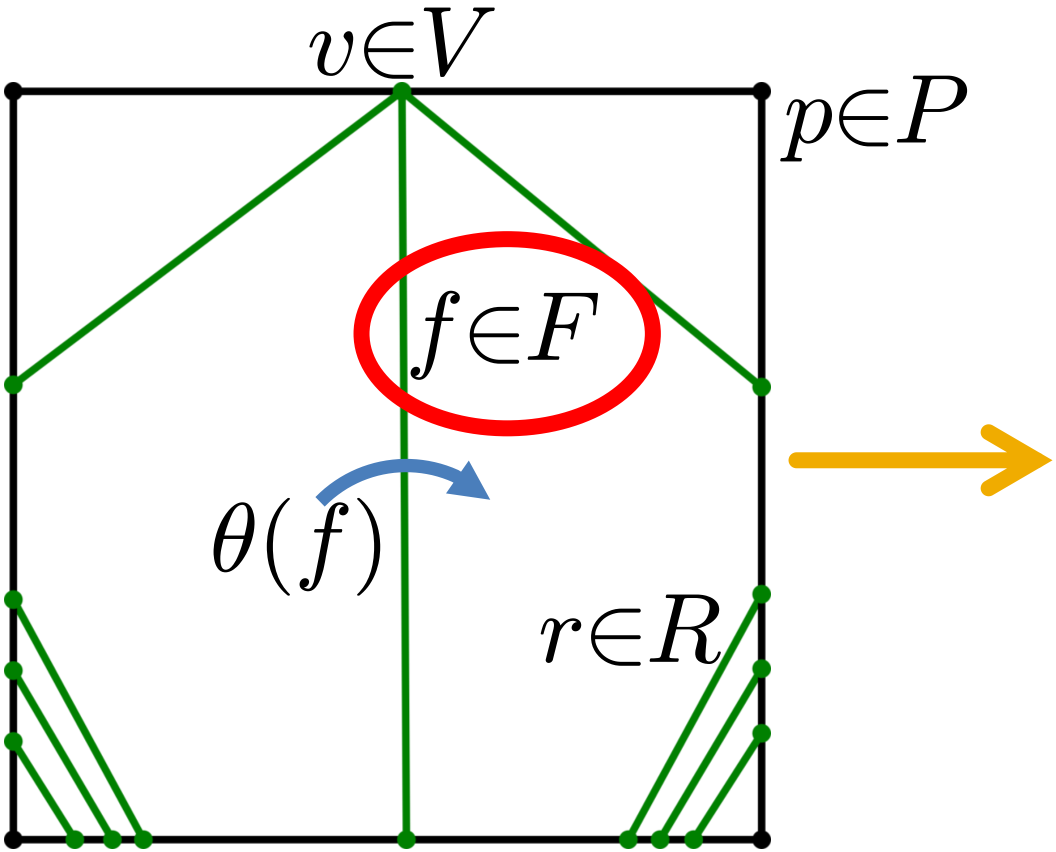
Polygonal domain

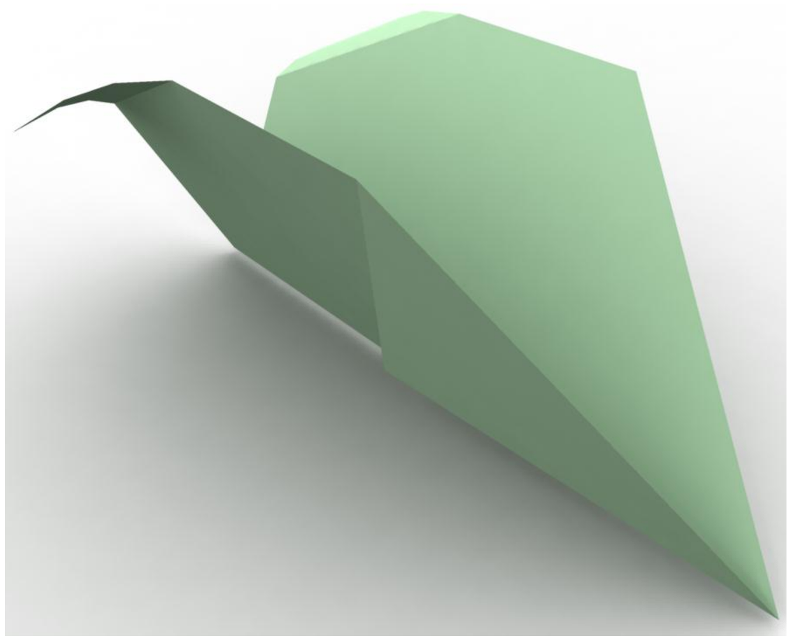
Discrete Developable Surfaces



Vertices

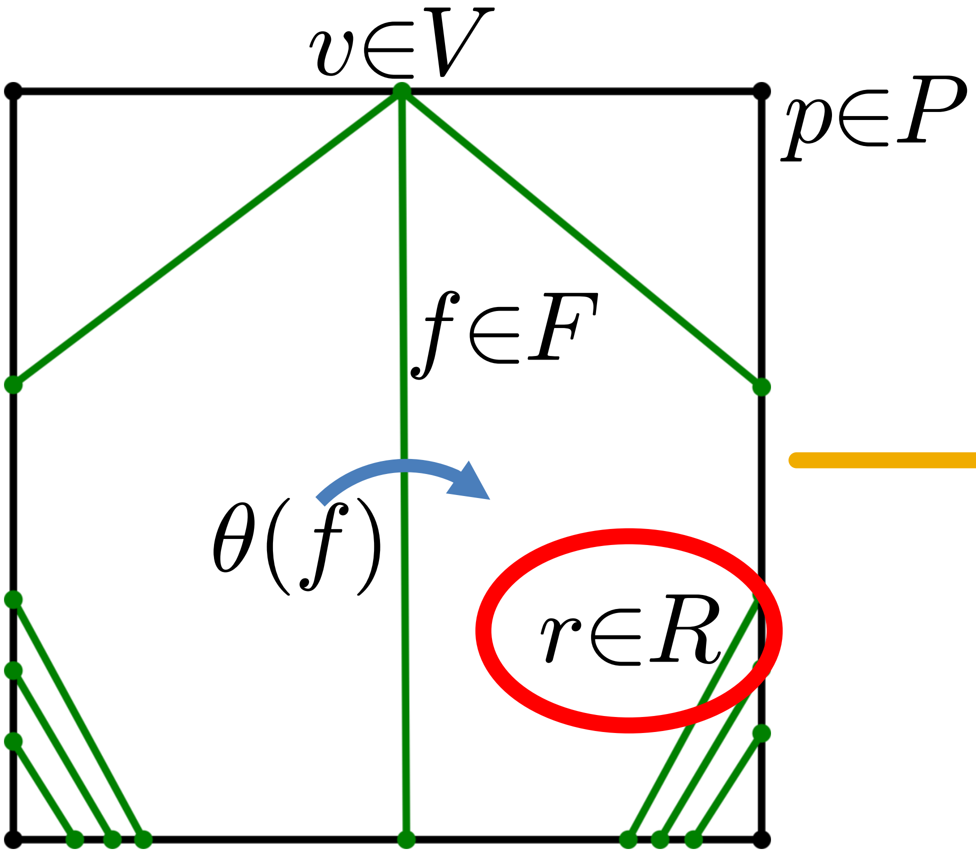
Discrete Developable Surfaces

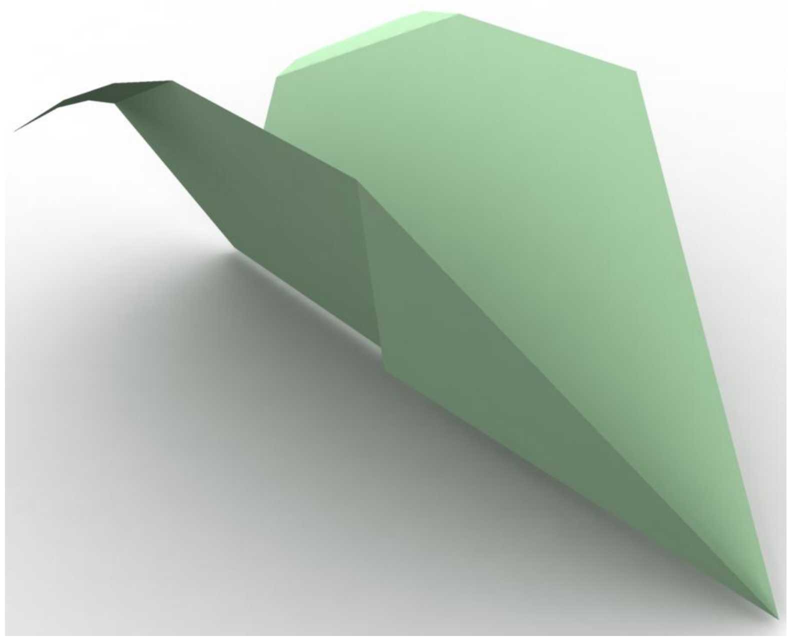




Folds

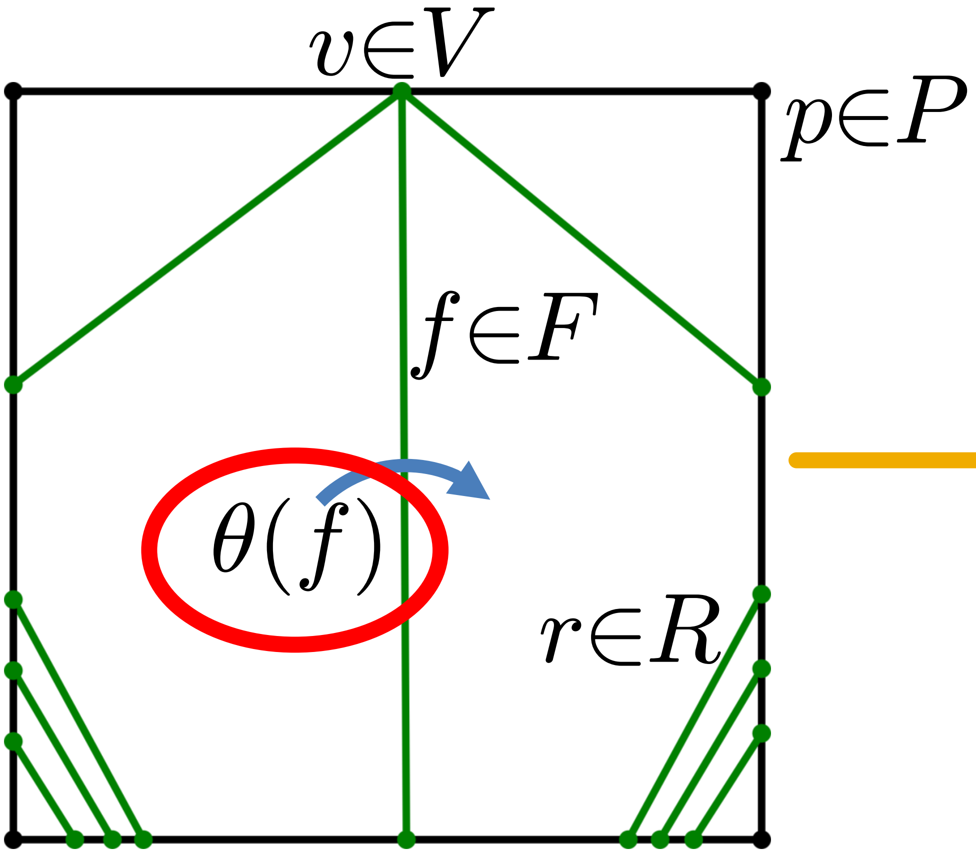
Discrete Developable Surfaces

