

**Euromech Colloquium EC565, May 6-9 2014**  
**Subcritical transition to turbulence**  
**Cargèse, France**



**BOOK OF ABSTRACTS**

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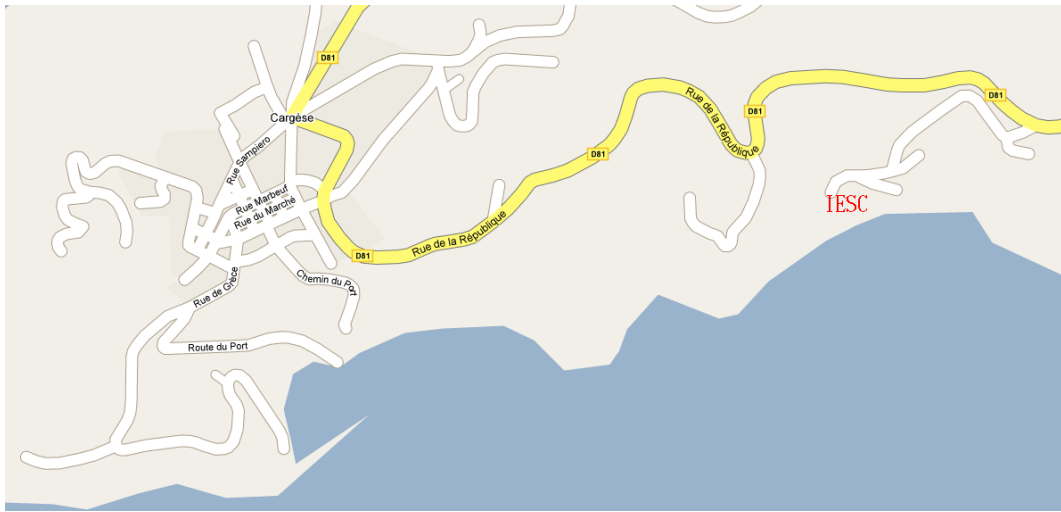
**Front cover picture:** side view of an experimental turbulent spot in plane Poiseuille flow using Time-resolved Stereoscopic PIV, published in Lemoult, Gumowsky, Aider & Wesfreid, Eur. Phys. J. E, 2014 (currently in press). Courtesy of Grégoire Lemoult.





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## TRAVELLING WAVES REVEALING THE STATE SPACE OF TURBULENT PIPE FLOW

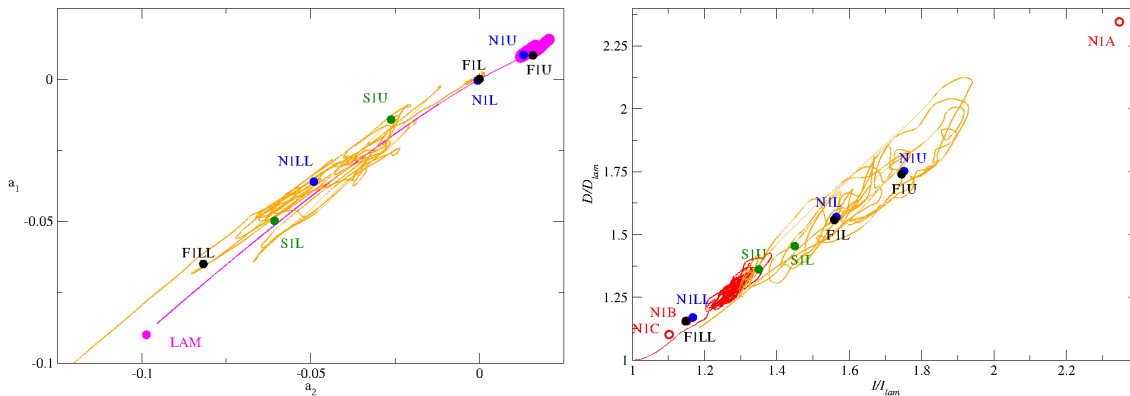
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Coherent structures like travelling waves are believed to provide the skeleton of the chaotic saddles that can explain the intermittent transition to turbulence in wall bounded shear flow. The essential idea is that turbulence evolves around such solutions and that a finite number of unstable states dominate the dynamics as supported by [1, 2, 2]. Thus TWs can give a description of turbulent flows in terms of key exact solutions. To find such states we apply the symmetry reduction methods of slices [3] to pipe flow to obtain a quotient of the streamwise translation and azimuthal rotation symmetries of turbulent flow states. Within the symmetry-reduced state space, all travelling wave solutions reduce to equilibria, and all relative periodic orbits (RPO) reduce to periodic orbits. Projections of these solutions and of its unstable manifolds from their infinite-dimensional symmetry-reduced state space onto suitably subspaces reveal their interrelations and the role they play in organizing of turbulence.



**Figure 1.** (Left) Projections of dynamics local to NIL travelling wave (0, 0) and (right) rate of energy input from the background pressure gradient  $I$  versus dissipation rate  $D$  for the invariant solutions NIU, NIL, NILL, S1U, S1L in D5, N1A, N1B, N1C in D40, and some typical turbulent orbits (orange:D5, red:D40, 'S' [N] for solutions in  $S$  [ $Z_2$ ] symmetry subspace).

In particular we restrict our investigations to dynamics for solutions only allowing one-fold (minimal) symmetry and either 'shift-and-reflect'  $S$  or 'rotate-and-reflect'  $Z_2$  symmetry subspace. For short pipe  $D5$  and restrictions to  $(S, Z_2)$  symmetry subspace we find most trajectories attracted to NIU state and the NIL travelling wave to be embedded within the laminar-turbulent boundary, the 'edge'. Within the  $S$  symmetry subspace trajectories spread out showing turbulent behavior and explore a far greater region of state space and appears to be representative of turbulence in full state space. While for quantities as the energy  $E$  and more representative dissipation rate  $D$  in contrast former investigations of two-fold symmetric states [3] here all states are visited frequently only the S1U, S1L and NILL are embedded in region associated with turbulence. At present we try to obtain a more complete picture of the dynamics by searching for periodic orbits embedded in the turbulent regime. At the same time the obtained solutions are continued to larger domains to identify the corresponding localised solutions.

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## INCREASING COMPLEXITY VIA CHAOTIC SADDLES IN PIPE FLOW

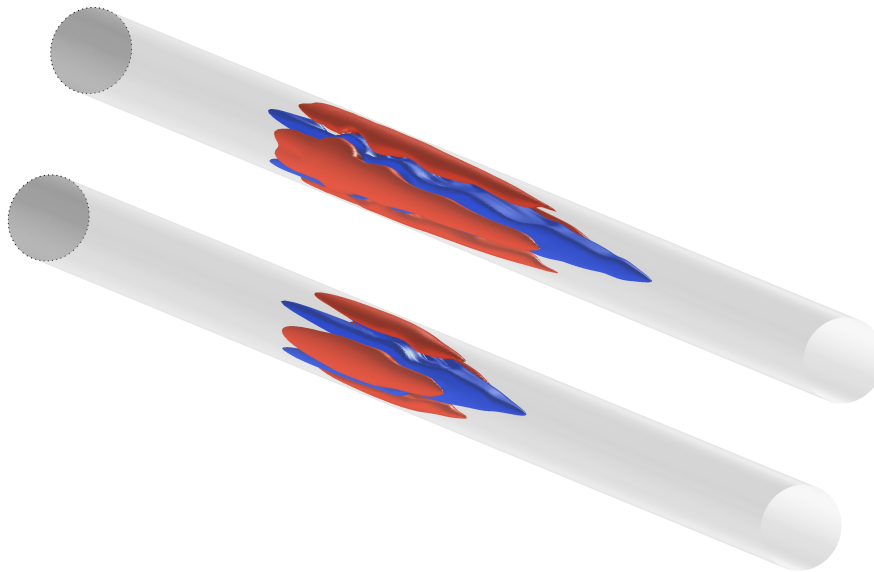
Paul Ritter<sup>1</sup> & Fernando Mellibovsky<sup>2</sup> & Marc Avila<sup>1</sup>

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In pipe flow transition occurs via so-called “puffs”, localized turbulent spots surrounded by laminar flow, whose generation requires a finite-sized perturbation amplitude. Currently, it is not understood how these localized turbulent structures arise from the equations of motion. Mounting evidence suggests that in analogy to low dimensional chaotic systems, turbulence is organized around relatively simple invariant solutions of the governing equations, which are coherent structures embedded in the turbulent state. Recently, periodic solutions which share structure and spatial complexity of intermittent turbulence were discovered. In addition, a bifurcation sequence emerging from one of these solutions and giving rise to chaotic transients was identified [1].

In this talk, we show that the last step in the bifurcation sequence corresponds to a transition from a chaotic attractor to a chaotic saddle (inverse boundary crisis). A lifetimes study of the transients close to the boundary crisis reveals a simple scaling of the form  $1/(\text{Re} - \text{Re}_{crit})$ , as observed in low dimensional models. However, the temporal complexity of the ensuing transients is still much lower than that of turbulent puffs seen in experiments at larger Reynolds number. Far from the boundary crisis a new type of transients, showing much more vigorous fluctuations, is observed. The emergence of these new transient structures, which coexist and interact with those arising at the boundary crisis, are linked to an increase of the lifetimes as observed experimentally. An attempt is made to explain these findings in terms of dynamical systems theory.



**Figure 2.** Visualization of the two different transient structures observed after the inverse boundary crisis. The upper one shows more vigorous fluctuations than the structure observed immediately after the crisis (bottom).

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## LOCALIZED AND EXPANDING STATES IN PIPE FLOW

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We exploit a surprising analogy between the subcritical transition to turbulence and the dynamics action potentials in order to understand the onset of turbulence in pipe flow. The focus here is the transition from localized to expanding turbulence. Analysis of fronts connecting laminar flow (quiescent state) to turbulent flow (excited state) gives the speeds of turbulent-laminar fronts. Combining with experiments and simulations we explain the various stages in the transition to turbulence [1].

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## LOCALIZED TURBULENT STATES AND RELAMINARIZATION TRANSITION IN MHD DUCT FLOWS IN UNIFORM MAGNETIC FIELDS

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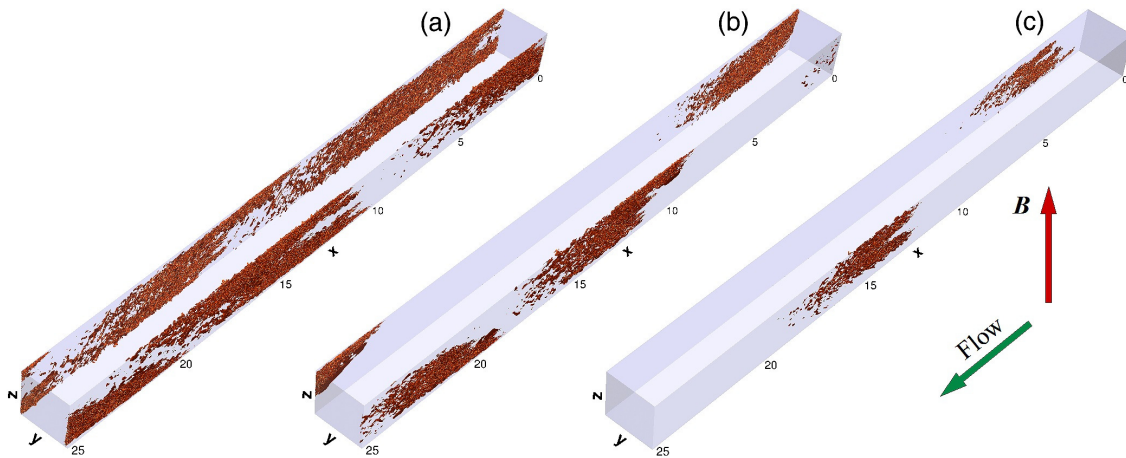
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Liquid-metal flows in rectangular ducts and pipes with insulating walls can be significantly transformed by a uniform transverse magnetic field. The velocity distribution becomes flat in the core because the induced Lorentz force balances the pressure gradient there, and MHD boundary layers appear at the walls. The major contribution to the total friction comes from the Hartmann layers, which appear at the walls perpendicular to the magnetic field. At the walls parallel to the field the so-called side layers appear. Their relative contribution to the friction is smaller, and diminishes as the magnetic induction increases. Transition detected by pressure drop measurements in experiments is therefore typically attributed to the Hartmann layers, and quantified by the Reynolds number  $R$  defined with the laminar Hartmann layer thickness.

Based on previous work on channel flows, we use DNS to study relaminarization in MHD flows in ducts and pipes. Our starting point are the classical experiments by Hartmann at Reynolds numbers  $Re$  below  $10^4$ , where the flow becomes laminar for  $R \approx 200$ . Using long domains with periodic boundary conditions we have reproduced the friction coefficients from these experiments [1]. Close to  $R \approx 200$  there exist localized turbulent states confined to the side layers, which are similar to the puffs and slugs found in hydrodynamic pipe flows. The puffs may appear as a staggered pattern between the two sides or isolated on one side only. To represent the experiments more closely we also conducted DNS with in- and outflow boundaries and a turbulent inflow, which show similar states and friction coefficients [2].

At present we are interested in the transition in strong fields. We have performed duct DNS at  $Re = 10^5$  and increased the magnetic induction until the flow turns laminar. This is again found near  $R \approx 200$ , where we see localized states spread across the full height of the side layers (see Fig. 55).



**Figure 3.** Patterned turbulence realized in DNS of duct flow at  $Re = 10^5$  and  $R = 250$  (a),  $R = 222$  (b), and  $R = 200$  (c). The isosurfaces of turbulent kinetic energy of transverse velocity components corresponding to 2% of the maximum energy in the domain are shown.

In such strong fields there should be a significant anisotropy on account of the Joule damping. The mean turbulent velocity distribution in the side layers can be represented in the framework of mixing-length theory by using a cut-off Joule dissipation length. A lower bound for this cut-off length can be assumed in order to predict a relaminarization threshold. It is consistent with values of  $R$  on the order of a few hundred but can only serve as a rough estimate [3].

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## SPATIO-TEMPORAL DYNAMICS OF 2 D EXPLODING DISSIPATIVE SOLITONS

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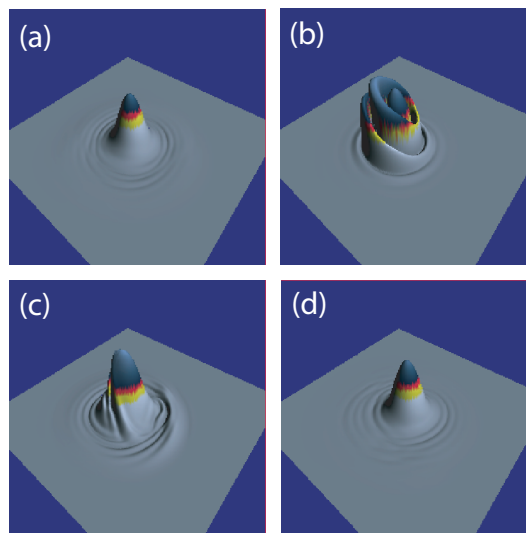
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We investigate a two-dimensional extended system showing chaotic and localized structures. We demonstrate the robust and stable existence of two types of exploding dissipative solitons.

Exploding dissipative solitons in one spatial dimension have been pioneered Akhmediev et al. for anomalous linear dispersion in the cubic-quintic complex Ginzburg-Landau equation [1] and have been found experimentally shortly thereafter [2]. The study of exploding dissipative solitons in two spatial dimensions is attracting attention only recently [3, 4, 5]. In our contribution we will in particular rely on refs. [4, 5].

Here we discuss the spatio-temporal dynamics of explosive dissipative solitons in the two-dimensional cubic-quintic complex Ginzburg-Landau equation with positive linear dispersion as a function of the bifurcation parameter  $\mu$  and the cubic nonlinear refractive index  $\beta_i$ .



**Figure 4.** Four snapshots are plotted characteristic of the time evolution of an asymmetric exploding dissipative soliton. The parameters chosen for these snapshots are  $\mu = -0.4$  and  $\beta_i = 1.0$

In Fig.55 we show four snapshots of the time evolution of asymmetric exploding dissipative solitons. In particular for large amplitudes as they prevail in Figs.55(b) and (c), the asymmetry is strongly pronounced and one can clearly see the qualitative differences to the case of azimuthally symmetric explosions. We note that in the long time limit this asymmetry is not reduced and that the shape of the asymmetric explosions as well as the orientation of the entire object vary even in the long time limit.

We show that the center of mass of asymmetric dissipative solitons undergoes a random walk despite the deterministic character of the underlying model. Since dissipative solitons are stable in two-dimensional systems we conjecture that our predictions can be tested in systems as diverse as nonlinear optics, parametric excitation of granular media and clay suspensions and sheared electroconvection.

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## TRANSITION TO TURBULENCE IN THE PRESENCE OF FINITE SIZE PARTICLES

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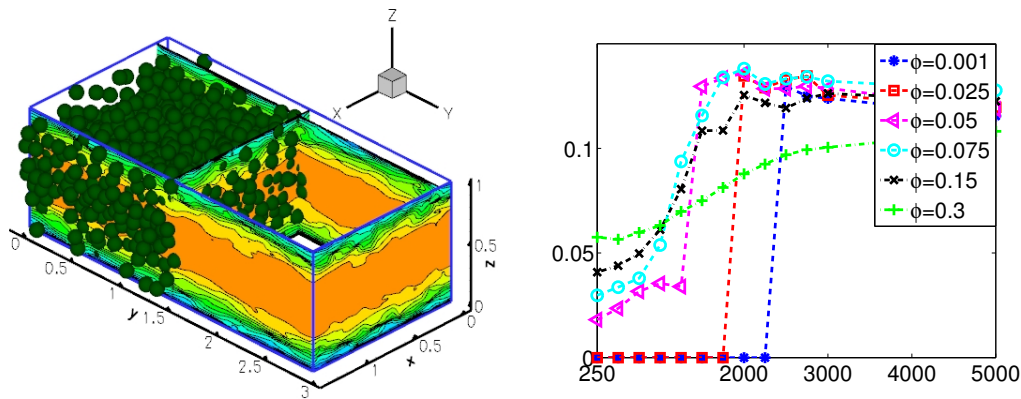
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We study the process of transition from the laminar to the turbulent regime in a channel flow suspended with finite-size neutrally buoyant particles via numerical simulations. A fixed ratio of 1/10 between the particle diameter and channel height is considered. The study is conducted in the range of Reynolds numbers  $500 \leq Re \leq 5000$  (defined using the bulk velocity and the channel half width) and particle volume fractions  $0.001 \leq \phi \leq 0.3$  (see visualization in figure 5).

The simulations reported are performed using the Immersed Boundary solver with second order spatial accuracy developed by Breugem [1]. The code couples the uniform Eulerian fixed mesh for the fluid phase with the uniform Lagrangian mesh for the solid phase. The Lagrangian mesh is used to represent the moving surface of the particles in the fluid. Lubrication forces and soft-sphere collision models have been implemented to address the near field interactions (below one grid cell), see also [3].

