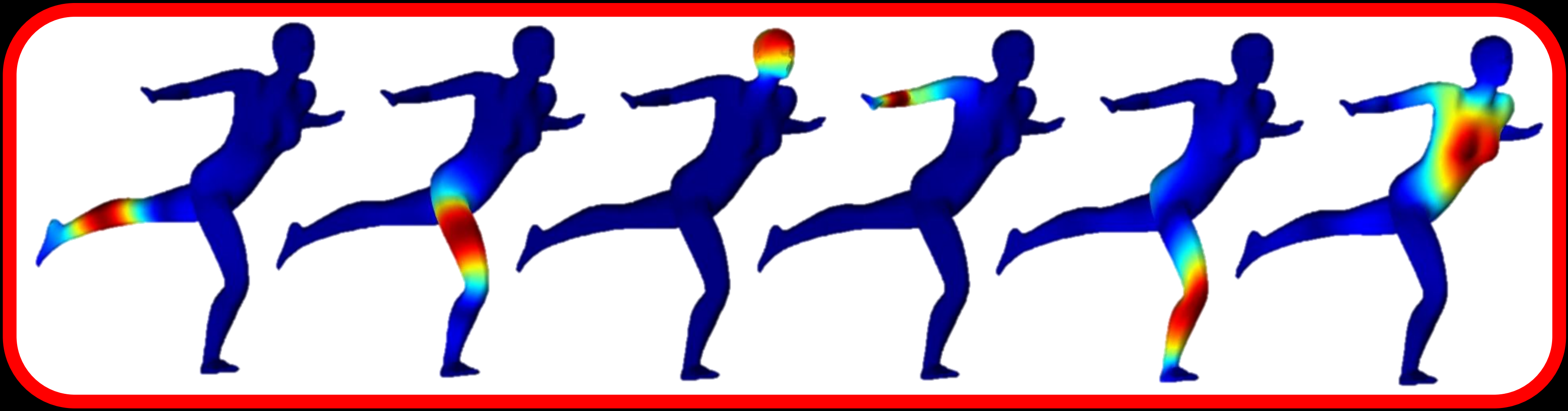
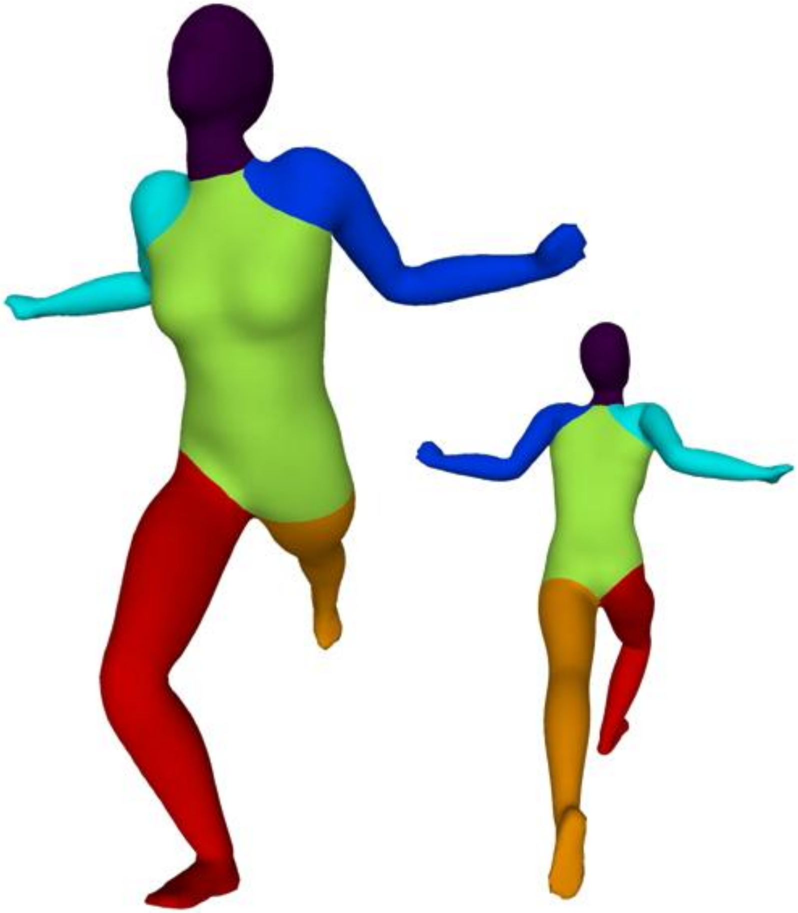