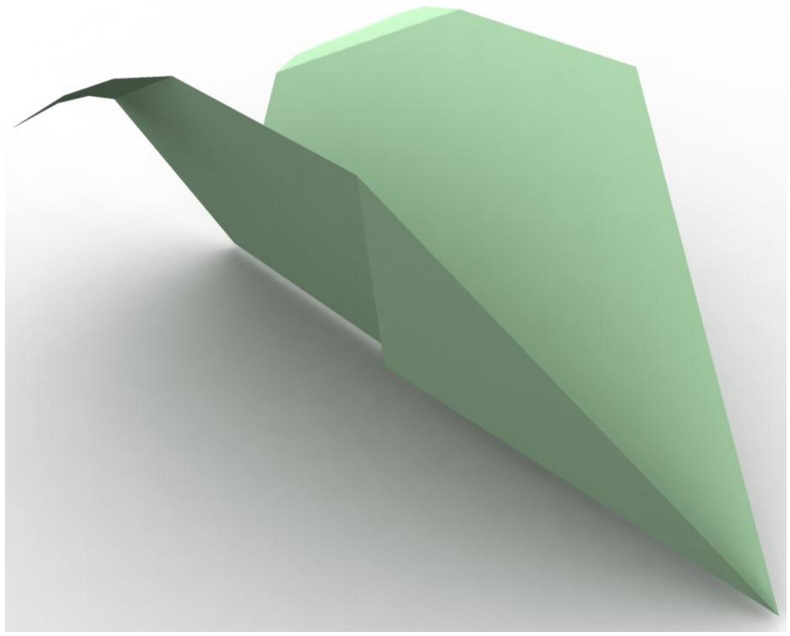




Rulings

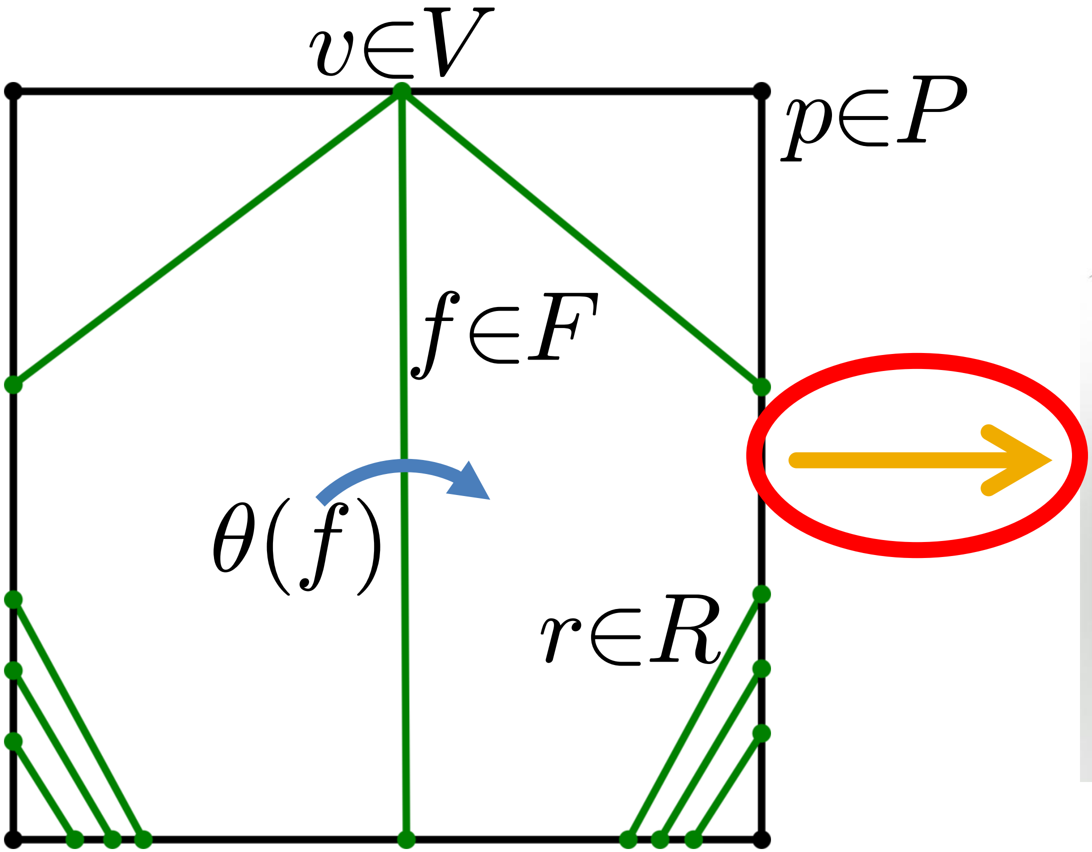
Discrete Developable Surfaces

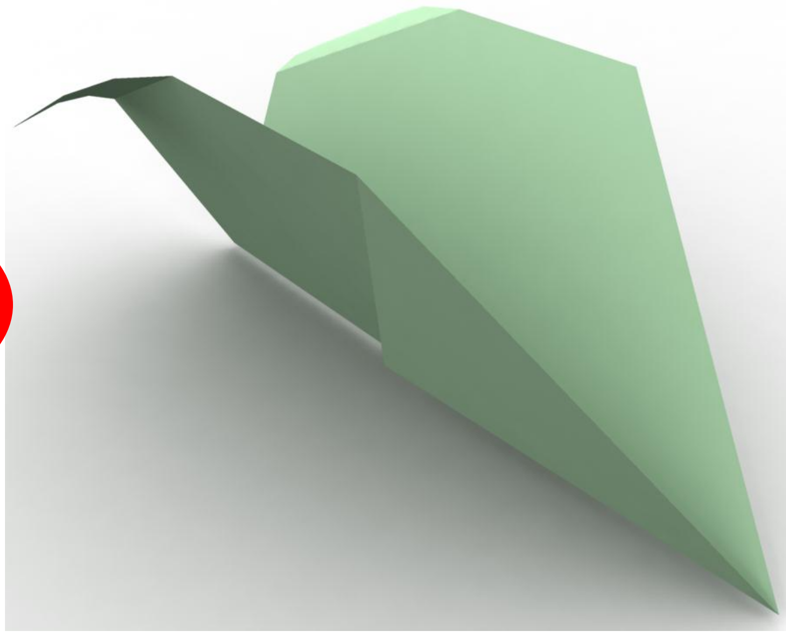




Folding angles

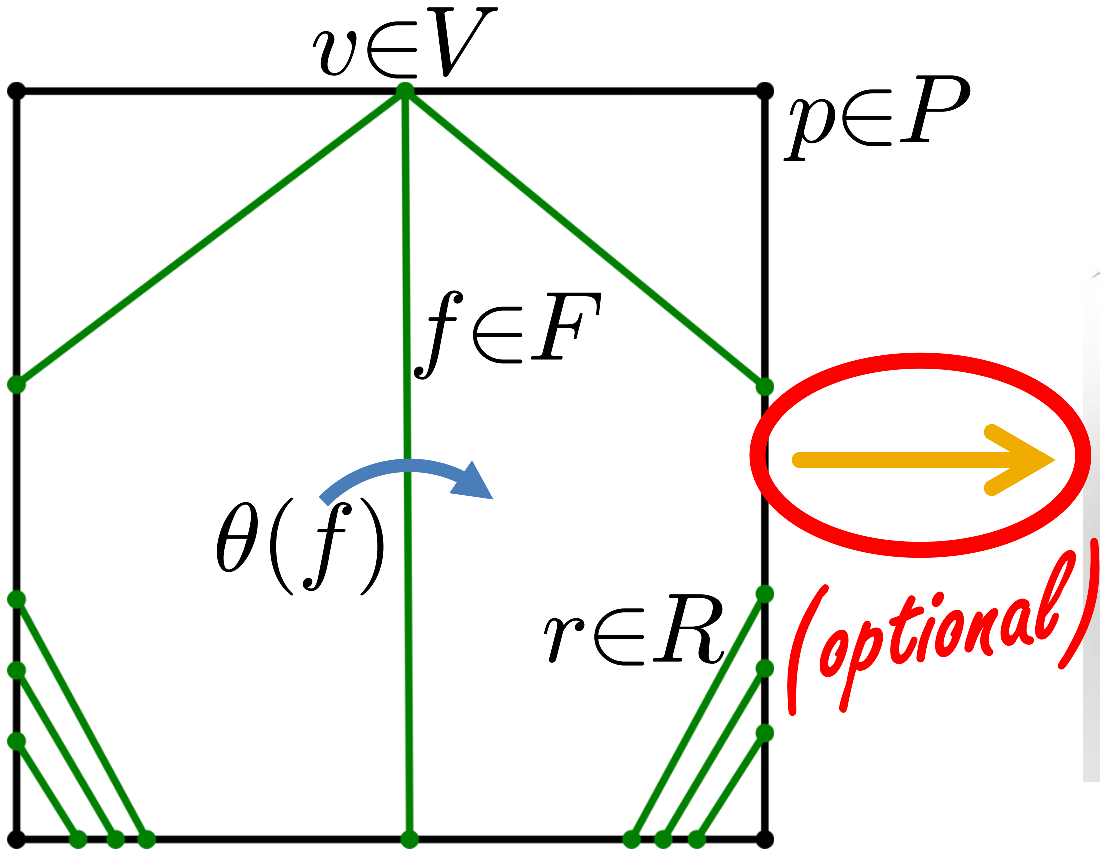
Discrete Developable Surfaces

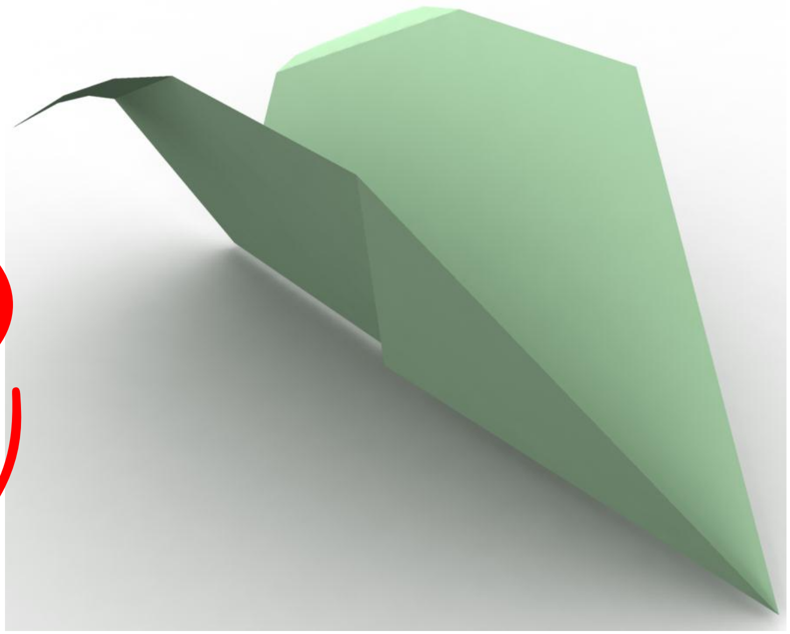




Rigid transformations