We find a non-monotonic behavior of the critical conditions for transition when increasing the volume fraction as in previous experiments [2]. To quantify the behavior of the flow in different regimes we examine the perturbation kinetic energy budget once the mean quantities are statistically converged. The volume-averaged fluctuation kinetic energy is depicted versus Reynolds number for different volume fractions in figure 5. For low volume fractions,  $\phi \leq 0.05$ , the transition threshold is evident through a sharp jump of the average kinetic energy. Interestingly, the critical Reynolds number for the onset of turbulence is decreasing when increasing the particle volume fraction. For  $0.05 < \phi < 0.3$ , the level of the fluctuations increases already at low Reynolds number and the transition becomes smoother. For  $\phi = 0.3$ , it is indeed difficult to identify a transitional Reynolds number and the perturbation kinetic energy only slightly increases with the flow inertia. In this case the level of fluctuations does not reach the one of the single phase turbulent flows even at high Reynolds numbers. At the same time, we record an increase of the wall friction and a decrease of the turbulent Reynolds stresses. This can be explained by an additional dissipation mechanism at high volume fractions, not connected to classic turbulence.



**Figure 5.** Left: Instantaneous flow field from the simulation with  $Re = 2500$  &  $\phi = 0.3$ . The rigid particles are displayed only on one half of the domain. Right: Average kinetic energy in the domain versus the Reynolds number for different volume fractions (See legend).

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## ELASTIC INSTABILITIES IN THE FLOW OF COMPLEX FLUIDS

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Viscoelastic fluids exhibit an intermediate behavior between elastic solids and viscous liquids, depending on the time scale at which they are perturbed. Elastic instabilities may occur at vanishing Reynolds number in the flow of viscoelastic fluids and a general criterion for the onset of such instabilities can be established based on the curvature of flow streamlines and fluid elasticity [1].

Our goal is to study experimentally the onset of elastic flow instabilities in viscoelastic (polymeric or wormlike micellar) solutions of various rheologies in different flow geometries, from microfluidic to macroscopic devices.

Experiments realized in microfluidic serpentine channels using diluted polymeric solutions showed that indeed the onset of elastic instabilities strongly depends on the channel curvature (which could be easily tuned in such devices) [2]. Besides, we recently found that an increase of the polymer concentration tends to stabilize the flow, shifting the elastic instability onset towards larger applied flows.

The onset of elastic instabilities was also experimentally reported for the Taylor-Couette flow of shear-banding wormlike micellar solutions [3]. We observed that the transition from a laminar to an elastically-driven turbulent flow regime exhibits significant hysteresis, which makes us presume that the bifurcation is subcritical.

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## DESIGNING A MORE NONLINEARLY STABLE LAMINAR FLOW VIA BOUNDARY MANIPULATION

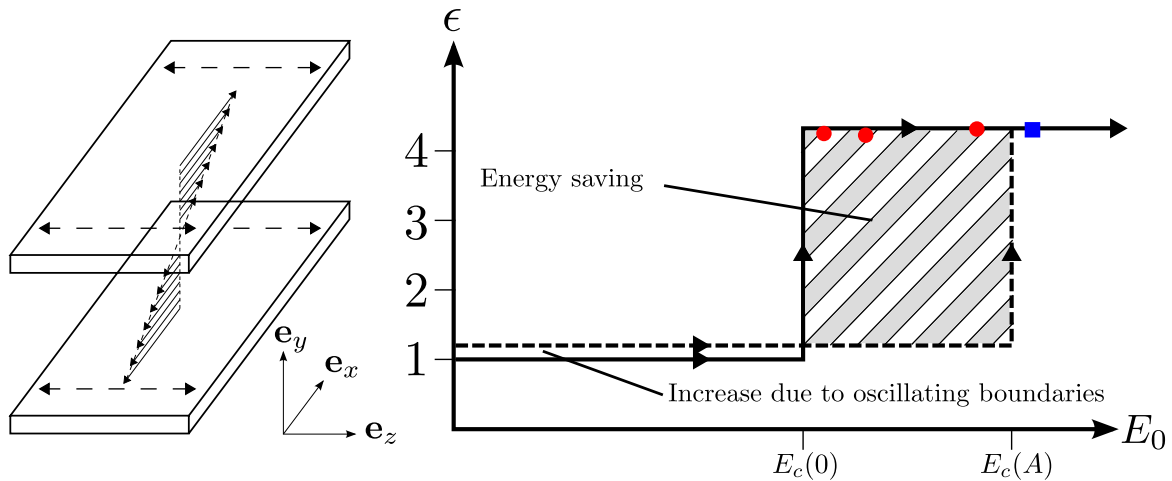
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We show[2] that a fully nonlinear variational method can be used to design a more nonlinearly stable laminar shear flow by quantifying the effect of manipulating the boundary conditions of the flow. Using the example of plane Couette flow (PCF), we demonstrate that by forcing the boundaries to undergo spanwise oscillations in a certain way with an appropriate and intermediate amplitude  $A$ , as shown schematically in figure 55a, it is possible to increase the critical disturbance energy  $E_c(A)$  for the onset of turbulence by 41%, compared to the value  $E_c(0)$  for the unoscillated PCF[1]. If this is sufficient to ensure laminar flow (i.e. ambient noise does not exceed this increased threshold  $E_c(A)$ ), nearly four times less energy is consumed than in the turbulent flow which exists in the absence of imposed spanwise oscillations, as shown in figure 55b.



**Figure 6.** a) Schematic flow geometry showing coordinate system, laminar flow profile (solid lines) and spanwise boundary oscillation of amplitude  $A$  (dashed lines). b) Schematic to illustrate how delaying transition can save energy. The symbols indicate actual average dissipation rate data (relative to the laminar unoscillated PCF value) from numerical simulations: red dots are for unoscillated PCF and the blue square is for spanwise-oscillated PCF (note the turbulent dissipation rate for oscillated PCF is very similar to that for unoscillated PCF).

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## SPANWISE-INHOMOGENEOUS FORCING OF PLANE COUETTE FLOW

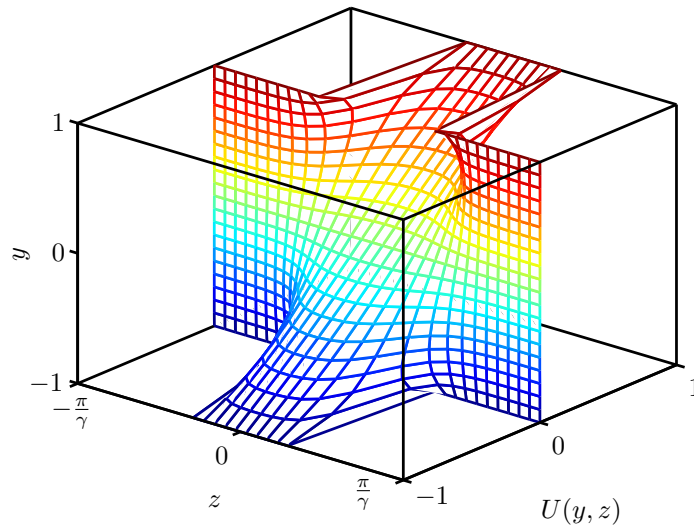
Matthew Chantry<sup>1</sup> & Rich R. Kerswell<sup>1</sup>

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Shear flow transition has been a rapidly moving field as increasing computational power enables the simulation of turbulence and discovery of exact solutions in domains of increasing size, moving towards physically realisable domains. In this work we consider adaptations of plane Couette flow where wall speed has spanwise dependence. We begin with the forcing

$$U(x, \pm 1, z) = \begin{cases} \pm 1 & |z| \leq \frac{1}{2}L, \\ 0 & |z| > \frac{1}{2}L, \end{cases} \quad (1)$$

which results in a downstream laminar flow now dependent upon  $y$  and  $z$  (see figure 55). These localized forcing profiles open up a wealth of questions concerning localization and the length-scales involved in supporting turbulence.



**Figure 7.** Resulting laminar flow for inhomogeneous forcing described in equation (1). Flow has an approximately linear upon  $y$  near  $z = 0$  and has zero flow away from the forcing region.

For these localized Couette flows (ICf) we calculate the energy stability, recovering for large  $L$  the plane Couette value,  $Re_E = 20.67$ . We demonstrate that as  $L$  decreases,  $Re_E$  increases, and the spanwise lengthscale becomes dominant. Building upon this we show that introducing a second shear direction introduces a linear instability. Finally we discuss the many exciting research avenues possible for spanwise localized forcing.

## LOW-ENERGY PERTURBATIONS RAPIDLY APPROACHING THE EDGE STATE IN A COUETTE FLOW

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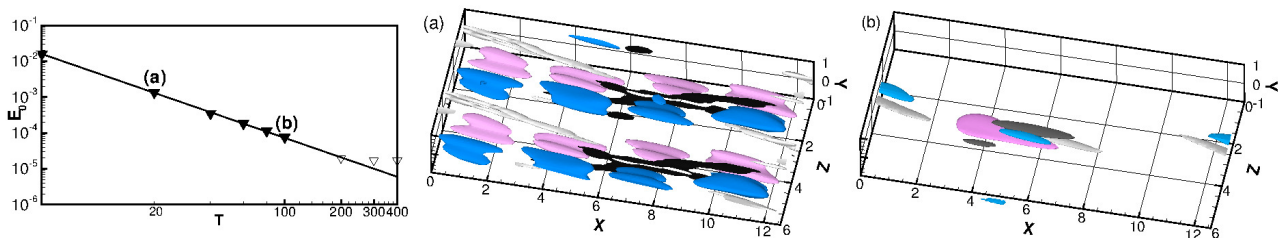
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Plane Couette flow (pCf) may undergo subcritical transition to turbulence when excited by finite-amplitude perturbations. The minimal threshold energy necessary to lead this flow to a chaotic state has been the focus of some recent studies [1, 2, 2], whose aim was to identify the *minimal seed* to trigger turbulent transition. By definition, the minimal seed is the perturbation of minimal energy lying close to the boundary between the laminar state and the turbulent one (either persistent or not), reaching in a rather large time a close neighborhood of the relative attractor on this boundary, called the edge state. However, there can exist perturbations with initial energy (larger but) close to the one characterizing the minimal seed, which are able to reach the vicinity of the edge state in a much smaller time. In this work, we are interested in finding the perturbations of minimum energy capable of leading the flow towards the edge state in a given (small) time, and comparing these minimal disturbances to the minimal seed obtained in the same configuration.

The pCf in a small computational box of dimensions  $4\pi \times 2 \times 2\pi$  at  $Re = 400$  is considered. In this configuration, the edge state is steady and of co-dimension one [3]. We directly target the edge state at a given target time  $T$ , minimizing: i) the norm of the difference between the edge state and the final state reached at time  $T$  by an initial perturbation superposed to the laminar flow; ii) the initial energy of such perturbation. The minimization is performed by means of a Lagrange multiplier method, where the functional to minimize depends on both the distance from the edge and the initial perturbation energy, and the Navier-Stokes equations are imposed as constraints. The minimal perturbations approaching the edge are computed for 6 different target times, from  $T = 5$  to  $T = 100$ , and they are compared to the minimal seed computed for  $T = 200, 300$  and  $400$ .

The minimal energy to approach the edge state is found to vary with the target time following the power law  $E_{min} \propto T^{-\frac{7}{4}}$  when  $T < 300$  (see the left frame in figure 1). For larger target times, it saturates to a value characterizing the minimal seed. For very short target times (see the middle frame of figure 1 for  $T = 20$ ), the minimal perturbations are characterized by spanwise-periodic patches of streamwise perturbation. For larger target times ( $T > 20$ ), the perturbation localizes in the streamwise and spanwise directions and the initial spanwise periodicity is broken. Inclined vortices along patches of streamwise disturbances recalling the shape of the minimal seed are recovered (see the right frame of figure 1 for  $T = 100$ ). This indicates that, even if such perturbations follow different trajectories, the same basic mechanisms leading the flow towards the edge state can be observed provided the time is high enough.



**Figure 8.** Minimal energy to approach the edge versus target time (left frame) and minimal perturbations for  $T = 20$  (middle frame),  $T = 100$  (right frame). Isosurfaces of the streamwise vorticity (black/white) and velocity (pink/blue) perturbation are shown.

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## LAMINAR-TURBULENT TRANSITION IN A FIBRE LASER

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We report our studies on a fibre laser that we have learned to operate in both laminar and turbulent regimes. We show that the laminar phase is analogous to a one-dimensional coherent condensate and the onset of turbulence is due to the loss of spatial coherence. We discovered a new mechanism of laminar-turbulent transition in laser operation: Condensate destruction by clustering dark and grey solitons. This is important for the design of devices exploiting coherence and conceptually new technologies based on systems operating far from thermodynamic equilibrium [1].

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## NEW EXPERIMENTS ON TRANSITIONAL PLANE COUETTE FLOW: EVIDENCE OF LARGE SCALE FLOWS

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Plane Couette flow is the shear flow that, ideally, develops between two infinite parallel plates moving at the same speed  $U$  in opposite directions. The laminar solution is known to be stable against infinitesimal perturbations for any Reynolds number  $R = Uh/\nu$ , where  $\nu$  is the kinematic viscosity of the fluid,  $U$  the velocity of the plates and  $h$  the half gap between the two plates. According to experimental investigations in the nineties [1, 2], a subcritical transition to turbulence is nevertheless achieved at moderate values of  $R$  that involves coexisting turbulent and laminar domains whose complex spatiotemporal dynamics was observed in some range  $[R_g, R_t]$ , with  $R_g \approx 325$  and  $R_t \approx 405$  as per Saclay's team work.

We have designed a new experimental plane Couette setup in order to investigate the transitional regime where laminar and turbulent flows coexist. Particle Image Velocimetry (PIV) and conventional visualizations have been implemented. Preliminary results allow us to study streaky structures typical of the transitional range but also open the way to quantitative statistical analysis of the growth of spots and of the decay of laminar-turbulent patterns. These growths can be linked to large scale flows at the laminar-turbulent interface that we have identified through PIV measurements. These flows that we have evidencing are compatible with the quadrupolar flows obtained numerically by Lagha and Manneville [3] and Duguet and Schlatter [4]. If such large scale flows have already been experimentally observed in channel flows by Lemoult *et al* [5], this is the first time that they can be captured in a plane Couette flow experiment.

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## NOISE IS YOUR FRIEND

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Computation of recurrent unstable solutions embedded in turbulence can be a prohibitively expensive effort, but as one requires only finite predictive precision, we are led to an information-theoretic question: *What is the best resolution possible for a given data set?*

Traditionally this has been estimated by what amounts to a global Shannon entropy averaged over entire data sets. We address the question by treating finite precision as noise that limits the resolution attainable in partitioning the state space. For chaotic flows, this resolution depends on the interplay of local stretching/contraction and the Gaussian smearing due to noise.

As it turns out, for nonlinear dynamical systems the noise itself is highly nonlinear; that means that the effective noise is different for different regions of system's state space. The best obtainable resolution depends on the observed state as well as the memory of its previous states, and has to be computed orbit by orbit. But noise also associates to each orbit a finite state space volume, helping us by both smoothing out what is deterministically a fractal strange attractor, and restricting the computation to a set of unstable periodic orbits of finite period. By computing the local eigenfunctions of the Fokker-Planck evolution operator for each periodic orbit, forward operator along stable linearized directions and the adjoint operator along the unstable directions, and using overlaps of eigenfunction widths as the criterion for the optimal partition, we determine the 'finest attainable' partition for a given hyperbolic dynamical system and a given weak additive white noise. The space of all turbulent spatio-temporal states is infinite, but noise kindly coarse-grains it into a finite set of resolvable states. Fokker-Planck evolution is then represented by a *finite* transition graph whose spectral determinant yields time averages of dynamical observables. [1]

Thus, the same theory that identifies the turbulent patterns also yields their attainable resolution: In nonlinear dynamics, all entropy is local.

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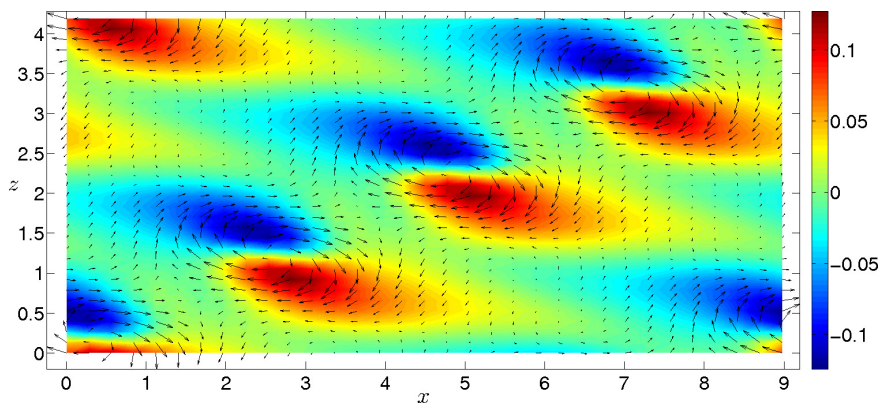
## OBLIQUE COHERENT STRUCTURES IN PLANE COUETTE FLOW

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It is conjectured that exact coherent structures, finite-amplitude solutions to the Navier-Stokes equations, are key components to an understanding of turbulent dynamics. Exact coherent structures can form an invariant set about which chaos is supported, and it is thought that turbulence related phenomena are a manifestation of these chaotic dynamics. By continuation of tertiary states from supercritical, spanwise rotating plane Couette flow, we find obliquely oriented structures which persist in subcritical plane Couette flow. The structures emerge in a saddle-node bifurcation in plane Couette flow, similar to previously found coherent structures in the subcritical shear flows. We discuss the symmetry properties of our coherent structures, and investigate their relevance to oblique turbulent phenomena such as turbulent stripes. We plot an  $(x, z)$ -projection of the flowfield an example structure in figure 9, where the oblique orientation of the fluid can clearly be seen.



**Figure 9.** Oblique exact coherent structure for  $Re = 330$ . We plot the streamwise ( $x$ ) and spanwise ( $z$ ) cross-section of the fluid at the wall-normal midpoint ( $y = 0$ ). The streamwise and spanwise velocities ( $u, w$ ) are depicted in a vector plot, while the colours correspond to the wall-normal velocity  $v$ . The figure has a true aspect ratio, for a solution with streamwise/spanwise wavenumbers  $\alpha = 0.7$ ,  $\beta = 1.5$  respectively.

