

Soft Maps Between Surfaces



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“Traditional” Mapping

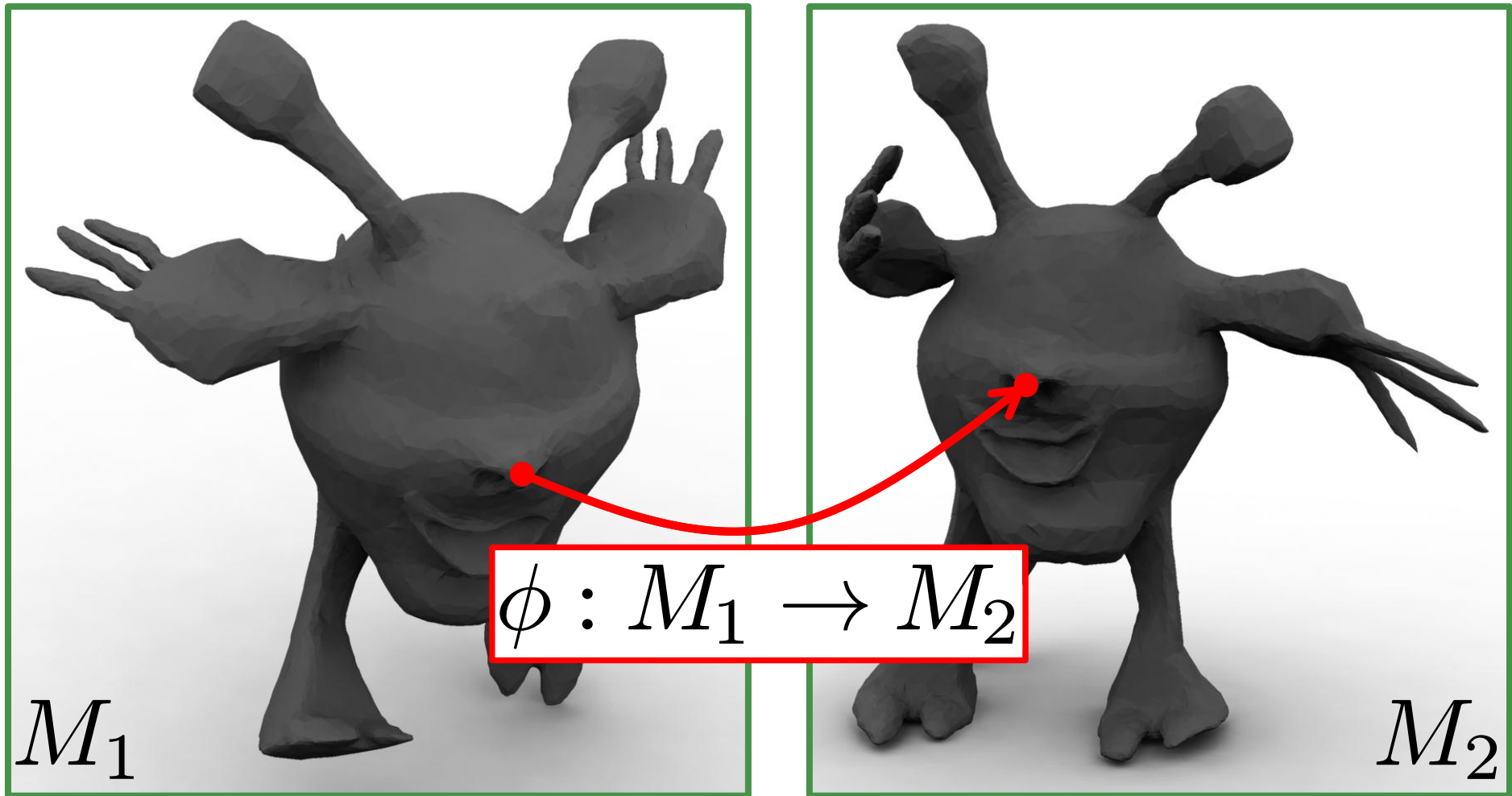


M_1

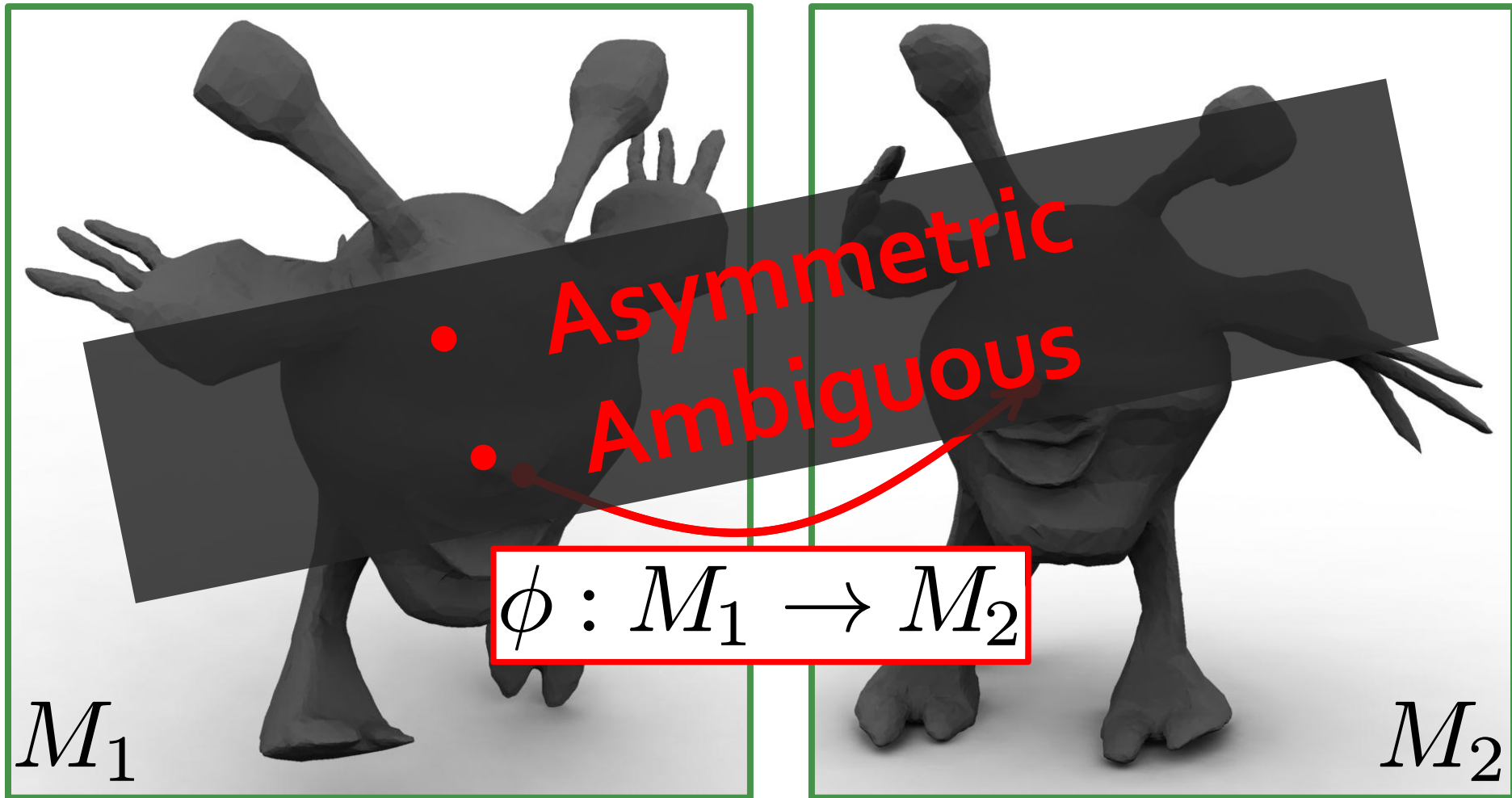


M_2

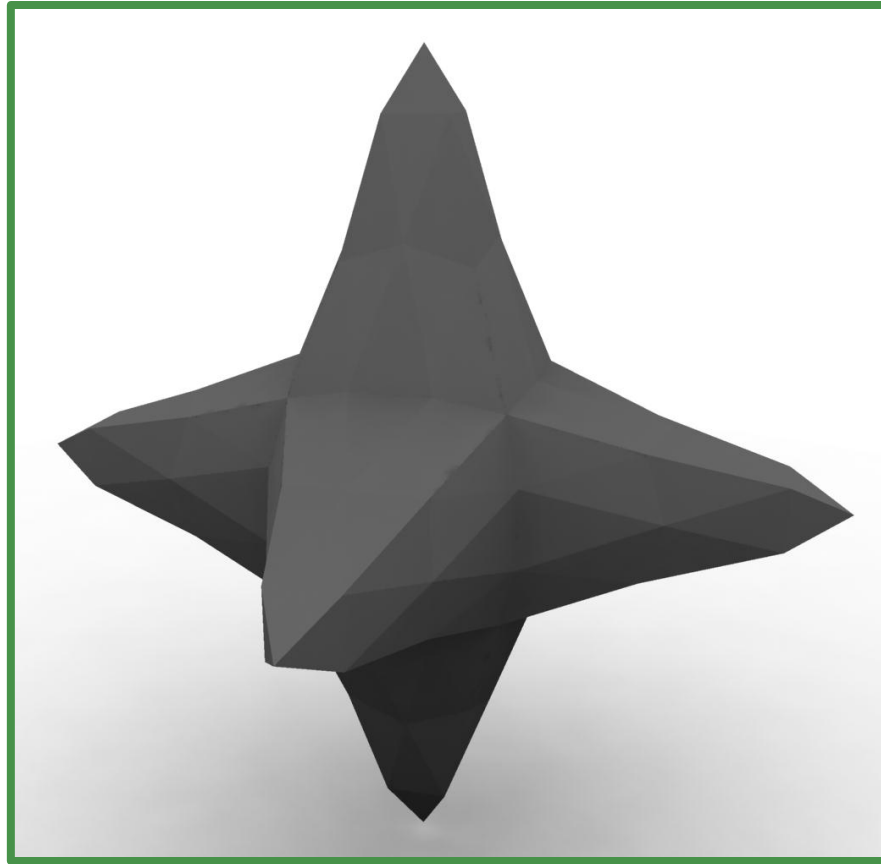
“Traditional” Mapping



“Traditional” Mapping

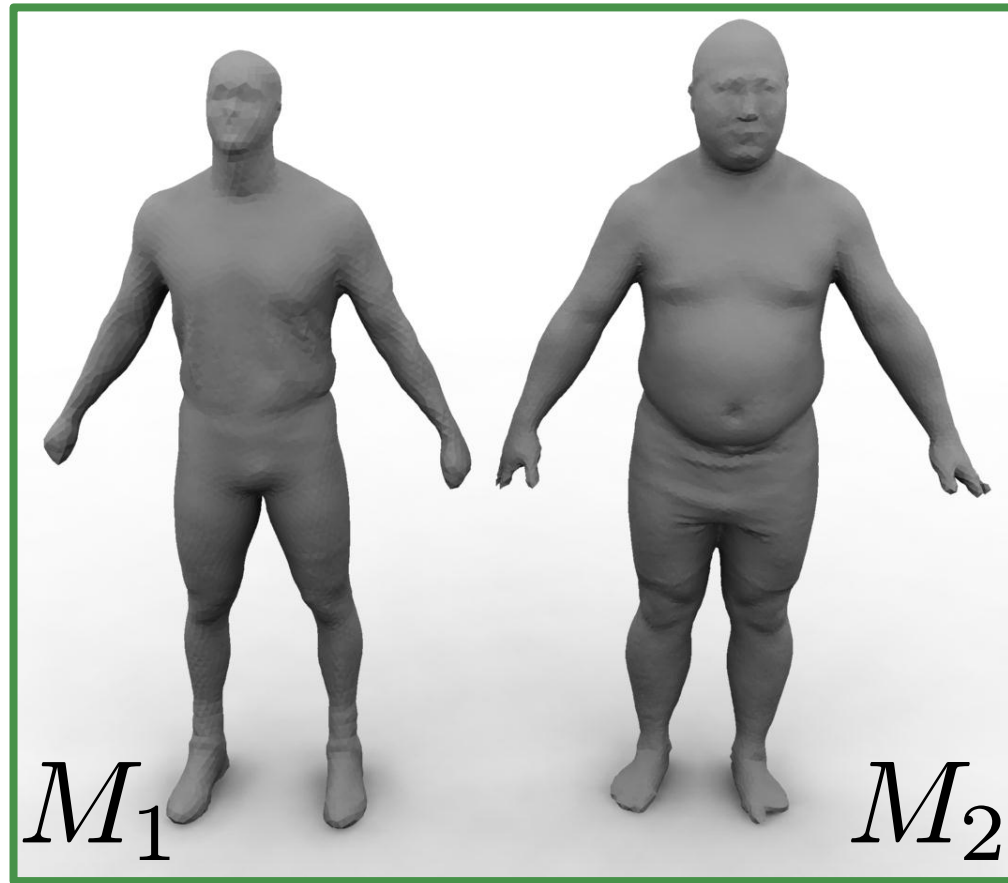