Discovery of Intrinsic Primitives on Triangle Meshes

Justin Solomon, Mirela Ben-Chen, Adrian Butscher, and Leo Guibas

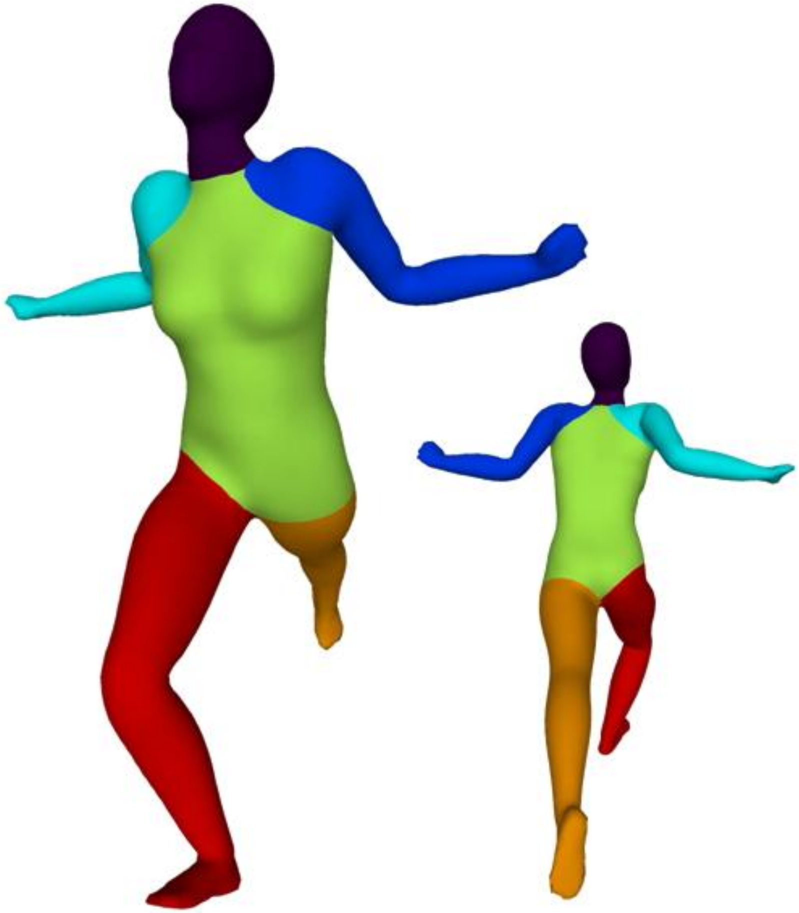
Stanford University

Two Related Problems

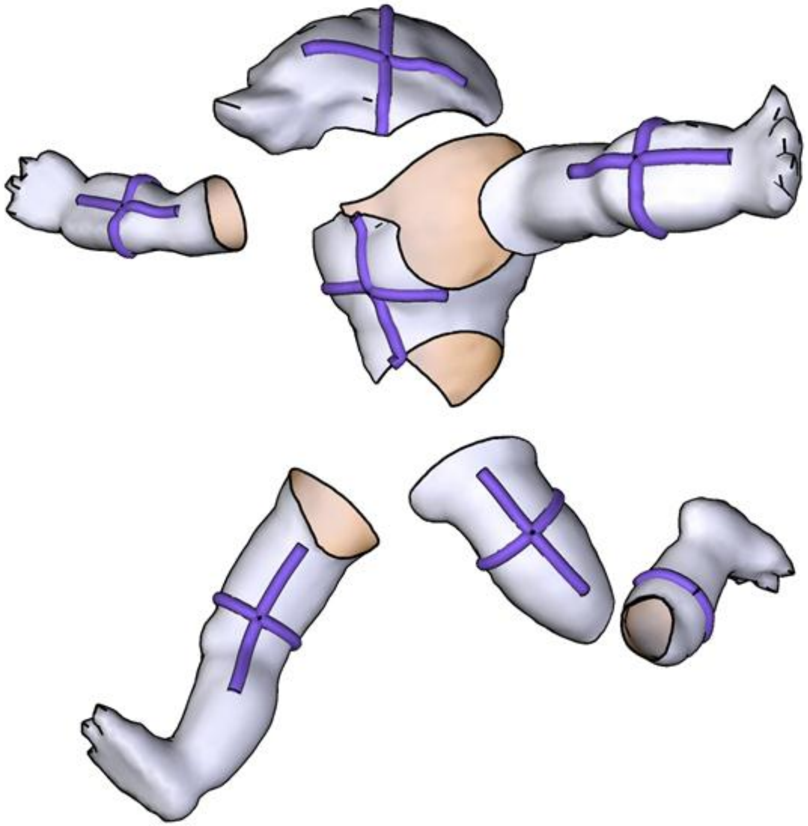


Segmentation

Two Related Problems



Segmentation



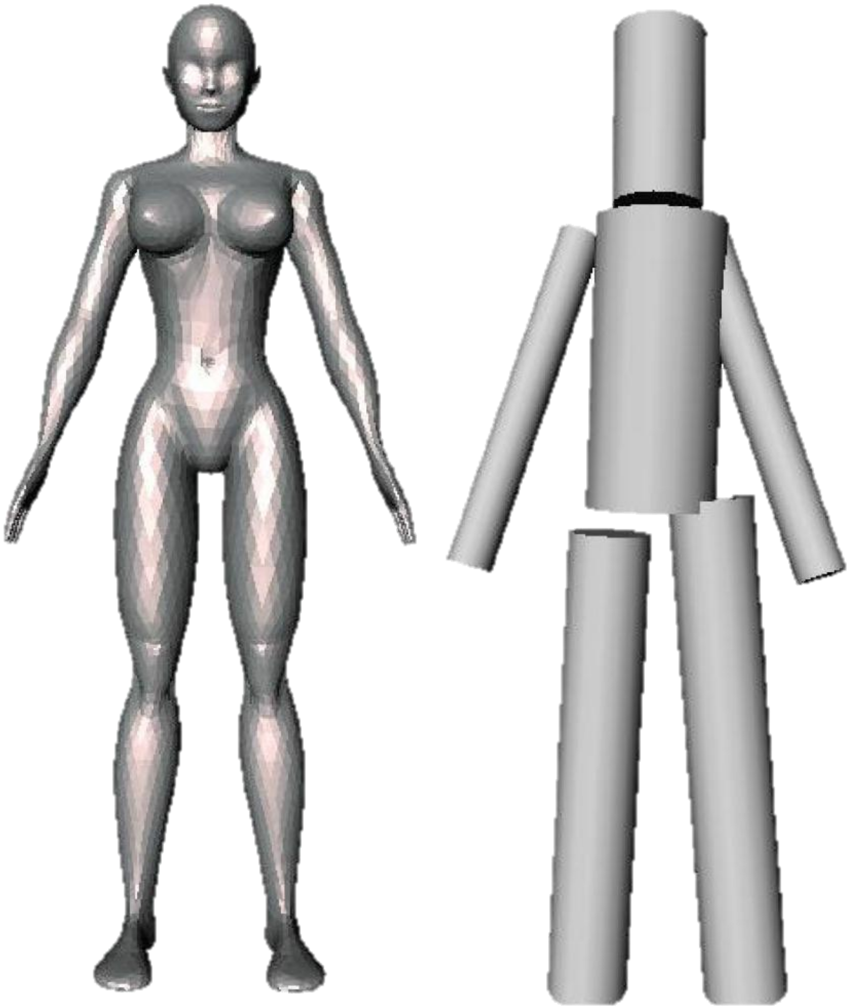
Part Discovery

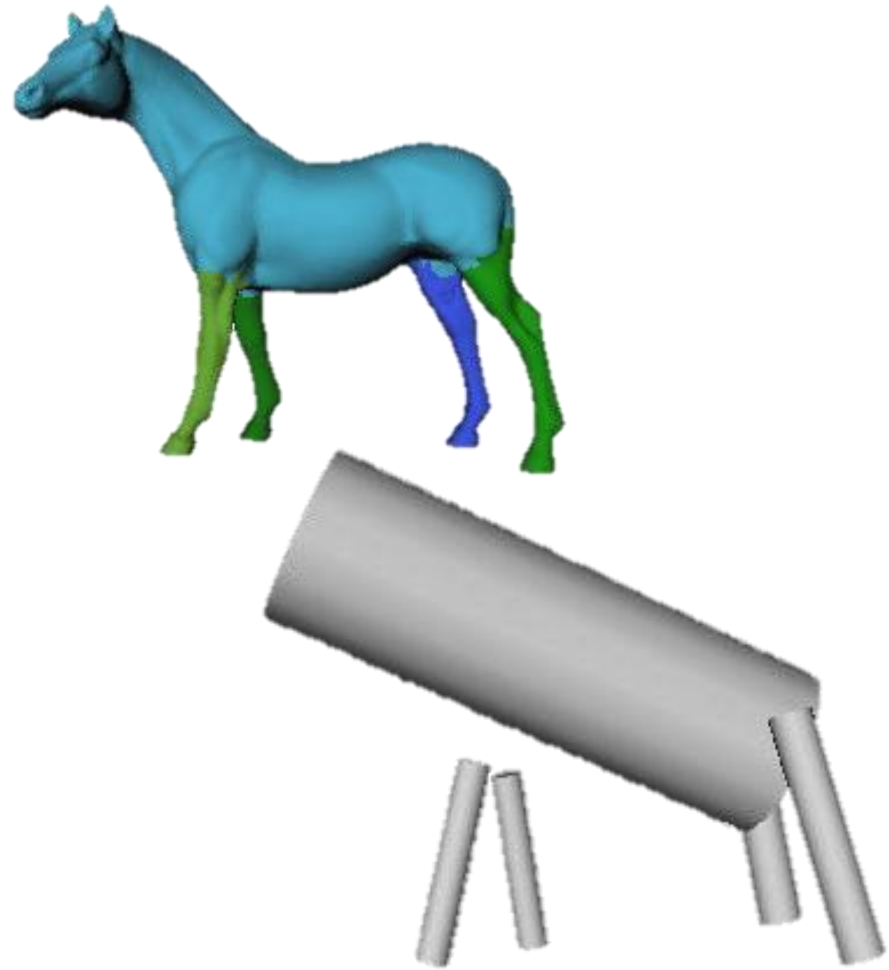
Two Related Problems

Segmentation

Part Discovery

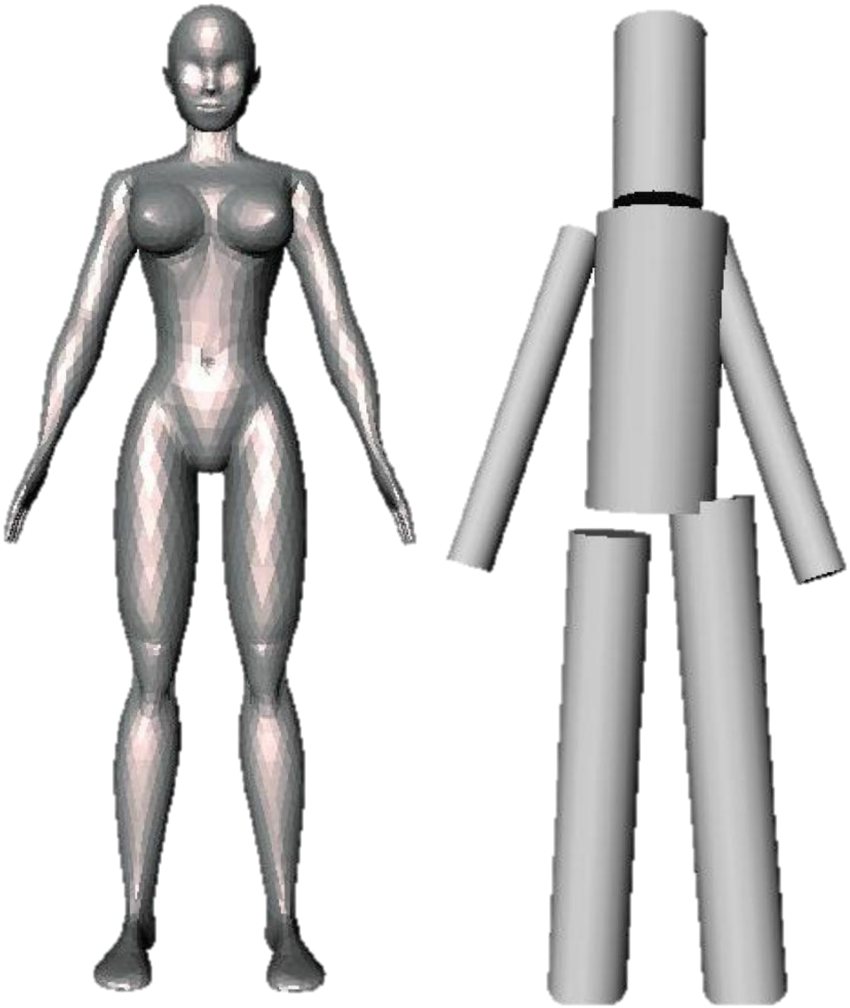
“Meaningful” Parts?



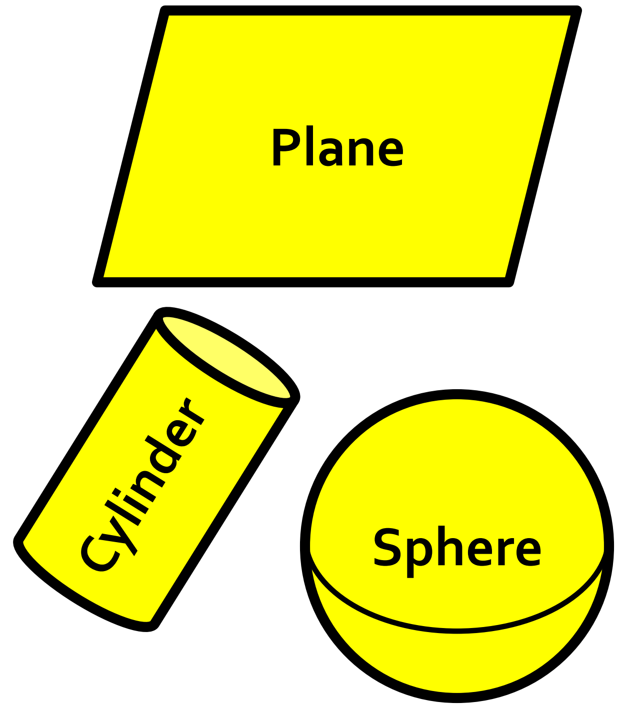


Hierarchical Mesh Segmentation Based on Fitting Primitives
Attene et al, Visual Computer 22.3 (2006)

“Meaningful” Parts?

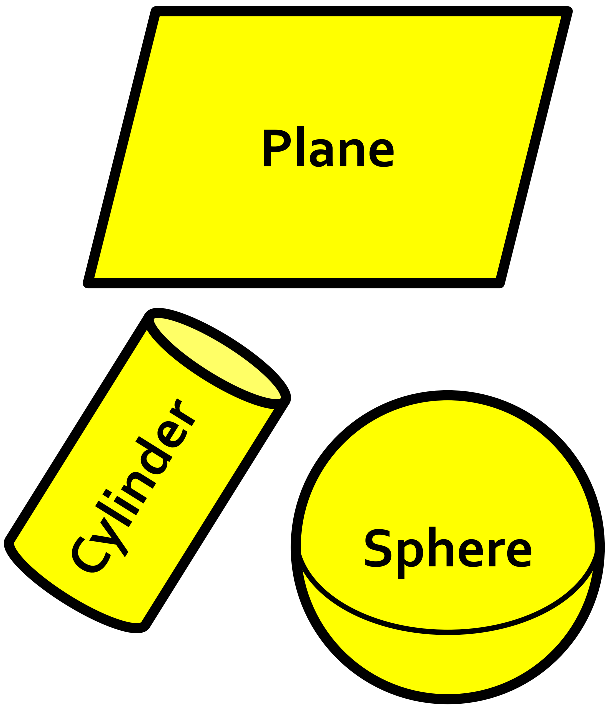


$$\approx \sum_{i=1}^{n}$$



Problem

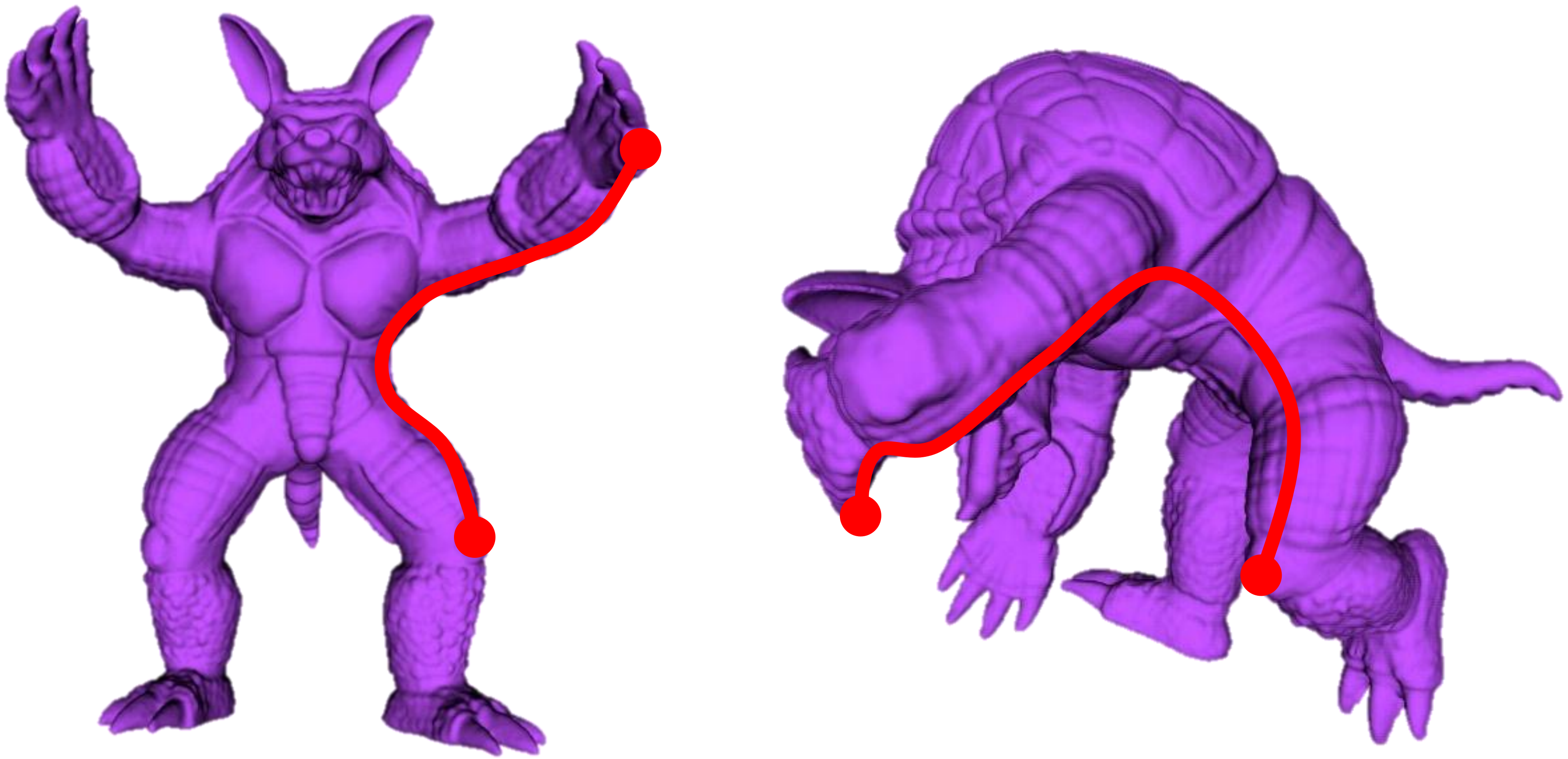
$$\stackrel{?}{\approx} \sum_{i=1}^{n}$$



How do you choose?

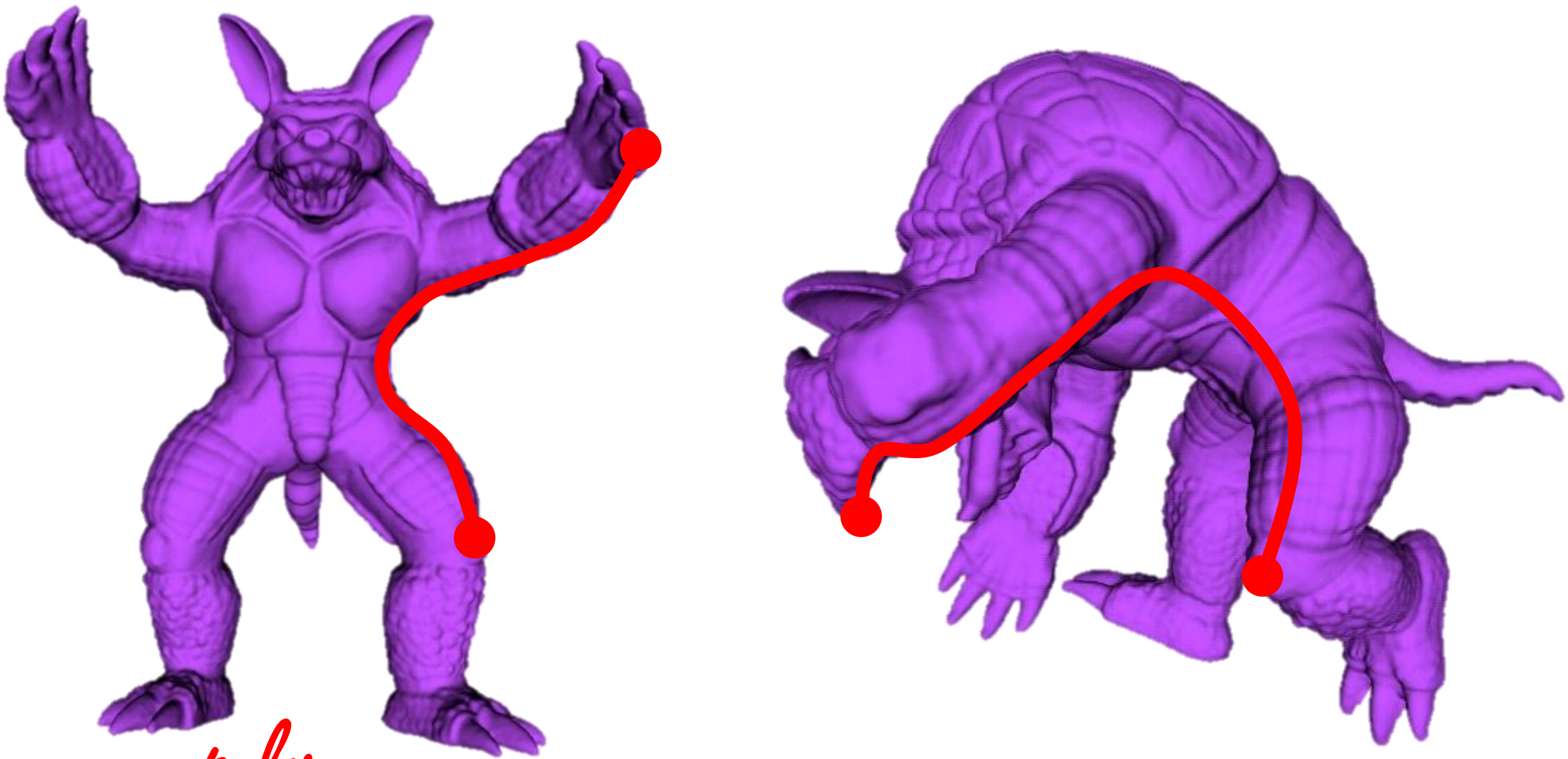
Idea: Let Parts Bend

Isometric Deformation



Preserves Pairwise Distances

Near- Isometric Deformation



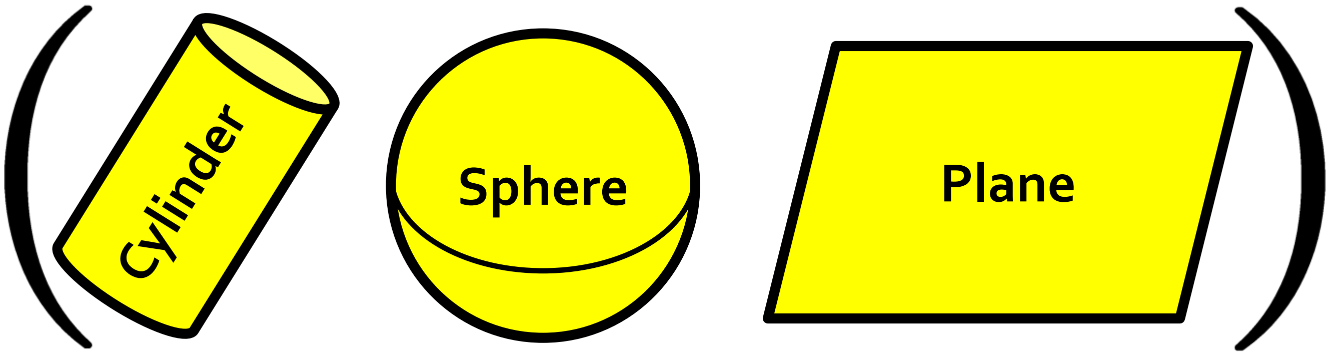
Approximately Preserves Pairwise Distances