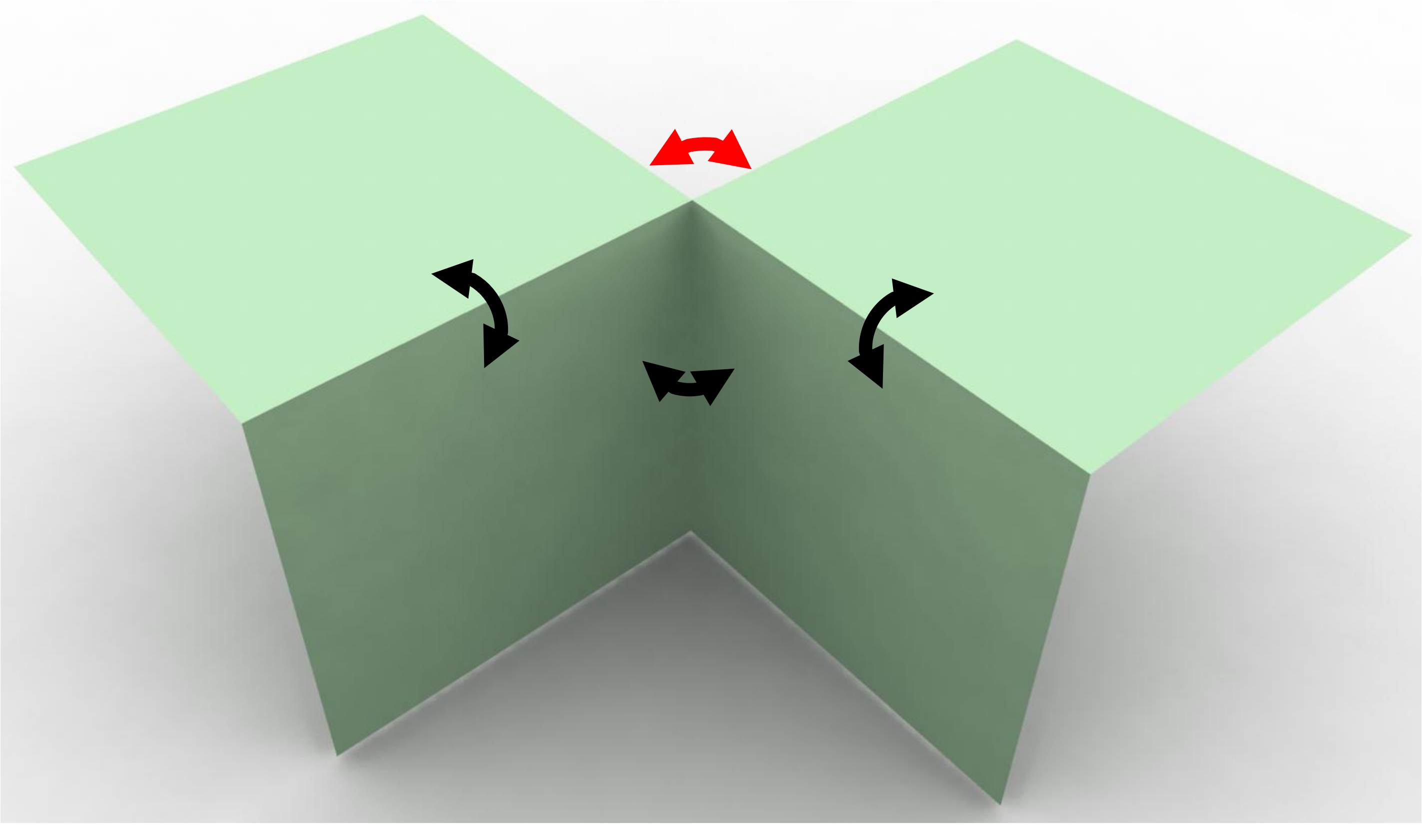
Discrete Developable Surfaces





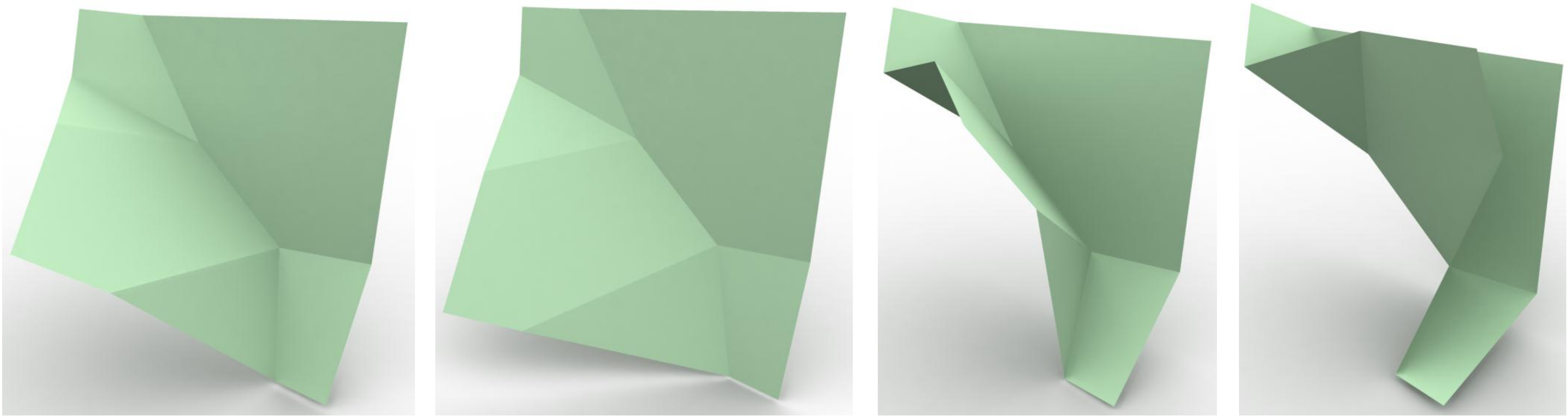
Rigid transformations

Problem



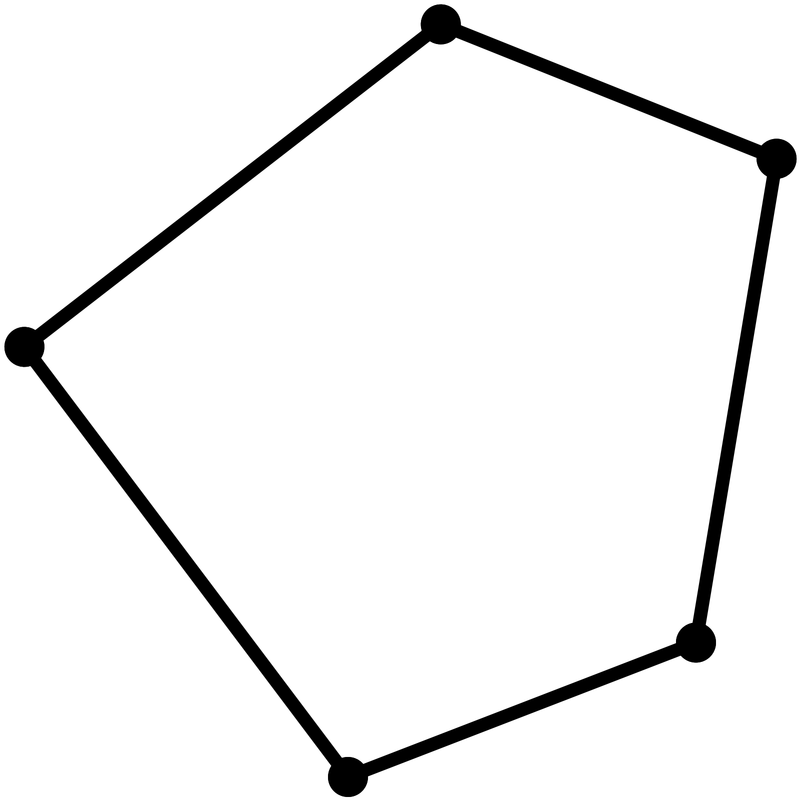
Lots of bad configurations!

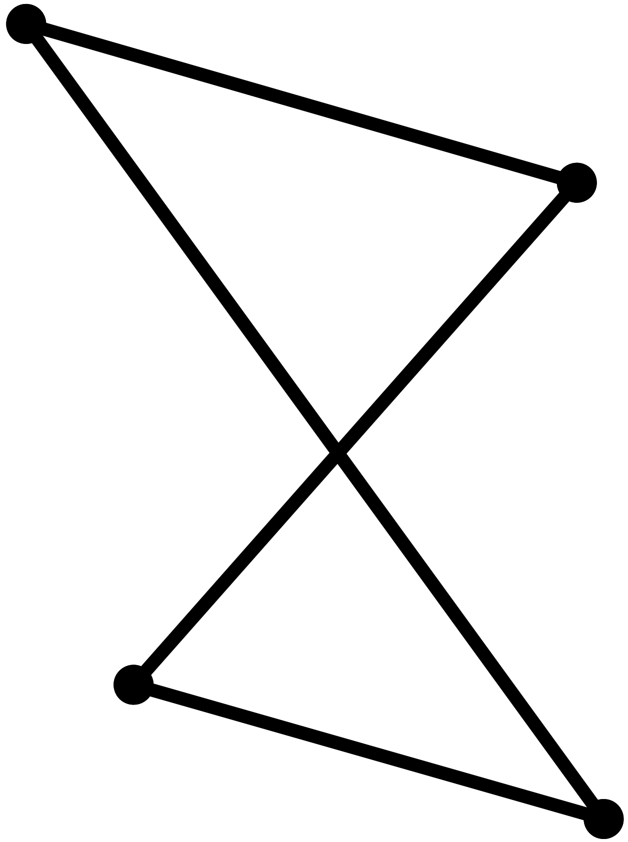
Goal



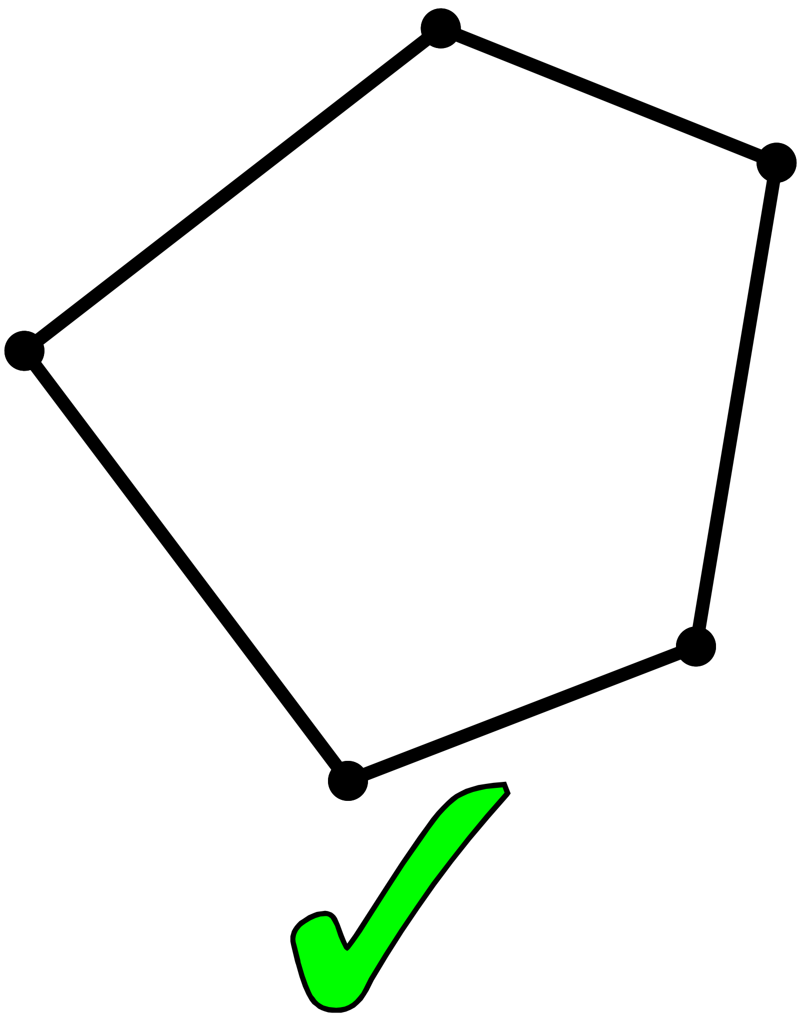
Characterize discrete developability constraints