## SUBCRITICAL TRANSITION TO TURBULENCE : A MODEL INSPIRED BY THE PHYSICS OF GLASSES

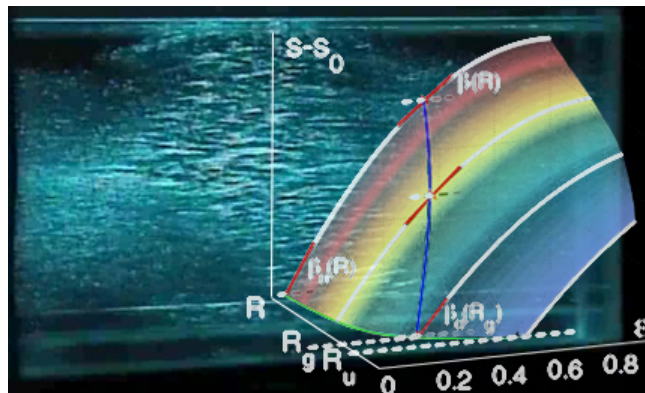
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Globally subcritical [1, 2] transition to turbulence is commonly observed in flow regimes lacking linear instability. It is particularly delicate to understand owing to its abrupt character and the complex spatio-temporal dynamics which involves the nucleation and the growth or decay of turbulent domains. A recent surge of interest resulted from the audacious proposal that shear flow turbulence could remain transient up to arbitrarily large Reynolds number [3]. This proposal has in turn motivated further experiments as well as the development of various models, and an impressive number of numerical studies (see [4] for a complete review). As a result, some comprehension of the mechanisms at play in the coexistence dynamics, as well as a better knowledge of the organization of phase space, involving many unstable solutions of the Navier-Stokes equation, has been gained.

In this work [4], we explore the analogy between the subcritical transition to turbulence and the glass transition from several viewpoints, noticing that both the presence in phase space of many unstable solutions and the existence of finite, yet extremely large, relaxation times, are reminiscent of the physics of glasses (see, e.g., [5, 6, 7]). We first discuss the limitations of fitting procedures in assessing the divergence of the turbulence lifetime, drawing inspiration from similar discussions in the glass literature. We then propose an adaptation of an oversimplified model, the so-called Random Energy Model [8], which has greatly inspired the physics of glass, in order to possibly gain insight into the statistical mechanisms at play in this transition. As a result, we obtain an estimate of the turbulence lifetime as a function of the Reynolds number close to the transition, an estimate which qualitatively agrees amazingly well with the observed phenomenology.



A precise mapping between the glass transition and the transition to turbulence should not be expected, and the proposed analogy should not be considered in a strict sense. However it will allow us to discuss in an original way the dependence of the turbulence lifetime on the Reynolds number. More generally, we hope that it could foster contributions from the statistical physics community to the old standing problem of the transition to turbulence, taking advantage of recently developed concepts in the statistical physics of glasses and disordered systems.

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## THE HIGH REYNOLDS NUMBER ASYMPTOTIC DEVELOPMENT OF NONLINEAR EQUILIBRIUM STATES IN SHEAR FLOWS

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Our concern is with nonlinear equilibrium solutions of the Navier-Stokes equations thought to underpin turbulent flows. Such solutions have been investigated in shear flows by a number of authors dating back to Nagata (1990) and more recently for example Waleffe (2001), Faisst and Eckhardt (2003), Wedin and Kerswell (2004), Wang *et al.* (2007), Gibson *et al.* (2009). Here the relationship between nonlinear equilibrium solutions of the full Navier-Stokes equations and the high Reynolds number asymptotic vortex-wave interaction (VWI) theory developed for general shear flows by Hall & Smith (1991) is investigated. Using plane Couette flow as a prototype shear flow, we show all solutions having  $O(1)$  wavenumbers converge to VWI states with increasing Reynolds number. The converged results here uncover an upper branch of VWI solutions missing in the calculations of Hall & Sherwin (2010).

For small values of the streamwise wavenumber, the converged lower branch solutions take on the long-wavelength state of Deguchi, Hall & Walton (2013) while the upper branch solutions are found to be quite distinct with new states associated with instabilities of jet-like structures playing the dominant role. Between these long-wavelength states, a complex ‘snaking’ behaviour of solution branches is observed. The snaking behaviour leads to complex ‘entangled’ states involving the long-wavelength states and the VWI states. The entangled states exhibit different scale fluid motions typical of those found in shear flows.

On the other hand, for large values of the wavenumbers results are given for the canonical VWI problem in an infinite region; the results confirm and extend the results for the infinite problem inferred by Blackburn, Hall and Sherwin (2013) from plane Couette flow. The results given define exact coherent structures in unbounded flows.

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## ELASTO-INERTIAL TURBULENCE IS THE ULTIMATE MAXIMUM DRAG REDUCTION STATE IN A FENE-P FLUID

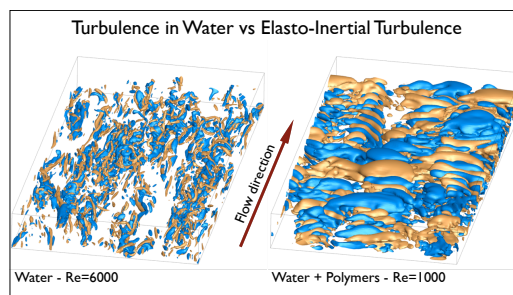
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The dilution of minute concentration polymers in a turbulent wall-bounded flows has two exceptional properties: (a) Polymers, although of much smaller scales than the typical eddies of turbulence, reduce friction drag up to 80% and (b) the drag reduction is bounded by an asymptotic state called maximum drag reduction or MDR. Whilst the mechanism of the former is now considered understood[2, 6], the latter property remains somewhat a controversial topic. Using numerical simulations, this presentation will demonstrate that MDR asymptotes to elasto-inertial turbulence (EIT) in FENE-P flows[7] for highly elastic flows and discuss the extrapolation of our findings to real polymer flows.

Elasto-Inertial Turbulence (EIT) [5] is a recently discovered new state of turbulence, where interactions between inertia and elastic effects can sustain a turbulence-like state in channel and pipe flows at Reynolds numbers much lower than the critical Reynolds at which Newtonian flows undergo a transition from laminar to turbulent state. The structure of EIT[3, 8] consists of thin sheets of stretched polymers that are tilted upward by the mean shear and produce spanwise coherent flow structures by opposition to the quasi-longitudinal vortices observed in Newtonian wall turbulence (See Fig. 40). The friction drag as a function of the Reynolds number of EIT experimental and numerical EIT flows[5] appears to transition smoothly from the laminar drag asymptote the MDR asymptote, for Reynolds numbers higher than the critical Reynolds numbers. However this initial study was insufficient to conclusively establish whether the flow state at MDR is EIT.

In order to demonstrate that MDR asymptotes to EIT, I will use numerical experiments and direct numerical simulations of highly elastic flows that clearly show how polymers inject energy into the flow to prevent relaminarization and create the flow topology shown in Fig. 40. The numerical experiments consist of minimal channel flow simulation similar to [9] and simulations with controlled modification of the governing equations[4]. I will conclude this presentation by showing how EIT provides closure to key discrepancies in current theories of MDR and by discussing the connection between EIT and a specific state of the transition to turbulence in wall-bounded flows: the state of breakdown of linear instabilities.



**Figure 10.** Isosurfaces of the second invariant  $Q$  of the velocity gradient tensor. Yellow and blue denote regions of positive and negative  $Q$  or regions dominated by the rotation rate and the deformation rate, respectively[1]. Left shows a snapshot of one half of a turbulent channel flow simulation in the absence of polymers. Right shows the structure of EIT at a subcritical Reynolds number.

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## BYPASS TRANSITION IN PLANE POISEUILLE FLOW AND BOUNDARY LAYERS

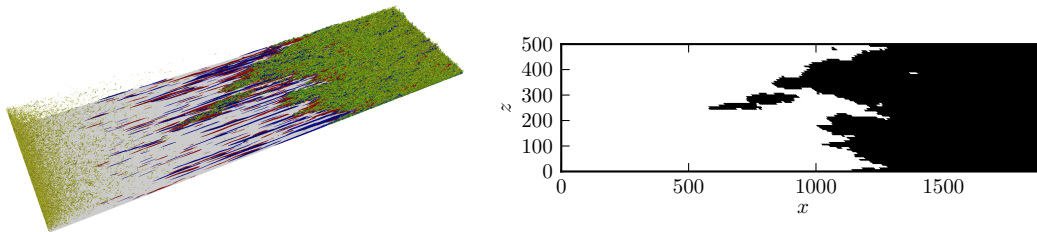
Bruno Eckhardt<sup>1</sup>, Tobias Kreilos<sup>1</sup>, Stefan Zammert<sup>1</sup>, Taras Khapko<sup>2</sup>, Philip Schlatter<sup>2</sup>,  
Yohann Duguet<sup>3</sup> & Dan S. Henningson<sup>2</sup>

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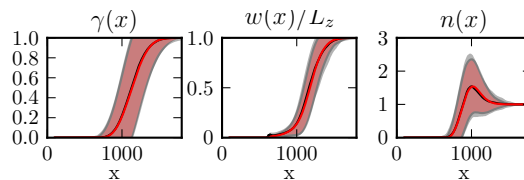
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Both plane Poiseuille flow and boundary layers show a linear instability to the formation of Tollmien-Schlichting waves and a transition at lower Reynolds numbers that is not connected with a linear instability of the laminar profile but triggered by finite amplitude perturbations, the bypass transition. In the case of plane Poiseuille flow we show how the two processes are arranged in the state space of the system and explain under which conditions the bypass process is the more dominant one, with the linear instability confined to a small set of initial conditions. For the case of the boundary layer, we appeal to the successful modelling of shear flows with probabilistic cellular automata and concepts from directed percolation. We fit a automaton model to data from large eddy simulations of boundary layers with different levels of free stream turbulence (Figure 1). The cellular automaton reproduces the statistics of the simulation data extremely well (Figure 2).



**Figure 11.** Visualization of turbulent spots in the LES data. (a) The untreated LES data. The colors indicate the level of turbulence by measuring the wall-normal velocity gradient at the wall. Dark blue indicates low intensities, the bright regions higher intensities. (b) Digitized LES data where only laminar (white) and turbulent (black) regions are distinguished. These data are then used to extract the parameters for the cellular automaton model.



**Figure 12.** Comparison of statistics between the LES data (black) and the probabilistic cellular automaton (blue). (a) Intermittency factor. (b) Width of independent spots in units of the domain width as a function of downstream position. (c) Number of spots at every downstream position.

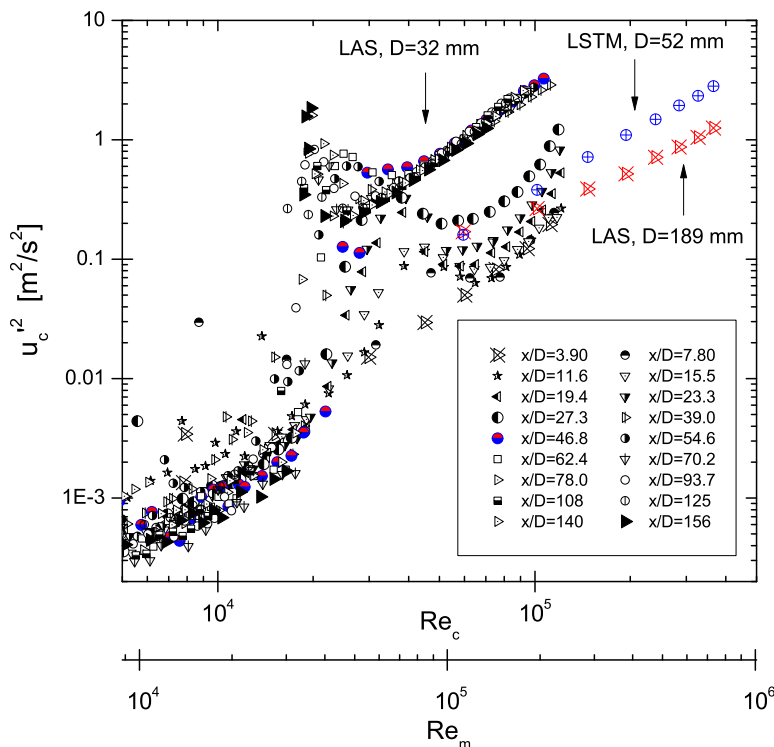
## THE COTTBUS LARGE PIPE (COLAPIPE) CENTRELINE FLOW MEASUREMENTS FOR RELATIVELY WIDE RANGE OF REYNOLDS NUMBER

E.-S. Zanoun<sup>1</sup> & F. König<sup>2</sup> & C. Egbers<sup>2</sup> & E. Öngüner<sup>2</sup>

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The present piece of work is an extension for earlier research paper by Zanoun, Kito, and Egbers [Journal of Fluids Eng. 131 061204, 2009] reporting observations on few aspects concerning the dependence of flow transition and development on the inlet flow condition and the entrance length in rectangular and circular ducts. Herewith, the experimental data is limited to the circular duct, however, for relatively wider range of Reynolds number,  $1.5 \times 10^5 \leq Re_m \leq 8.5 \times 10^5$ , compared to the pipe data of Zanoun et al. [2009]. The hot-wire anemometer was used to carry out measurements at various locations within the range  $0 \leq x/D \leq 146$ , where  $x$  is the length of the pipe section and  $D$  is the pipe diameter, see Zimmer [2011] for more details. Particular considerations were given to the bulk flow velocity, the mean velocity profile, and the centreline turbulence statistics to the fourth order. The transition criteria were discussed, reflecting effects of the inlet flow conditions, and the entrance length on the transition Reynolds number and flow development. Samples of the centreline flow measurements without triggering the flow at the pipe inlet are illustrated in Fig. 1. The figure represents the centreline-velocity fluctuations squared ( $u_c'^2$ ) versus the centreline velocity and the bulk velocity based Reynolds numbers,  $Re_c$  &  $Re_m$ , respectively. For the low range of the Reynolds number and depending on the measuring location (i.e.  $x/D$ ), various values for the transition Reynolds number were obtained, see Fig. 1. Early transition, however, was observed at a larger downstream distance from the pipe inlet. The figure shows also that after reaching the fully developed state it was observed that the centreline velocity fluctuations are different for different pipe diameters.



**Figure 13.** The pipe centreline-velocity fluctuations for various  $x/D$  versus the centreline-average velocity and/or the bulk-velocity based Reynolds numbers,  $Re_c$  &  $Re_m$ , respectively.

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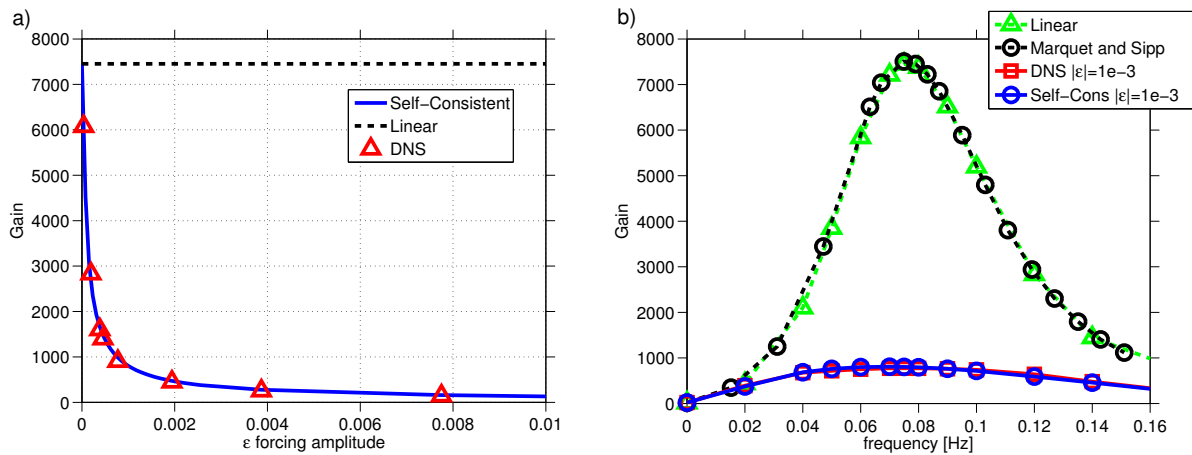
## TOO BIG TO GROW: THE MEAN FLOW SATURATION MECHANISM OF THE BACKWARD-FACING STEP RESPONSE TO HARMONIC FORCING CAPTURED BY A SELF-CONSISTENT MODEL

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Certain flows denominated as amplifiers are characterized by their global stability while showing large amplifications to sustained linear perturbations. For instance, the well known 2D backward-facing step flow presents a maximum sustained energy linear growth of order  $10^7$  at the frequency 0.075 and  $Re = 500$ , which is related to convective instabilities growing in the shear layer of the separated flow[1].

However, a simple physical picture describing the saturation of the response to higher amplitudes of the sustained forcing is still missing. We present a simple self-consistent model that captures the saturation mechanism of the response to harmonic forcing in a simple manner. The amplifier is forced at one frequency with an optimal body forcing structure calculated from the receptivity analysis around the base flow. As the forcing amplitude increases, a strong saturation of the response appears when compared to the linear prediction.



**Figure 14.** (a) Gain saturation with forcing amplitude increase at frequency 0.075. Forcing shape fixed as linear optimal of the base flow,  $Re=500$ . (b) Gain variation for different forcing frequency and Gain saturation of the model compared to DNS for a forcing amplitude of  $\epsilon = 10^{-3}$ ,  $Re=500$ . Forcing shape optimal around base flow at each frequency.

The model consists of a decomposition of the full nonlinear Navier-Stokes (NS) equations in a mean flow equation plus a linear perturbation equation around the mean flow. This linear response equation feeds back onto the mean flow through the Reynolds stress. The full fluctuating response and its corresponding Reynolds stress are approximated by only the first harmonic calculated from the linear response of the forcing around the aforementioned mean flow. This set of coupled equations is closed and solved in an iterative way as partial nonlinearity is still preserved in the mean flow equation despite the assumed simplifications.

The results show an accurate prediction of the response energy when compared to DNS, thus capturing the saturation process, as seen in Figure 1. It seems that the aforementioned coupling is enough to retain the main nonlinear effects of the saturation process. Hence, a simple physical picture is revealed, wherein the response modifies the mean flow through the Reynolds stress in such a way that the correct response energy is attained. Moreover, the saturated mean flow and fluctuating velocity fields and the Reynolds stress spatial structure are thereby well approximated in a self-consistent manner, without resorting to DNS. These results call for a generalization to externally forced globally unstable flows (oscillators), where the partial restabilization harmonically forced unstable flows was seen to coincide with a strong mean-flow distortion[2].

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## EXPERIMENTAL INVESTIGATION OF ELASTO INERTIAL TURBULENCE AND OF THE MAXIMUM DRAG REDUCTION ASYMPTOTE.

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The addition of small amounts of long chain polymers or surfactants to turbulent flows can lead to a reduction of skin friction more than 70%. The maximum drag reduction that can be achieved is surprisingly independent of the type of polymer used and its dependence on the Reynolds number is described by an empirically found limit, the so called maximum drag reduction (MRD) asymptote. It has recently been shown that polymers postpone the transition to Newtonian turbulence but at the same at high enough shear rates (i. e. large Weissenberg number) will promote an elastic instability. The ensuing disordered motion has been dubbed Elasto Inertial Turbulence (EIT). At high shear rates (i. e. in small diameter tubes) the onset EIT is observed at much lower Reynolds numbers ( $Re < 1000$ ) than Newtonian turbulence. At the same time it has been observed that the MDR asymptote can be approached directly from the elasto inertial instability without ever visiting Newtonian turbulence. Equally EIT when starting from Newtonian turbulence EIT is observed during the approach of the MDR asymptote. We here aim to characterize EIT and its connection to the MDR asymptote. In particular we carry out detailed velocity measurements (PIV) in order to investigate coherent structures and velocity spectra in the different flow regimes. Velocity spectra of EIT at low  $Re$  ( $< 2000$ ), where no Newtonian turbulence exists are compared to those at high Reynolds numbers ( $Re$  5000-10000), where the MDR asymptote has been closely approached. The spectra will be contrasted to Newtonian turbulence data. Finally some data from micro tubes and channels will be presented where EIT (due to the high shear and therefore high Weissenberg number) sets in at very low Reynolds numbers. Possible connections to elastic turbulence [Ref. [1]] will be explored.