Goal

Segmentation and part-finding invariant to near-isometry

Goal

Segmentation and part-finding invariant to near-isometry



Approach

- Find symmetries using **approximate Killing vector fields**
- Compute and cluster **isometry-invariant point signatures**

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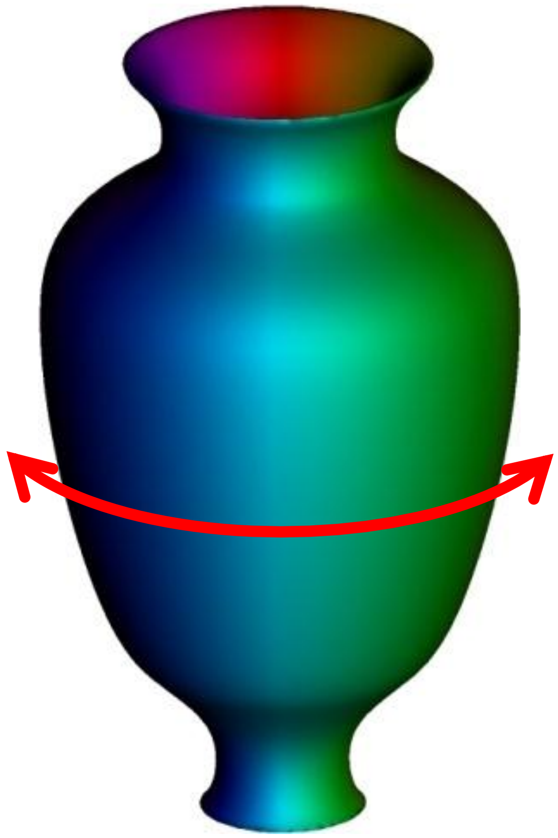
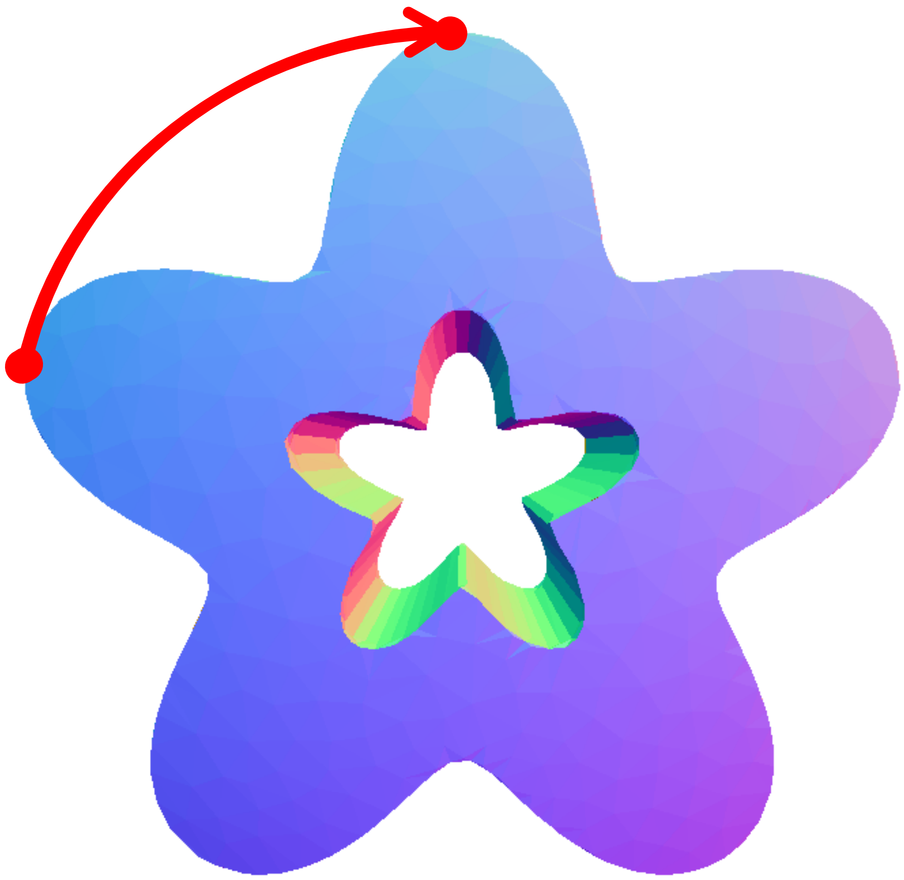
Killing Vector Fields

<digression>

Killing Vector Fields

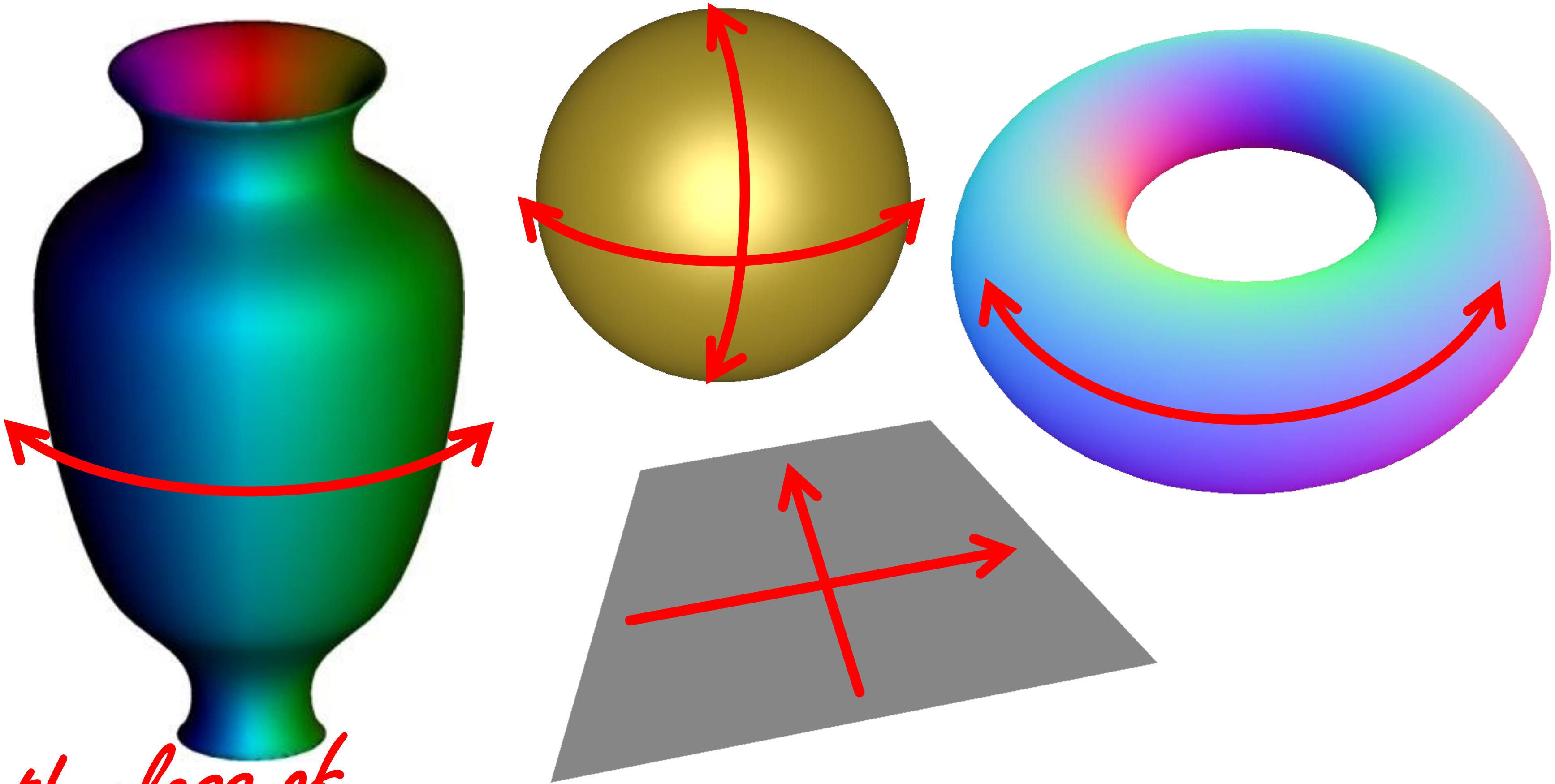
Wilhelm Killing
1847-1923

Self-Isometry

Distance-Preserving Self Map

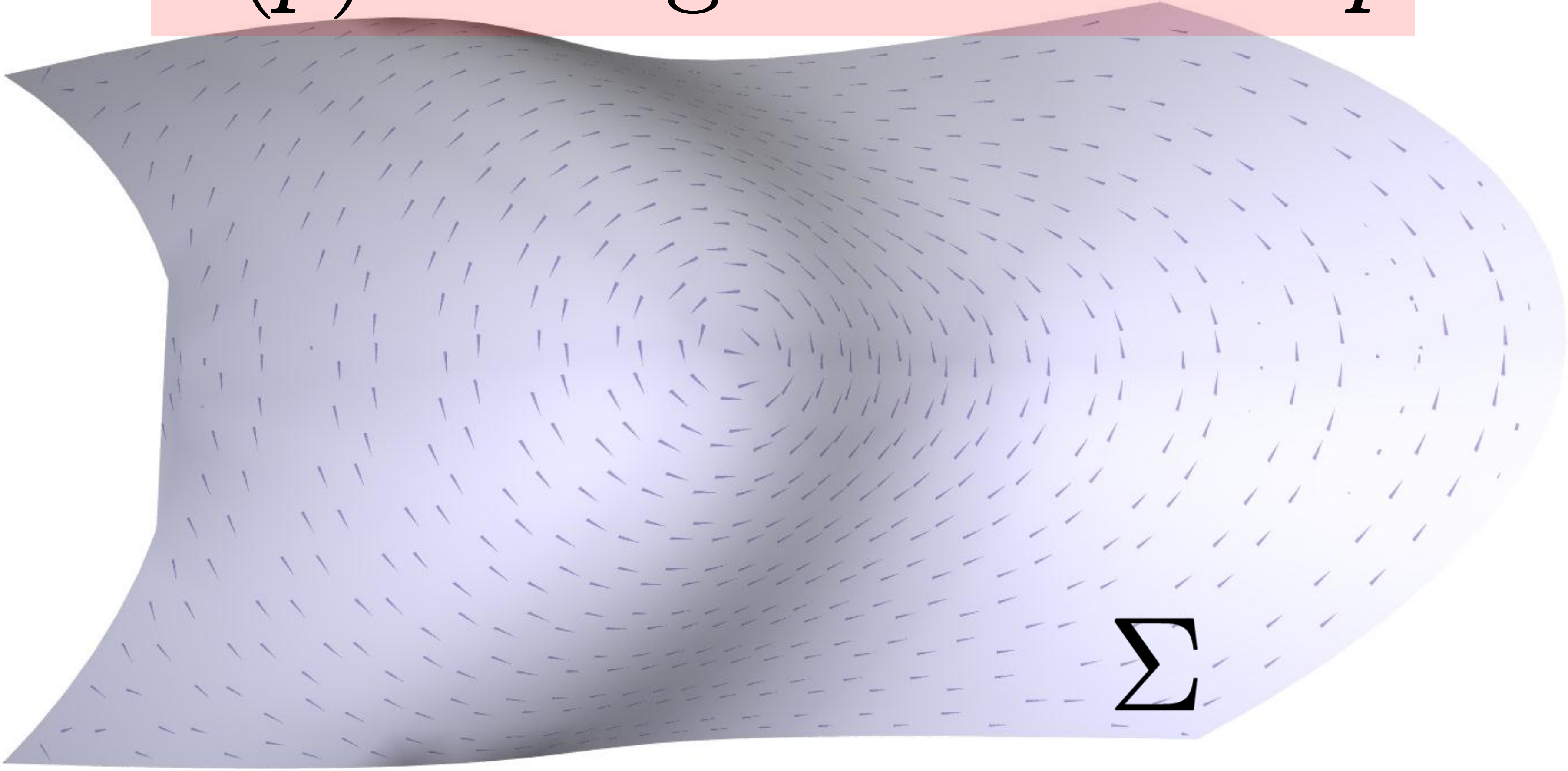
Continuous Self-Isometry



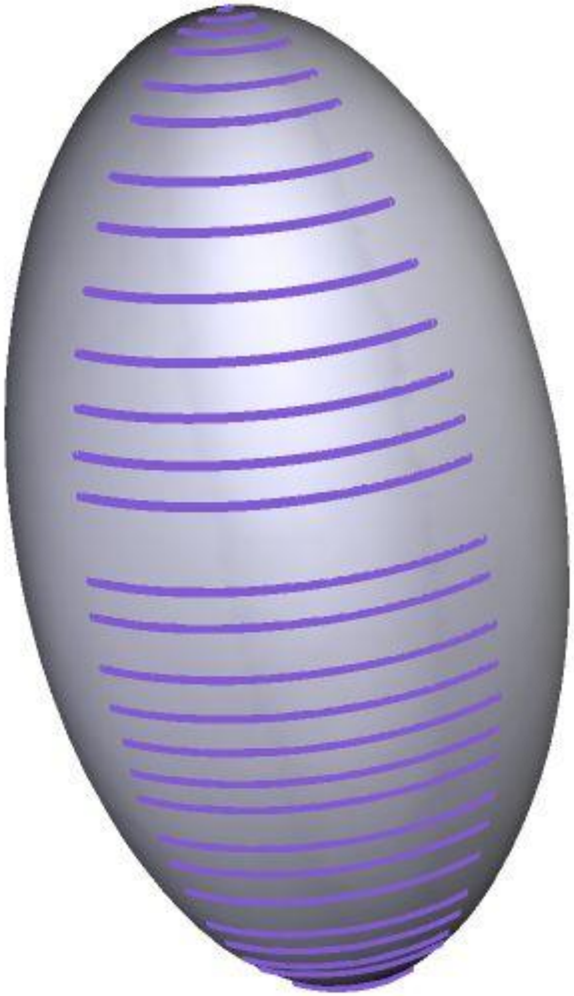
Smooth class of ^ Distance-Preserving Self Maps