Mapping Ambiguities



Global Ambiguity

Mapping Ambiguities

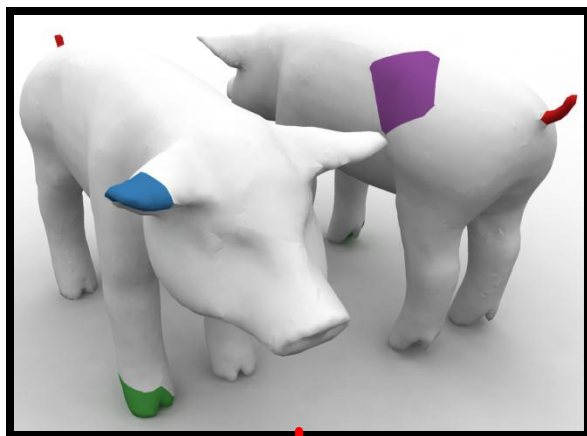


Local Ambiguity

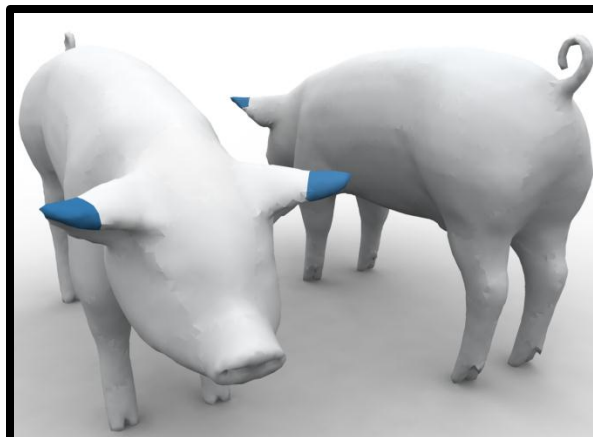
New Idea

**Represent maps
probabilistically**

New Idea



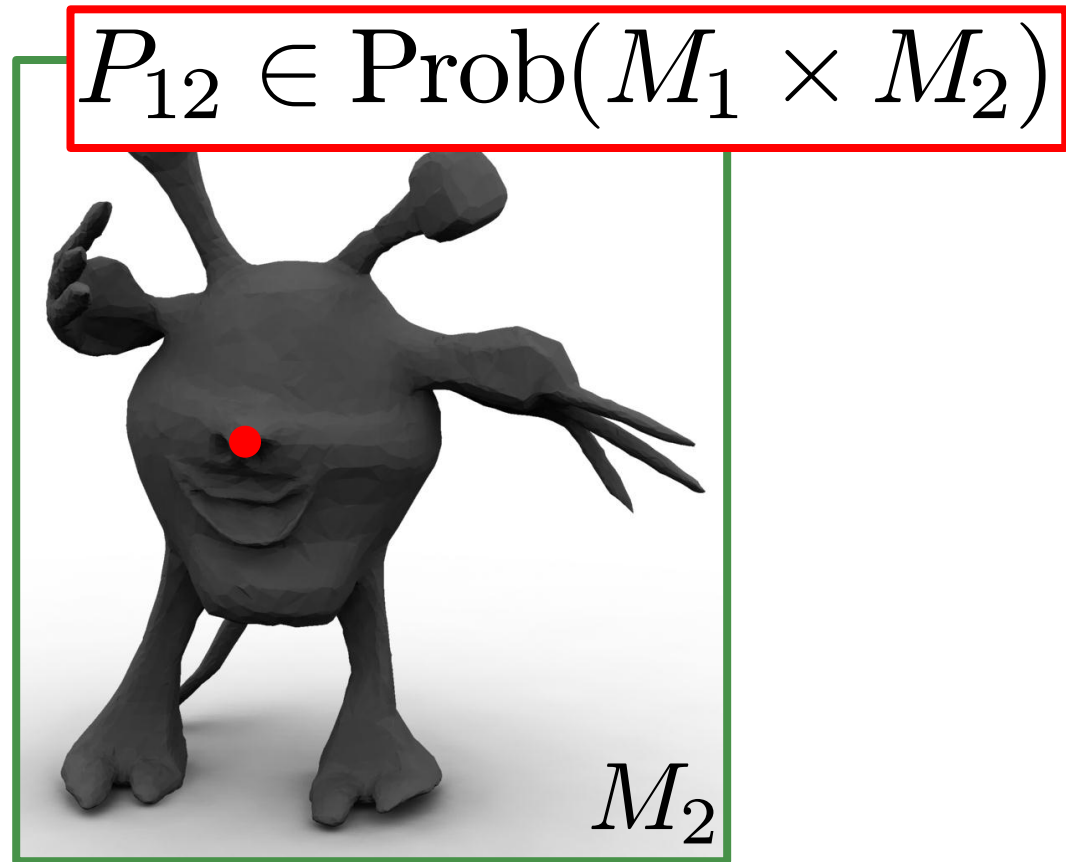
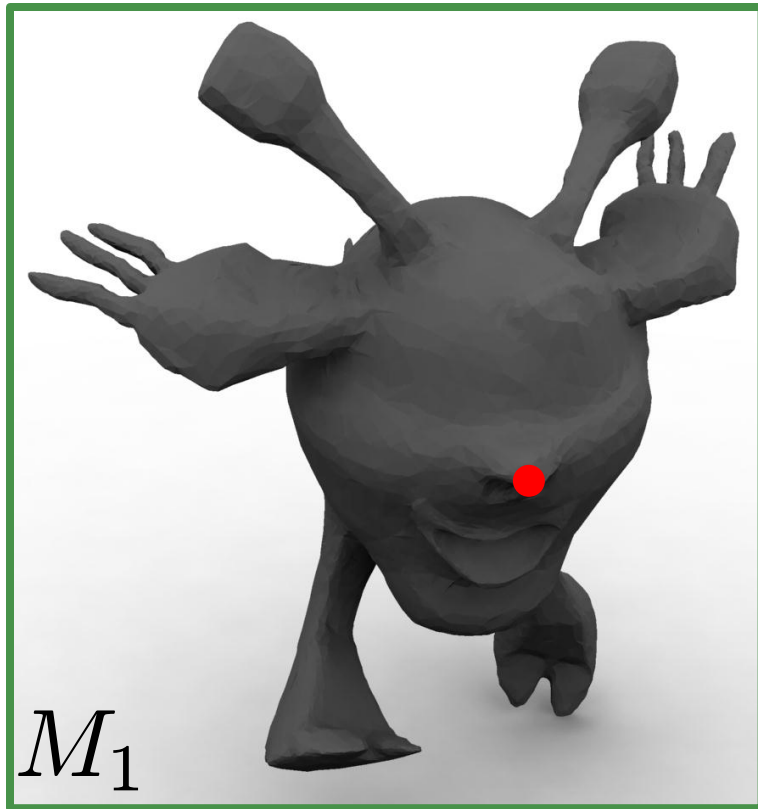
M_1