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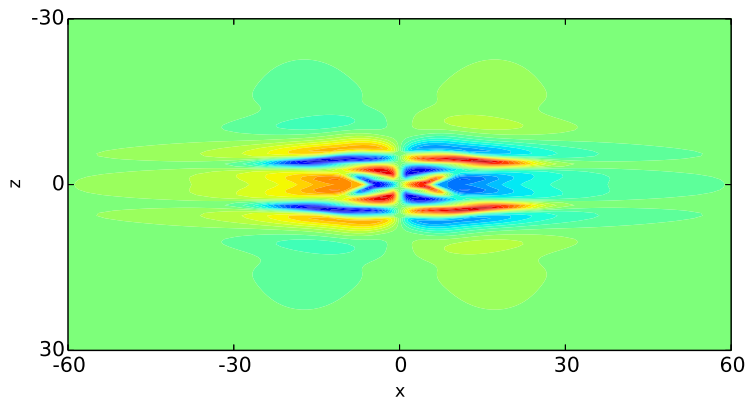
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## A DOUBLY-LOCALIZED EQUILIBRIUM SOLUTION OF PLANE COUETTE FLOW

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We present an equilibrium solution of plane Couette flow that is exponentially localized in both the spanwise and streamwise directions (Ref. [1]). The solution is similar in size and structure to previously computed turbulent spots (Ref. [2]) and localized, chaotically wandering edge states of plane Couette flow (Ref. [2], [3]). A linear analysis of dominant terms in the Navier-Stokes equations shows how the exponential decay rate and the wall-normal overhang profile of the streamwise tails are governed by the Reynolds number and the dominant spanwise wavenumber. Perturbations of the solution along its leading eigenfunctions cause rapid disruption of the interior roll-streak structure and formation of a turbulent spot, whose growth or decay depends on the Reynolds number and the choice of perturbation.



**Figure 15.** A doubly-localized equilibrium of plane Couette flow. The streamwise  $u$  velocity is shown in the  $y = 0$  midplane, with colormap [blue, green, red] spanning  $u = [-0.5, 0, 0.5]$ . A  $120 \times 60$  detail of the full  $200 \times 120$  computational domain is shown.

The solution is constructed from applying a tanh-based windowing function (Ref. [4]) in the streamwise  $x$  and spanwise  $z$  directions to a doubly-periodic equilibrium solution and refining with a Newton-Krylov-hookstep search algorithm. The solution decays exponentially to laminar flow in both  $x$  and  $z$ . Numerical evaluation of the terms of the Navier-Stokes equations for the solution shows that the dominant terms in the streamwise tails are

$$y u_x = Re^{-1}(u_{yy} + u_{zz}). \quad (2)$$

where  $u$  is the deviation of the streamwise velocity from laminar flow and subscripts indicate differentiation. A good approximation to the exponential decay in  $x$  and the wall-normal overhang profile in the tails can be obtained by assuming a solution of the form  $u(x, y, z) = \hat{u}(y) \exp(i\gamma z + \mu x/R)$  where  $\gamma$  is given by the dominant spanwise wavenumber in the tails. This leads to a boundary value problem for  $\hat{u}(y)$  whose Airy-function solution determines the streamwise decay rate  $\mu$  as a function of  $Re$  and  $\gamma$ .

The solution appears in a saddle-node bifurcation at  $Re \approx 228$  and continues smoothly to at least  $Re = 400$ . At low  $Re$  the solution has just a few unstable eigenfunctions in each of four allowed symmetry groups, but the number of unstable eigenfunctions increases rapidly with  $Re$ . For  $Re < 360$ , all eigenfunction perturbations decay to laminar flow; for  $Re > 400$  all grow to produce long-lived turbulent spots, suggesting that the solution sits on the boundary between laminar flow and turbulence for  $360 < Re < 400$ .

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## PREDATOR-PREY INTERACTIONS DRIVE THE LAMINAR-TURBULENCE TRANSITION INTO THE UNIVERSALITY CLASS OF DIRECTED PERCOLATION

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We argue how the superexponential scaling of the turbulent lifetime in pipe flow is related to extreme value statistics. There are two related arguments, one based on the threshold of energy needed to sustain a turbulent puff[1], the other based on the mapping of turbulence to directed percolation[2]. We show how directed percolation in 3+1 dimensions reproduces the principal features of the phenomenology of the transition in pipe flow. Finally, we describe our recent work that attempts to explain the directed percolation universality class from the fluid equations of motion, using a mapping between laminar and turbulent regions as predator-prey ecosystem dynamics. We explain the evidence for this mapping, and propose how a unified picture of the transition emerges in systems ranging from turbulent convection to magnetohydrodynamics.

### ACKNOWLEDGEMENT

Part of this work benefited from discussion with Tsung-Lin Hsieh. It is a pleasure to thank Maksim Sipos, Nicholas Guttenberg and Gustavo Gioia for their participation in References 1 and 2. This work was partially supported by the US National Science Foundation through grant NSF-DMR-1044901.

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## DRAG REDUCTION AND THE DYNAMICS OF TURBULENCE IN SIMPLE AND COMPLEX FLUIDS

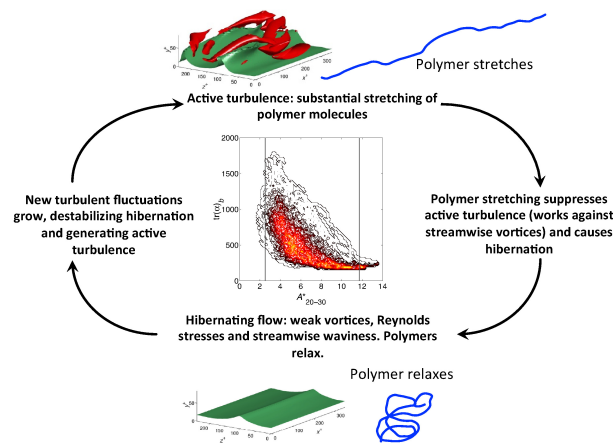
Michael D. Graham<sup>1</sup>, Li Xi<sup>2</sup>, Jae Sung Park<sup>1</sup> & Sung-Ning Wang<sup>1</sup>

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Addition of a small amount of very large polymer molecules or micelle-forming surfactants to a liquid can dramatically reduce the energy dissipation it exhibits in the turbulent flow regime. The most striking feature of this phenomenon is the existence of a so-called maximum drag reduction (MDR) asymptote: for a given geometry and driving force, there is a maximum level of drag reduction that can be achieved through addition of polymers. Changing the concentration, molecular weight or even the chemical structure of the additives has no effect on this asymptotic value. This universality is the major puzzle of drag reduction.

We describe direct numerical simulations of turbulent channel flow of Newtonian fluids and viscoelastic polymer solutions. Even in the absence of polymers, we show that there are intervals of “hibernating” turbulence that display very low drag as well as many other features of the MDR asymptote observed in polymer solutions [2, 2, 4]. As viscoelasticity increases, the frequency of these intervals also increases, leading to a stochastic cycle between active and hibernating turbulence, as shown in Figure 16, and to flows that increasingly resemble MDR [2, 7]. A simple theory captures key features of the intermittent dynamics observed in the simulations. Additionally, simulations of “edge states”, dynamical trajectories that lie on the boundary between turbulent and laminar flow, display characteristics that are similar to those of hibernating turbulence and thus to the MDR asymptote, again even in the absence of polymer additives [3]. Furthermore, a family of Newtonian nonlinear traveling solutions in channel is found whose mean velocity closely resembles MDR [6]. Based on these observations, we propose a tentative unified description of rheological drag reduction.



**Figure 16.** Schematic of the stochastic cycle displayed by viscoelastic turbulent channel flow in a minimal flow unit [4]. The plot in the center is the joint probability distribution function of spatially-averaged polymer stretch  $\text{tr } \alpha$  and instantaneous log-law slope  $A^*$ . The isosurface plots show streamwise velocity (green) and vortex strength (red) at active and hibernating instants.

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## WEAK TURBULENCE IN A TWO-LAYER KOLMOGOROV FLOW

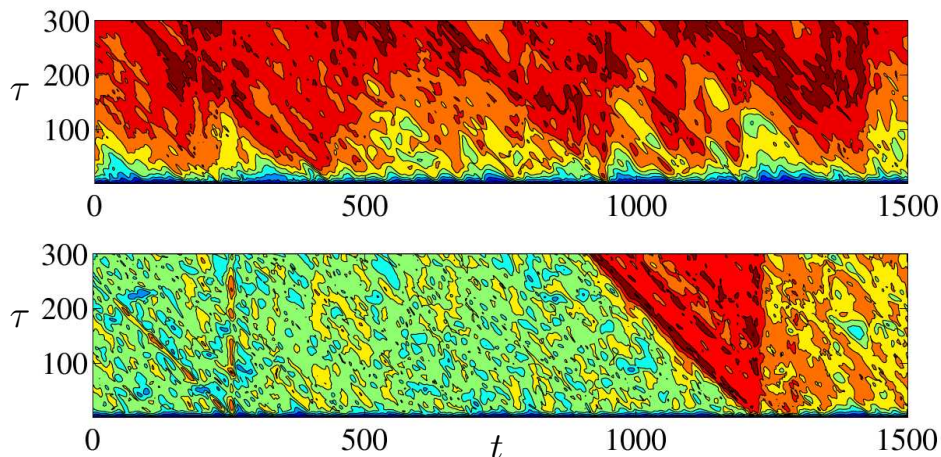
Roman O. Grigoriev<sup>1</sup> & Ravi Pallantla<sup>1</sup> & Balachandra Suri<sup>1</sup> & Jeffrey Tithof<sup>1</sup> & Radford Mitchell, Jr.<sup>1</sup> & Michael F. Schatz<sup>1</sup>

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Over the past decade novel numerical methods facilitated major advances in developing a dynamical description of weakly turbulent fluid flows based on the concept of exact coherent structures (ECS). This promising approach has been partially validated in numerical simulations of flows in various geometries, however, the connection between theory and experiment remains tenuous. While experiments offer glimpses of flow structures reminiscent of numerically computed ECS, proper experimental validation of the theory remains elusive. Most of the research in this field is currently focused on three-dimensional flows, which are both expensive to simulate and hard to quantify experimentally. Two-dimensional flows, which are both easier to simulate and to observe, offer an unprecedented opportunity to connect numerical results with experimental observations in a quantitative manner.

This presentation describes a combined experimental and numerical investigation of weakly turbulent flows in shallow, stratified horizontal layers of two immiscible fluids: The top layer is an electrolyte which is driven by Lorentz force and the lubricating bottom layer is a dielectric fluid. An array of long magnets with alternating polarity produces a sinusoidal forcing profile which has (locally) continuous translational symmetry in the longitudinal direction and discrete translational symmetry in the transverse direction, giving rise to what is known as a Kolmogorov flow. Experimental flows reconstructed using particle image velocimetry were compared with numerical solutions obtained using codes with doubly periodic boundary conditions in the lateral directions (mimicking typical 3D simulations) as well as those with proper no-slip boundary conditions corresponding to the experimental setup.

Although the system is quite large (about ten times the period of the forcing, i.e., the width of a pair of magnets), numerical simulations show that the dynamics of weakly turbulent flows depend rather sensitively on the system size and the choice of boundary conditions, as the recurrence plots shown below illustrate. In particular, near onset of turbulence the dynamics of the doubly periodic flow are dominated by relative time-periodic solutions with low translation velocity, while the dynamics in the bounded case are dominated by time-periodic solutions which have a completely different structure. Furthermore, preliminary analysis has shown that experimental flows, rather unexpectedly, frequently visit neighborhoods of ECS found using the doubly periodic simulation.



**Figure 17.** Recurrence plot for the doubly periodic flow (top) and bounded flow (bottom). System size and current are the same, but the values of  $Re$  are slightly different due to the difference in boundary conditions. The color scale (from blue to red) represents the normalized values of the recurrence function  $E(t, \tau) = \min_{g \in G} \|\omega(x, y, t - \tau) - g\omega(x, y, t)\|$ .  $E(t, \tau)$  measures the difference between the states of the system (represented by vorticity  $\omega$ ) at times  $t$  and  $t - \tau$  in the symmetry-reduced representation;  $g$  is an element of the group  $G$  of spatial symmetries of the flow.

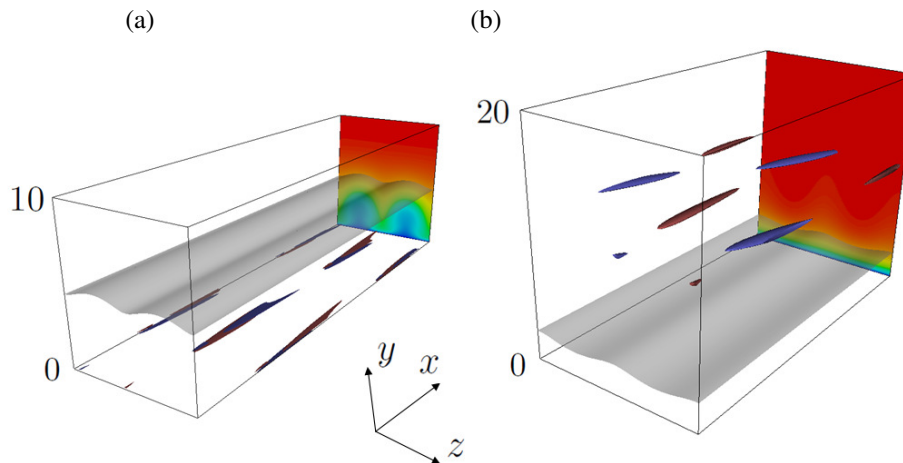
This work is supported in part by the National Science Foundation under grants No. CBET-0853691, CBET-0900018, and CMMI-1234436.

## FREESTREAM COHERENT STRUCTURES IN BOUNDARY LAYERS AND VORTEX DOMINATED FLOWS

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Our concern is with high Reynolds number nonlinear equilibrium solutions of the Navier-Stokes equations for boundary-layer flows. The first part of our investigation concerns the asymptotic suction boundary-layer which we take as a prototype parallel boundary-layer. Solutions of the equations of motion are obtained using a homotopy continuation from two known types of solutions for plane Couette flow. At high Reynolds numbers, it is shown that the first type of solution takes the form of a vortex-wave interaction state, see Hall & Smith (1991), Hall & Sherwin (2010), and is located in the main part of the boundary-layer. On the other hand, the second type is found to support an equilibrium solution of the unit Reynolds number Navier-Stokes equations in a layer located a distance of  $O(\ln R)$  from the wall. Here  $R$  is the Reynolds number based on the freestream speed and the unperturbed boundary-layer thickness. The streaky field produced by the interaction grows exponentially below the layer and takes on its maximum size within the unperturbed boundary-layer. An asymptotic description of the new state is given and the crucial effects of non parallelism show that the new state in a growing boundary layer can exist only intervals of finite length. We also show the new state applies to viscous models of vortex sheets. The results suggest the possibility of two distinct types of streaky coherent structures existing, possibly simultaneously, in turbulent boundary-layers.



**Figure 18.** Total flow comparison of (a) wall mode and (b) free stream coherent structure. The red/blue surfaces are the 80% maximum/minimum of streamwise vorticity.

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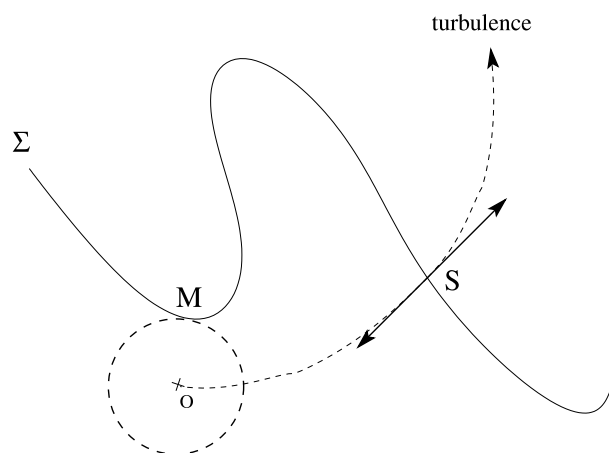
## MINIMAL TRANSITION THRESHOLDS IN PLANE COUETTE FLOW

Dan S. Henningson<sup>1</sup> & Yohann Duguet<sup>2</sup> & Antonios Monokrousos<sup>1</sup> & Luca Brandt<sup>1</sup>

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Subcritical transition to turbulence requires finite-amplitude perturbations. Using a nonlinear optimisation technique in a periodic computational domain, we identify the perturbations of plane Couette flow transitioning with least initial kinetic energy for  $Re < 3000$  [2, 2]. We suggest a new scaling law  $E_c = O(Re^{-2.7})$  for the energy threshold vs. the Reynolds number, in quantitative agreement with experimental estimates for pipe flow. The route to turbulence associated with such spatially localised perturbations is analysed in detail for  $Re = 1500$ . Several known mechanisms are found to occur one after the other: Orr mechanism, oblique wave interaction, lift-up, streak bending, streak breakdown, and spanwise spreading. The phenomenon of streak breakdown is analysed in terms of leading finite-time Lyapunov exponents of the associated edge trajectory.



**Figure 19.** Sketch of phase space.  $M$ : minimal perturbation,  $O$ : laminar fixed point,  $\Sigma$ : laminar-turbulent boundary,  $S$ : edge state.

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**SPATIO-TEMPORAL INTERMITTENCY IN COUNTER-ROTATING TAYLOR-COUETTE FLOW**Shreyas V. Jalikop<sup>1</sup> & Kerstin Avila<sup>2</sup> & Grégoire Lemoult<sup>1</sup> & Björn Hof<sup>1</sup><sup>1</sup>*Institute of Science and Technology Austria*<sup>2</sup>*Max Planck Institute for Dynamics and Self-organization, Göttingen, Germany*

Spatio-temporal intermittency (STI) usually refers to the state where turbulent patches coexist with laminar background for a fixed value of the control parameter. STI is observed during the sub-critical transition to turbulence in many shear flows, most notably pipe flow and Couette flows. We experimentally study its dynamics in the counter-rotating Taylor-Couette (TC) flow to show that the transition to sustained turbulence in the sub-critical regime is a second-order phase transition.

Our TC setup has an aspect ratio and an azimuthal length of more than 300 gap-widths [1]. These large dimensions minimise the finite-size effects, and render the setup suitable to experimentally distinguish between first and second order phase transitions. We have measured mean turbulent fraction and the size distributions of the laminar gaps above the critical point. By quenching the system below the critical point, we have also measured the lifetimes of turbulent patches. Based on these measurements we show that this phase transition is second-order in nature.