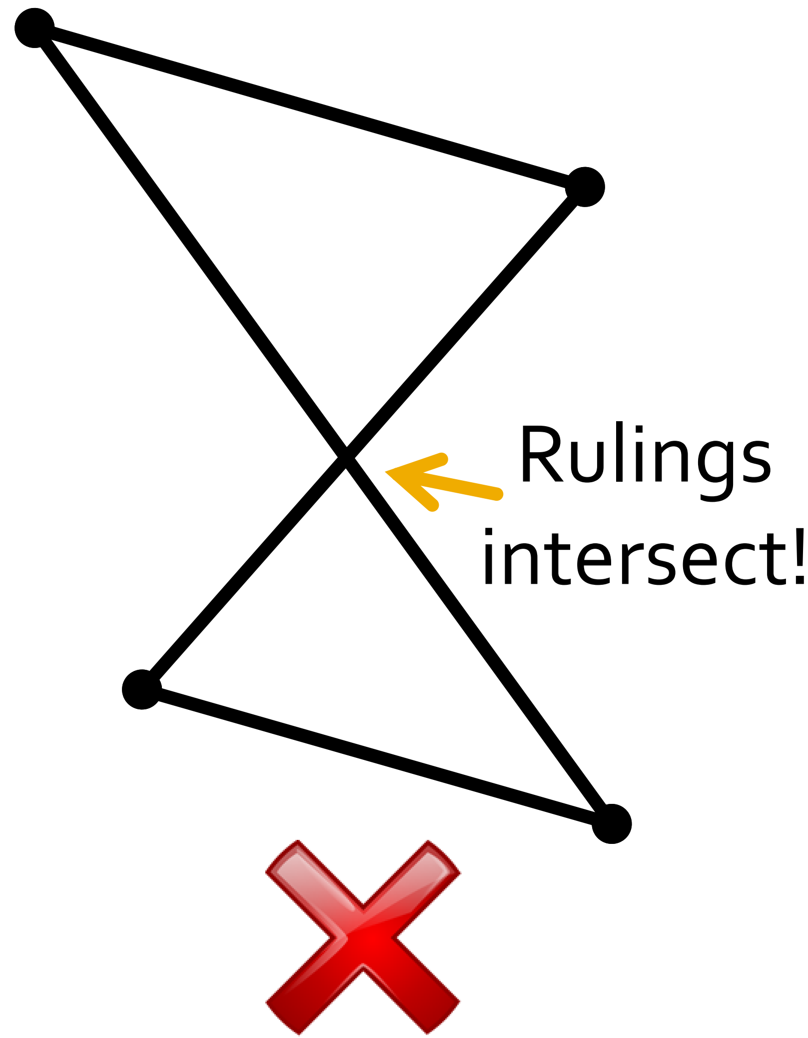
Graph Constraints



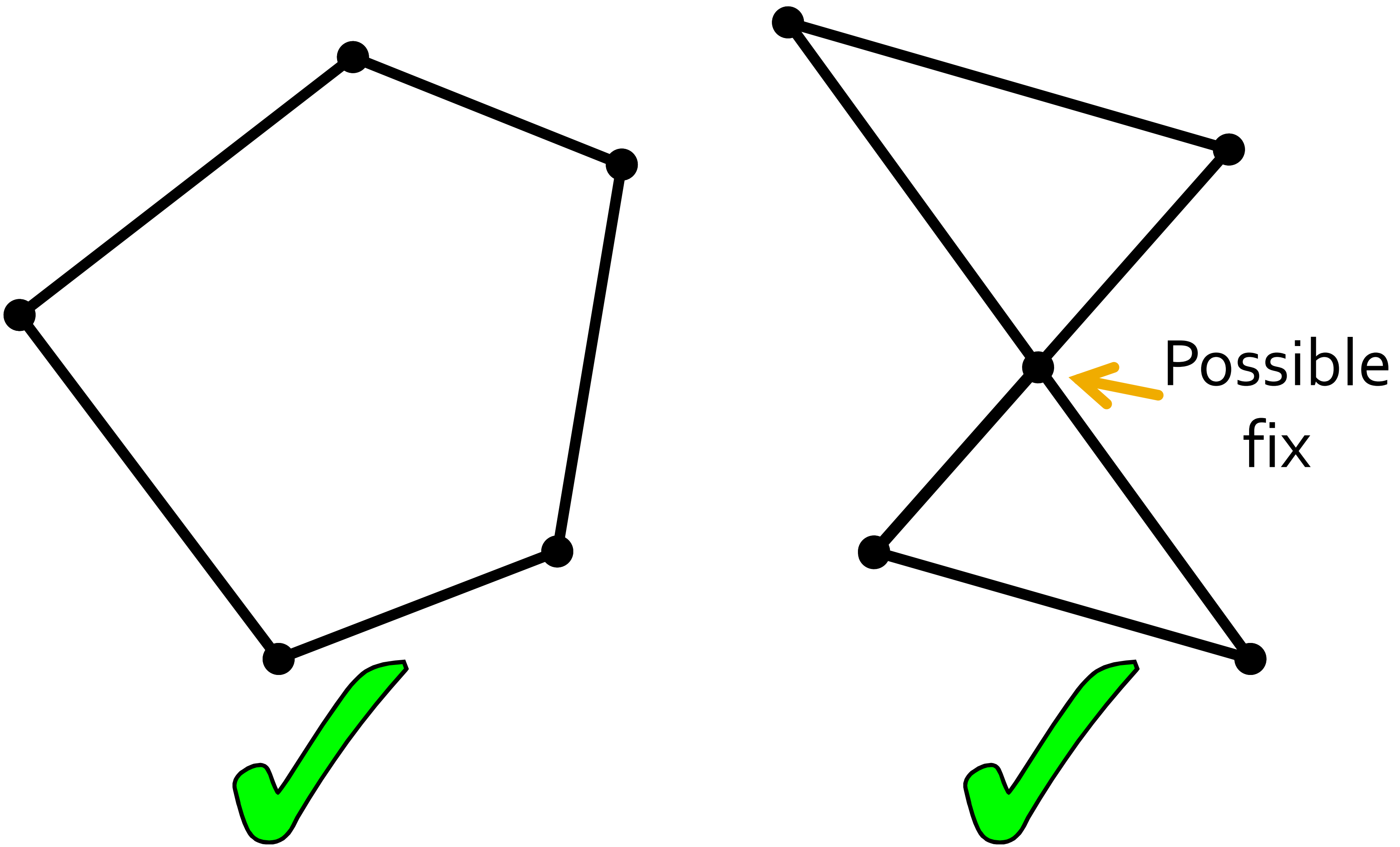


Graph Constraints

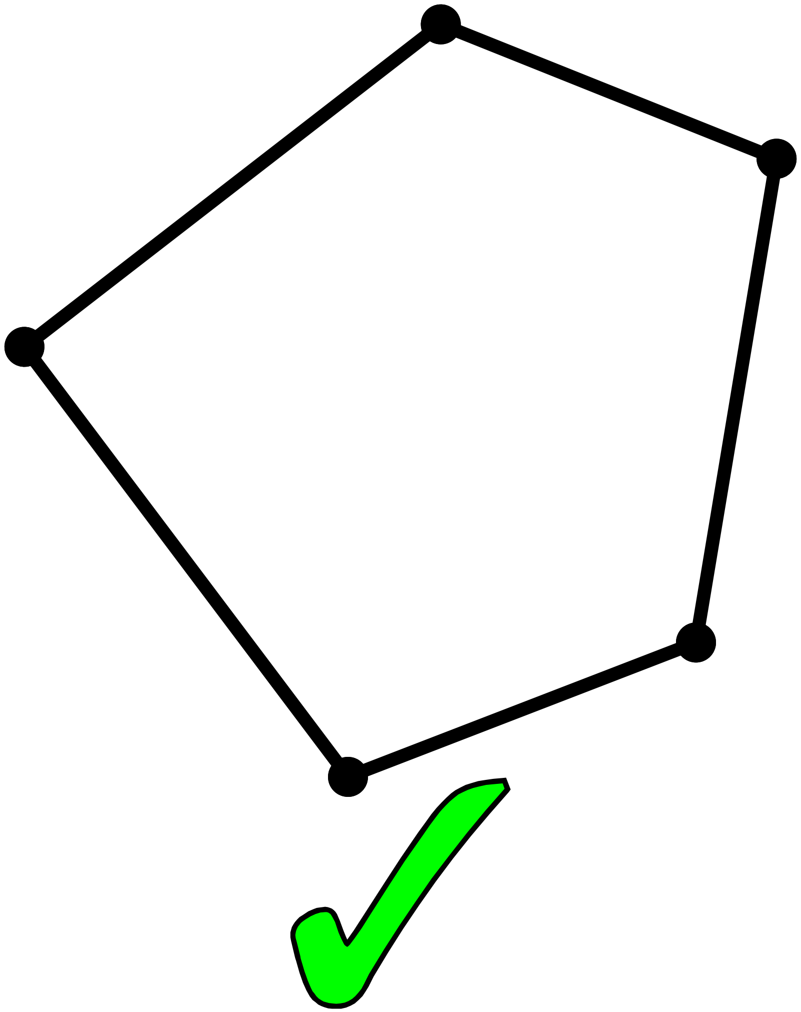


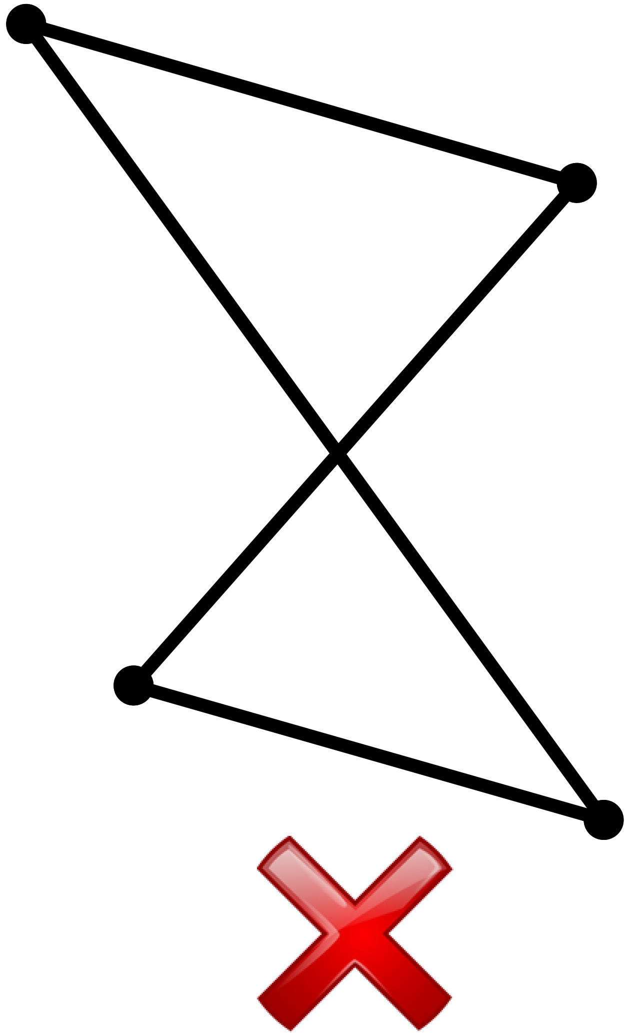


Graph Constraints

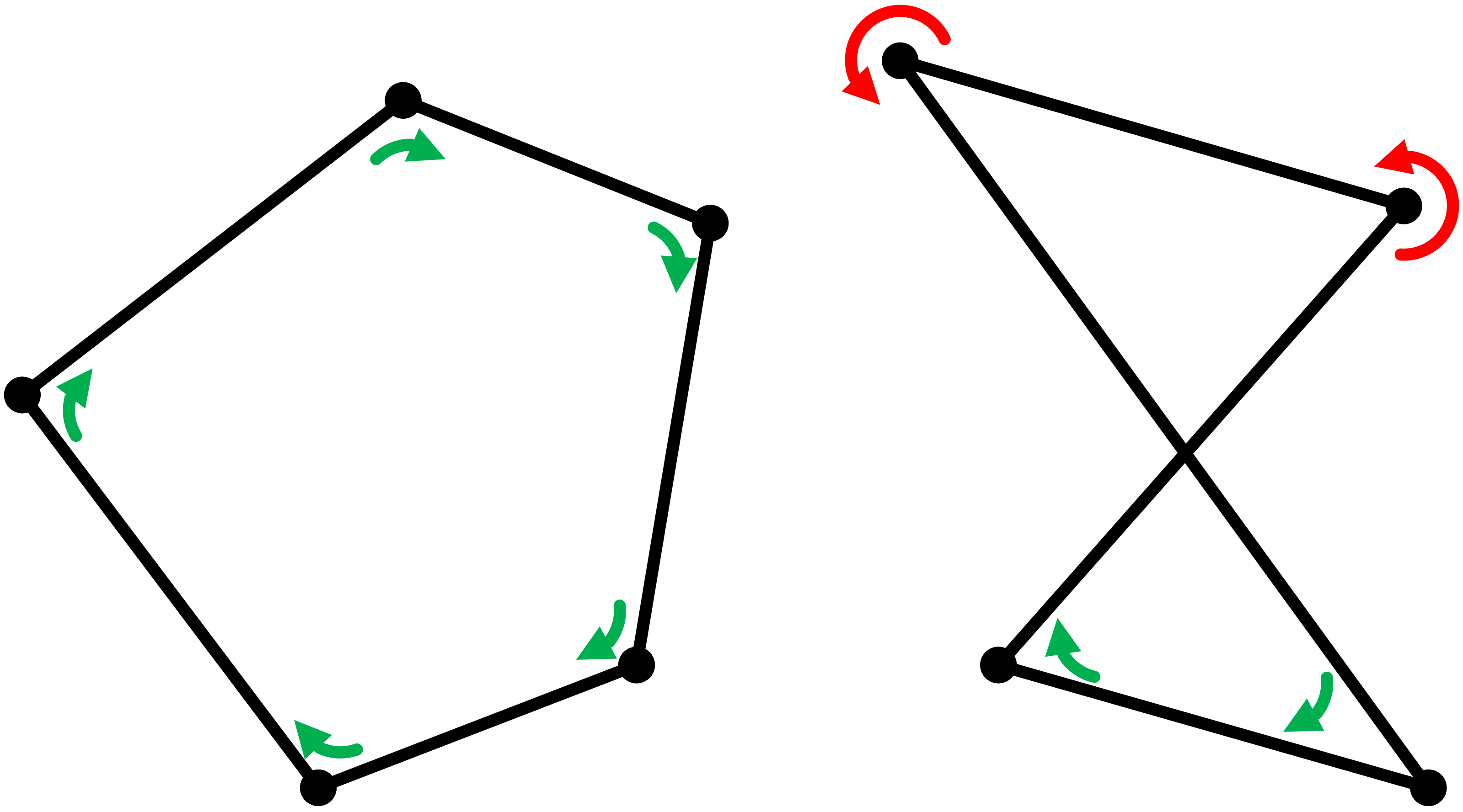