M_2

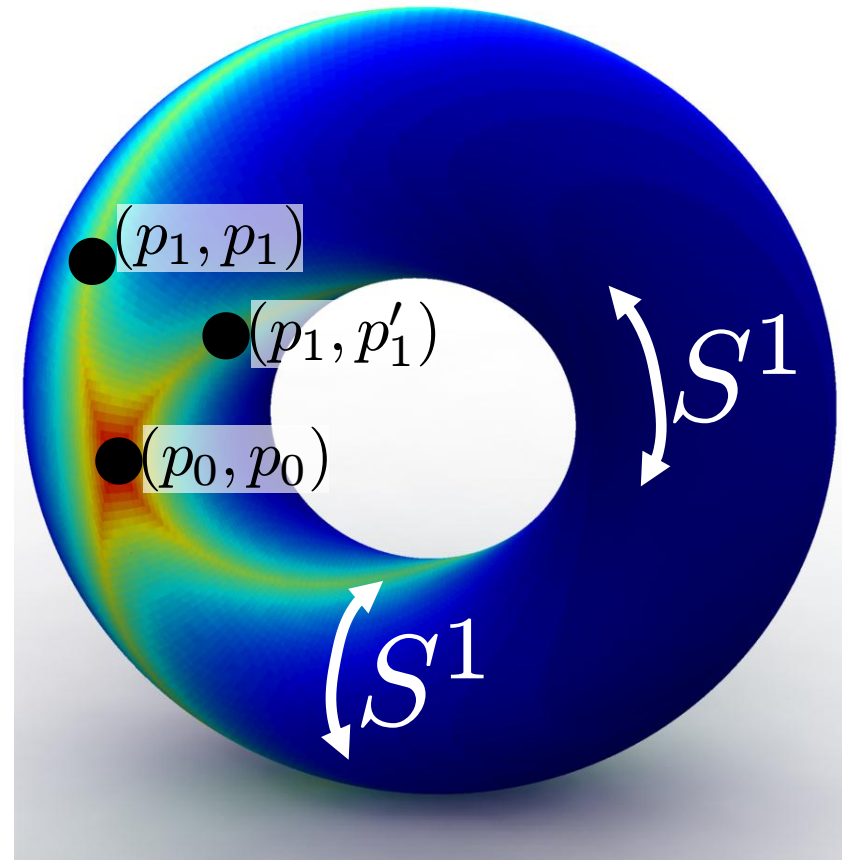
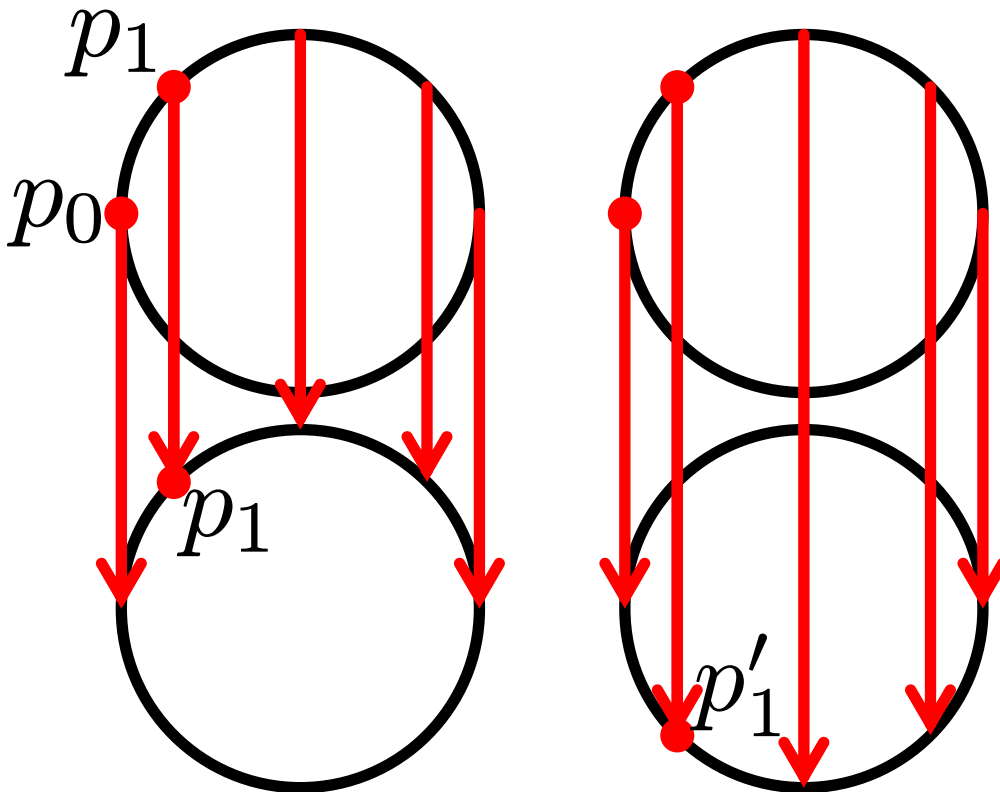
Represent maps probabilistically