Tangent Vector Field

$$\omega(p) = \text{tangent vector at } p$$



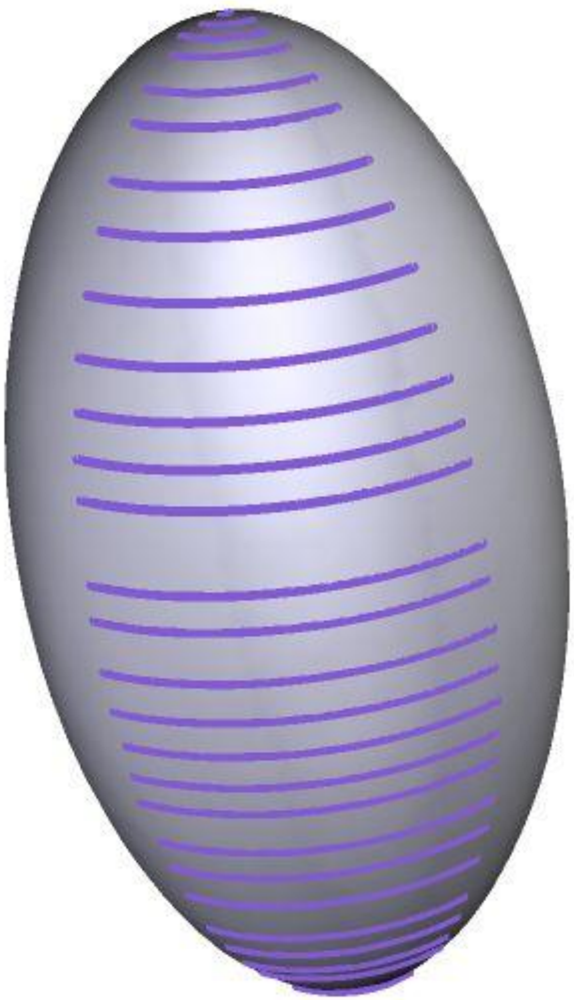
Killing Vector Fields (KVFs)



Continuous self-isometry

$$\phi_t : \Sigma \to \Sigma$$

Killing Vector Fields (KVFs)



Continuous self-isometry

$$\phi_t : \Sigma \to \Sigma$$

↓

Killing vector field

$$\frac{d\phi_t}{dt} : \Sigma \to T\Sigma$$

Discrete Approximate KVFs

Eurographics Symposium on Geometry Processing 2010
Olga Sorkine and Bruno Lévy
(Guest Editors)

Volume 29 (2010), Number 5

On Discrete Killing Vector Fields and Patterns on Surfaces

Mirela Ben-Chen Adrian Butscher Justin Solomon Leonidas Guibas

Stanford University

Abstract

Symmetry is one of the most important properties of a shape, unifying form and function. It encodes semantic information on one hand, and affects the shape's aesthetic value on the other. Symmetry comes in many flavors, amongst the most interesting being intrinsic symmetry, which is defined only in terms of the intrinsic geometry of the shape. Continuous intrinsic symmetries can be represented using infinitesimal rigid transformations, which are given as tangent vector fields on the surface – known as Killing Vector Fields. As exact symmetries are quite rare, especially when considering noisy sampled surfaces, we propose a method for relaxing the exact symmetry constraint to allow

AKVFs: Main Idea

Vector field: $\omega \in \mathbb{R}^E$

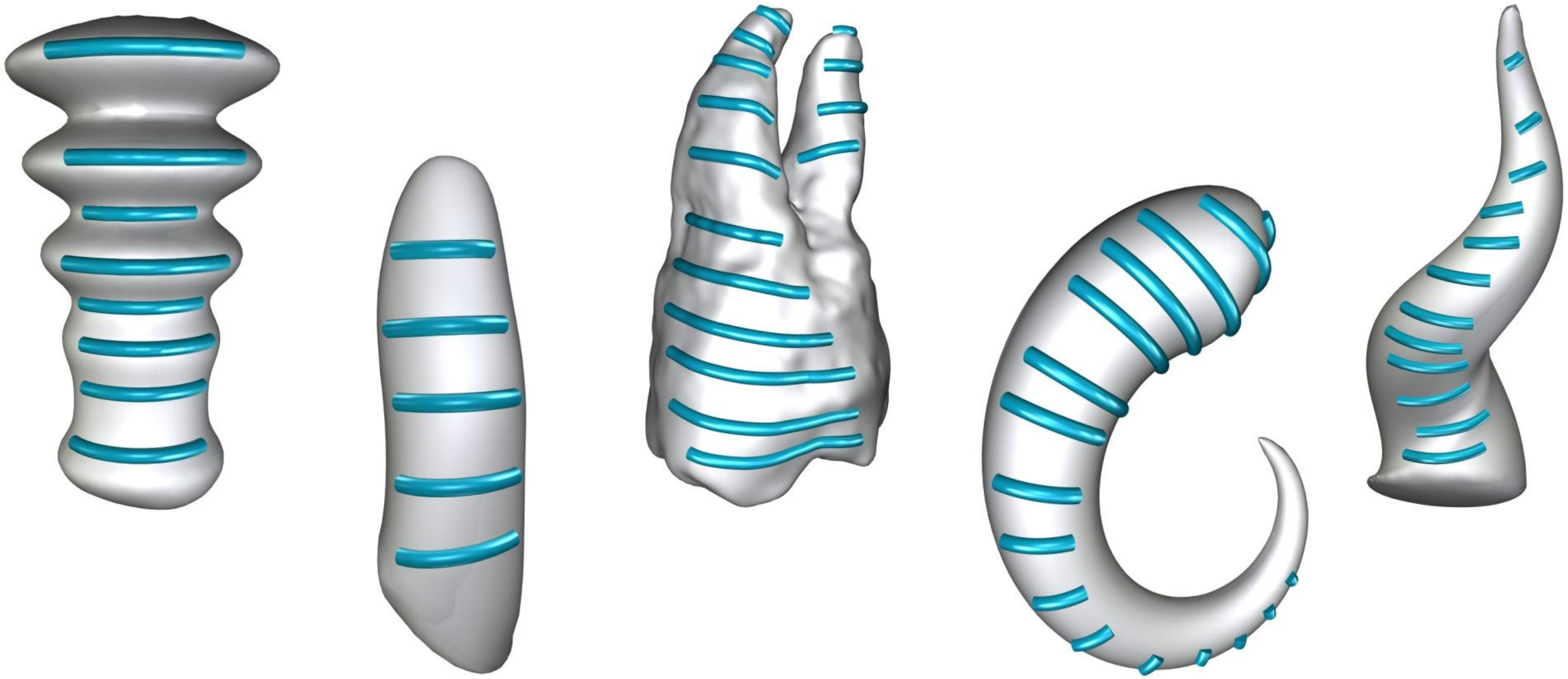
Operator (matrix) K measuring deviation from isometry

Want to minimize $\|K\omega\|^2$ subject to $\|\omega\| = 1$

$$\Updownarrow$$

Find eigenvectors ("eigenfields") of K

AKVF Examples



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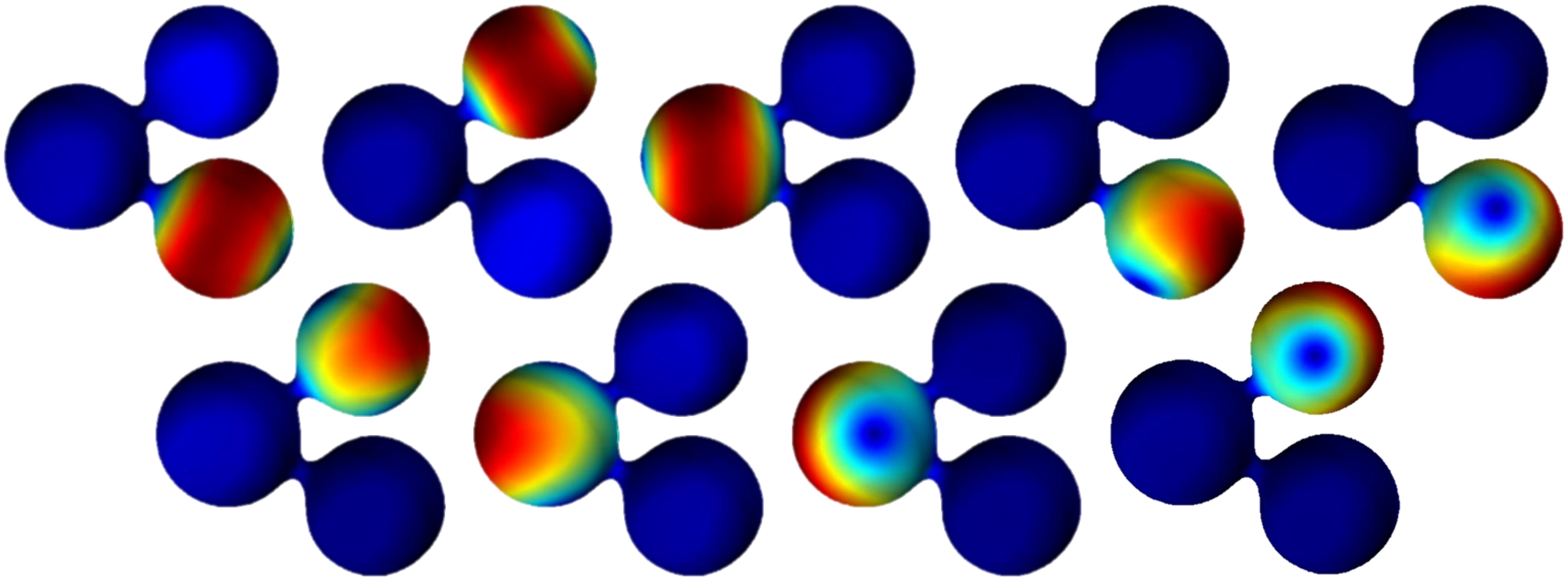
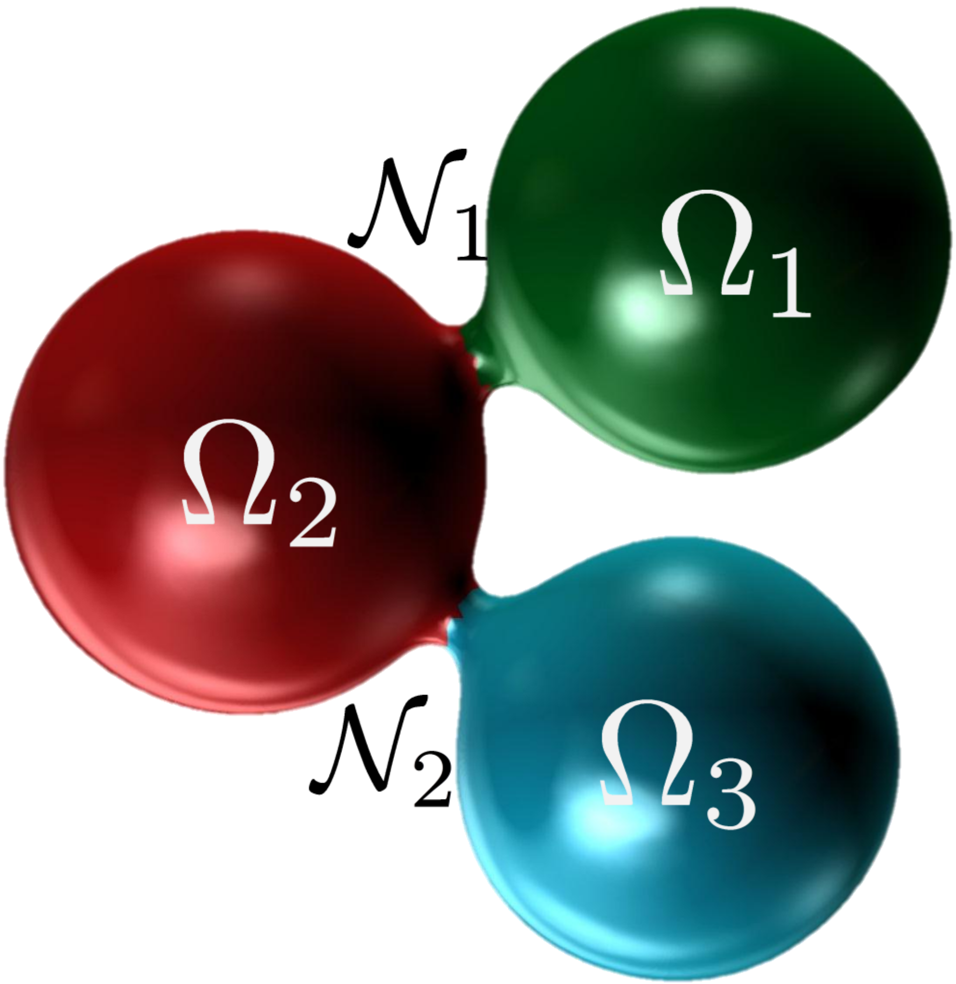
Observation

Eigenfields of *K* are localized on parts!



Composite Shape



- $_i = \text{component } i$
- $_i \subseteq \text{surface } \Sigma_i$

$\mathcal{N}_i = \text{neck } i$

Motivating Theorem

Proposition 1. *There exist constants* $\varepsilon_0, C > 0$ *depending only on the eigenvalues of* Σ_1, Σ_2 *and a number* $M(\varepsilon)$ *with* $\lim_{\varepsilon \to 0} M(\varepsilon) = \infty$ *so that the spectral data of* $P^* P$ *satisfies:*

1. *If* $\varepsilon < \varepsilon_0$ *then for all* n *s.t.* $\lambda_n < M(\varepsilon)$ *we have*

$$|\lambda_n - \mu_n| \leq C / |\log(\varepsilon)| .$$

2. *Let* $P_{n,\delta}$ *be the* L^2*-orthogonal projector onto the subspace* $\mathcal{W}_{n,\delta} := span\{\tilde{u}_k : k \text{ s.t. } |\lambda_n - \mu_k| \leq \delta\}$. *If* $\varepsilon < \varepsilon_0$ *then for all* $\delta > 0$ *and* n *s.t.* $\lambda_n < M(\varepsilon)$ *we have*

$$\omega_n = P_{n,\delta}(\omega_n) + \eta$$

where $\eta \perp \mathcal{W}_{n,\delta}$ *and satisfies* $\|\eta\|_{L^2}^2 \leq C / (\delta^2 |\log(\varepsilon)|)$.

Motivating Theorem

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Composite eigenfields come from the parts.