Graph Constraints



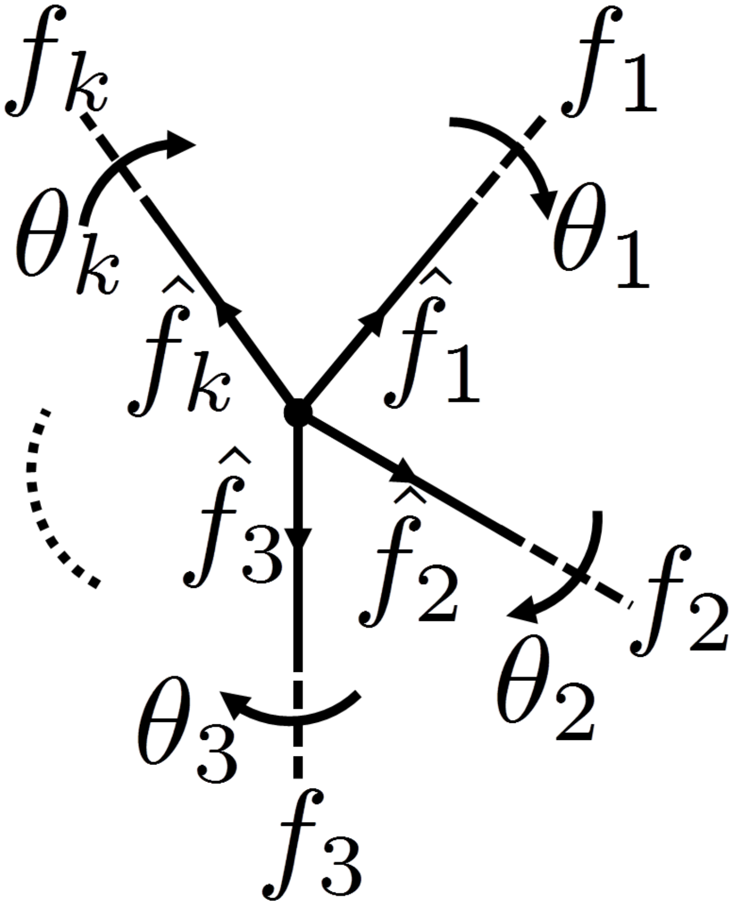


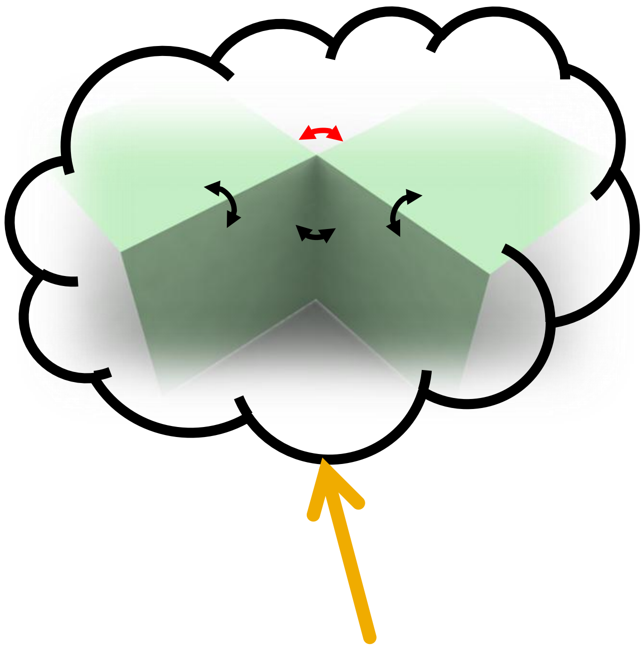
Graph Constraints



Turning angles in $[0, 2\pi]$

Compatibility Constraints

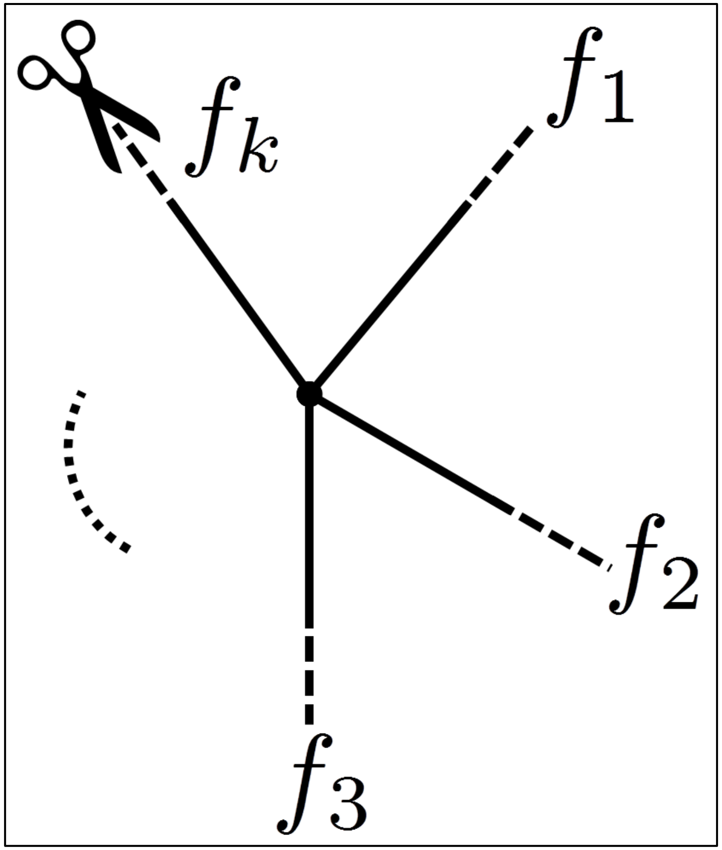


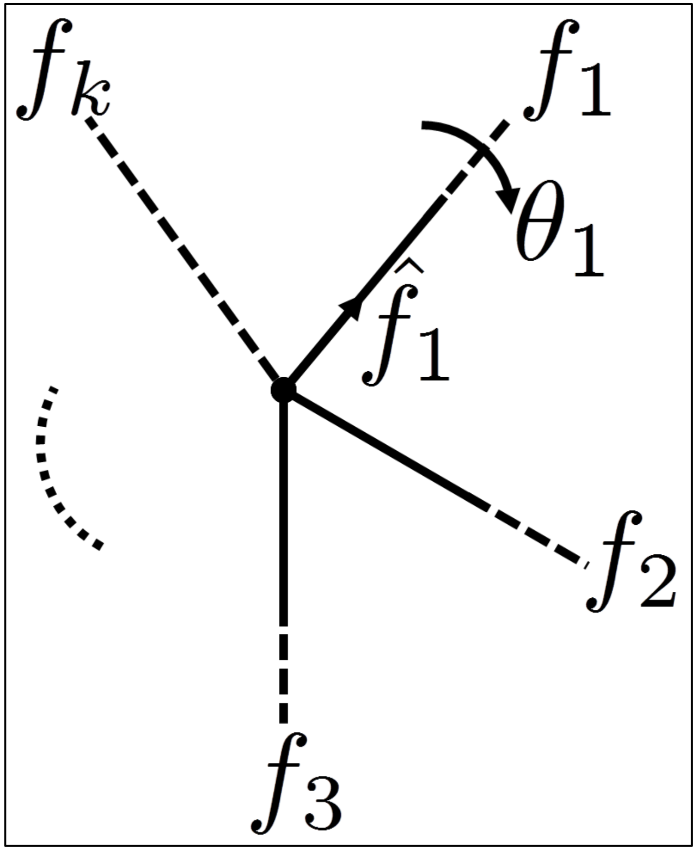


Avoid tearing!