Soft Correspondence



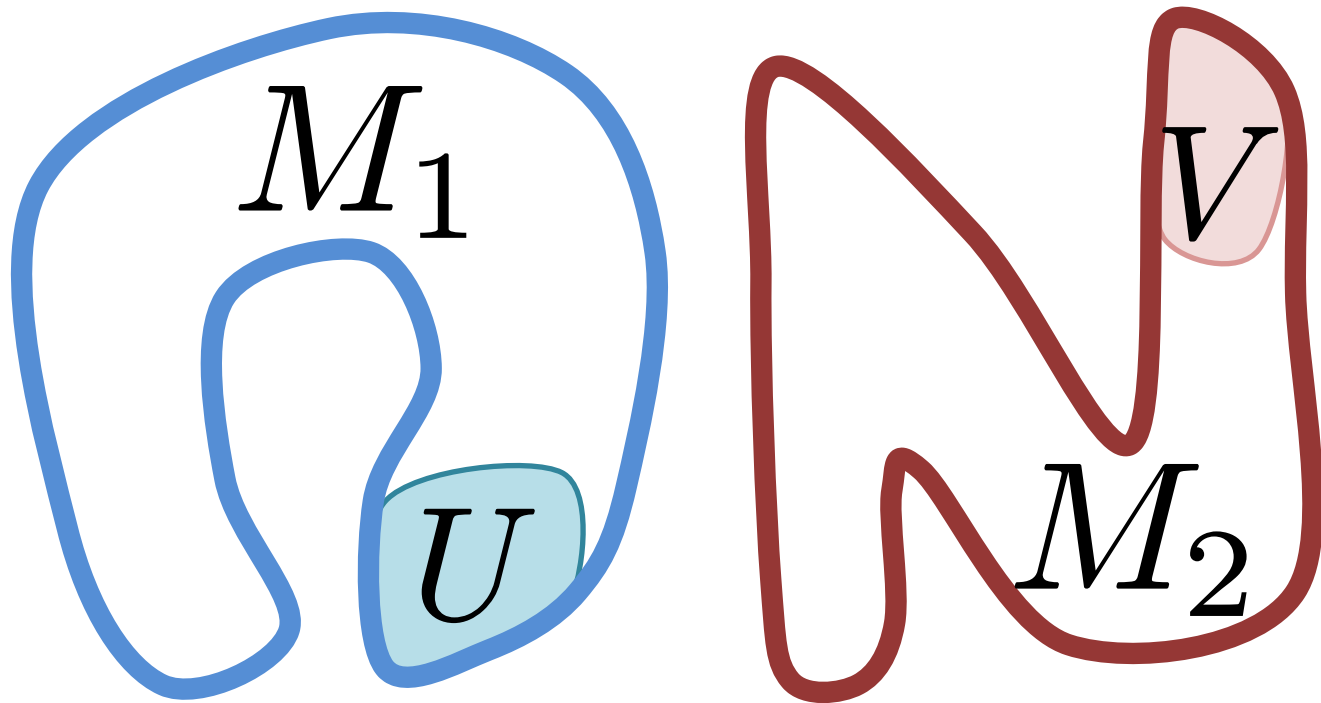
Likelihood of a matched pair

Soft Correspondence



Likelihood of a matched pair

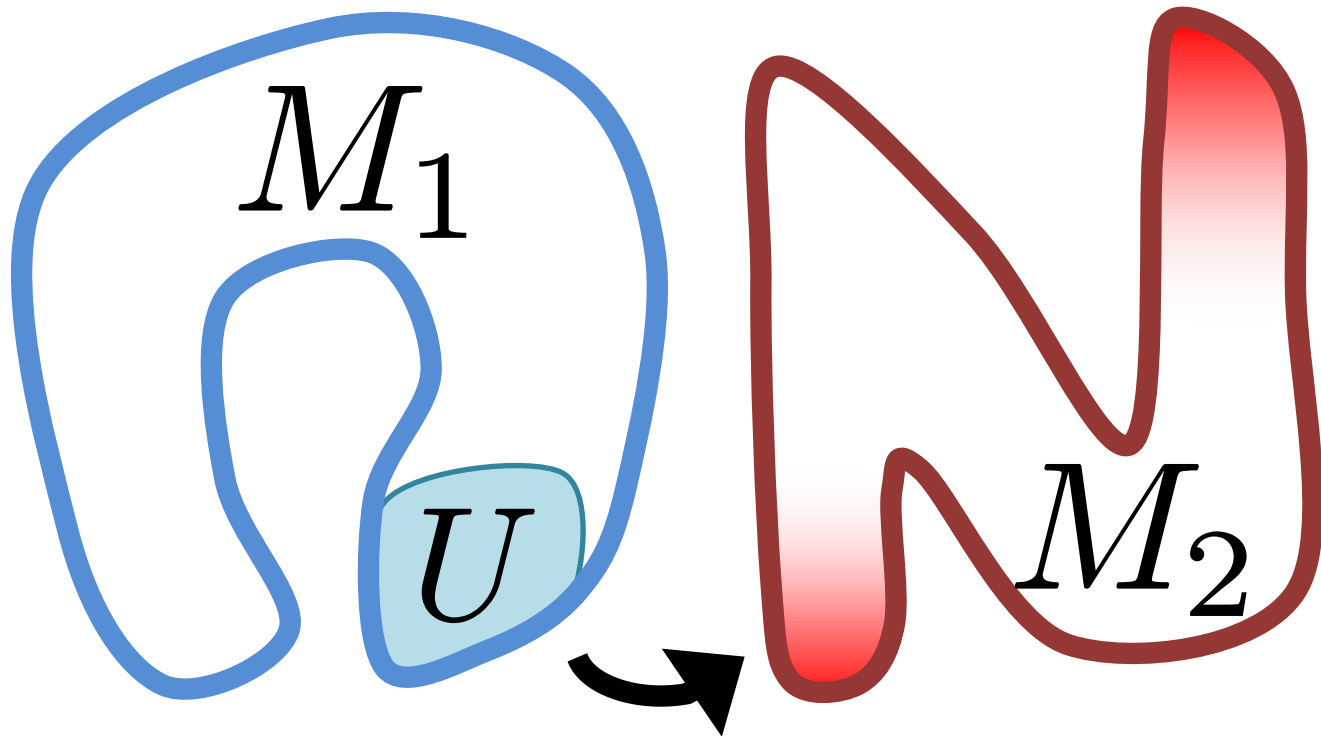
Soft Correspondence



$$P_{12}(U \times V) \in [0, 1]$$

Likelihood of a matched pair

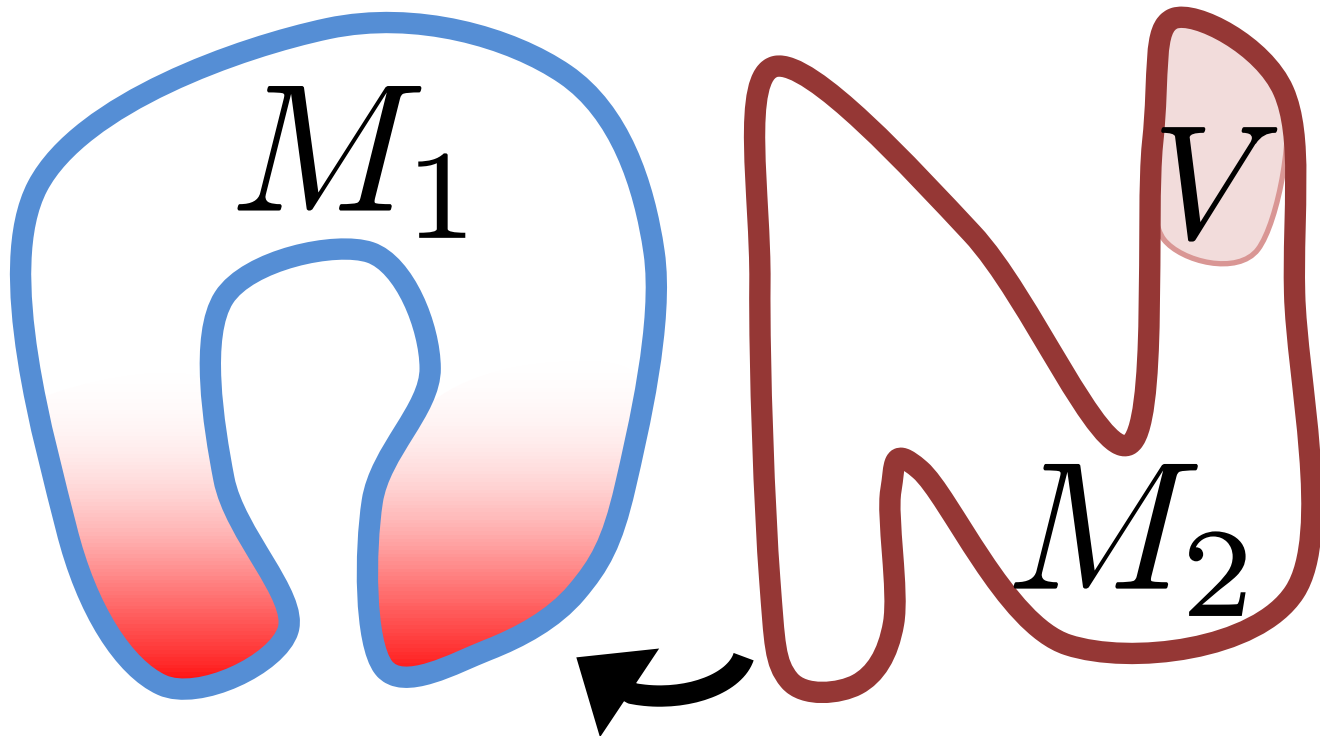
Soft Maps



$$P_{U_2}(B) \in [0, 1]$$

Conditional probability distribution

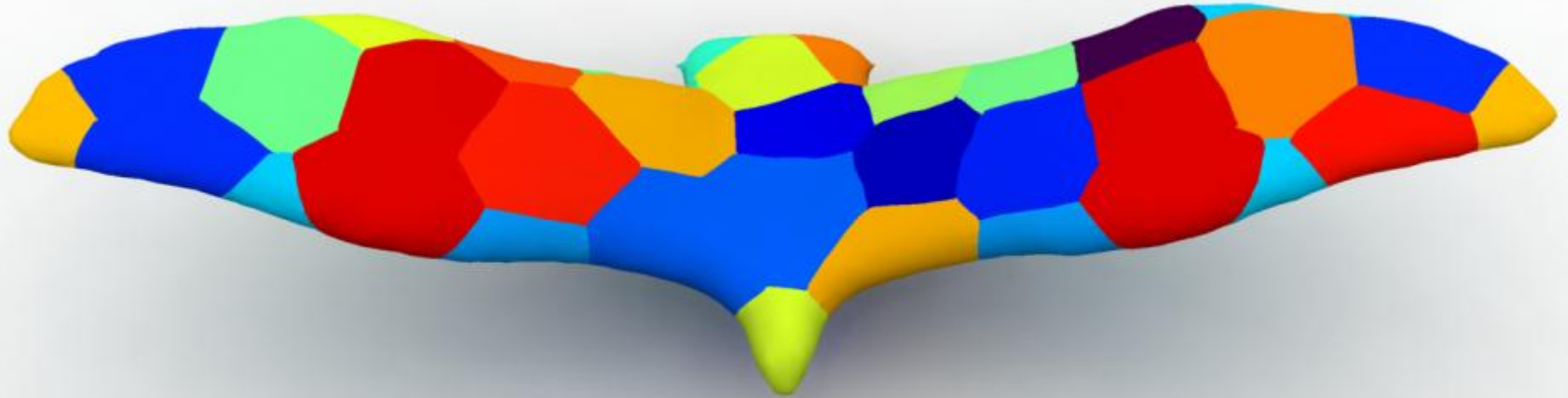
Soft Maps



$$P_{1V}(A) \in [0, 1]$$

Conditional probability distribution

Discretization



$$M_1 = U_1 \cup \dots \cup U_{N_1}$$

$$M_2 = V_1 \cup \dots \cup V_{N_2}$$

Voronoi cells

Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$

**Descriptor
matching**



Continuity



Sharpness



Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$



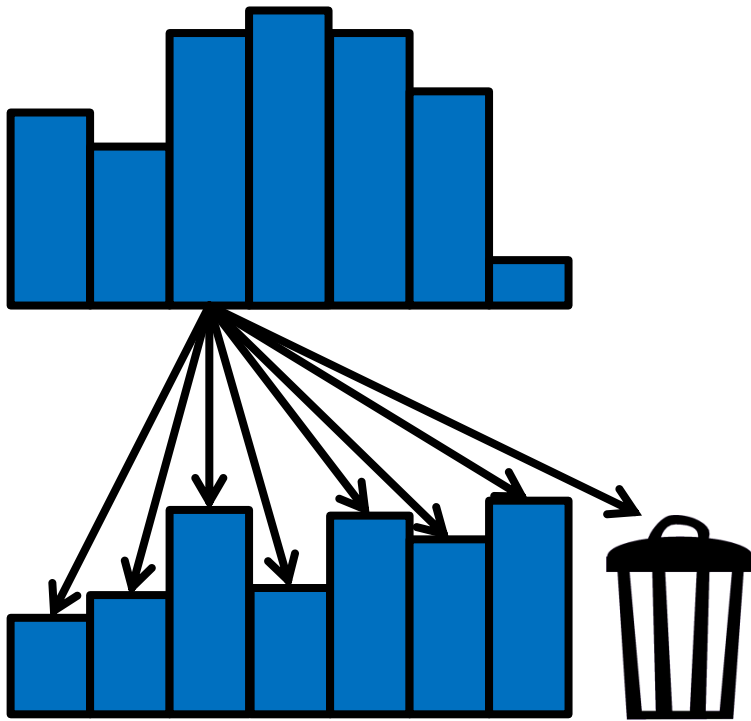
$$E_{\phi}(A) = \sum_{ij} a_{ij} \phi_{ij}$$

<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6130444>

Descriptor matching

Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$

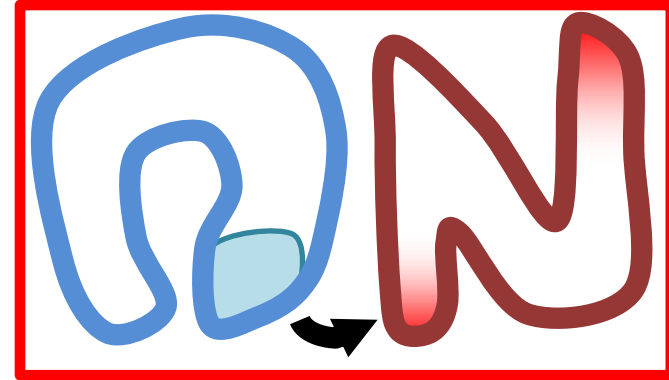
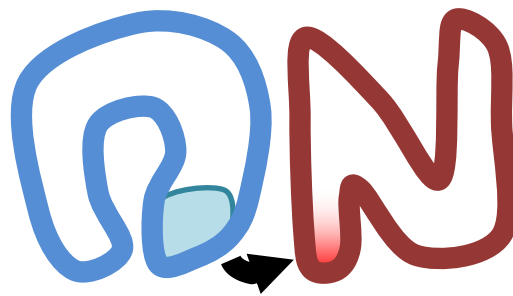
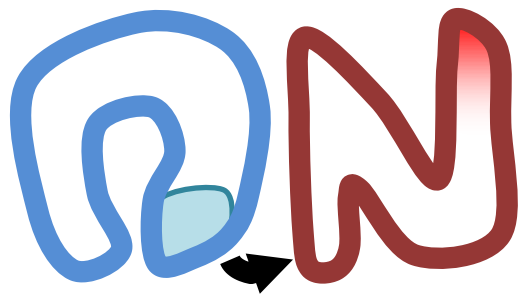