Further, the large radius-ratio ( $\eta$ ) of 0.98 in our setup allows us to characterise STI that occurs in the region of phase space where the transition to turbulence is not sub-critical. In this regime, STI appears in the background of vortex rolls in the form of intermittent bursts of laminar wavy-patches [2], which gradually become turbulent as the degree of counter rotation of the cylinders is increased. This transition from wavy to turbulent patches, we believe, has not been reported before in Taylor-Couette system, and has been only observed in rotating plane Couette experiments [2]. We have mapped the onset curve of these wavy patches in the Reynolds number phase space, and find that it connects to the curve for the onset of sustained turbulence in the sub-critical regime. We also find that the lifetimes of these patches are exponentially distributed and that the mean lifetime increases with increasing degree of counter rotation.

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## WEAKLY NONLINEAR STABILITY OF STREAKS IN PIPE FLOW OF SHEAR-THINNING FLUIDS

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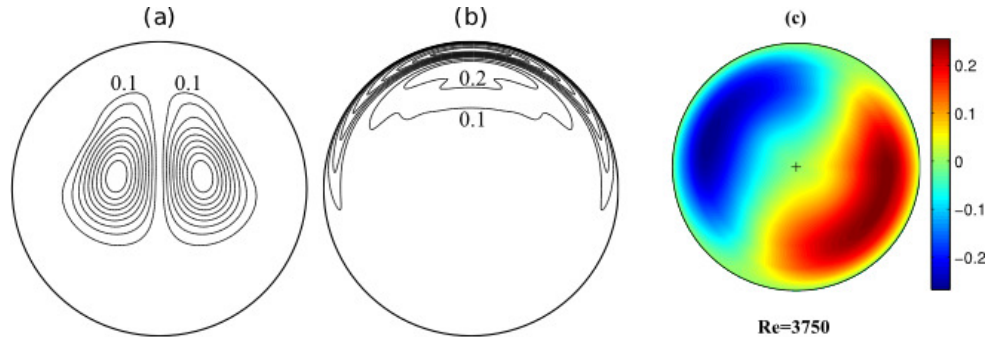
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This study is motivated by experimental results dealing with the transition to turbulence in a pipe flow of shear-thinning fluids, where a streaky flow with an azimuthal wave number  $n = 1$  is observed in the transitional regime as shown on the figure (c) [1, 2]. A linear stability analysis of pipe flow of shear-thinning fluids modulated azimuthally by finite amplitude streaks was performed in [3]. The shear-thinning behavior of the fluid is described by the Carreau model.

The streaky base flows considered are obtained from two-dimensional direct numerical simulation using finite amplitude longitudinal rolls as the initial condition and by extracting the velocity field at time  $t_{max}$ , where the amplitude of the streaks reaches its maximum, denoted by  $A_{max}$ . It is found that the amplitude  $A_{max}$  increases with increasing Reynolds number as well as with increasing amplitude  $\epsilon_0$  of the initial longitudinal rolls. For sufficiently large streaks amplitude, *i. e.* for  $A_{max}$  greater than a threshold amplitude  $A_c$ , streamwise velocity profiles develop inflection points, leading to instabilities.

Depending on the threshold amplitude  $A_c$ , two different modes may trigger the instability of the streaks. If  $A_c$  exceeds approximately 41.5% of the centerline velocity, the instability mode is located near the axis of the pipe, *i. e.*, it is a "center mode" (Fig. a). For weaker amplitude  $A_c$ , the instability mode is located near the pipe wall, in the region of highest wall normal shear, *i. e.*, it is a "wall mode" (Fig. b). The threshold amplitude  $A_c$  decreases with increasing shear-thinning effects. The energy equation analysis indicates that (i) wall modes are driven mainly by the work of the Reynolds stress against the wall normal shear and (ii) for center modes, the contribution of the normal wall shear remains dominant; however, it is noted that the contribution of the Reynolds stress against the azimuthal shear increases with increasing shear-thinning effects.



**Figure 20.** (a): Critical mode for Newtonian and weakly shearthinning fluids. (b): Critical mode for shearthinning fluids. (c): Streaky flow observed in shearthinning fluids.

A weakly nonlinear study is performed for streaky shear-thinning fluids flow where the linear unstable mode is a wall mode. The objective is to determine how the nonlinear terms sustain the streaky flow and to elucidate the role of the nonlinear viscous terms, *i. e.*, those arising from the viscosity perturbation. Preliminary results show that the bifurcation is supercritical and it is dominated by the nonlinear inertial terms. The nonlinear viscous terms tend to promote subcritical bifurcation. Deeper analysis is under consideration.

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## ON SECONDARY INSTABILITY OF LINEAR TRANSIENT GROWTH IN COUETTE FLOW

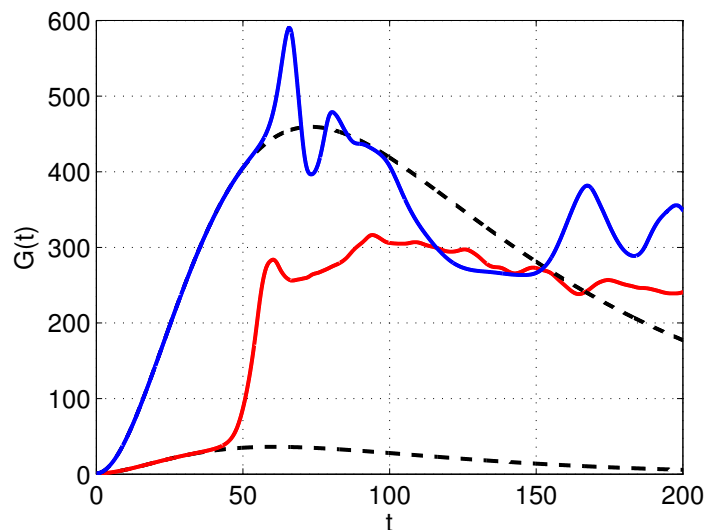
Michael Karp &amp; Jacob Cohen

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It is well known that Couette flow is stable with respect to infinitesimal wavy disturbances. An alternative possible explanation for the flow instability and the consequent transition to turbulence may lie in the linear mechanism of transient growth. Accordingly, a small disturbance can achieve a significant non-modal transient growth that can trigger nonlinear mechanisms before its eventual long-time decay owing to viscous effects. The linear optimal achieving the maximal growth corresponds to a pair of nearly streamwise independent counter rotating vortices (CVPs) which create streamwise streaks varying along the spanwise direction (with a spanwise wavenumber  $\beta$ ) [1]. The streaks may become unstable with respect to infinitesimal disturbances and undergo secondary instability.

In this study we propose an analytical approximation to the linear transient growth consisting of 4 modes. The analytical approximation allows performing a two dimensional (wall-normal and spanwise directions) secondary linear stability analysis of the base-flow undergoing transient growth (namely, Couette flow with streamwise streaks, e.g. [2]). The secondary stability analysis is verified by checking whether transition occurs when simulating the secondary disturbance in the modified baseflow. The simulation used in this study is Gibson's well-tested 'Channelflow' direct numerical simulation (DNS) software [2]. It is shown analytically that the inclusion of nonlinear interactions between the base-flow and the CVPs is required in order to obtain eigenvalues that correspond to transition scenarios obtained using DNS.

The stability analysis allows comparing the transition scenarios of the symmetric (2 vortices) and anti-symmetric (4 vortices) transient growth and to determine the relative dominance of the inflection points in the wall-normal and spanwise directions. An example of the normalized kinetic energy gain evolution,  $G(t)$ , is presented for  $Re=1000$  in the figure for symmetric (blue solid curve) and anti-symmetric (red solid curve) transient growth based transition scenarios. It can be seen that the transitions occur at comparable times. In the symmetric transient growth transition most of the growth occurs during the transient growth stage whereas in the anti-symmetric transient growth transition the growth is mainly attributed to the secondary instability. After transition both cases attain similar energy values.



**Figure 21.** Energy growth for symmetric (blue,  $\beta = 2$ ) and anti-symmetric (red,  $\beta = 1$ ) transient growth based transitions for  $Re=1000$  and initial energy of  $E_0 = 2.66 \cdot 10^{-4}$ . The secondary disturbance in both scenarios has same fraction of energy ( $0.09\% E_0$ ) and it is based on a secondary stability analysis at  $t = 20$  for  $\alpha = 1$ . The corresponding unperturbed transient growth curves are given for reference by the dashed lines.

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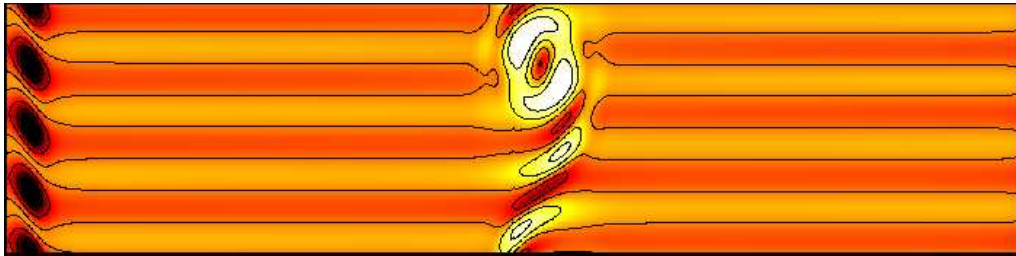
**SPATIOTEMPORAL DYNAMICS IN 2D KOLMOGOROV FLOW OVER LARGE DOMAINS**

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Kolmogorov flow [1] in two dimensions - the 2D Navier-Stokes equations with a sinusoidal body force

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p &= \frac{1}{Re} \Delta \mathbf{u} + \sin 4y\hat{x}, \\ \nabla \cdot \mathbf{u} &= 0, \end{aligned}$$

- is considered over extended periodic domains (typically  $(x, y) \in [0, 8\pi] \times [0, 2\pi]$ ) to reveal localised spatiotemporal complexity. The flow response mimicks the forcing at small  $Re$  but beyond a critical value, the system develops into 1D flow regions joined by localised 2D transition regions ('kinks' and 'antikinks'). We will discuss how these kinks and antikinks act as building blocks for multiple attractors which emerge as  $Re$  increases further. Some have spatially localised chaotic time variation and present an inviting target for recurrent flow analysis (e.g. [2] and references herein).



**Figure 22.** The vorticity field for a typical solution displaying a kink (centre) and antikink (left). Colour code is vorticity  $\in [-5, 5]$  with 5 evenly spaced contours across  $[-4, 4]$ . For details see [2].

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## LOCALISED EDGE STATES IN THE ASYMPTOTIC SUCTION BOUNDARY LAYER

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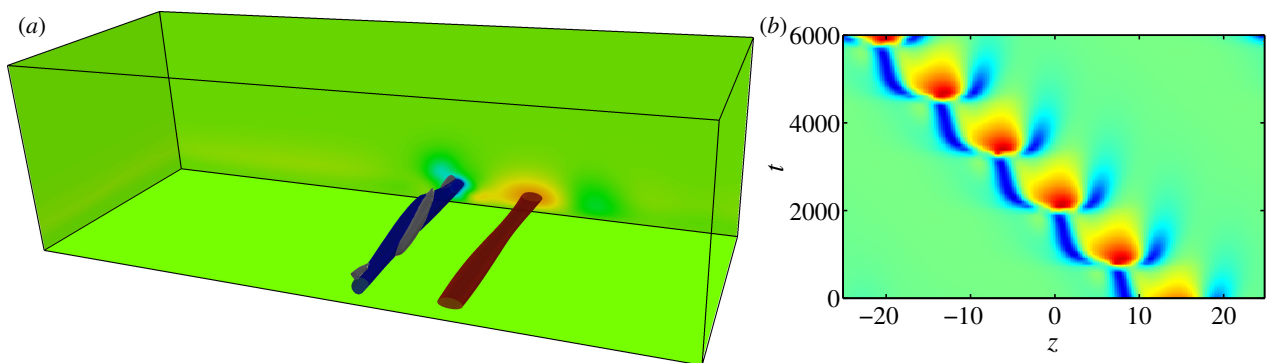
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Despite having large theoretical and practical importance subcritical transition to turbulence is still not completely understood. One of the recent ideas coming from Dynamical Systems field is the concept of edge states, relative attractors within the laminar–turbulent separatrix [1]. They serve as simple examples of non-trivial dynamics of the flow and can help with understanding processes occurring during nonlinear stages of transition.

We study edge states in the asymptotic suction boundary layer, which is a boundary-layer flow over a flat plate subject to constant homogeneous suction. Suction counteracts the usual slow spatial growth of the boundary layer, resulting in a parallel flow. When considering spanwise-extended domains the attracting structures on the edge inevitably localise [2]. Depending on the streamwise extent of the domain their behaviour is either periodic or erratic in time, with long calm and short bursting phases in both cases. At calm phases all identified edge states structurally consist of an active pair of low- and high-speed streaks (see figure 23(a)). During bursts this structure is destroyed and re-created with a shift in the spanwise direction. Different states are distinguished based on the regularity and direction of this shift (see figure 23(b)). In all cases a clear regeneration cycle is identified, bearing strong similarities to the original self-sustaining process in the near-wall turbulence [2].

Considering streamwise length of the domain as the bifurcation parameter a number of different phenomena is observed [3]. Among those are multistability, intermittency and period doubling.



**Figure 23.** (a) Three-dimensional representation of one of the edge states in ASBL during a calm phase. (b) Time–space diagram of the streamwise velocity fluctuations averaged in the streamwise direction at a fixed wall-normal plane showing the left-shifting edge state.

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## A REDUCED MODEL FOR EXACT COHERENT STATES IN SHEAR FLOW

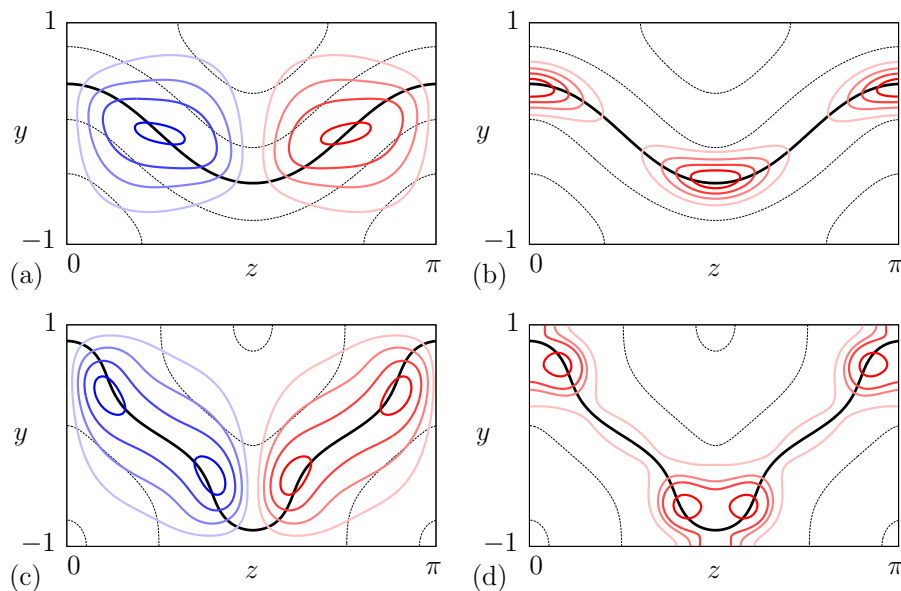
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In plane Couette flow, the lower branch Nagata solution has been shown to follow simple streamwise dynamics at large Reynolds numbers. A decomposition of this solution into Fourier modes in this direction yields modes whose amplitudes scale with inverse powers of the Reynolds number, with exponents that increase with increasing mode number [1]. In this work, we use this scaling to derive a reduced model for exact coherent structures in general parallel shear flows and apply the theory to Waleffe flow [2, 4]. The reduced model describes the dynamics of the streamwise-averaged flow and of the fundamental fluctuations, and prominently features terms related to the self-sustaining process identified in [2]. The model is numerically regularized by retaining higher order viscous terms for the fluctuations [4, 7] and numerical methods are designed to find good approximates of nontrivial solutions which are then converged using a preconditioned Newton method. For Waleffe flow the procedure captures both lower branch (Fig. 1(a,b)) and upper branch (Fig. 1(c,d)) solutions [2] and demonstrates that these branches are connected via a saddle-node bifurcation, thereby extending related results by Hall & Sherwin [3] and Blackburn *et al.* [7] for plane Couette flow beyond the lower branch.



**Figure 24.** Solutions computed at  $Re \approx 1500$  in Waleffe flow [2]. (a,b) The lower branch solution, represented by contours of constant streamwise streamfunction, and a measure of the spanwise fluctuations  $\|(v_1, w_1)\|_{L_2}$ , respectively. In each plot red indicates positive values while blue indicates negative values; contour values are equidistributed. Each representation is superposed on top of the streaks shown in black with the solid line representing the critical layer and the dashed lines showing equidistributed contours of the streamwise velocity. (c,d) Corresponding plots for the upper branch solutions.

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## THE ONSET OF CHAOS IN PLANE COUETTE FLOW

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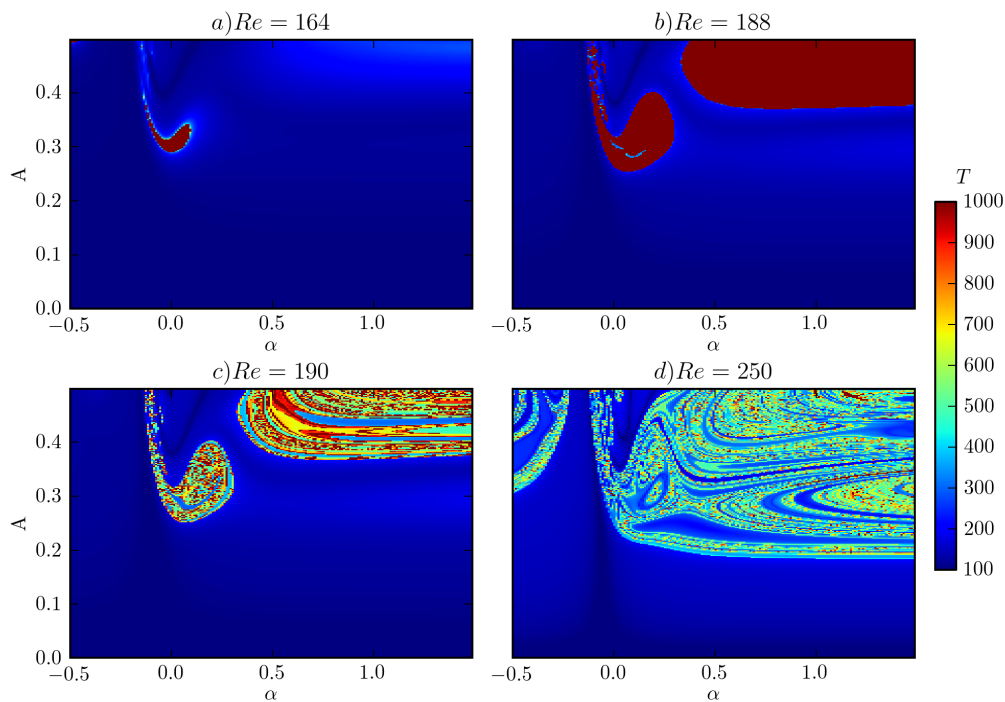
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We track the onset of chaos in plane Couette flow. The formation of a chaotic saddle through a boundary crisis is discussed. We show that inside the chaotic saddle, new saddle-node bifurcations create stable pockets of turbulence, which subsequently undergo boundary crisis. This leads to a non-monotonic variation of characteristic lifetimes with Reynolds number.