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Composite eigenfields come from the parts.*

*** When the necks are small**

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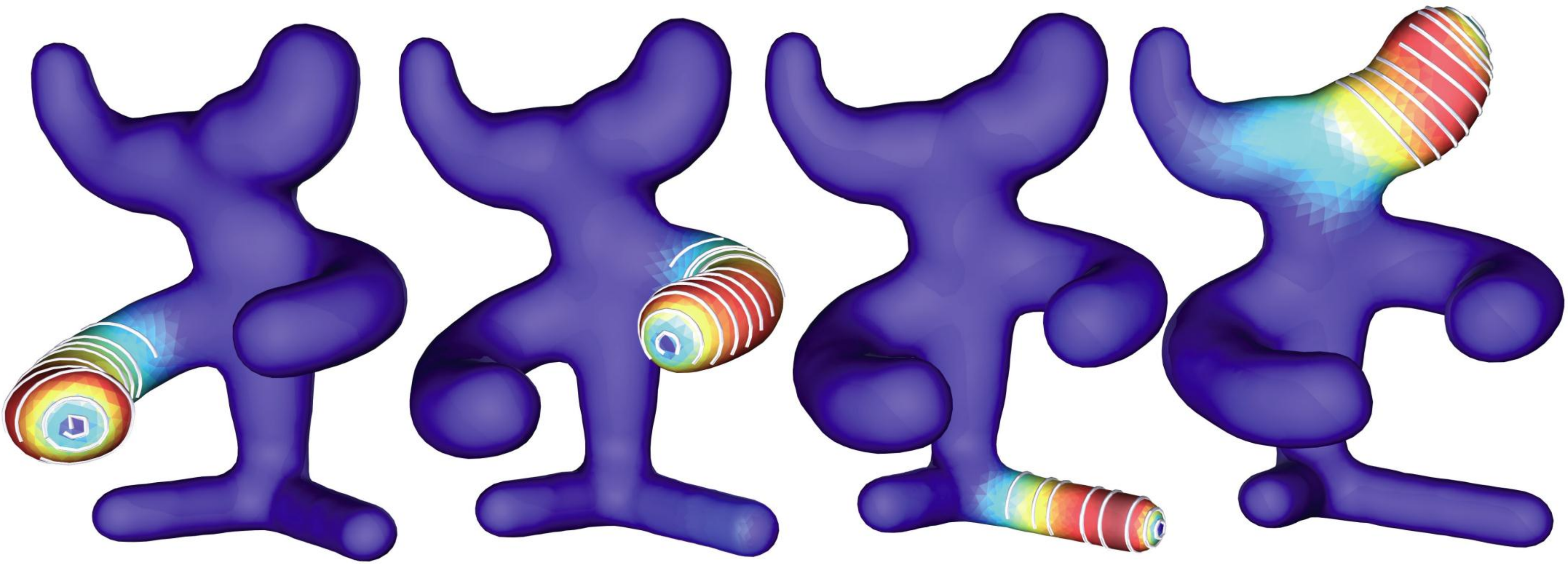
Composite eigenfields come from the parts.*,**