Earth Mover's
Distance

Continuity

Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$



$$E_s(A) = \sum_{ij} a_{ij}^2 = \|A\|_{\text{Fro}}^2$$

Sharpness

Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$

**Descriptor
matching**



Continuity



Sharpness



Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$

Descriptor
matching

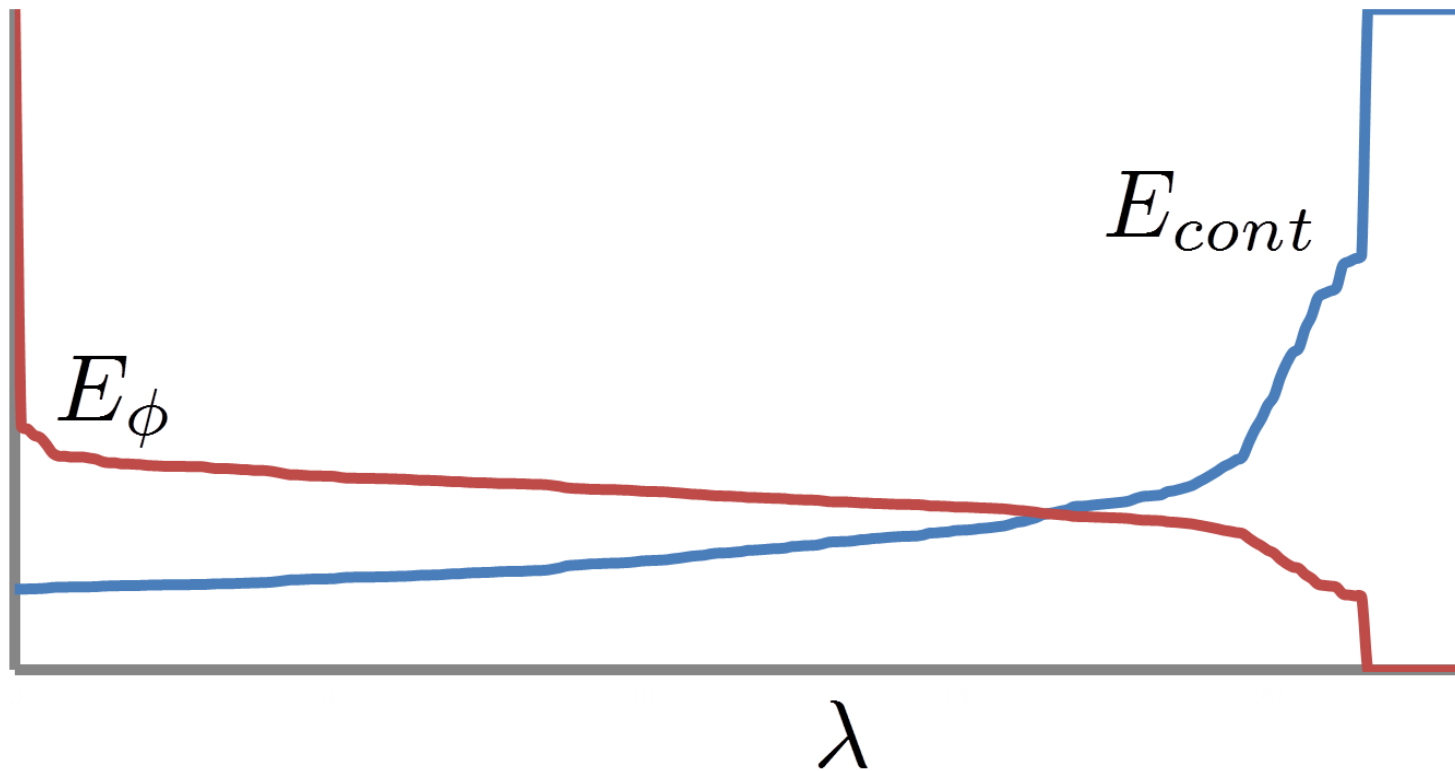
Continuity

Sharpness

Linear program!

Finding Soft Correspondences

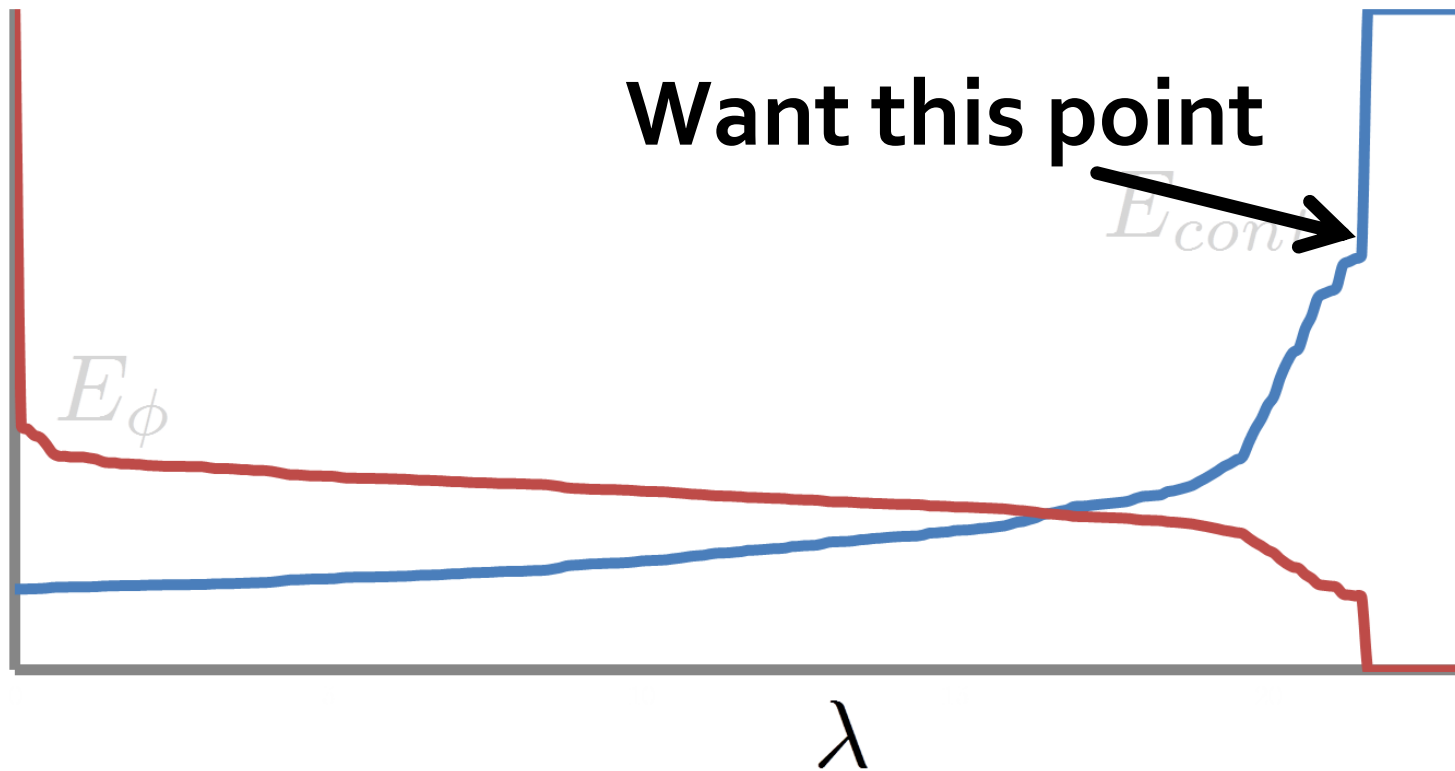
$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$