Folds meet at interior point

Compatibility Constraints





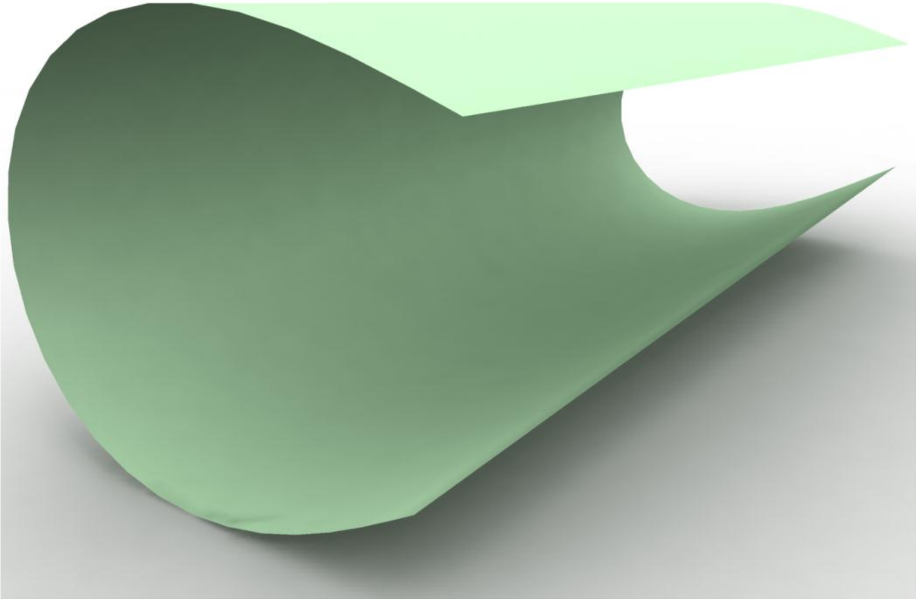
Composing rotations yields id.

Additional Constraints

- 3D configuration
- Pin positions in 2D/3D
- Pin folding angles
- Keep vertex on edge

Bending Energy

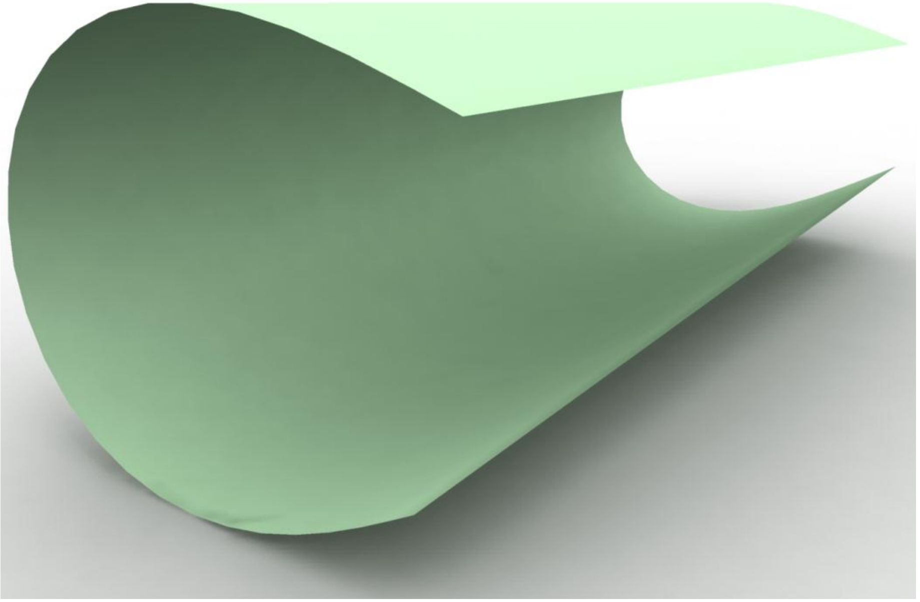
$$\boxed{E = \int_{\Sigma} H^2 dA}$$



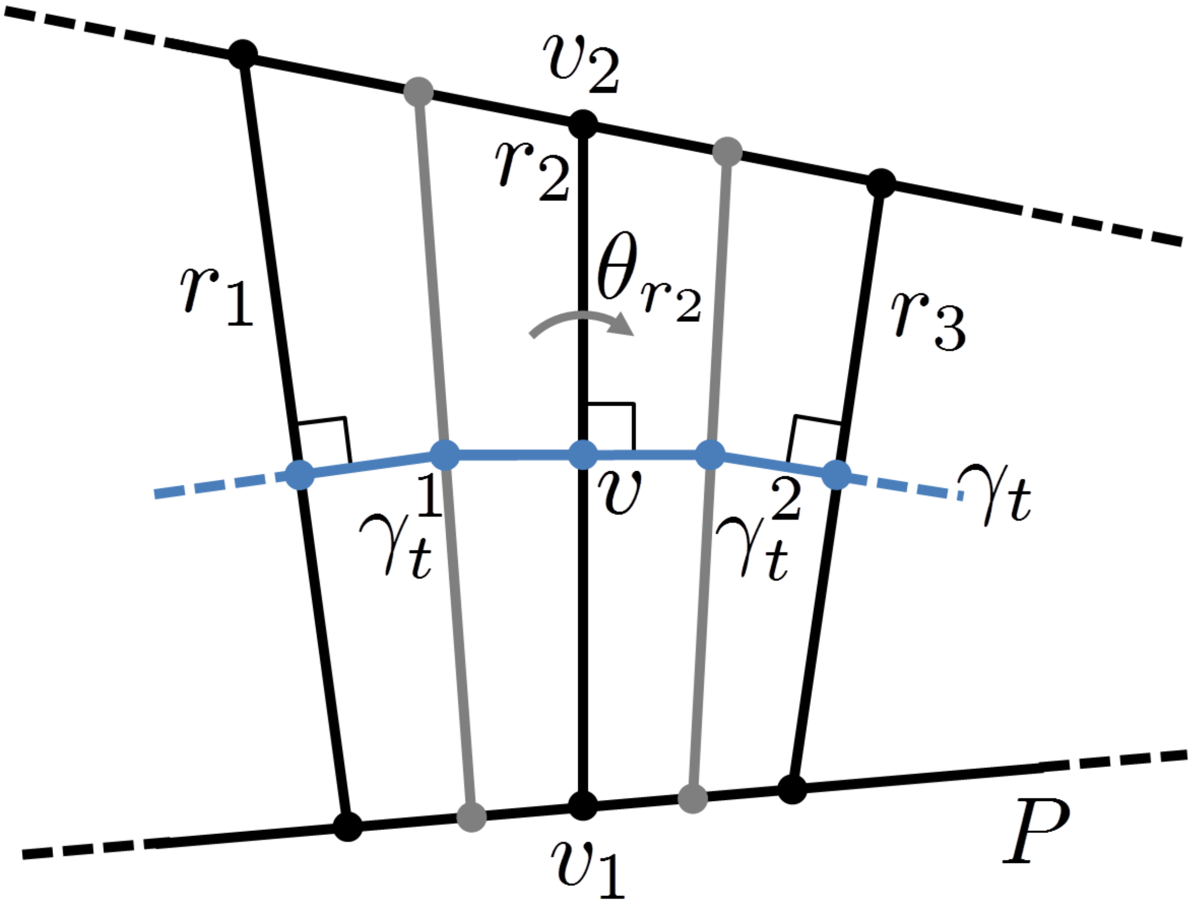
Bending Energy

$$\boxed{E = \int_\Sigma H^2 dA}$$

$$\left(K = \kappa_1 \kappa_2 \equiv 0, H = \frac{\kappa_1 + \kappa_2}{2} \Rightarrow H^2 = \frac{1}{4}\kappa_1^2\right)$$