### The Onset of Chaos

Studies of the transition to turbulence in linearly stable shear flows, such as pipe flow and plane Couette flow, have revealed a route to turbulence that passes through the formation of a chaotic saddle [1, 2]. We here present the analysis of the state-space structures and the bifurcations that lead to the creation of this saddle. The chaotic saddle supports long transients preceding sustained turbulence [4].



**Figure 25.** Two-dimensional slices of the state-space of transitional plane Couette flow. Coordinates are chosen such that the x-axis is an interpolation between the lower- and upper-branch Nagata-Busse-Clever states. The y-axis is the amplitude of the interpolated state. The structure of state-space is shown by the lifetimes  $T$  of initial conditions. (a) Just after the first saddle-node bifurcation. (b) Just before the boundary crisis. (c) Just after the boundary crisis. (d) At higher Reynolds number.

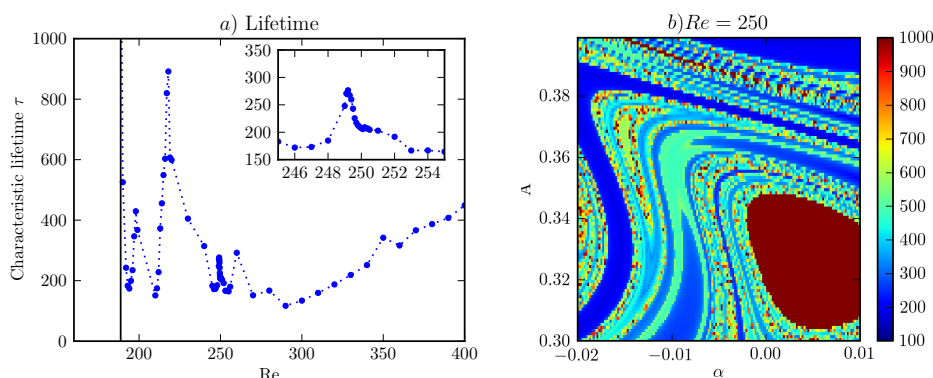
We study plane Couette flow in a small periodic domain of size  $2\pi \times 2 \times \pi$  in the downstream, wall-normal and spanwise directions with imposed shift-and-reflect symmetry. In this cell, the Nagata state [5] is created as the lower-branch of a saddle-node bifurcation at Reynolds number  $Re = 163.8$ . It has exactly one unstable direction, while the corresponding upper-branch solution is stable. The latter one is hence an attractor that coexists with the laminar attractor. Figure 25(a) illustrates the situation by showing the lifetimes of initial conditions in a two-dimensional slice of the state-space at  $Re = 164$ . The small dark-red region corresponds to the basin of attraction of the upper-branch, the blue region to states that decay to the laminar state. The fixed-point undergoes a series of bifurcations that result in a chaotic attractor [2]. Its basin of attraction grows with increasing  $Re$ , figure 25(b).

At  $Re = 188.7$ , the chaotic attractor has expanded so much that it collides with the lower-branch state, leading to a boundary crisis. At this point, the basin of attraction opens up and the chaotic attractor turns into a chaotic saddle. By comparing the situation before (figure 25(b) at  $Re = 180$ ) and after (figure 25(c) at  $Re = 190$ ) the crisis, we still see the chaotic saddle as a region of rapidly varying lifetimes.

As Reynolds number increases further, the chaotic saddle grows to fill more and more parts of the state-space, figure 25(d) at  $Re = 250$ .

### Creating More Chaos

Inside the chaotic saddle the distribution of lifetimes follows an exponential scaling, with a characteristic lifetime  $\tau$ . Figure 26(a) shows  $\tau$  as a function of Reynolds number, with each value of  $\tau$  calculated from 50000 trajectories. Just above the crisis point in  $Re$ , the lifetime decays quickly. For higher  $Re$  it varies irregularly, before increasing more steadily for  $Re$  above 300.



**Figure 26.** Non-monotonic lifetimes. (a) The characteristic lifetime as a function of Reynolds number varies non-monotonically. (b) A “pocket of turbulence”, associated with a saddle-node bifurcation of periodic orbits, immersed in the chaotic saddle.

We are able to associate the non-monotonic variation of lifetimes to the creation of “pockets of turbulence”: in a small region in state-space a stable orbit and a hyperbolic point are created in a saddle-node bifurcation; this pocket grows, opens up and merges with other pockets, leading to an increase in lifetimes.

The crisis-bifurcation discussed above gives a first example of such a pocket. As a second example we study the peak around  $Re = 250$ , magnified in the inset of figure 26(a). This peak can be related to a pair of periodic orbits which are created in a saddle-node bifurcation at  $Re = 249.01$ . In this bifurcation, the upper-branch orbit is an attractor, the lower-branch orbit has one unstable direction. The situation is hence similar to the one described above for the Nagata-solutions, only that now the saddle node bifurcation occurs inside the chaotic saddle. We see this chaotic pocket as the dark-red region in figure 26(b), where states never decay to the laminar state, immersed in the colorful saddle. The attractor is destroyed in a new crisis at  $Re = 250.13$ , leaving behind a larger chaotic saddle than before.

The creation of invariant solutions in saddle-node bifurcations and the destruction in crisis bifurcations provides one mechanism underlying the generally observed increase of turbulent lifetimes with  $Re$ . Note however that lifetimes can vary non-monotonically.

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## SUBCRITICAL VERSUS SUPERCRITICAL TRANSITION IN CURVED PIPES

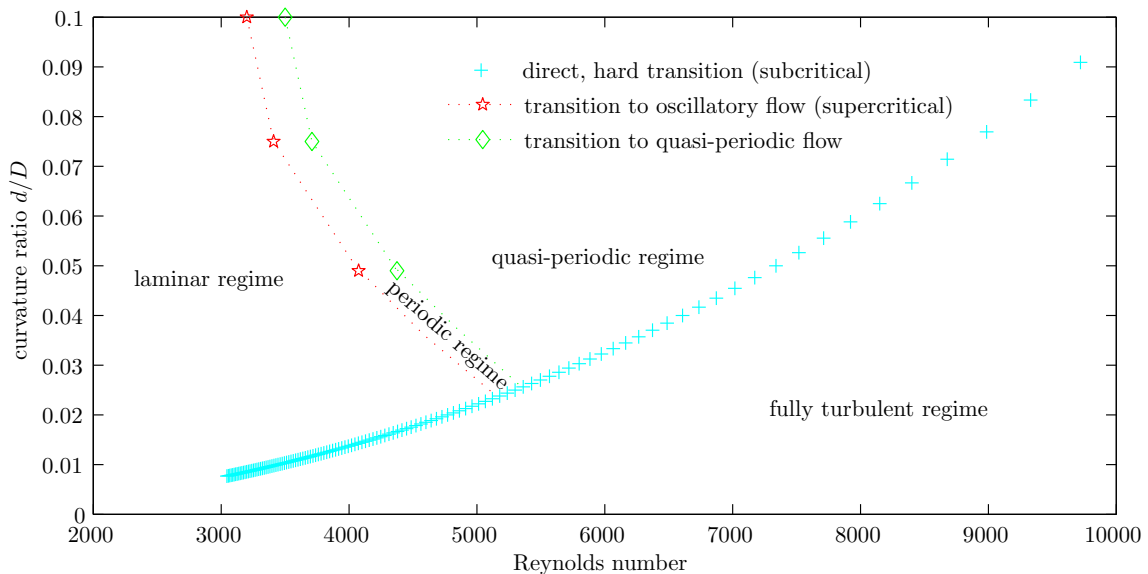
Jakob Kühnen<sup>1</sup> & Philipp Braunschier<sup>1</sup> & Hendrik Kuhlmann<sup>2</sup> & Björn Hof<sup>1</sup>

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It is a characteristic of straight pipe flow that transition occurs in spite of the linear stability of the laminar Hagen–Poiseuille flow if the flow perturbations exceed a minimum threshold (subcritical transition). In curved pipes the process of transition to turbulence can differ qualitatively from that in straight pipes, as the steady axisymmetric basic flow is influenced by centrifugal effects. If the curvature ratio (tube-to-coiling diameter  $d/D$ ) exceeds a certain threshold, transition is dominated by these centrifugal effects. Hence, on an increase of the Reynolds number we find a sequence of bifurcations (supercritical transition). For intermediately and strongly curved pipes an oscillatory flow is found in which waves travel in the streamwise direction with a phase velocity slightly faster than the mean flow. The oscillatory flow is, at a further increase of the Reynolds number, superseded by a presumably quasi-periodic flow before turbulence sets in [1].

High-speed stereoscopic particle image velocimetry (S-PIV), laser-Doppler velocimetry as well as pressure drop measurements have been used to investigate and capture the development of the transitional flow in curved pipes and to characterize the transitional behavior as a function of the curvature ratio. Our measurements reveal the threshold of curvature ratio where the two different transition scenarios meet. We present a map of the sub-critical and super-critical transition points in parameter space and determine how the two competing instability scenarios influence each other.



**Figure 27.** Different transition scenarios in dependence of the curvature ratio. The data are preliminary.

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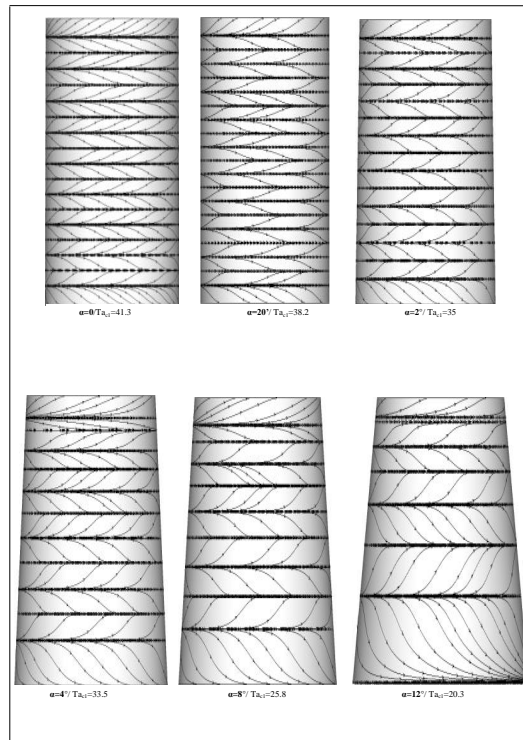
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### 3D CFD OF THE ONSET OF TAYLOR VORTEX FLOW BETWEEN COMBINATIONS OF CIRCULAR AND CONICAL CYLINDERS

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Taylor- Couette system consists of the flow in the annular space between two concentric rotating bodies is a convenient fluid system to study the laminar turbulent transition. Further, the study of this flow is of basic importance in the field of hydrodynamic stability. The fluid motion in an annular gap between cylinder-cone combinations is investigated using CFD, Ansys Fluent for a three dimensional viscous and incompressible flow. Particular attention is given to the transition regime and the onset of instabilities when outer cylinder is replaced with a cone. The transition phenomena that appear in this flow are discussed under the effect of the cone angles of outer body. The numerical calculations are carried out over a range of cone angle  $\alpha$  from 0 (classical case) up to  $12^\circ$ . The main goal it is to show how operates the change in the structure of the movement when changing the geometry of the flow through angular deviation, i.e., from the coaxial cylindrical rotating system to the coaxial conical rotating flow system. The onset of Taylor vortex flow, the various mechanisms of the laminar-turbulent transition and the flow behavior in combined Taylor-Couette apparatus are examined and described in detail for a wide range of cone angles. The critical Taylor number,  $Ta_{c1}$ , characterizing the onset of Taylor vortices in the flow, decreases dramatically. It is established that the first instability mode of transition is advanced from  $Ta_{c1} = 41.33$ , corresponding to the classical case (cylindrical Taylor-Couette system) until  $Ta_{c1} = 20.3$  when the cone angle reaches  $12^\circ$ . The appearance of the first instability is then substantially advanced with respect to classical case. The various cell patterns and the velocity profiles are predicted. The wavelengths of the vortices for different cone angles are also determined. The number of vortices occurring in the gap between rotating cylinder in a cone is inversely proportional to the cone angles.



**Figure 28.** Onset of Taylor vortices between a rotating cylinder in a cone for various apex angle.

#### References

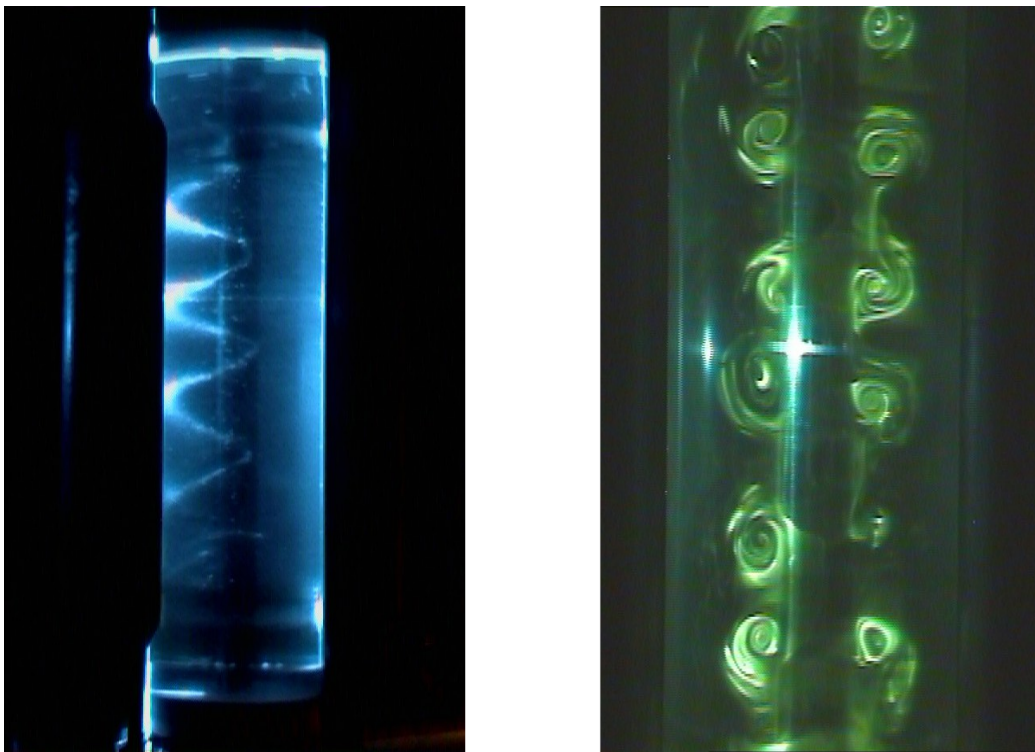
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## A NOT YET EXPLAINED INSTABILITY IN AN ELLIPTICALLY DEFORMED CYLINDRICAL COUETTE FLOW.

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In 2007, we experimentally discovered a new instability in a cylindrical Couette apparatus that possesses an elliptically deformed rotating outer wall [1]. The deformation of this wall is produced by applying two stationary rollers in a similar way to Malkus [2] and later to Eloy [2] who explored the elliptical instability. The inner cylinder is maintained fixed so that the traditional stable cylindrical Couette flow is only slightly disturbed by the elliptical deformation of the rotating outer wall. The following figure shows Kalliroscope (a) and dye visualizations (b) of the unstable flow. From the examination of the symmetry of the pattern, we concluded that the instability should be of centrifugal type. We will review our different attempts to explain and describe this instability that today still resists our understanding and remains a challenge for theoreticians.



**Figure 29.** Kalliroscope (a) and dye visualizations (b) of the unstable flow.

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## EXPERIMENTAL OBSERVATION OF EXACT COHERENT STRUCTURES IN TRANSITIONAL CHANNEL FLOW

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According to dynamical systems theory the disordered dynamics of turbulence as well as of its *edge* are organized around unstable solutions of the Navier-Stokes equations. Within the past two decades, the computation of exact solutions of the Navier-Stokes equation has attracted considerable attention. The discovery of these exact coherent structures has opened a new approach to understanding the dynamics of unsteady flows in transitional Re range[1].

We measure the internal structure of a turbulent spot with Time Resolved Stereoscopic Particle Image Velocimetry and reconstruct the instantaneous three-dimensional velocity field. We isolate travelling wave like structures at the trailing edge of the spot and compare it with known computed exact coherent structures. Eventually, we use this experimentally measured velocity field as starting guess in a Newton algorithm in order to converge to an exact coherent structure.

To our knowledge it corresponds to the first observation of exact coherent structures in transitional channel flow.

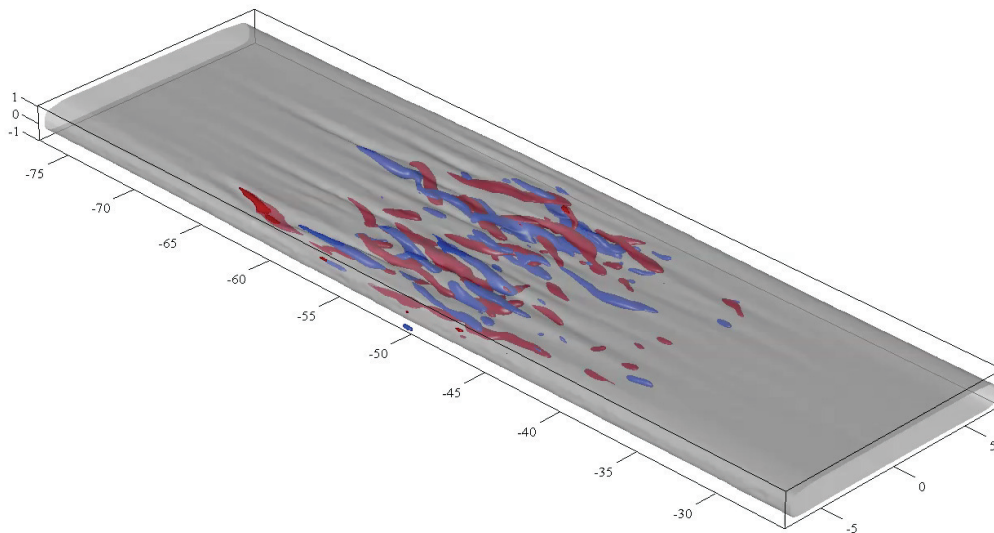


Figure 30. Isosurfaces of streamwise vorticity.