Sharp transition

Finding Soft Correspondences

$$E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A)$$



Sharp transition

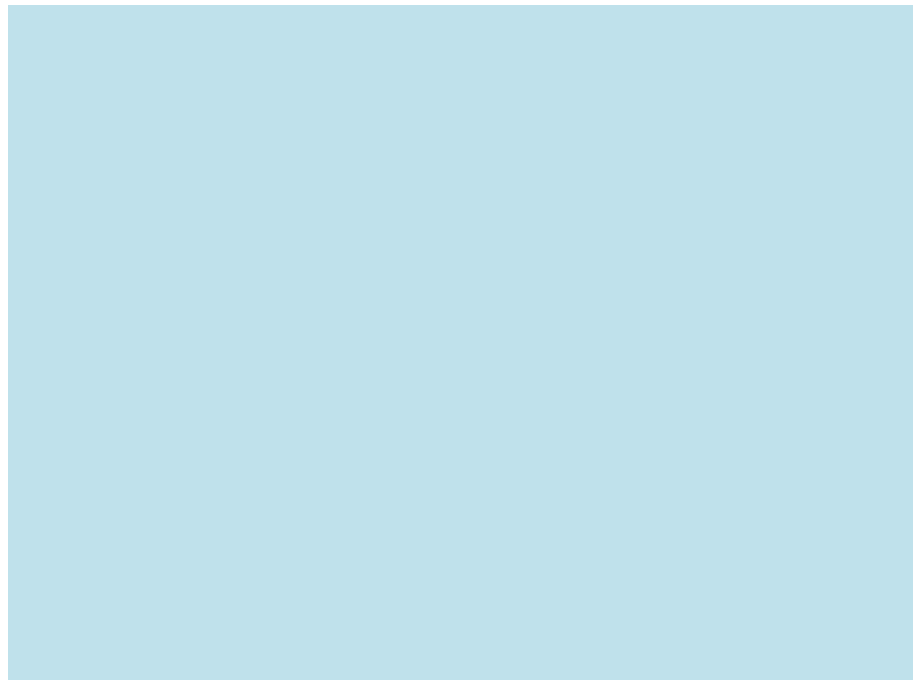
Finding Soft Correspondences

$$\min E_{\phi}(A) + \lambda E_{cont}(A)$$

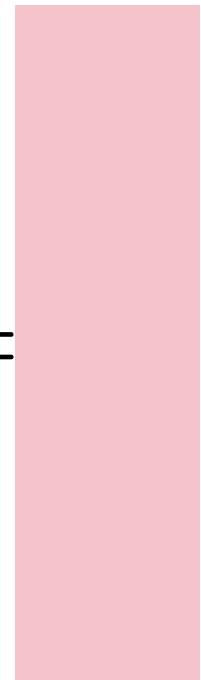
Finding Soft Correspondences

$$\min E_{\phi}(A) + \lambda E_{cont}(A)$$

such that



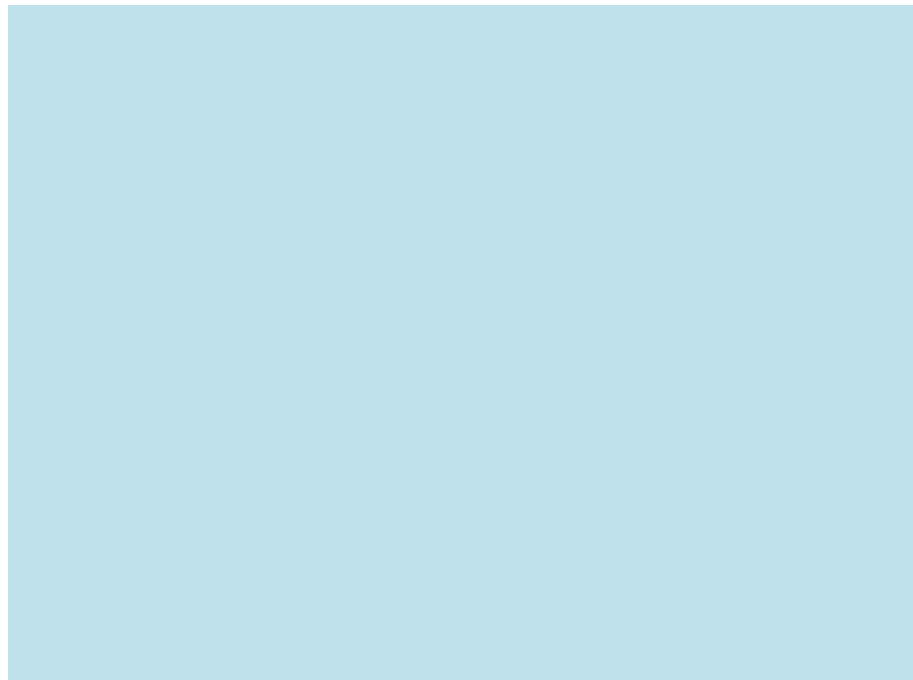
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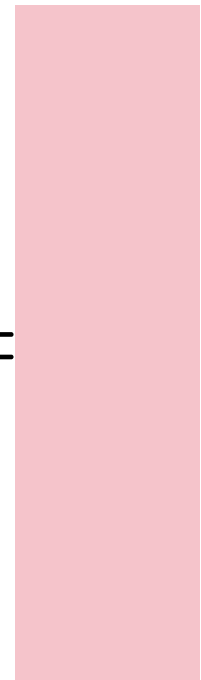
Finding Soft Correspondences

$$\min E_{\phi}(A) + \lambda E_{cont}(A)$$

such that



=



Primal

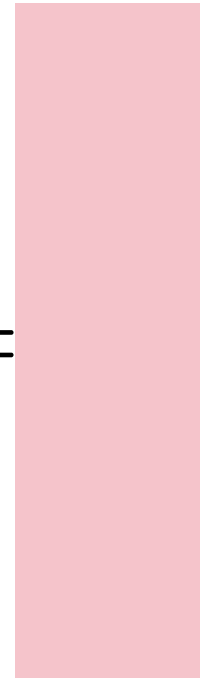
Finding Soft Correspondences

$$\min E_{\phi}(A) + \lambda E_{cont}(A) \rightarrow E_{\phi}(A^*)$$

such that

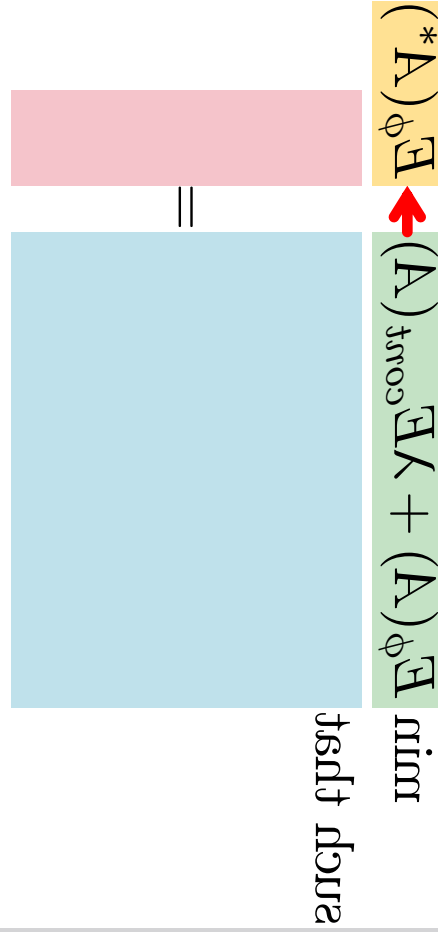


=



Primal

Finding Soft Correspondences



Dual

Finding Soft Correspondences

$$\max \text{ [red box] } \rightarrow E_{\phi}(A^*)$$

such that

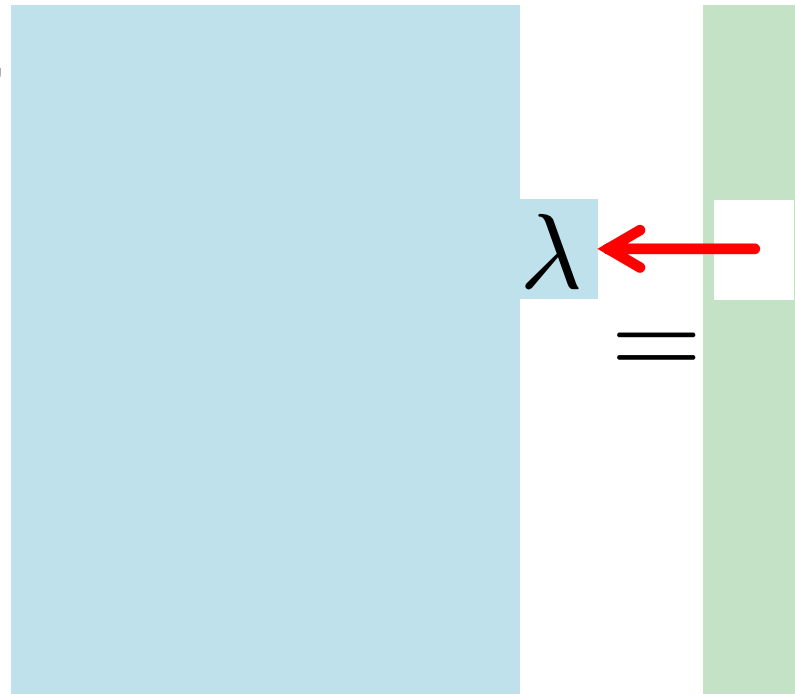
$$\text{[blue box]} = \lambda \text{ [green box]}$$