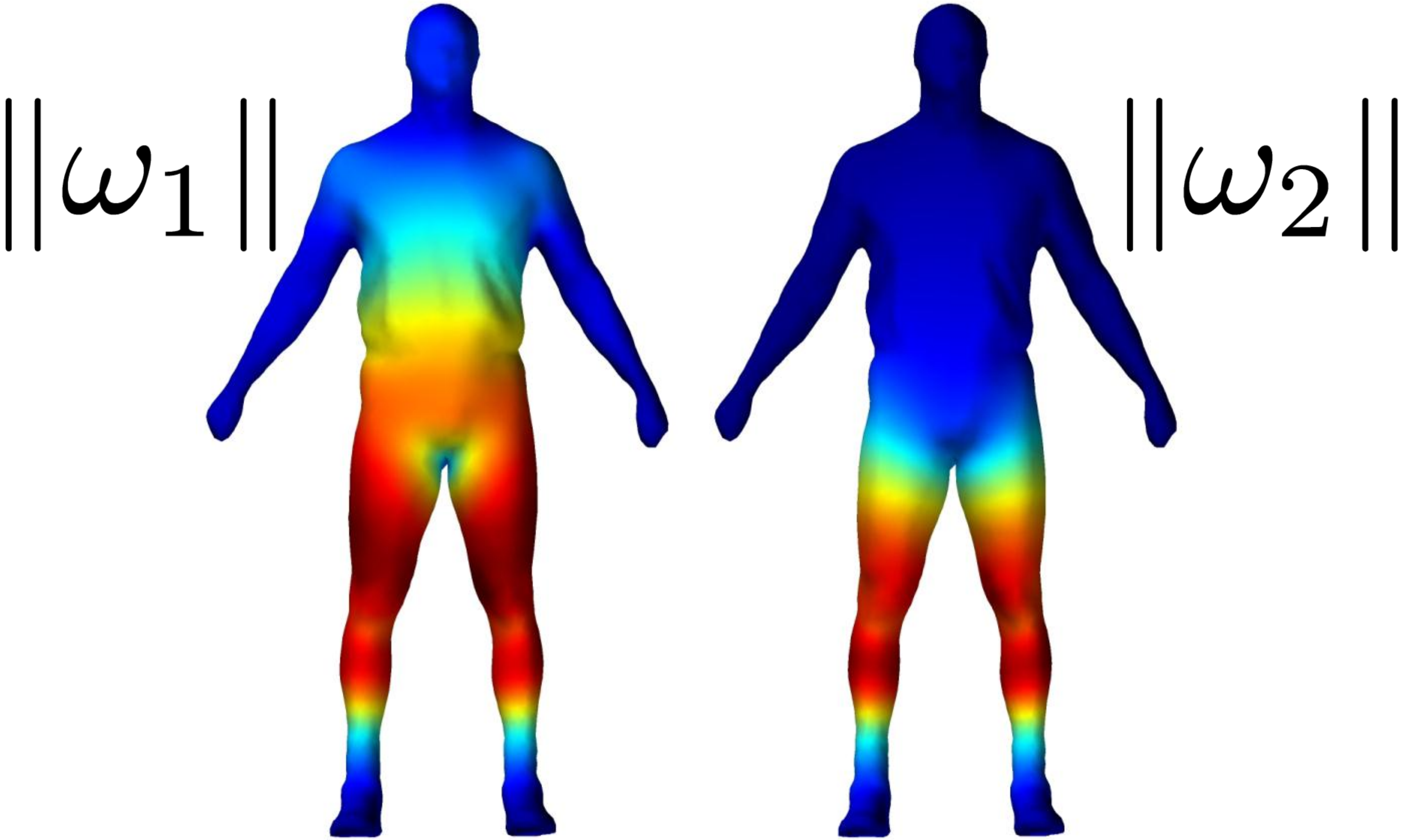
** See supplemental document

* When the necks are small

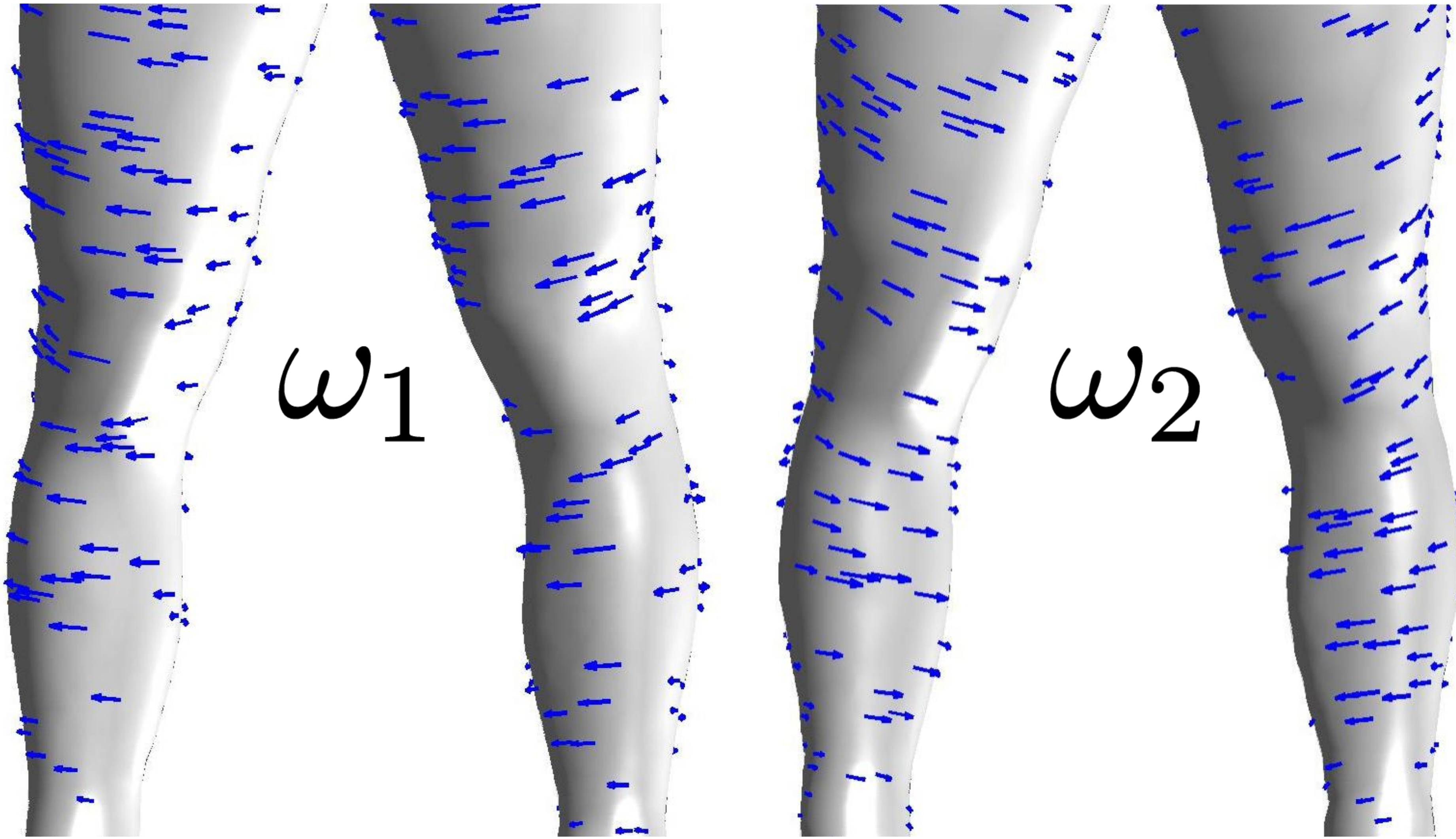
Larger Necks



Problem: Linear Combination

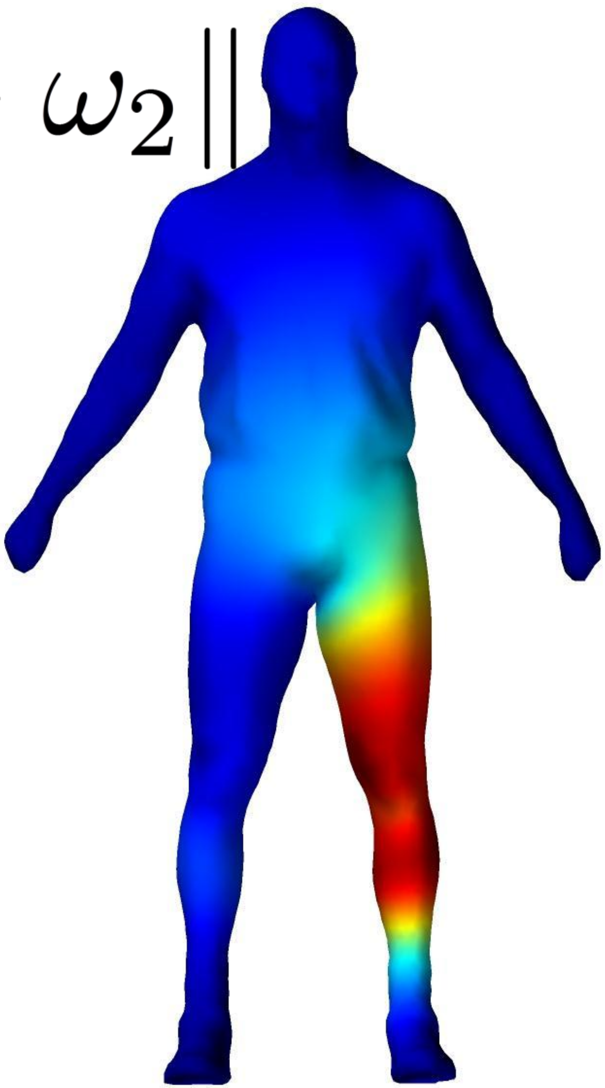


Problem: Linear Combination

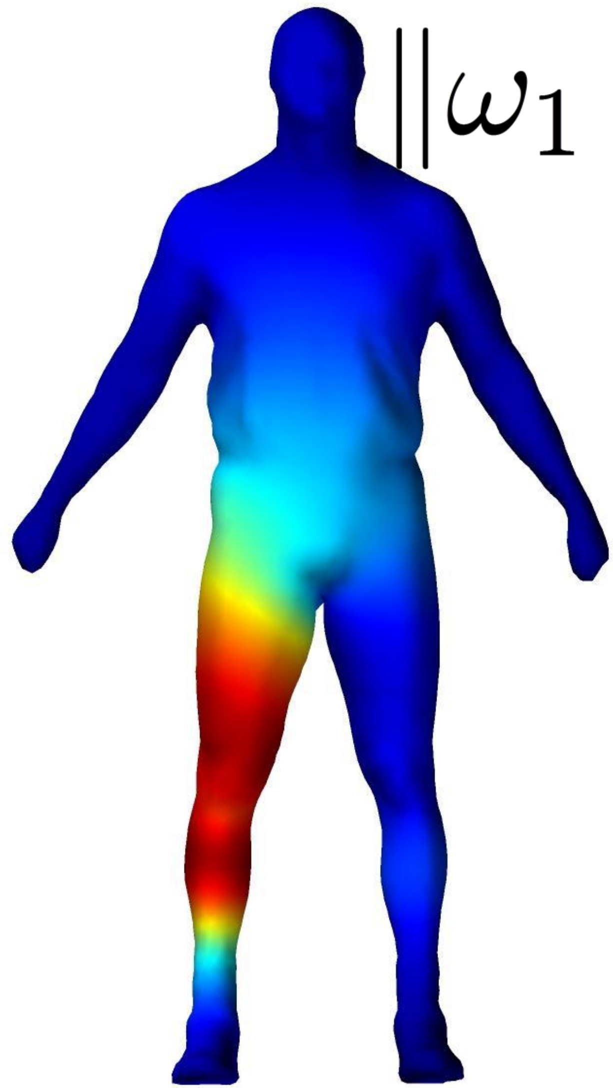


Problem: Linear Combination

$$\|\omega_1 - \omega_2\|$$



$$\|\omega_1 + \omega_2\|$$



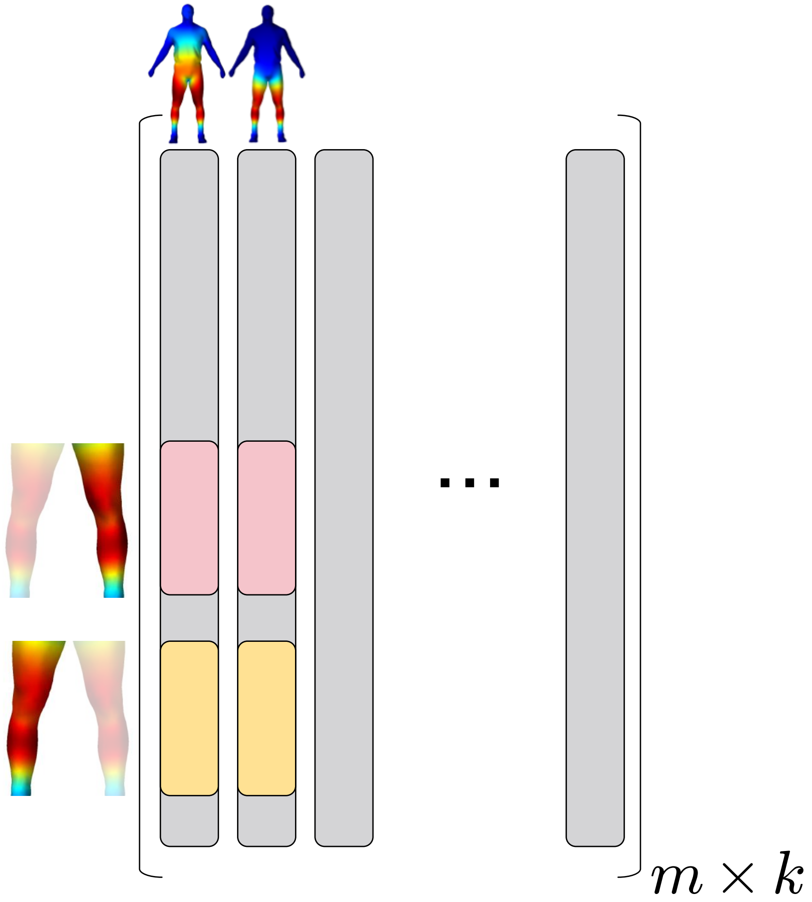
Tangling Energy

$$E(\omega_1, \omega_2) = \int_\Sigma \|\omega_1\|^2 \|\omega_2\|^2$$

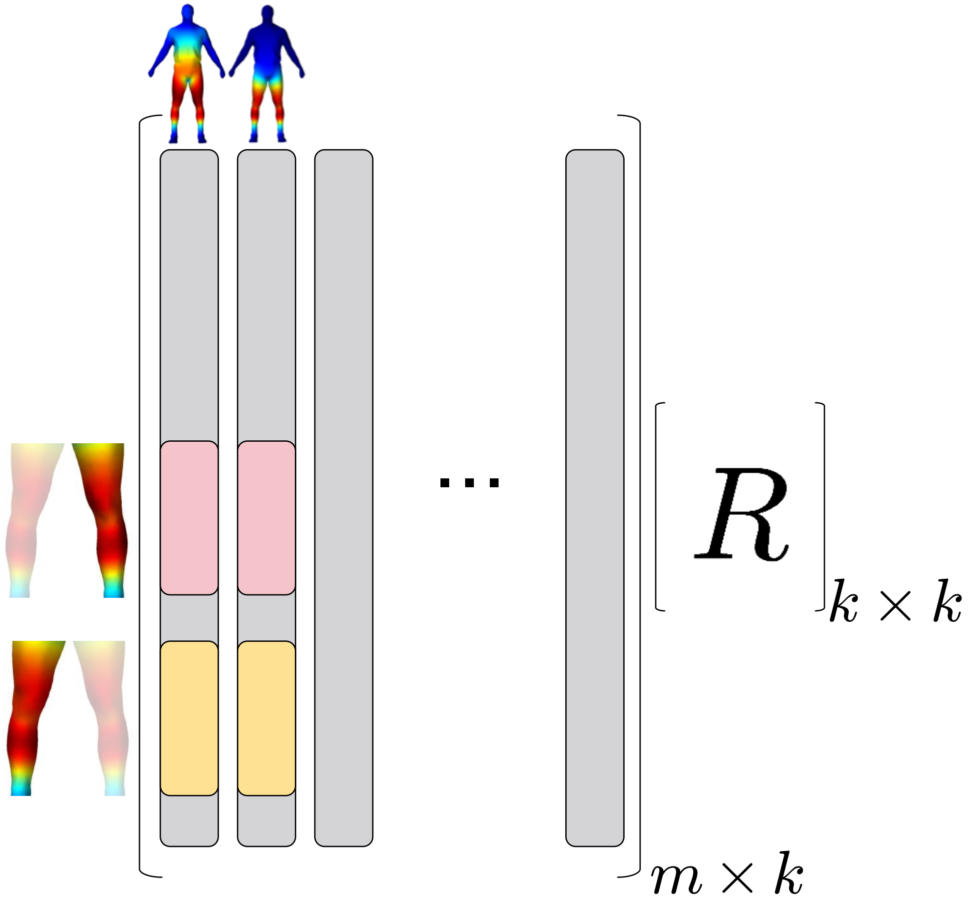
$$\downarrow$$

$$E(\omega_1, \ldots, \omega_N) = \sum_i \sum_{j>i} E(\omega_i, \omega_j)$$

How to Untangle



How to Untangle



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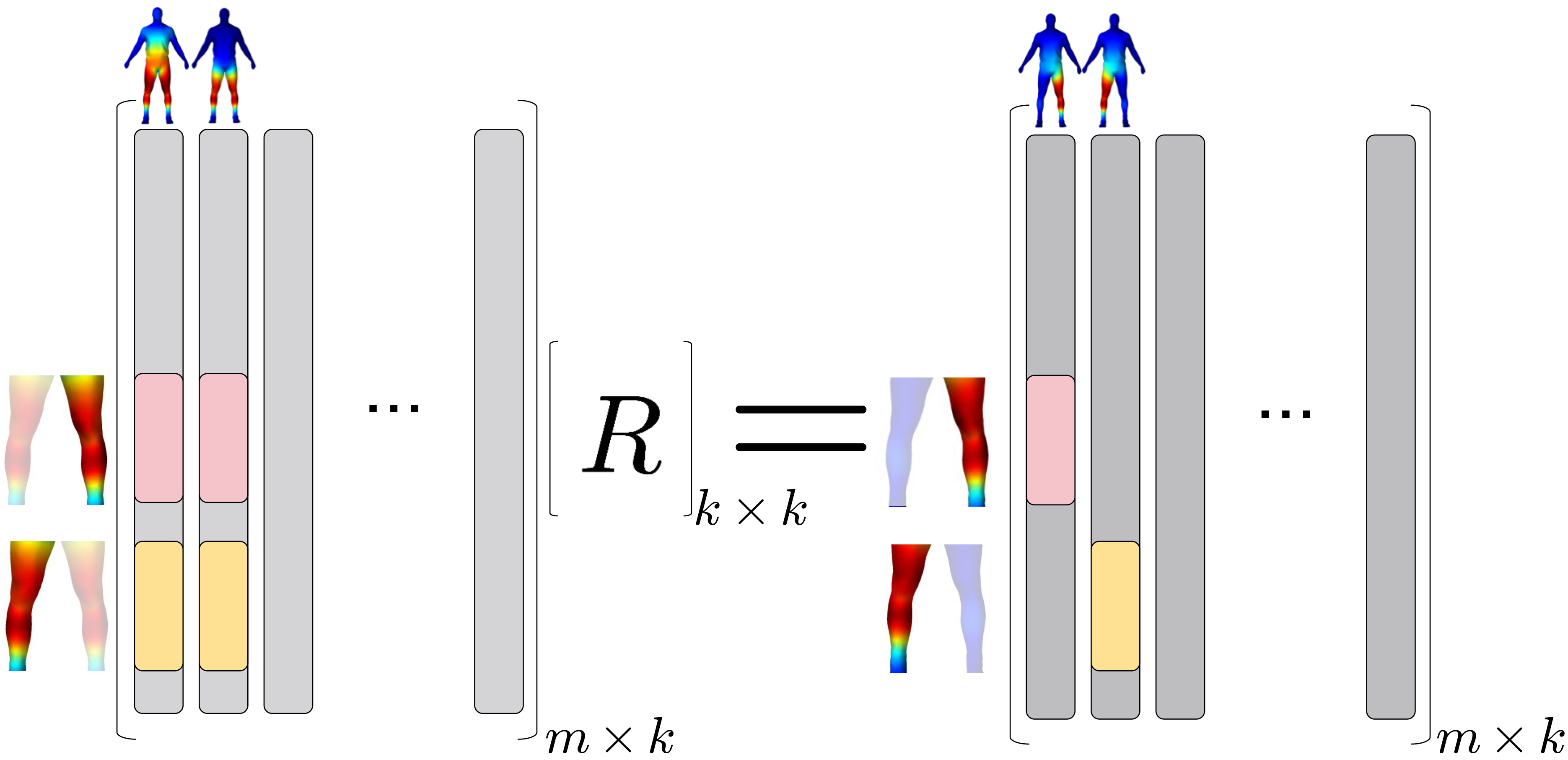
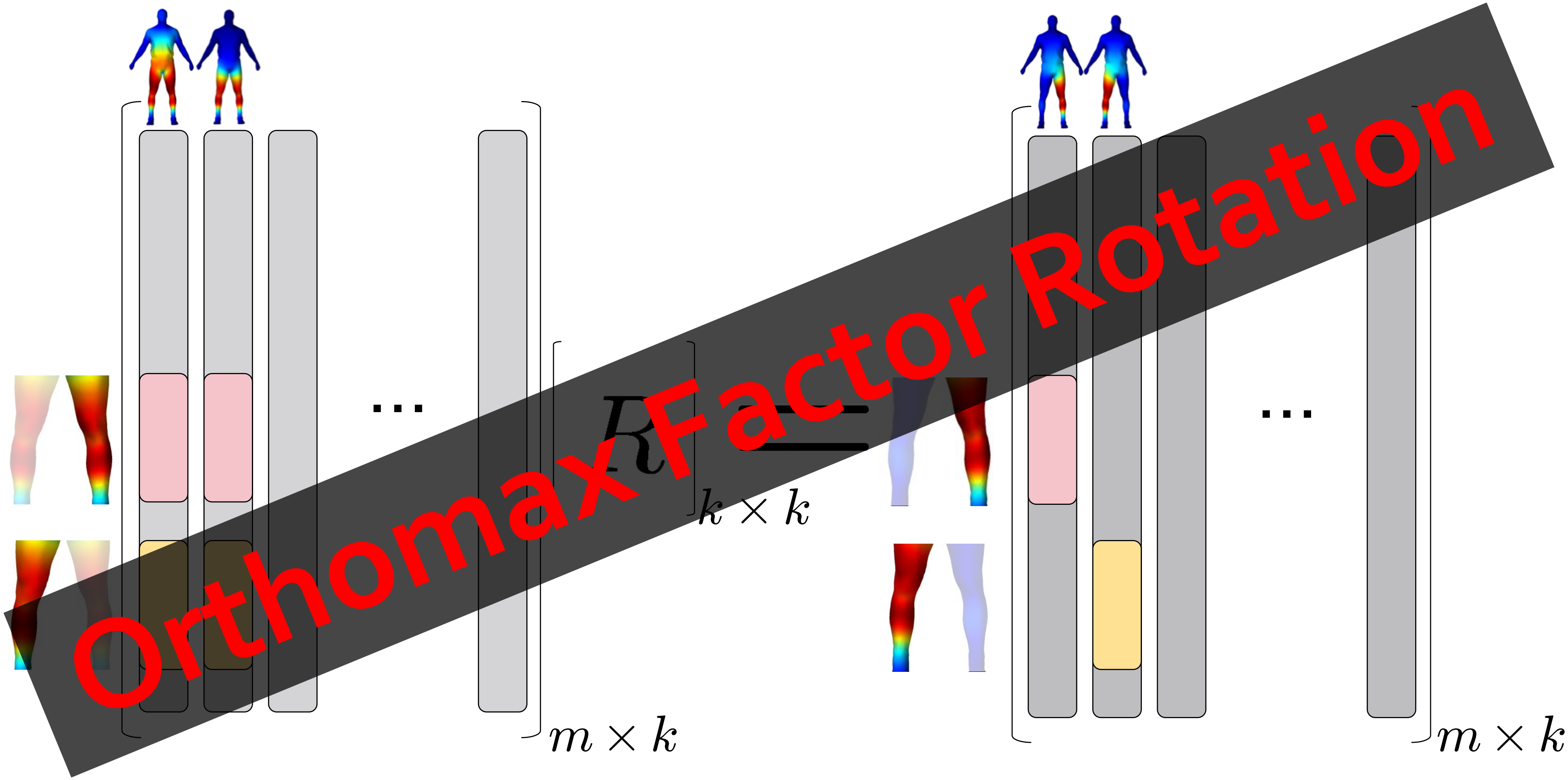
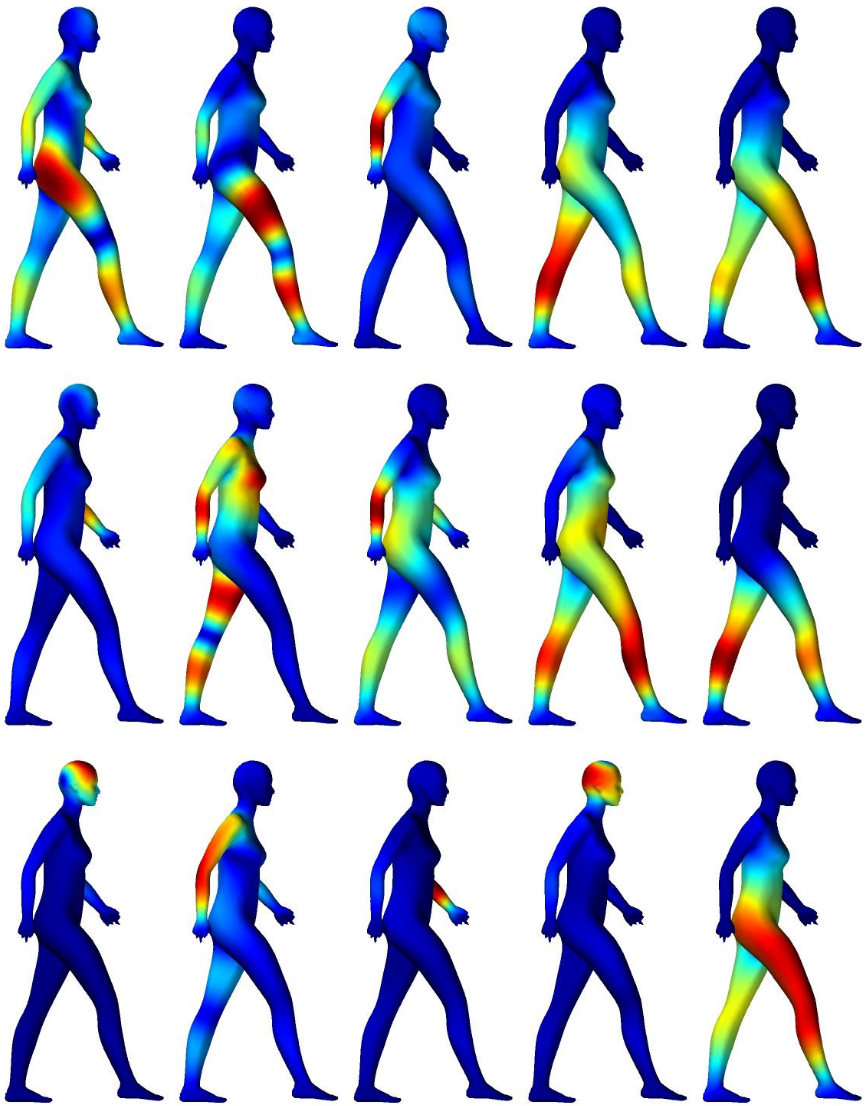