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### THREE-DIMENSIONAL CHIMERA STATES

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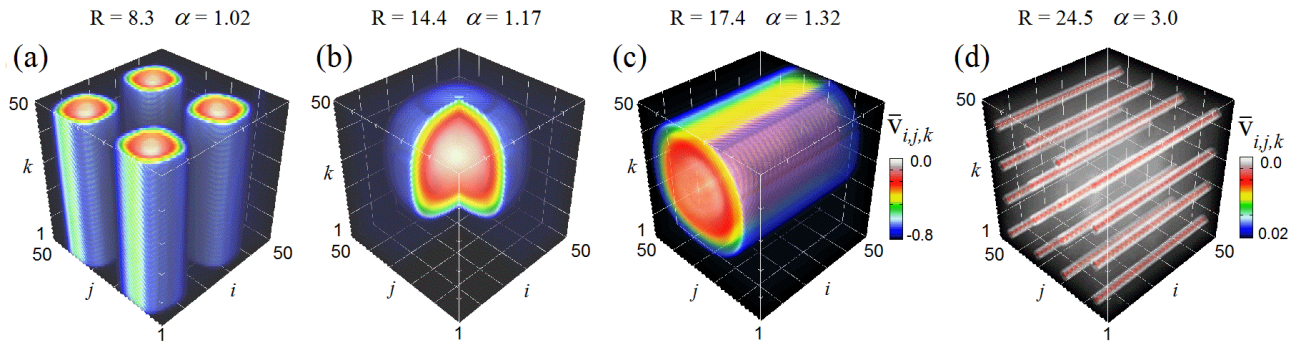
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Chimera states represent remarkable patterns of spatially separated domains of coherence and incoherence. Intriguingly, they can robustly develop in homogeneous media or oscillatory networks. Chimera states were discovered in 2002 for 1-D complex GLU and its phase approximation known as Kuramoto model [1] and soon after, got a lot of attention, both theoretically and experimentally. Recently, novel two-dimensional chimera states were explored in the form of incoherent spots or strips as well as spirals with incoherent core [2].

In this contribution, we report the first observation of three-dimensional chimera states of the following types: *incoherent streaks*, *incoherent ball*, and *incoherent tubes* (Fig.1) to compare the phenomenon with laminar-turbulent patterns in fluid. The dynamical system of concern is governed by Kuramoto-like network of  $N^3$  oscillators:

$$\dot{\varphi}_{ijk} = \omega + \frac{K}{R^3} \sum_{(i-i')^2+(j-j')^2+(k-k')^2 < R^2} \sin(\varphi_{i'j'k'} - \varphi_{ijk} - \alpha), \quad (3)$$

where  $\varphi_{ijk}$  are phase variables uniformly distributed in a 3-D torus, indexes  $i, j, k$  are periodic mod  $N$ ,  $R$  stays for coupling range. We take the natural frequency  $\omega$  of all oscillators identical, and set it to zero without loss of generality. The coupling in Eq. (1) is assumed long-ranged and isotropic: each oscillator  $\varphi_{ijk}$  is coupled with equal strength  $K$  to all its nearest neighbors within range  $R$  which may be interpreted as an analog of Reynolds number in hydrodynamics. Parameter  $\beta = \pi/2 - \alpha$  indicates some kind of "viscosity" in the model (1), it is positive when phase lag parameter  $\alpha < \pi/2$ , negative for  $\alpha > \pi/2$ ; and vanishes as  $\alpha \rightarrow \pi/2$ . Velocity  $V = \dot{\varphi}_{ijk}$ . Note that Figs (a),(b), and (c) are for positive "viscosity", (d) - for negative.



**Figure 31.** 3-D chimera states: (a) four spiral streaks with incoherent core, (b) incoherent ball, (c) incoherent tube, (d) 16 incoherent spiral streaks. Average velocities  $\bar{V}$  shown by color gamma are higher inside the chimera regions (red and white colors). Calculations up to  $T = 10000$  time units, average velocities for  $\Delta T = 100$ ,  $N = 50$ .

We find that 3-D chimera states exist in a wide regions of the parameter space  $(R, \alpha)$ , where they co-exist with other space-time regimes (regular or/and turbulent) as it is typical for laminar-turbulent transition in fluids. Cascades of 3-D chimera states with increasing number of streaks and other chimera types are obtained in analogy with 1-D case [3]. The massive calculations were performed at the computer cluster "CHIMERA" (<http://nll.biomed.kiev.ua/cluster>).

The contribution comprises video presentation illustrating the origin and evolution of the 3-D chimera states.

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## CHIMERA STATE: A NEW SCENARIO FOR LAMINAR-TURBULENT COEXISTENCE?

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Chimera state is a recently discovered phenomenon that displays spatially-temporal disorder (“turbulence”) surrounded by coherent (“laminar”) flow. Intriguingly, this fascinating behavior develops robustly in homogeneous media or oscillatory networks as a spontaneous symmetry breaking caused by pure internal nonlinear interactions. Chimera states emerge in wide domains of system parameters close to the transition between coherence and incoherence. In many cases, they co-exist with fully coherent or rotating flows (developed from other initial conditions).

Chimera states were found twelve years ago for complex Ginzburg-Landau equation and its phase approximation known as Kuramoto model. Currently, this is an area of intense theoretical research. Recently, an experimental evidence of the chimera state has been proven for optical, chemical, mechanical, and optoelectronic systems.

In this talk, I will give an overview of some features of chimera state such as chaotic wandering, Lyapunov spectrum, cascades of multi-headed chimeras with decreasing length scales etc. In two-dimensions, chimera states are obtained in the form of incoherent or coherent spots and strips, also spirals with incoherent core. Finally, I will present the first observation of three-dimensional chimera states: spirally-rotating streaks with incoherent core, incoherent ball within a regular flow, and others.

The purpose of this talk is to discuss with the audience, if chimera state can serve as a prototype of laminar-turbulent patterns in fluids? In particular, how could one relate the Kuramoto model parameters, coupling range and interaction phase shift, with Reynolds number and viscosity in fluids?

## SUBCRITICAL TURBULENCE IN TWO-DIMENSIONAL MAGNETOHYDRODYNAMIC PLANE SHEAR FLOWS – SELF-SUSTENANCE VIA INTERPLAY OF LINEAR TRANSIENT GROWTH AND NONLINEAR TRANSVERSE CASCADE

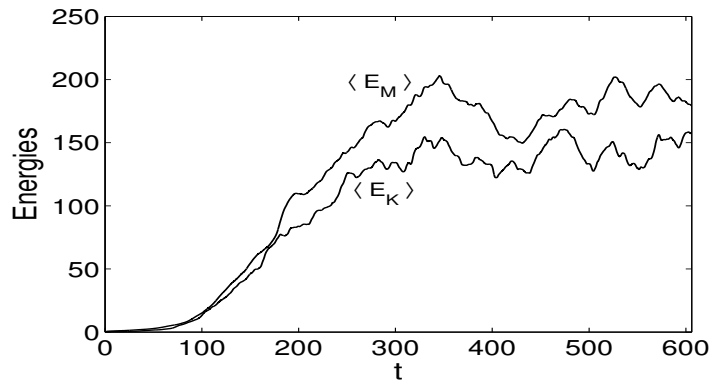
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The direct nonlinear cascade – a central process in Kolmogorov’s phenomenology – is a consequence of the existence of, so-called, inertial range in spectral/wavenumber space ( $\mathbf{k}$ -space), which is free from the action of linear energy-exchange processes and occupied by nonlinear cascades only. In the Kolmogorov’s scheme, long wavelength perturbations grow exponentially, which are being transferred by the direct cascade, through the inertial range, to shorter wavelengths and ultimately to the dissipation region. In this course of events, the direct cascade together with unstable and dissipative linear phenomena constitute the well-known scheme of forced turbulence. The concept of the direct cascade took root in the turbulence theory and acquired dogmatic coloring. However, in spectrally stable shear flows, in which transient growth of perturbations is the only possibility, the balance of processes leading to the self-sustenance of turbulence should be completely different. The shear-induced transient growth mainly depends on the orientation (and, to a lesser degree, on the value) of wavevector of perturbations: the spatial Fourier harmonics (SFHs) of perturbations having a certain orientation of the wavevector with respect to shear flow, draw flow energy and get amplified, whereas harmonics having another orientation of the wavevector give energy back to the flow and decay. In other words, the linear energy-exchange processes in hydrodynamic (HD) shear flows are strongly anisotropic in  $\mathbf{k}$ -space, cover an entire space without leaving a free room (i.e., inertial range) for the action of nonlinear processes only, unlike that in Kolmogorov’s phenomenology. The strong anisotropy of the linear processes, in turn, leads to anisotropy of nonlinear processes in  $\mathbf{k}$ -space. In this case, as shown in Ref. [1], even in the simplest HD shear flow with linear shear, the dominant nonlinear process turns out to be not a direct, but a *transverse cascade*, that is, transverse (angular) redistribution of perturbation harmonics in  $\mathbf{k}$ -space.



**Figure 32.** Evolution of the domain-averaged perturbed kinetic,  $\langle E_K \rangle$ , and magnetic,  $\langle E_M \rangle$ , energies. In the beginning, they steadily grow as a result of shear-induced transient amplification of separate SFHs. Then, at about  $t = 250S^{-1}$ , the amplification saturates to a quasi-steady turbulent state that persists till the end of the run. The magnetic energy is a bit larger than the kinetic one.

We present the results of our direct numerical simulations (DNS) [2], which demonstrate the dominance of the nonlinear transverse cascade of spectral energy in magnetohydrodynamic (MHD) shear flows too. Specifically, we consider the dynamics of two-dimensional perturbations in an incompressible MHD flow with a constant/linear shear of velocity,  $\mathbf{U}_0 = (0, Sx)$ , and uniform magnetic field,  $\mathbf{B}_0 = (0, B_{0y})$ , parallel to it. Our goals are: (i) to investigate subcritical transition to turbulence by DNS in this magnetized flow, (ii) to describe the general behavior of nonlinear processes – transverse cascade – in wavenumber plane that appears to be a keystone of the turbulence’s self-sustaining dynamics in this simple open system. The DNS were performed using the spectral SNOOPY code [3]. White noise perturbations of velocity and magnetic field with rms amplitudes 0.84 (in Alfvén speed,  $v_A$ , units) are applied initially; the subsequent time-development is shown in Fig. 1. In our simulations, we take the domain size  $L_x = L_y = 400$  (in Alfvén lengths,  $v_A S^{-1}$ , units) and resolution  $N_x \times N_y = 512 \times 512$ . These parameters ensure the presence of harmonics in the simulation domain that can undergo efficient transient growth.

To understand the sustaining mechanism of the turbulence, we Fourier transformed basic MHD equations and derived

evolution equations for the perturbed kinetic and magnetic spectral energies in  $\mathbf{k}$ -plane. In these spectral equations, using the simulation results, we calculated individual terms, which are divided into two types – terms of linear and nonlinear origin. The terms of linear origin – the Maxwell and Reynolds stresses – are responsible for energy exchange between the turbulence and the mean flow through transient amplification of perturbation harmonics due to shear. However, as we showed, only the positive Maxwell stress appears to be a dominant (magnetic) energy injector for the turbulence; it is much larger than the Reynolds stress, which has a negative sign and therefore cannot contribute to turbulent energy gain. Another linear term due to shear in these equations makes the spectral energies drift in the spectral plane parallel to the  $k_x$ -axis. The nonlinear terms, which do not directly draw the mean flow energy, act to redistribute this energy in spectral plane so as to continually repopulate perturbation harmonics that can undergo transient growth. Thus, we demonstrated that in spectrally stable shear flows, the subcritical MHD turbulent state is sustained by the interplay of linear and nonlinear processes – the first supplies energy for turbulence via shear-induced transient growth mechanism of magnetic field perturbations (characterized by the Maxwell stresses) and the second plays an important role of providing a positive feedback that makes this transient growth process persist over long times and compensate for high- $k$  dissipation due to viscosity and resistivity. This balance/cooperation of energy injecting linear and redistributing nonlinear transfer terms relies on their anisotropy (i.e., dependence on wavevector angle) in  $\mathbf{k}$ -plane. The obtained energy spectra are, consequently, anisotropic in  $\mathbf{k}$ -plane. The studied self-sustenance scheme is consistent with the *bypass* concept of subcritical turbulence in spectrally stable shear flows and differs fundamentally from the existing concepts of (anisotropic direct and inverse) cascade processes in MHD shear flows.

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## MODELING TRANSITIONAL PLANE COUETTE FLOW: WHY AND HOW?

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Understanding the abrupt transition to/from turbulence in wall-bounded flows remains a challenge that can be tackled by turning from Navier–Stokes equations (NSE) to supposedly enlightening simplified systems called models. In the transitional range around the global stability threshold (below which turbulent flow unconditionally return to laminar in the long time limit) such models have to account for (i) the transient character of chaos at the local scale [1] and (ii) the spatially extended character of the flow in all situations of experimental interest. Two broad types of model can be considered, either analogical and conceptual – aiming at a qualitative insight in the phenomena at stake – or derived from primitive equations, with the ambition of more quantitative realism besides mechanistic explanations [2].

The recognition of the subcritical character of the change of regime straightforwardly leads from Landau amplitude equations to Ginzburg–Landau (GL) envelope formulations with cubic and quintic nonlinear terms able to deal with the coexistence of laminar and turbulent flow in separate domains. This was early suggested by Pomeau [3] who, in addition, fostered the concept of spatiotemporal intermittency and the relevance of directed percolation to account for the chaotic nature of the flow within turbulent domains. The latter statistical-physics minded approach can be developed using coupled maps [4]. Interesting developments along these two paths include the study of the self-limitation of the width of the turbulent spiral regime of the cylindrical Couette–Taylor system using a GL model with nonlocal feedback by Hayot & Pomeau [5] and the statistical approach to the nucleation of laminar domains within featureless turbulence in plane Couette flow by Bottin & Chaté [6]. Limitations of the latter works clearly need to be overcome by going beyond analogy and having more input from hydrodynamic equations.

As shown by Waleffe [7], Galerkin expansions offer a neat framework to analyze couplings between the different ingredients of processes sustaining turbulence away from laminar flow. They typically yield ordinary differential equations governing the amplitudes of a few relevant elementary structures into which the flow pattern can be decomposed. This low-dimensional phase-space approach can be extended to account for the high-dimensional dynamics developing in physical space of interest in applications. When applied to NSE, the procedure leads to sets of partial differential equations governing the space dependence of the amplitudes introduced above. The so-obtained models can be studied for themselves, analytically and numerically. Having extracted the mechanisms at stake from the primitive equations, they can also serve as the starting point of further reductions allowing one to "understand" large-scale features such as spatiotemporal intermittency, turbulent plugs/spots growth/decay, or patterning. In this spirit, Barkley's approach of transitional pipe flow [8] builds a kind of bridge toward the deductive approach: He introduced phenomenological coupled-map lattices or stochastic partial differential equations in one dimension. By-passing the projection stage inherent in the Galerkin method he constructed the local dynamics of two coupled well-chosen variables after a careful scrutiny of the mechanisms controlling the destiny of turbulent puffs.

Extending previous work with Lagha [9], I shall present a Galerkin model of plane Couette flow directly derived from NSE. The obtained set of partial differential equations explicitly introduces dependence on streamwise and spanwise coordinates, and time. It governs the amplitudes of modes with polynomial wall-normal profiles adapted to the no-slip boundary conditions. Numerical simulations of the model reproduce the pattern of alternately laminar and turbulent bands displayed by this flow on its way to turbulence [10]. Work in progress involves the separation of the local and pattern scales and an adiabatic reduction to slow variables. It intends to give microscopic support to the reaction-diffusion approach to the transitional range promoted by Barkley [8] and more specifically to the interpretation of patterning in terms of a Turing instability of the featureless regime proposed earlier [11].

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## TAYLOR-COUETTE HIGHLY SUBCRITICAL SOLUTIONS

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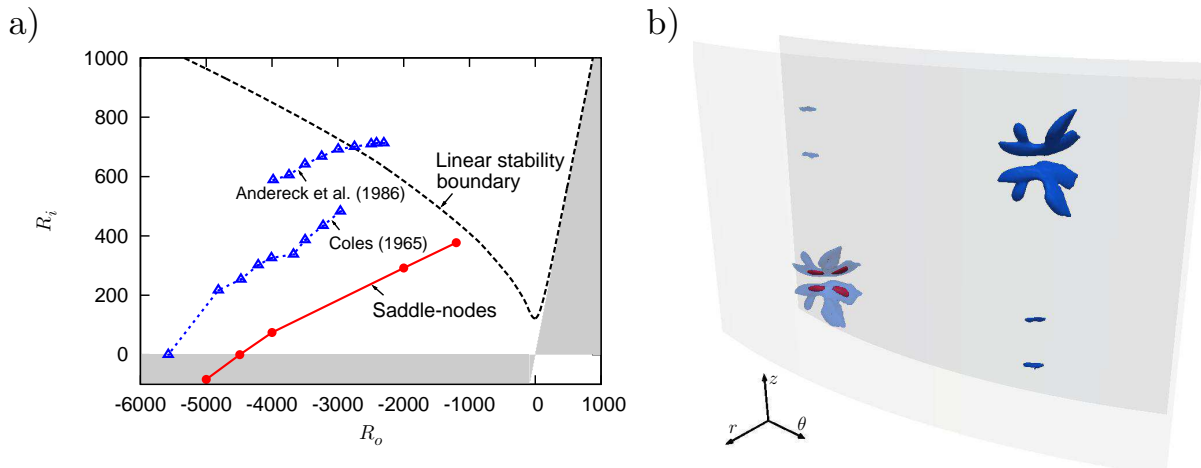
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We present a family of highly subcritical rotating waves in counter-rotating Taylor-Couette flow with an aspect ratio of  $\eta = 0.883$ . These nonlinear states are characterized by strongly localized vortex pairs (Fig. 40b) and their region of existence is consistent with the critical threshold for relaminarization observed in the classical experiments by Andereck et al [1] and Coles [2] (Fig. 40a). For sufficiently rapid outer cylinder rotation the solutions extend beyond the static inner cylinder case to co-rotation, thus exceeding the boundary defined by the inviscid Rayleigh's stability criterion.

These solutions, driven by a centrifugal instability at onset, where they connect with Taylor vortices, are purely shear driven as they approach the Rayleigh stable regime. Their high subcriticality suggests that they may comprise the backbone of the chaotic attractor set that is responsible for subcritical turbulence sustainment.



**Figure 33.** a) Stability diagram of Taylor-Couette flow for  $\eta = 0.883$ . Subcritical turbulence experimental thresholds (blue) and saddle node location of new solutions (red) are also shown. b) Vortical structure of highly subcritical solution at  $(R_i, R_o) = (0, 6600)$ .

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## SUBCRITICAL TRANSITION TO TURBULENCE IN TAYLOR-COUETTE FLOW FOR VANISHING INNER CYLINDER

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Turbulent structures in concentric rotating Taylor-Couette flow (TC) and its dependency on different parameters form the scope of this investigation. Depending on the rotation rate of the cylinders, one is able to have super critical transition to turbulence or sub critical transition. Eckhardt et al. [1] pointed recent analogies between the Taylor-Couette flow, Rayleigh-Bénard flow as well as pipe flow. The analogies in Nusselt number scaling with Rayleigh Bénard has been already studied (c.f. [2]). Subcritical transition in Taylor-Couette flow is mainly investigated for narrow gaps (radius ratio of inner and outer cylinder  $R_1/R_2 \rightarrow 1$ ), where the analogy to plane shear flows becomes more obvious [3]. In the present work we study the case where the inner cylinder vanishes ( $(R_1/R_2 \rightarrow 0)$  experimentally. When the inner cylinder vanishes completely the flow remains as a solid body rotation. For the case of small cylinders rotating at different speed as the outer cylinder a shear comes into account and disturbs the solid body rotation. Doing this with a small radius ratio ( $R_1/R_2 = 2/7$ ) for counter rotating cylinders one can observe small turbulent structures transferred to the outer cylinder. These turbulent structures than can grow to larger packages of turbulence or they decay.