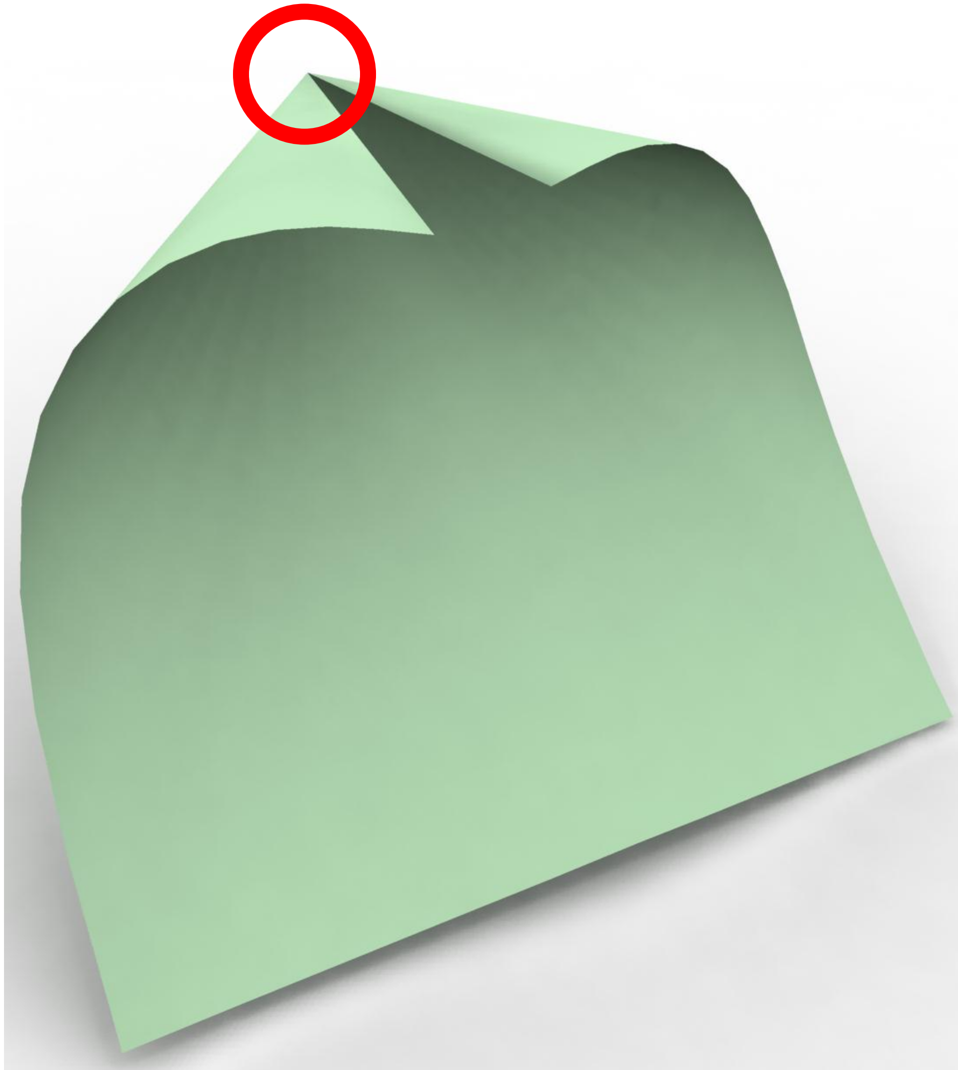


Discrete Bending Energy



Closed-form energy

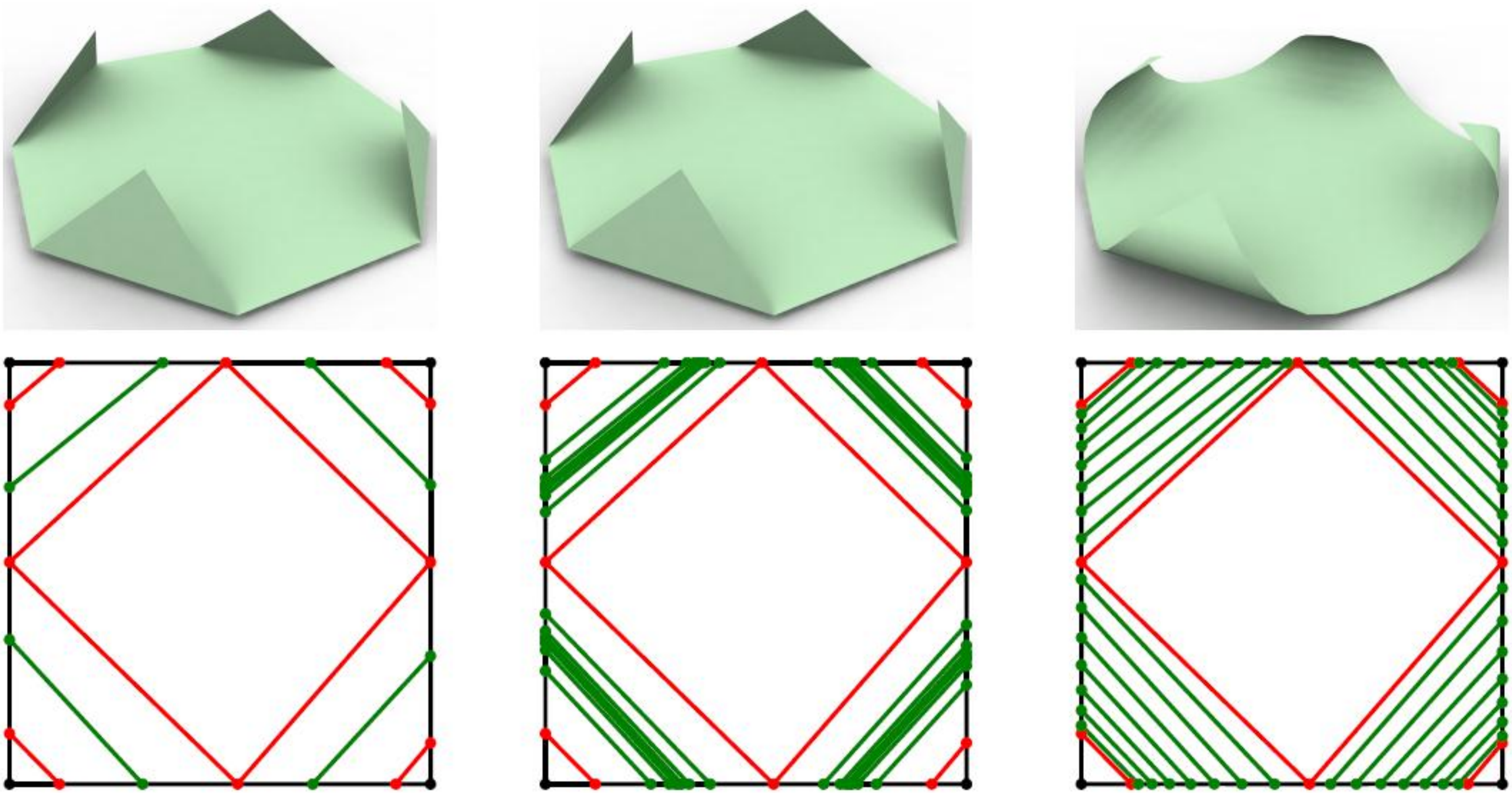
Singularities



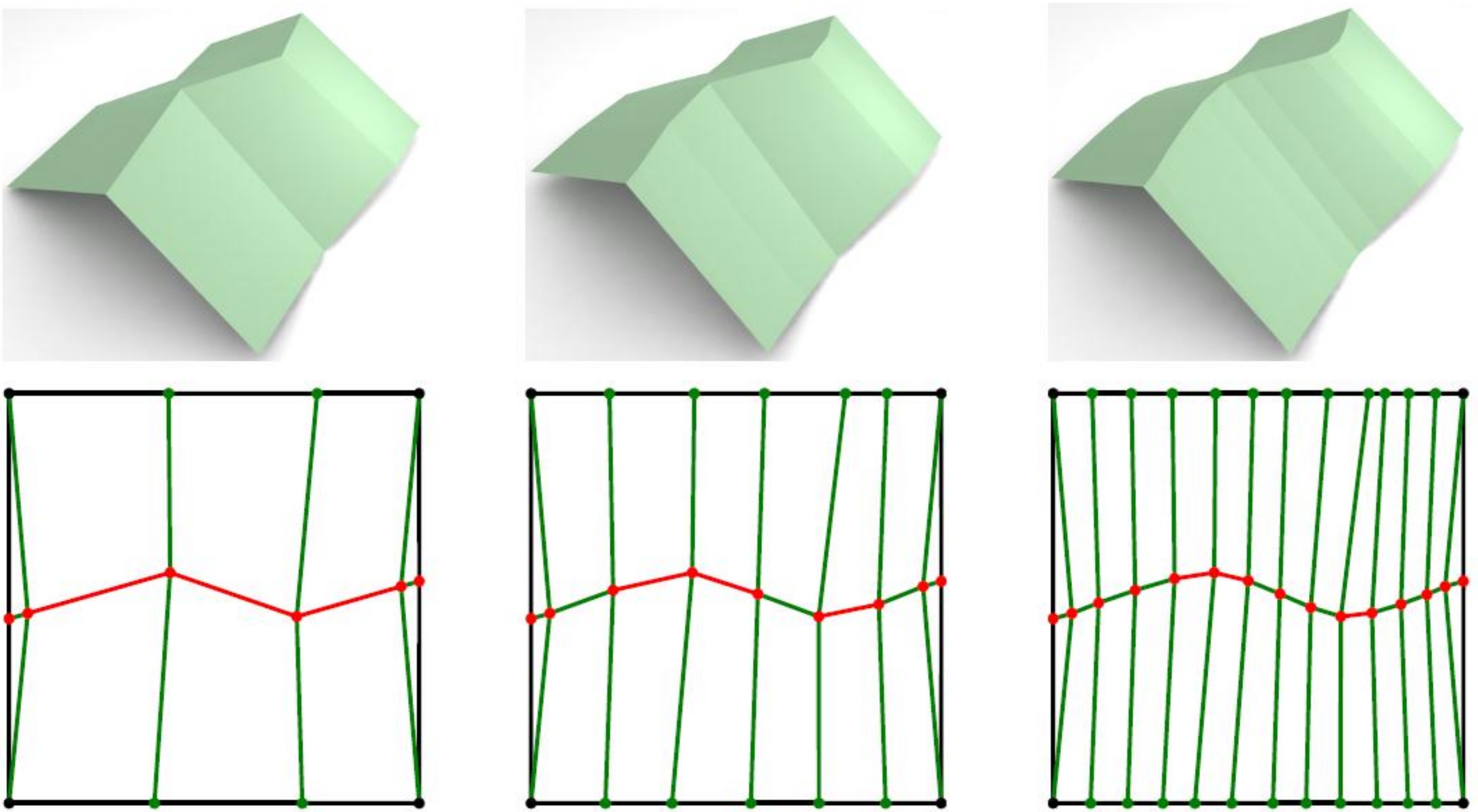
$$H \to \infty$$

Cut hole around singularity when computing bending energy

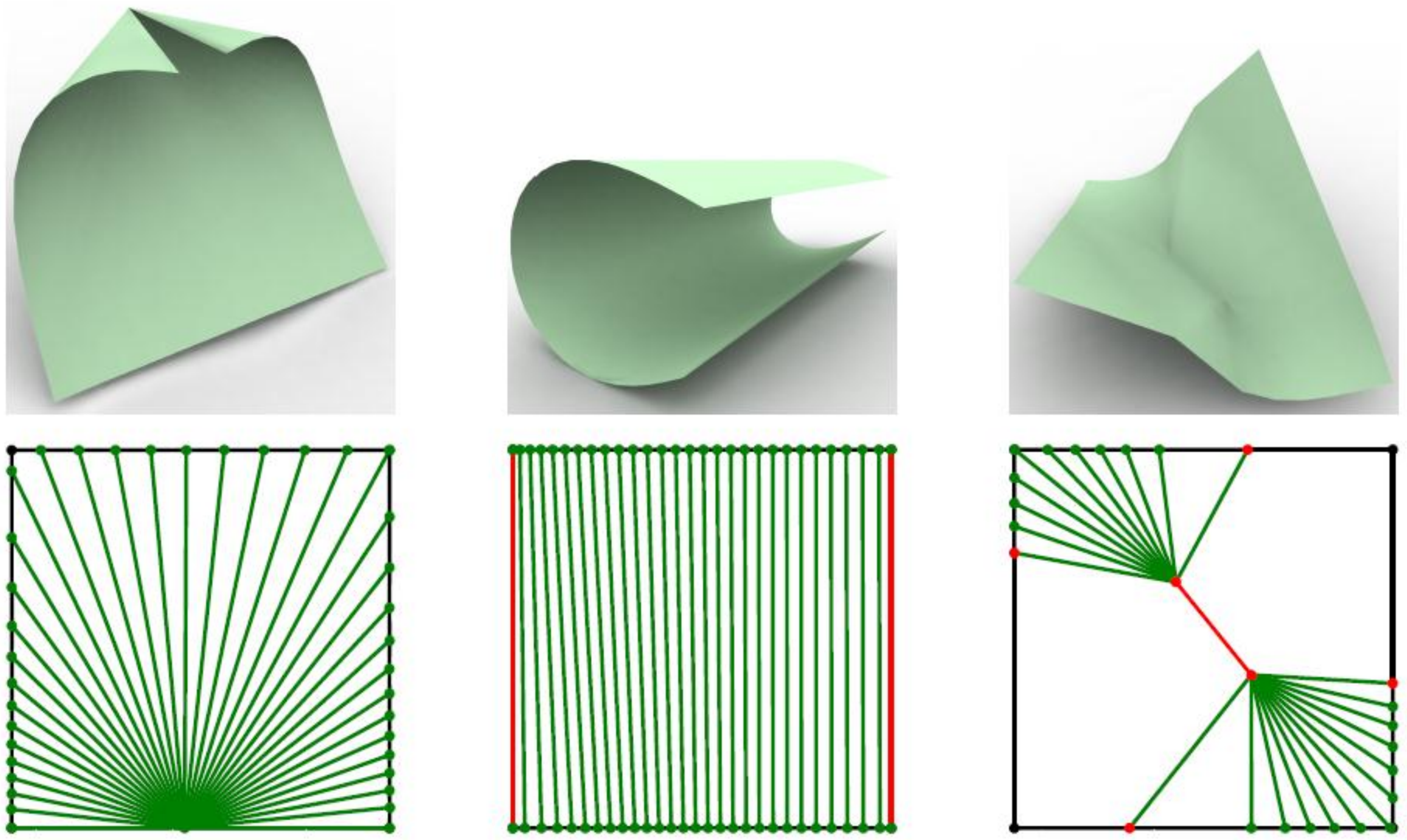
Mean Curvature Relaxation



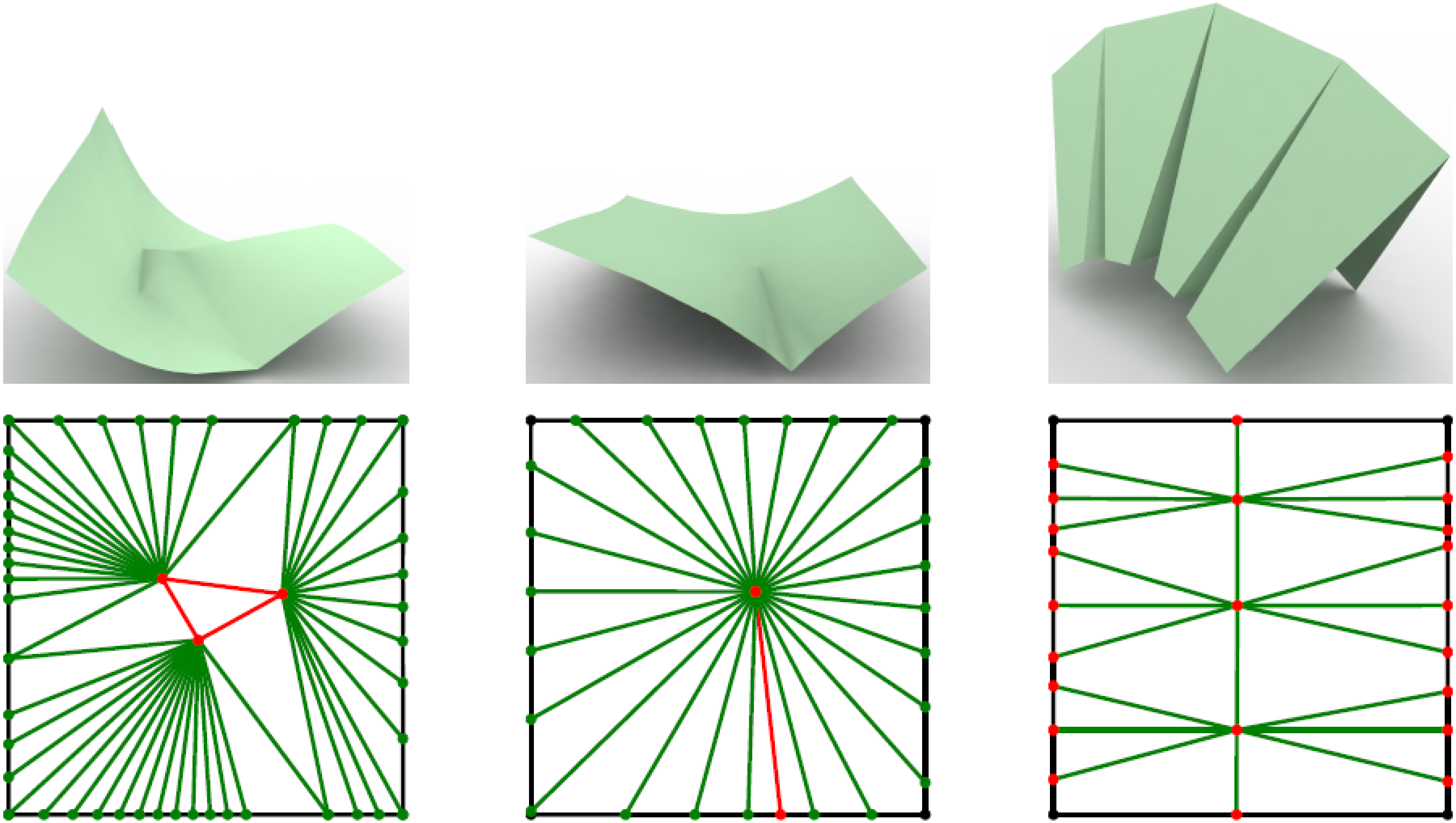
Curved Folding



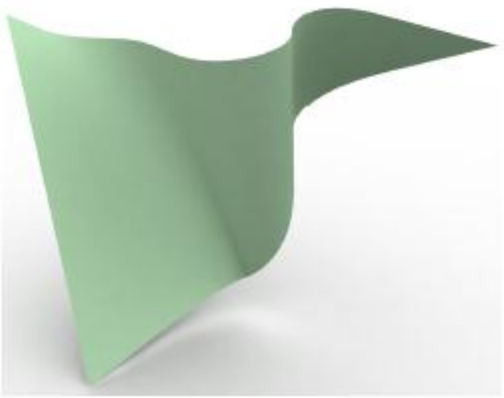
Additional Examples

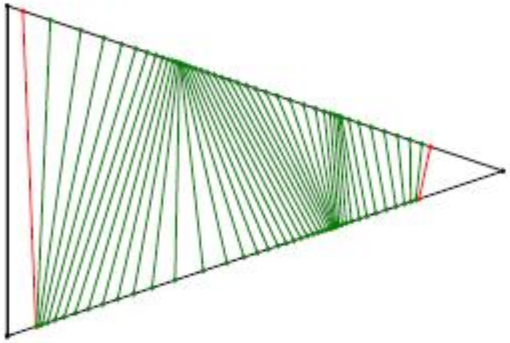


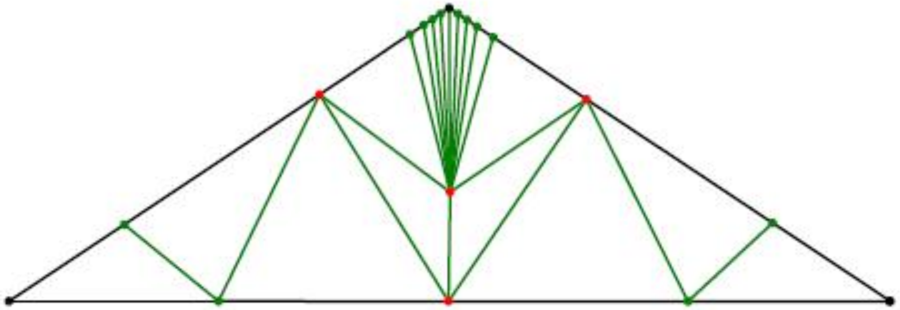
Additional Examples



Additional Examples







Interactive Tool