Dual

Finding Soft Correspondences

$$\max \text{ [red box] } \rightarrow E_{\phi}(A^*)$$

such that

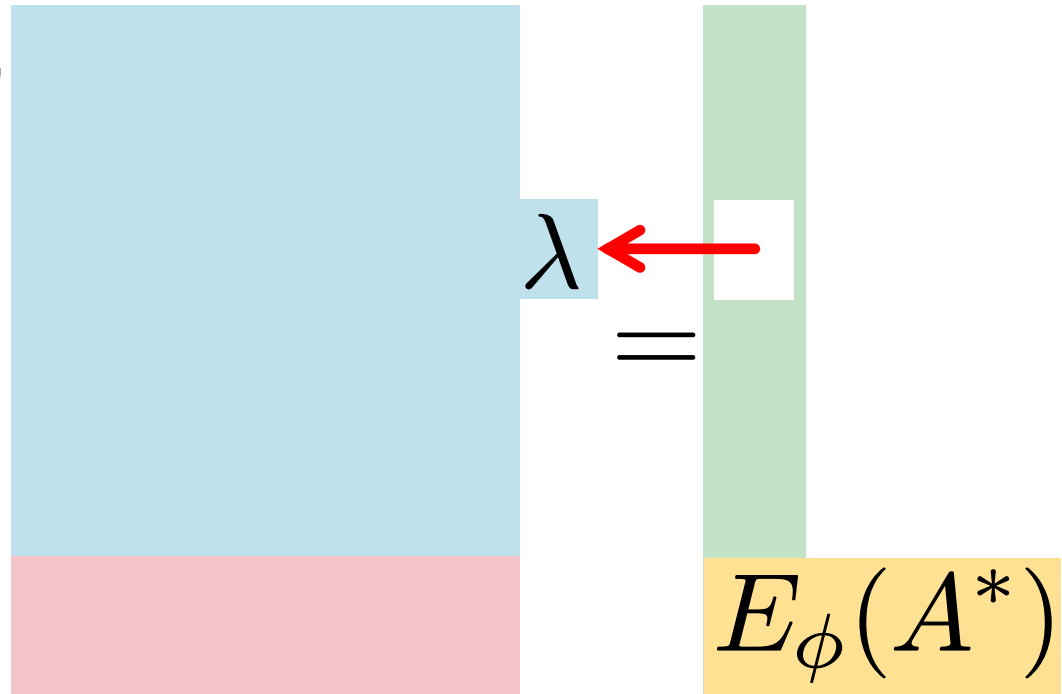


Dual

Finding Soft Correspondences

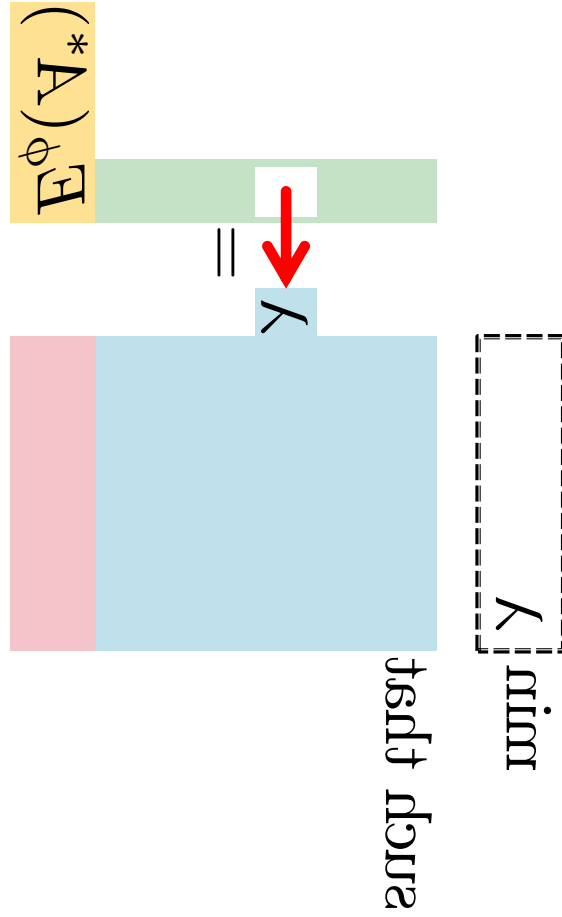
$$\min \lambda$$

such that



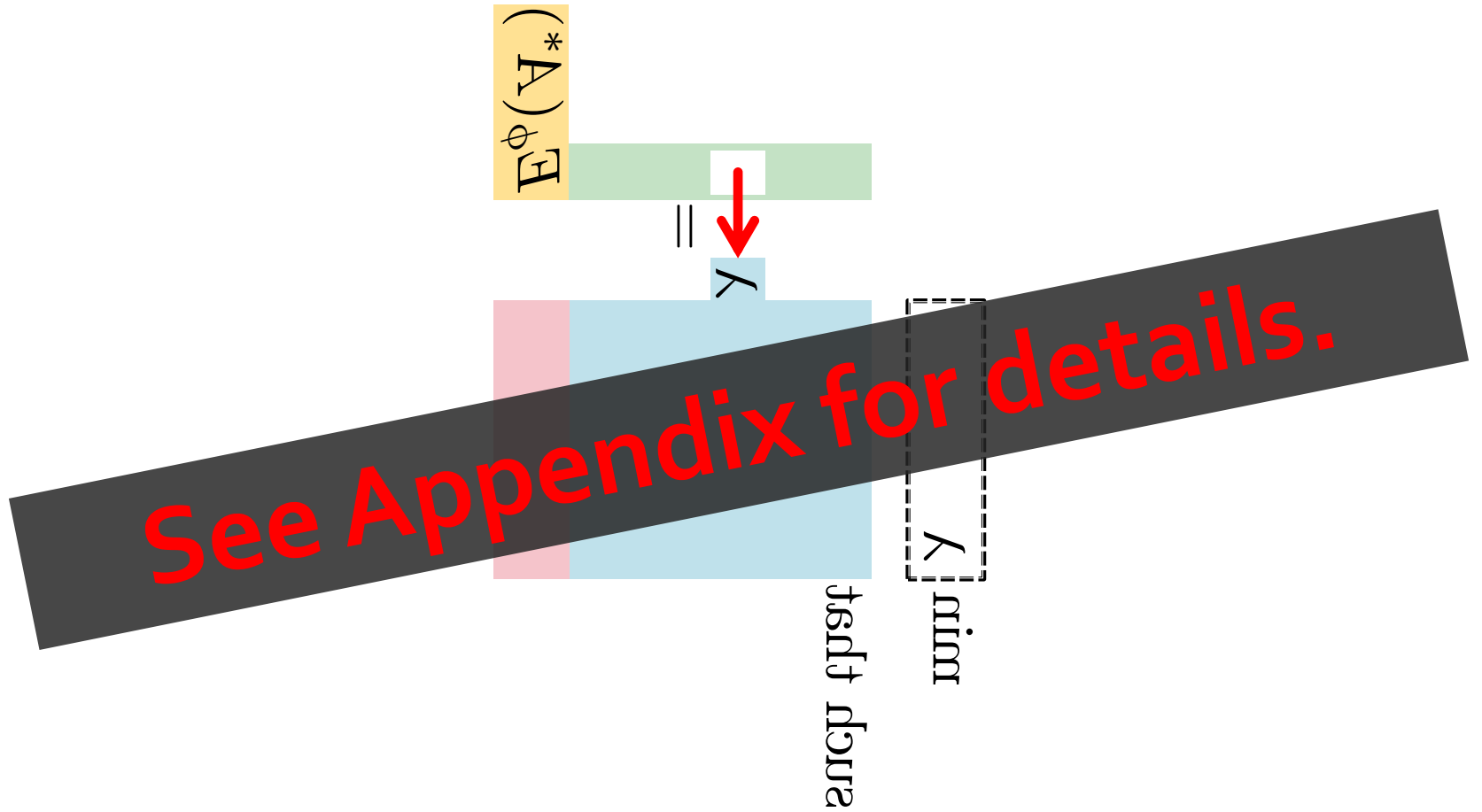
Dual

Finding Soft Correspondences



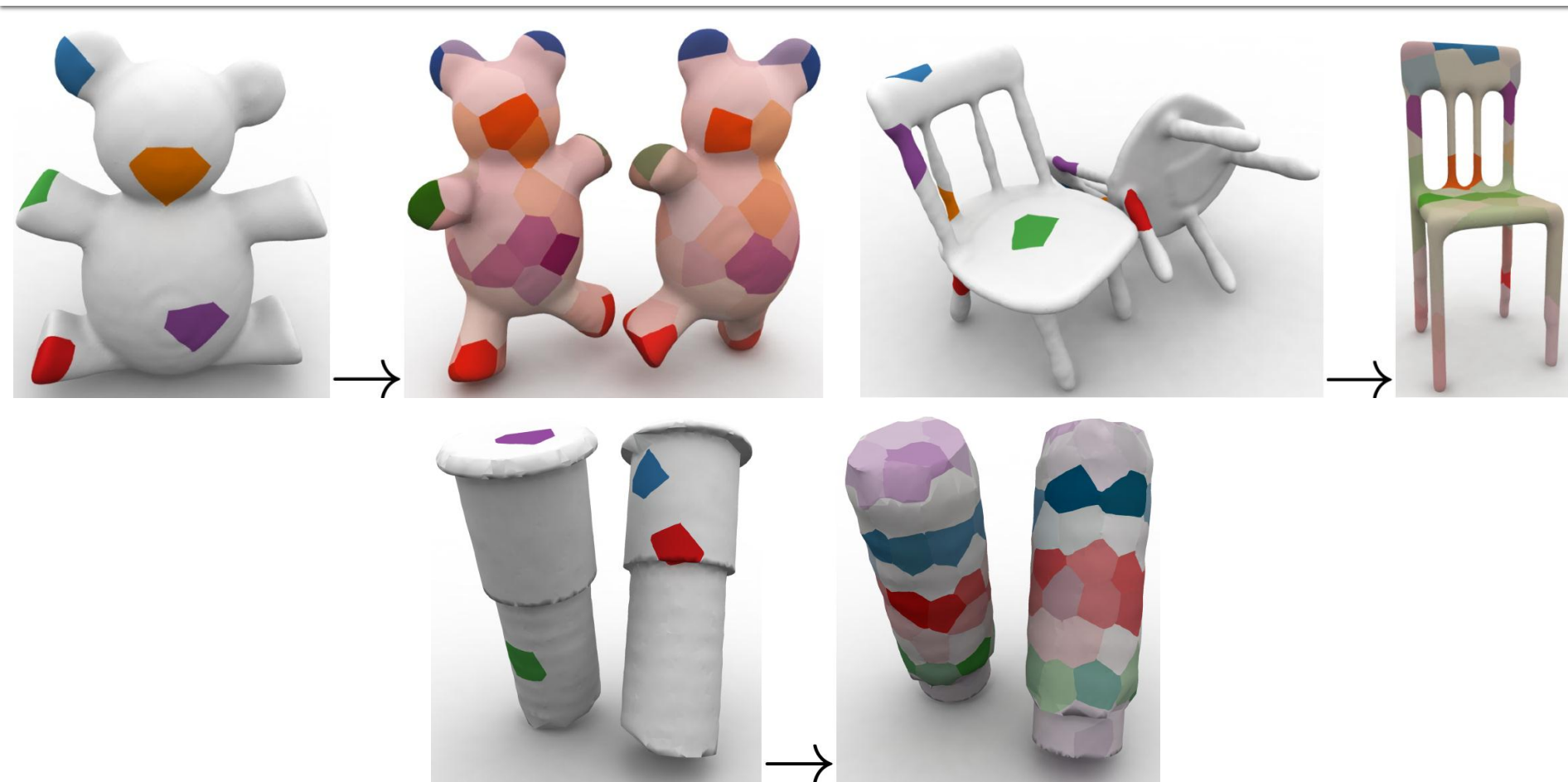
New primal

Finding Soft Correspondences



New primal

End Result



Continuous soft maps

PCA Analysis