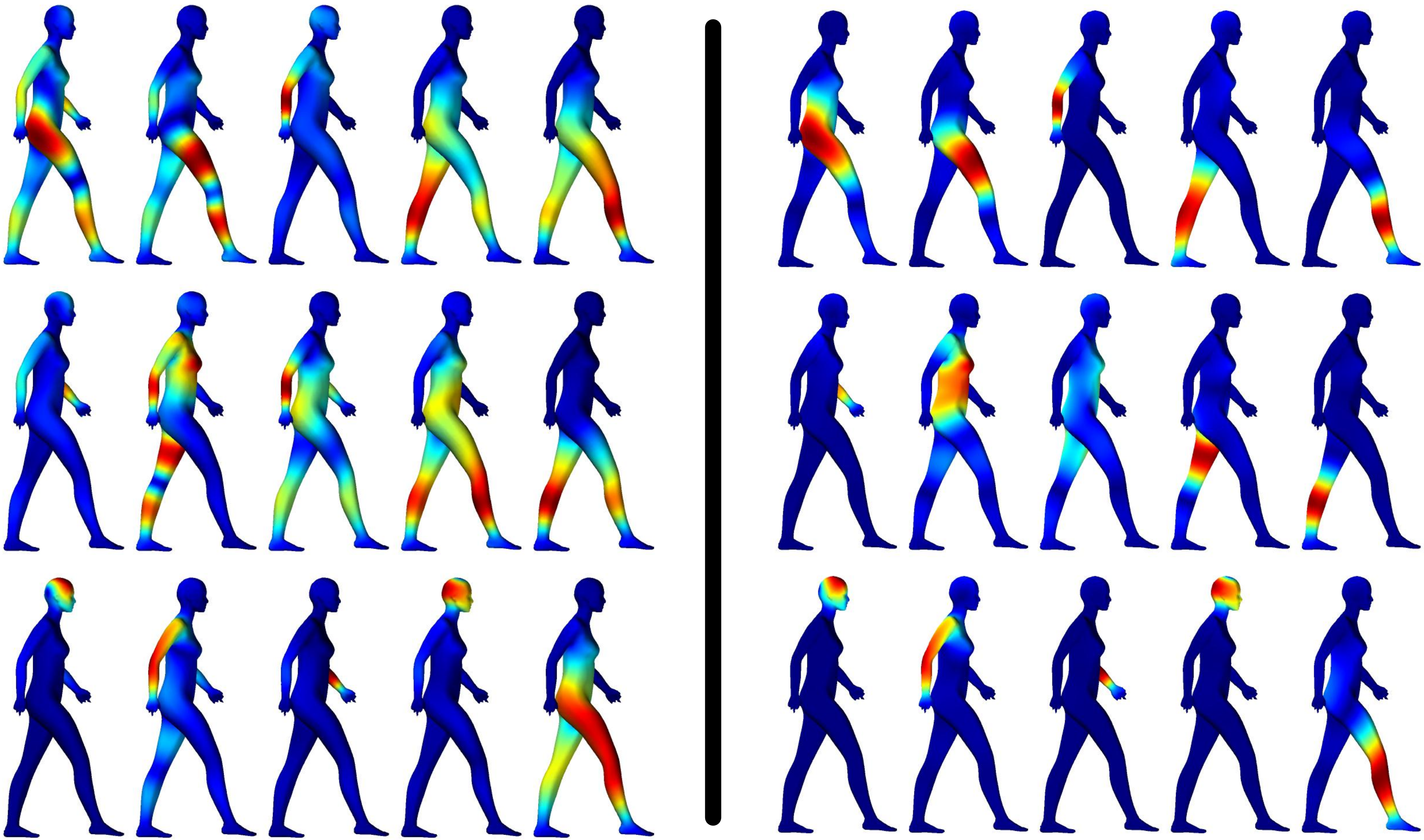
How to Untangle



Untangling Example



Untangling Example

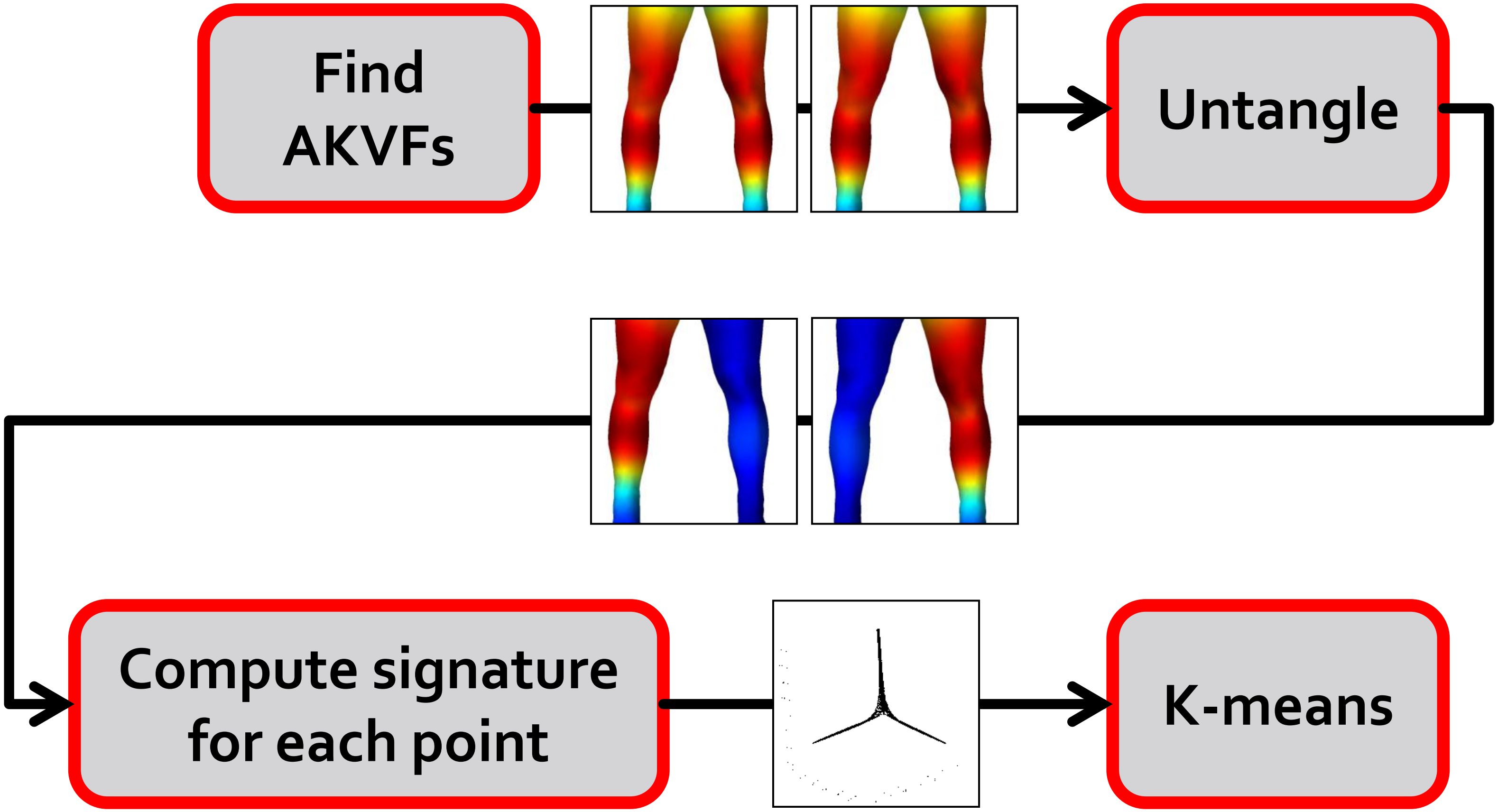


Signatures for Part Discovery

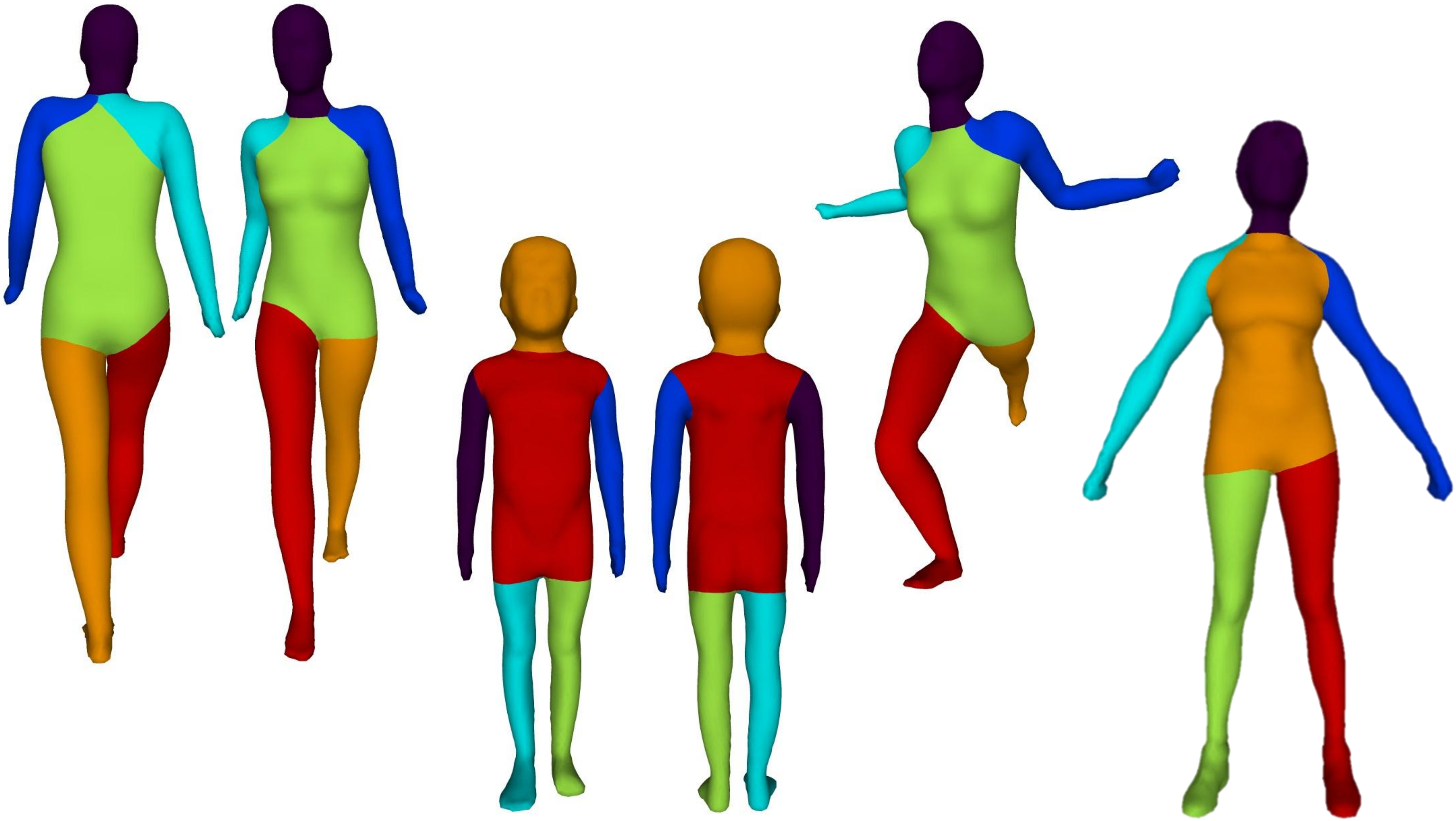
$$\varphi(p) = (\|u_1(p)\|, \|u_2(p)\|, \ldots, \|u_N(p)\|)$$

Untangled vector fields

Segmentation Algorithm



Segmentation Results

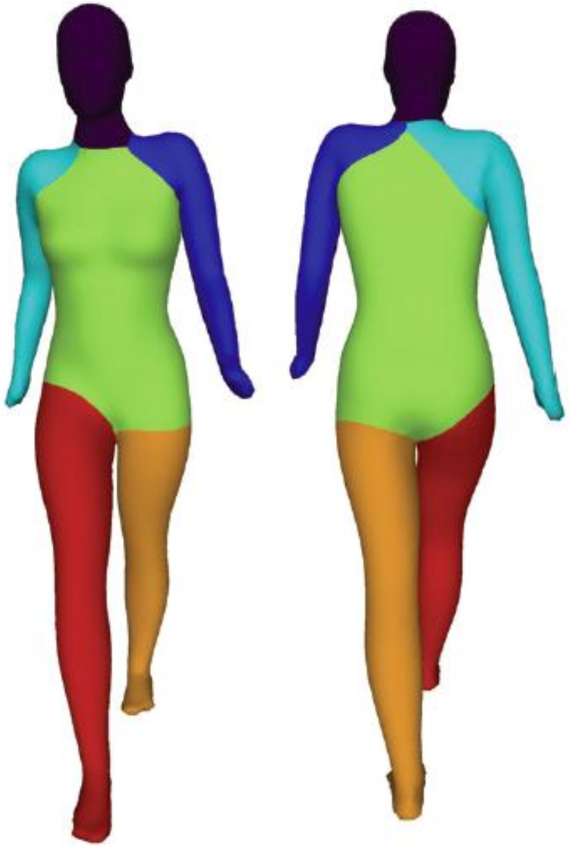


Segmentation Results

Comparison

Shape Diameter
[Shapira et al. 2008]