- Stepwise primal-dual **interior point** (lpopt)
  - Projection
  - Relaxation

- **Subdivision**: curved folds, curved pieces

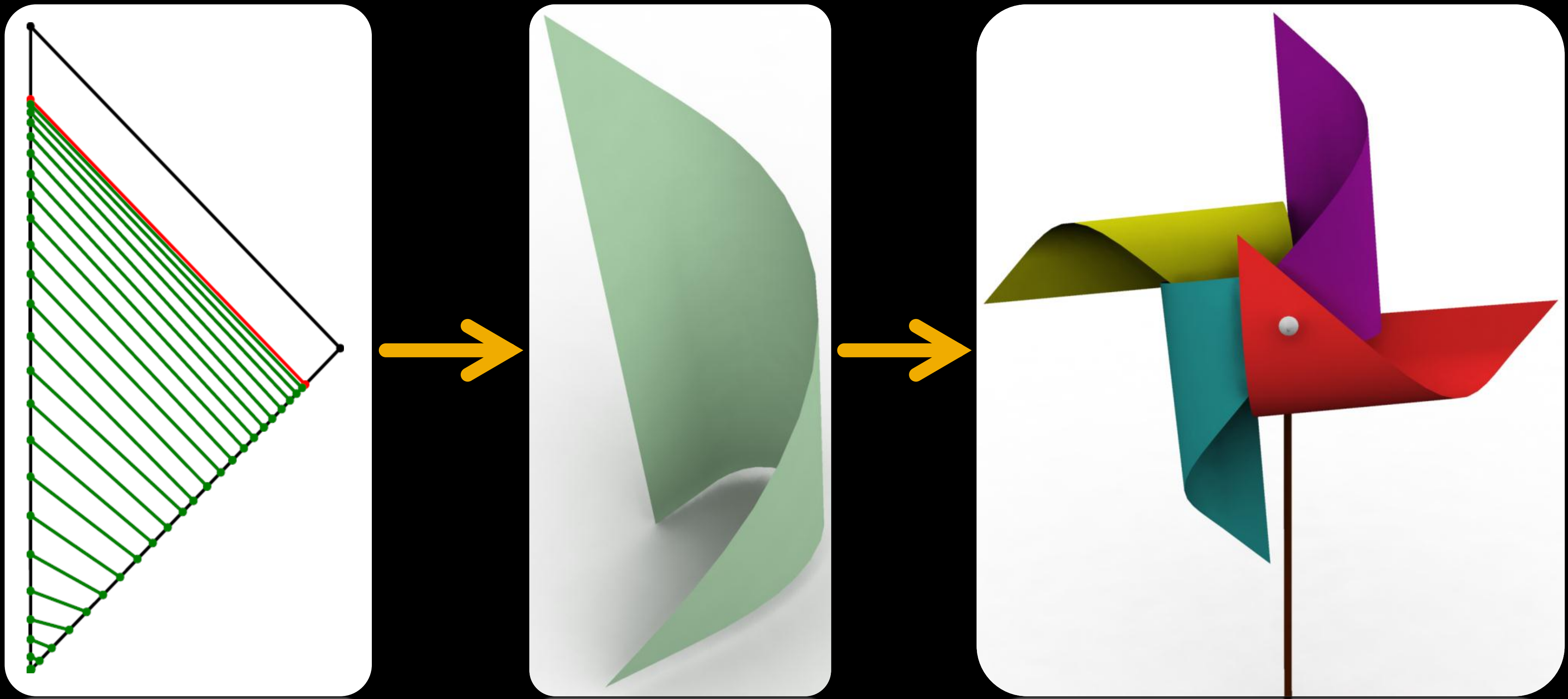
- Special case for **rounding corners**

Limitations

- **Exploring a highly nonlinear space**
  - Can deal with small to medium size
  - Incremental computation
- **Need to study best interaction**
  - Presenting allowable deformations
  - Requires insertion of rulings for basic structure

Potential Applications

- **Manufacturing**
- **Architecture**
- **Simulation**
  - Remove locking
  - Localized stretching?



Flexible Developable Surfaces

Questions?