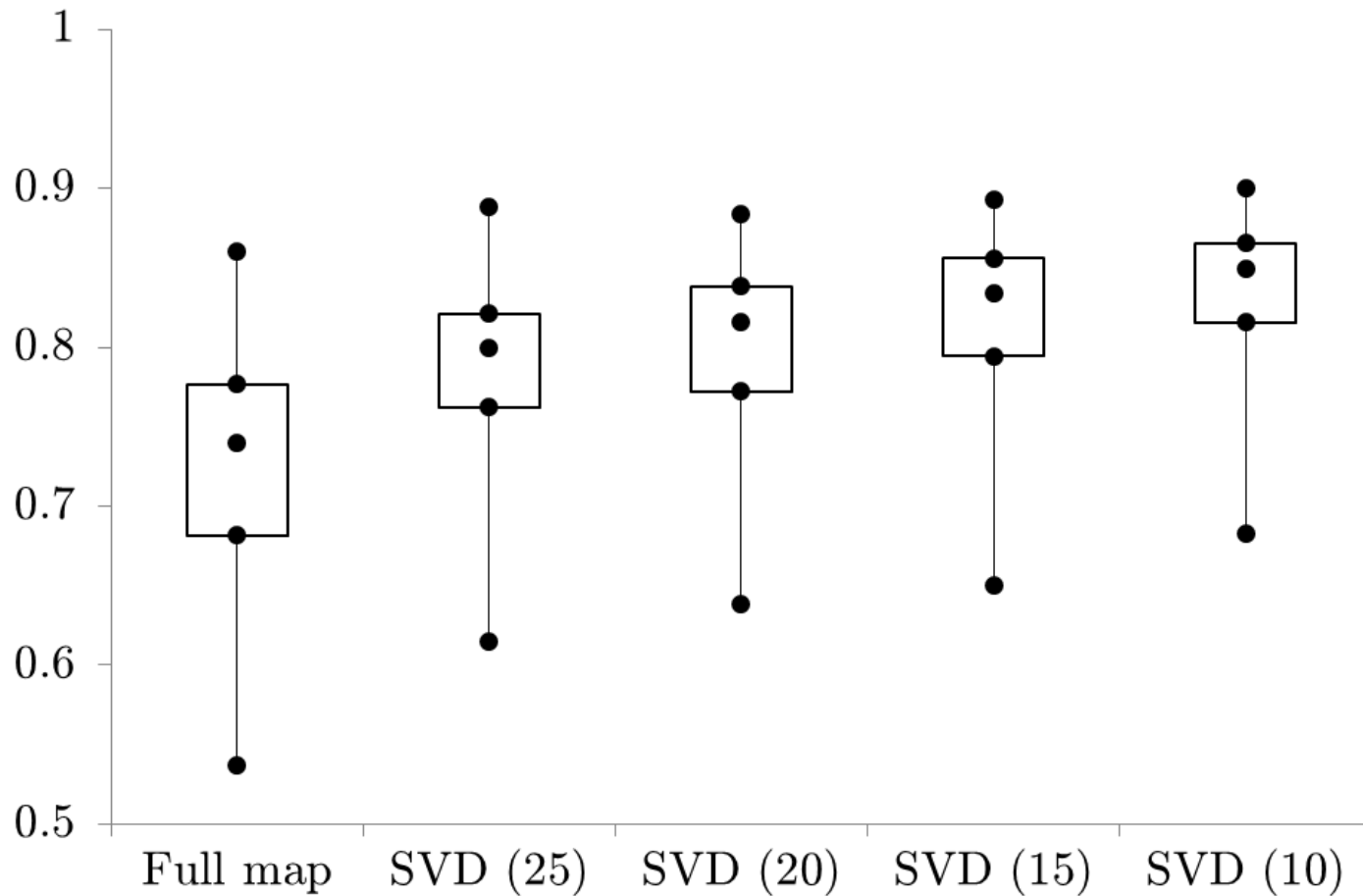
Basis for probability measures

PCA Analysis



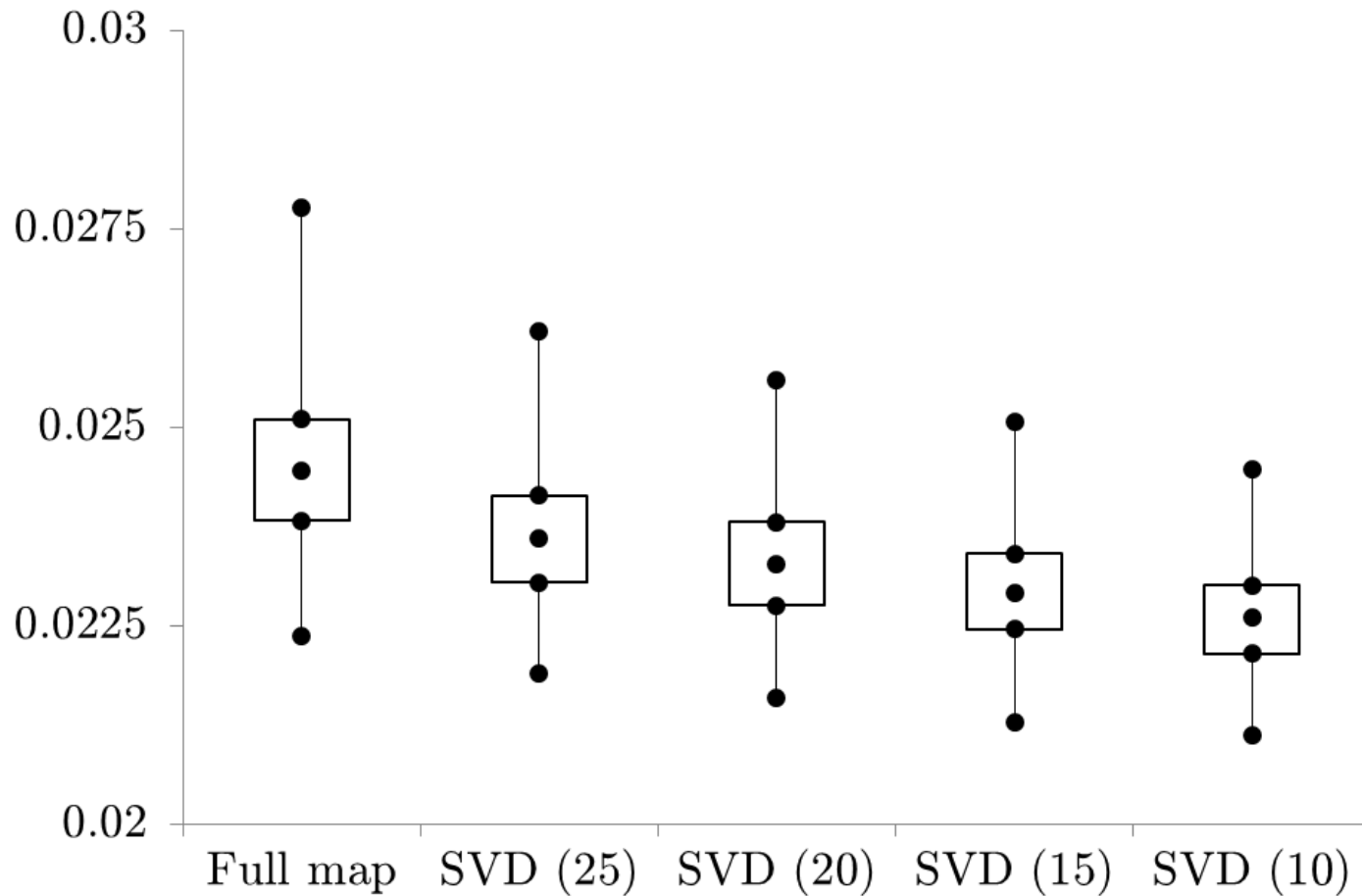
Basis for probability measures

Projection onto Basis



Validation improves

Projection onto Basis



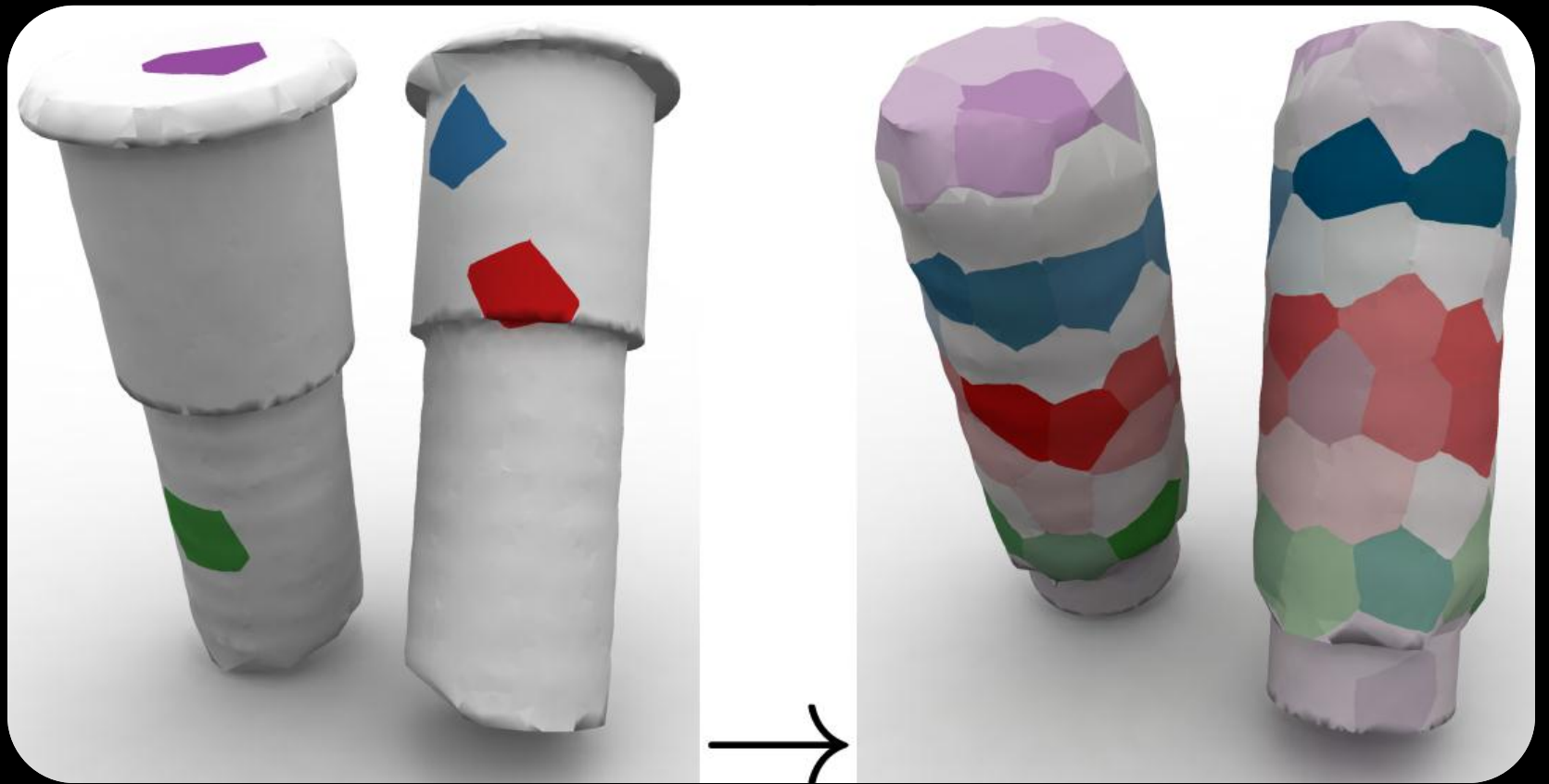
Sharpness decreases

Future Work

- **Computing denser maps**
- **Applications to graphics and other fields**
- **Map collections and composition**

Special Thanks





Soft Maps Between Surfaces

Questions?