Randomized Cuts
[Golovinskiy et al. 2008]



Intrinsic Primitives

Comparison

Shape Diameter
[Shapira et al. 2008]

Randomized Cuts
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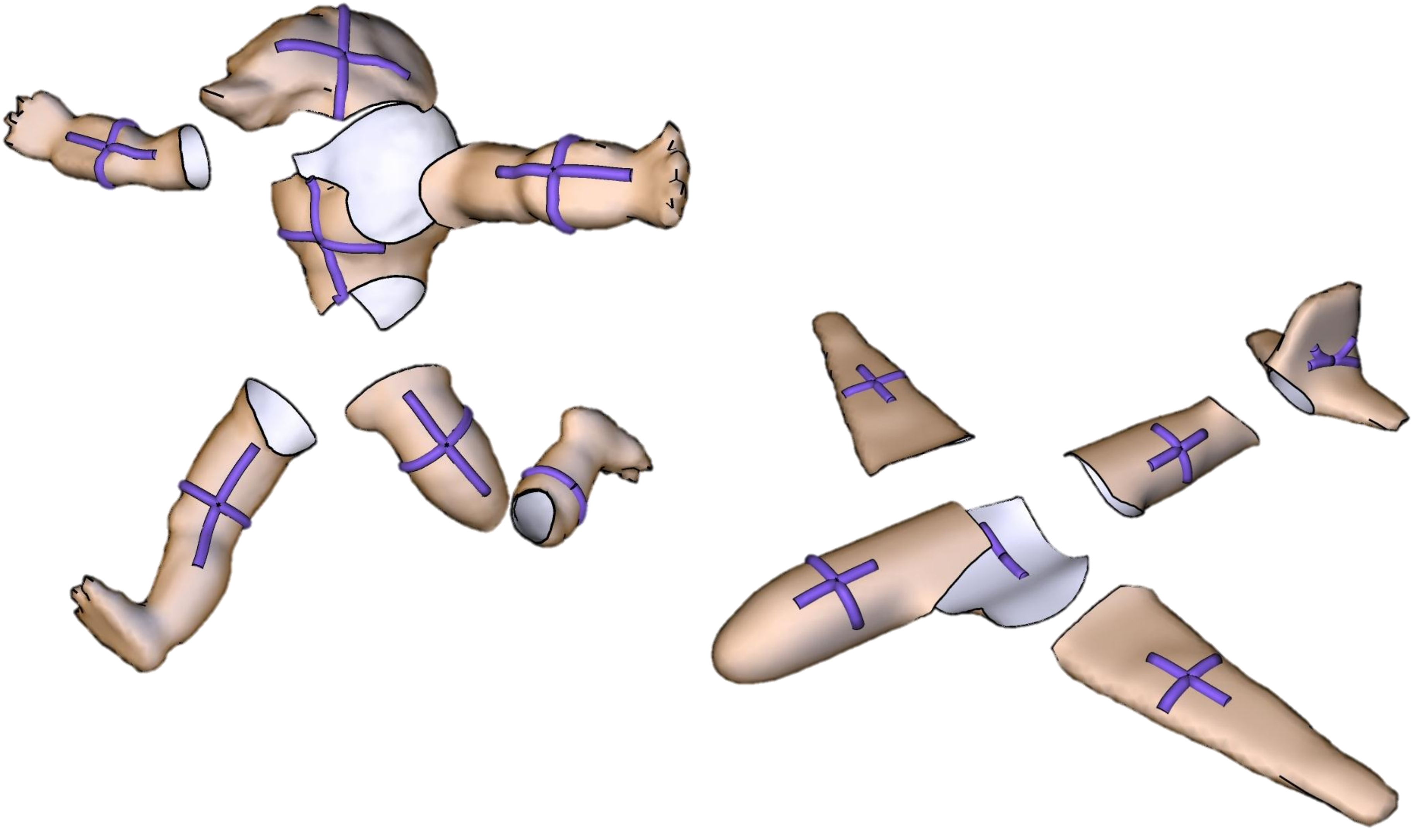
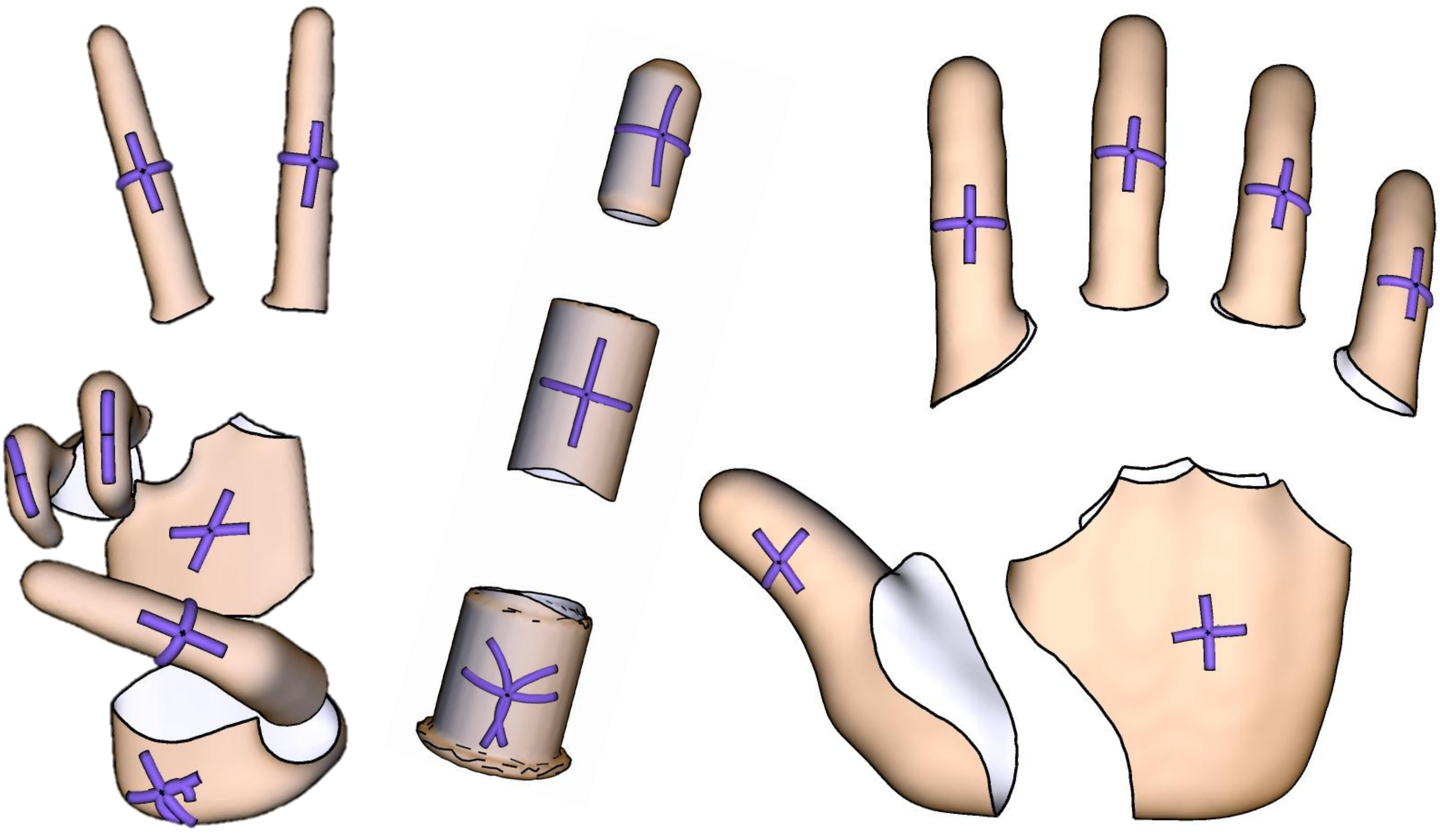
Intrinsic Primitives

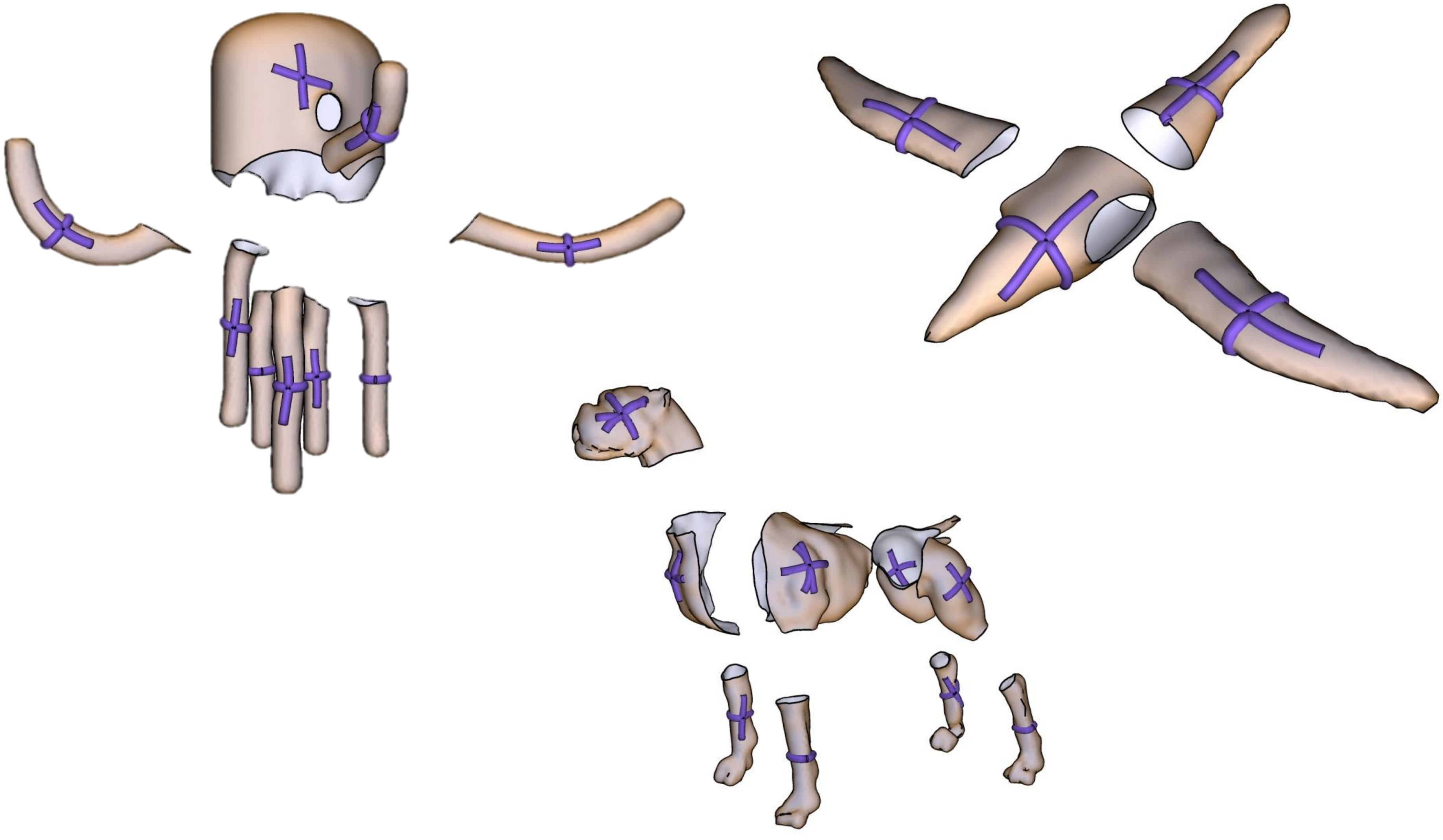
Part Discovery



Part Discovery



Part Discovery



Conclusions

1. Segmentation into **intrinsically symmetric parts**
2. **KVFs of a composite** come from KVFs of its parts
3. **Untangle** KVFs for better localization

Special Thanks

NSF

the Hertz FOUNDATION
freedom to innovate

DEPARTMENT OF DEFENSE
UNITED STATES OF AMERICA

FULBRIGHT

STANFORD
COMPUTER SCIENCE

מכון ויצמן למדע
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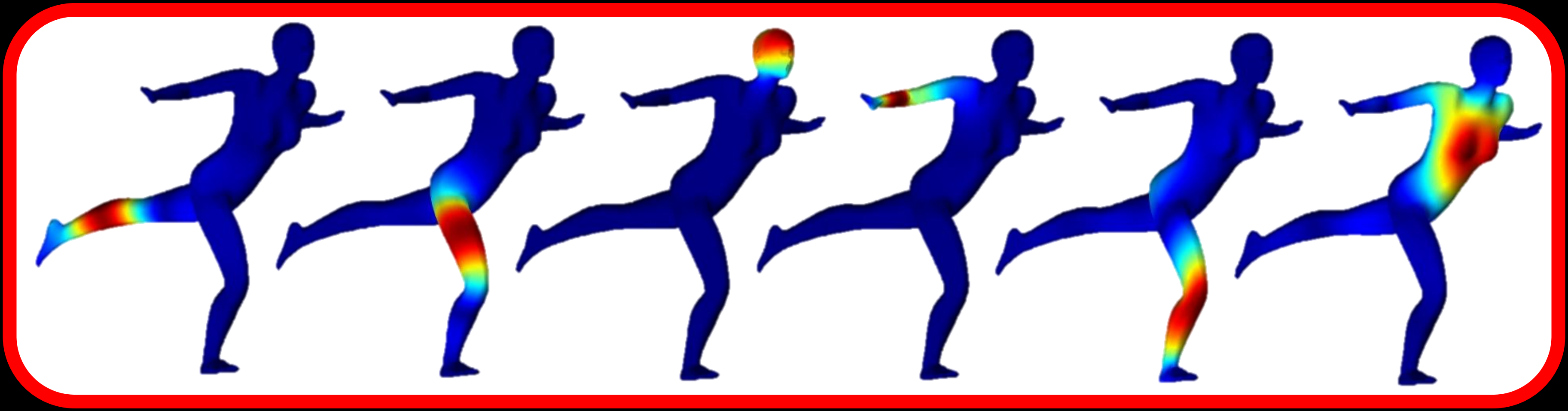

Questions?