Thus we use a Taylor-Couette experiment with inner cylinders of small radius. The smallest inner cylinder wich is technical possible in our case has the radius ratio of  $R_1/R_2 = 0.1$ . By visualisation using Kaliroscope and dye in water we can observe the turbulent states and capture time series using a camera. Processing the visualisation time series than can be used to determine the type of structures as well as the behaviour of growth and decay. Additional Laser Doppler Velocimetry should give the fluid velocity time signals to get a deeper understanding of the structures.

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## OPEN SHEAR FLOW ARCHETYPAL STREAMWISE LOCALIZATION

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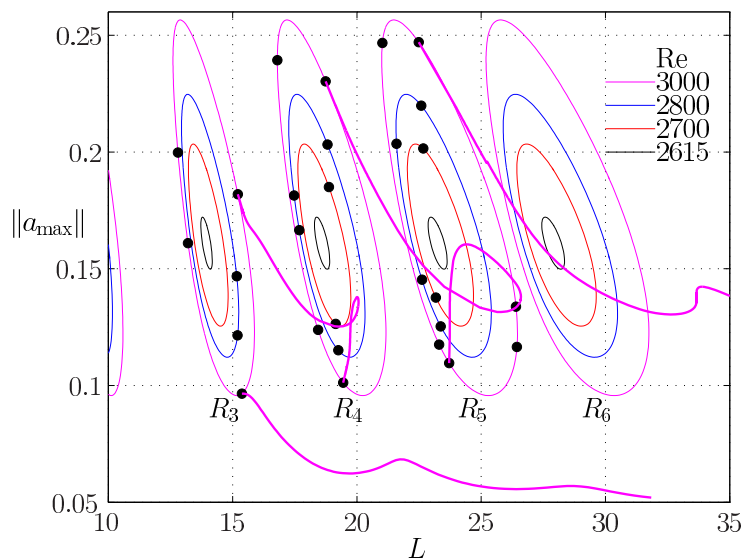
Streamwise localized subcritical turbulence is a generic phenomenon in open shear flows. Understanding the mechanisms underlying this type of pattern formation requires a deterministic study of the topological features of the phase space corresponding to the Navier-Stokes dynamical system associated with the shear flow under exploration.

A first approach to this problem was addressed in [1] by studying subharmonic linear instabilities in two-dimensional plane Poiseuille flow. In the aforementioned work, it was already suggested that localized waves may emerge from subharmonic Hopf bifurcations of Tollmien-Schlichting travelling waves (TSW). More recently, numerical studies in pipe flow have reported the mechanism of streamwise localization as a result of secondary subharmonic bifurcations from travelling waves [2].

In this work we follow similar lines of the aforementioned studies in two-dimensional plane Poiseuille flow (PPF), which allows a more comprehensive numerical exploration of localized modulated waves (LMW). The study is carried out within a range of subcritical Reynolds numbers ( $Re \leq 3000$ ) where the basic Poiseuille profile is still linearly stable. In this study, linear stability analyses of the TSW have been carried out from 2 up to 5 replications of the basic TSW wavelength. Newton-Krylov-Poincaré methods, embedded within arclength continuation schemes, have been devised to identify and track LMW in the  $(L, Re)$  parameter space, being  $L = 2\pi R/k_0$  the length of the computational box,  $k_0$  the fundamental TSW wavenumber and  $R \in [1, 5]$  the replication factor.

Different families of LMW have been identified. In some cases, these modulated waves do emerge from Hopf bifurcations of the TSW, but they also may remain disconnected from them, forming isolas. The ones that emerge from TSW may also have different fates. In some cases they depart from a  $R_k$  replicated TSW to eventually reconnect with its  $R_{k\pm 1}$  counterpart. These connections are found to be quite intricate sometimes and no simple mechanisms of localization and de-localization have yet been studied in detail. Inner connections, in which the LMW connects upper and lower branches of the same  $R_k$ -TW have also been identified.

The figure below shows the norm of replicated Tollmien-Schlichting TW (ellipses) for different aspect ratio boxes. The black dots are the location of the subharmonic Hopf bifurcations (saddle-node points are omitted). Some of the LMW connecting different branches are depicted (thicker curves) for  $Re = 3000$ .



**Figure 34.** Maximum 2-norm of Tollmien-Schlichting TW (ellipses) and LMW solutions (thicker curves).

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## A SIMPLE CARICATURE PDE MODEL OF THE TRANSITION TO TURBULENCE

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We present a simple model based on two coupled PDE's that is constructed to exhibit a subcritical instability to a chaotic state. We show that its phenomenology is in many aspects similar to the observed features of the transition to turbulence in parallel shear flows. We demonstrate the existence of exact coherent states, spontaneous relaminarisation and double-exponential-like behaviour of the turbulent lifetimes. We show that typical dynamics in extended domains in this model exhibits frequent relaminarisation attempts and we present a mechanistic explanation for the dynamics during such events. Based on our observations, we show that it can be explained by a non-equilibrium statistical mechanics-type model that belongs to the directed percolation universality class but is different from what was proposed in the literature. We make connection between our scenario and the recently observed 'hibernating' turbulence.

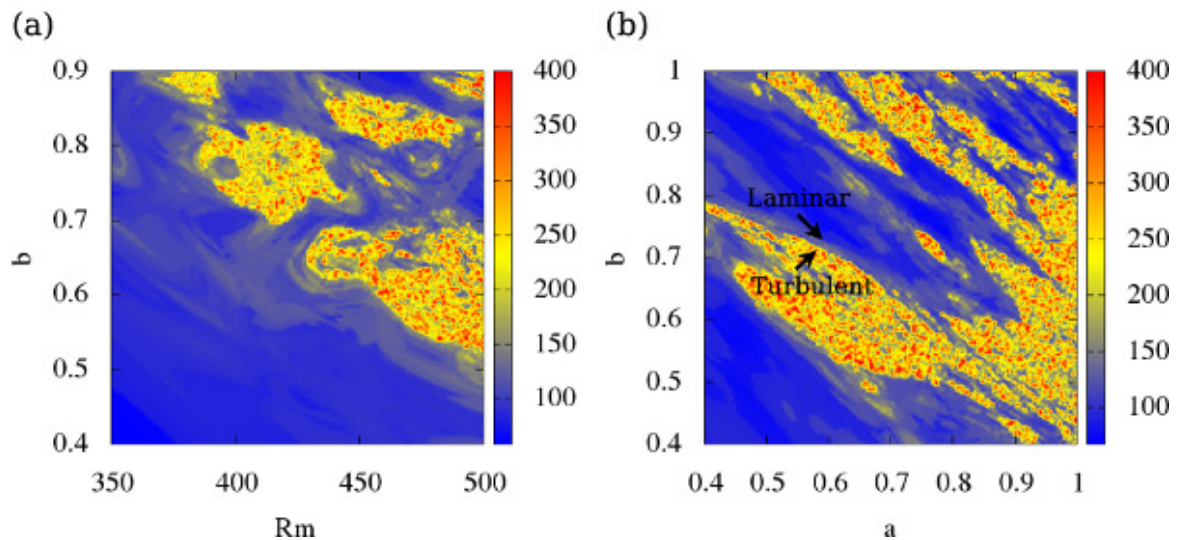


## EDGE OF CHAOS IN KEPLERIAN MHD DISKS

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Turbulent motion due to magnetorotational instability (MRI) is believed to be responsible for the efficient angular momentum transport and the consequent accretion of matter to a central body in accretion disks. However, a recent study by Rempel et al. [1] showed that MHD turbulence triggered by MRI in a three-dimensional shearing box model is a transient phenomenon, with turbulent states always decaying back to the laminar state. Thus, an important problem to be solved in magnetized accretion disks is how the turbulence becomes stable. In the last years, numerical and experimental evidence in neutral fluids indicates that the boundary separating laminar from turbulent motions in the phase space, called edge of chaos, plays a fundamental role in the relaminarization process. In the present work, these concepts have been applied to study transition to MHD turbulence in accretion disks using a shearing box numerical model [2] for the configuration of [1] and also for the restricted problem studied in Ref. [3].

Numerical results show that the size and fractality of the turbulent region in the phase space increase with the magnetic Reynolds number  $Rm$ , as shown in Fig. 35(a), in agreement with previous studies showing a type-II supertransient law [4] for the turbulence lifetime. We study the lifetime function for a fixed  $Rm$ , generating a two dimensional projection of the phase space using a linear combination of two turbulent initial conditions. Figure 35(b) shows the characteristic separation between regions with short and long lifetimes in the phase space, related to laminar and turbulent behavior, respectively. The unstable solution which lies at the laminar-turbulent boundary in the phase space, the edge state, is found by using a tracking method, and its properties are discussed.



**Figure 35.** (a) Lifetime of initial conditions as a function of  $Rm$ . (b) Lifetime function in a two dimensional projection of the phase space, fixing  $Rm = 500$ .

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## VISCO-ELASTIC INSTABILITY IN DIFFERENTIALLY COROTATING COUETTE-TAYLOR SYSTEM

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Transition to turbulence in viscoelastic flows remains one of unsolved problems in Hydrodynamics despite their many industrial applications (drag reduction, food or cosmetics, etc.). The understanding of viscoelastic flows, their predictability and control, are mainly hindered by the lack of universal equations governing them. While the Navier-Stokes equations reproduce most of the flow features for Newtonian fluids; for viscoelastic flows the momentum balance equation has to be complemented with constitutive equations (for elastic stresses), for which only approximate models, such as Oldroyd-B or Giesekus models, apply to particular solutions (constant viscosity or shear-thinning solutions). Many studies have been achieved in last decades in the viscoelastic solutions in the Couette-Taylor system (i.e. a flow between two coaxial differentially rotating cylinders). New types of modes were discovered : inertio-elastic, elastic instability and elastic turbulence. A special attention has focused on the case when the inner cylinder was rotated while the outer was fixed [1, 2, 3, 4, 5]. Our investigation of the viscoelastic instability (VEI) in the corotating Couette-Taylor system is motivated by the prediction of Ogilvie [5, 6] that such an instability is analogous to the MRI (magneto-rotational instability) which is believed to play a key role in the angular momentum transport in accretion disks and in planetary dynamos. This analogy is supported by stretched spring argument developed in [7] which is similar to that of the polymer stretching model in viscoelastic solutions [8]. In fact the polymer stress tensor  $\mathbf{T}_p = \mathbf{T} + \mu_p \mathbf{1}/\tau$  and the magnetic stress tensor  $\mathbf{T}_m = \tilde{\mathbf{B}}\tilde{\mathbf{B}}/\mu_0$  are described by similar equations :

$$\partial_t \mathbf{T}_p + (\mathbf{u} \cdot \nabla) \mathbf{T}_p - (\nabla \mathbf{u})^t \cdot \mathbf{T}_p - \mathbf{T}_p \cdot \nabla \mathbf{u} = -\frac{1}{\tau} (\mathbf{T}_p - \frac{\mu_p}{\tau} \mathbf{1}) \quad (4)$$

$$\partial_t \mathbf{T}_m + (\mathbf{u} \cdot \nabla) \mathbf{T}_m - (\nabla \mathbf{u})^t \cdot \mathbf{T}_m - \mathbf{T}_m \cdot \nabla \mathbf{u} = (\lambda/\mu_0) \left[ \tilde{\mathbf{B}} \nabla^2 \tilde{\mathbf{B}} + (\nabla^2 \tilde{\mathbf{B}}) \tilde{\mathbf{B}} \right] \quad (5)$$

where  $\mathbf{T}$  and  $\tilde{\mathbf{B}}$  represent the liquid stress tensor and the magnetic field respectively,  $\mu_0$  is the vacuum permeability,  $\mu_p$  is the polymer viscosity,  $\tau$  is the polymer relaxation time and  $\lambda$  is the magnetic viscosity. These equations show that in the limits of large relaxation time and small magnetic viscosity (i.e.  $\tau \rightarrow \infty$  and  $\lambda \rightarrow 0$ ) corresponding to the large values of Weissenberg number  $Wi = \tau \dot{\gamma}$  and magnetic Reynolds  $Re_m = \dot{\gamma} d^2 / \lambda$ , the polymeric and magnetic stress tensors satisfy similar equations. This similarity was discussed in [5, 6] to favor the analogy between the MRI and the VEI. To our best knowledge, only one experiment for the search of the analogy VEI-MRI has been reported [9]. We will present both theoretical and experimental results obtained in the viscoelastic Couette-Taylor system when both the cylinders are constrained to rotate along the Keplerian line. The polymer solutions are prepared from an aqueous mixture of PEG of viscosity  $\mu_s$  in which polyoxyethylene polymer powder has been dissolved for a longtime. The resulting solution has a constant viscosity  $\mu = \mu_s + \mu_p$  and its rheology can be described by the Oldroyd-B model. The control parameters are the aspect ratio  $\Gamma$ , the radius ratio  $\eta$ , the Reynolds number  $Re$ , the elastic number  $E = Wi/Re$  and the viscosity ratio  $S = \mu_p/\mu$ . Linear stability analysis for infinite aspect ratio has shown that critical modes are oscillatory and non-axisymmetric for all covered values of parameters. Within the experimental precision, the observed modes are subcritical and time dependent modes. Moreover, in case of rotating only the outer cylinder, we have observed subcritical elastic modes.

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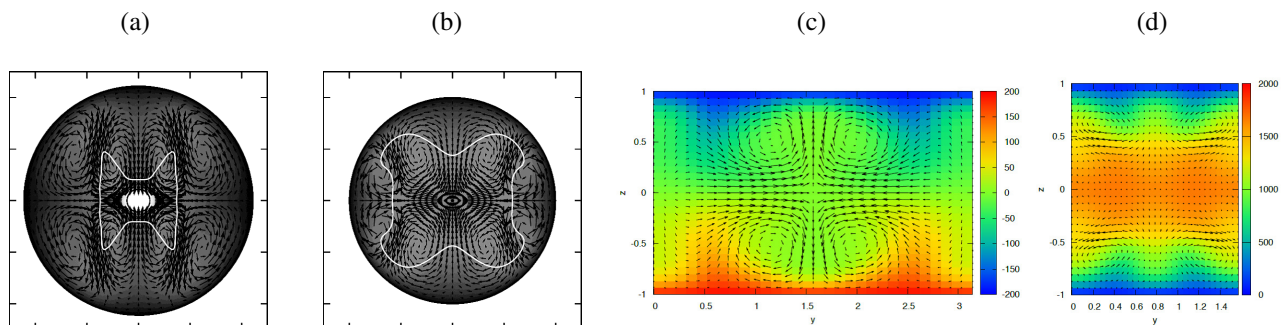
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## SYMMETRY CLASSIFICATION OF EXACT COHERENT STRUCTURES IN SHEAR FLOWS

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For the past two decades invariant sets of Navier-Stokes equations, often called exact coherent structures, have played important roles in understanding subcritical transition from laminar state to turbulence. Several exact coherent structures for various shear flows are known to date and it would be of interest to examine connections among them in terms of their symmetries and to investigate common flow structures that appear during transitional states. For this aim we first consider annular Poiseuille-Couette flow because it could serve as an intermediary between two canonical shear flows without linear instabilities, namely plane Couette flow and pipe flow. Being linearly stable for all Reynolds numbers they are bound to undergo subcritical transition. Although plane Couette flow and pipe flow are very different geometrically, annular Poiseuille-Couette flow recovers plane Couette flow by taking the narrow gap limit and also pipe flow by taking the limit of vanishing inner cylinder though the regularity condition at the centre must be taken care of.

Recently, by using the steady three-dimensional plane Couette flow solution, which is often referred to as Nagata solution [1], as a seed, a homotopy continuation with respect to the radius ratio of the cylinders has successfully been conducted [2] to obtain non-axisymmetric solutions in sliding Couette flow, i.e. annular Poiseuille-Couette flow without axial pressure gradient, via transformation from plane geometry. In the course of the homotopy continuation, solutions possessing a double-layered mirror-symmetric flow structure are found to bifurcate from those non-axisymmetric solutions continued from Nagata solution. These double-layered mirror-symmetric solutions in sliding Couette flow are traced back to the previously reported mirror-symmetric solutions [2], [3] in plane Couette flow. Applying further homotopy continuations which concern the adjustment of the axial velocity to a parabolic profile and a smooth change of the basis functions from no-slip to regularity conditions at the centre, we demonstrate that only the double-layered mirror-symmetric solutions in annular Poiseuille flow, as shown in Fig.1(a), successfully reach the pipe flow limit, reproducing the double-layered mirror-symmetric solution classified previously as M1 in pipe flow [4] (see Fig.1(b)). Also, we present a study on the symmetry connection of invariant sets between plane Couette flow and plane Poiseuille flow and demonstrate the existence of double-layered mirror-symmetric solutions in plane Poiseuille flow [6] (see Fig.1(c, d)). The double-layered mirror-symmetric exact coherent structures existing in shear flows in common may play an active role in subcritical transition to turbulence [7].



**Figure 36.** Cross-sectional flow field with a double-layered mirror-symmetric structure. (a): Annular Poiseuille flow. (b): Pipe flow. (c): Plane Couette flow. (d): Plane Poiseuille flow. The gray and colour scales represent the streamwise velocity component.

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## SUBCRITICAL TRANSITION STUDIES IN COLA-PIPE

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Turbulent pipe flow has been investigated at different Re-numbers as a traditional example of a shear flow that is linearly stable. For a subcritical transition some perturbations are needed to trigger turbulence. Turbulence can be achieved in natural or artificial methods and some structures called puffs and slugs can be observed. The first most detailed study of transition in a turbulent pipe was performed by Wygnanski and Champagne (1973) [1] in the Re-number range 1000-50 000. The hairpin vortices are also observable in wall bounded flows. Adrian (2007) [2] reports about this vortex packets at different conditions. In the outer layer the so called large-scale motions (LSM) are composed of detached eddies with wide range of azimuthal scales. The very-large-scale motions (VLSM) have radial scales, are concentrated around a single azimuthal mode and make a smaller angle with the wall compared to the LSM. These results are obtained by using hot wire measurements.

These phenomenas will be investigated at high Reynolds numbers in the pipe facility CoLa-Pipe (Cottbus-Large Pipe), which provides a Reynolds number of  $Re_m \leq 1.5 \times 10^6$ . Zimmer et. al. (2011) [2] and König et. al. (2013) [3] provide us an outline for conditions of fully developed turbulent flow state with natural and artificial transition. Considering these fully developed flow conditions at CoLa-Pipe, next investigations will be primarily focused on the hairpin structures in boundary layer in terms of LSM and VLSM by using hot wire anemometry. Additionally PIV-measurements will be performed in order to identify these structures optically. The main purpose of this work will be comparing the structure types of high Re-numbers with those of low Re-numbers in terms of subcritical transition.

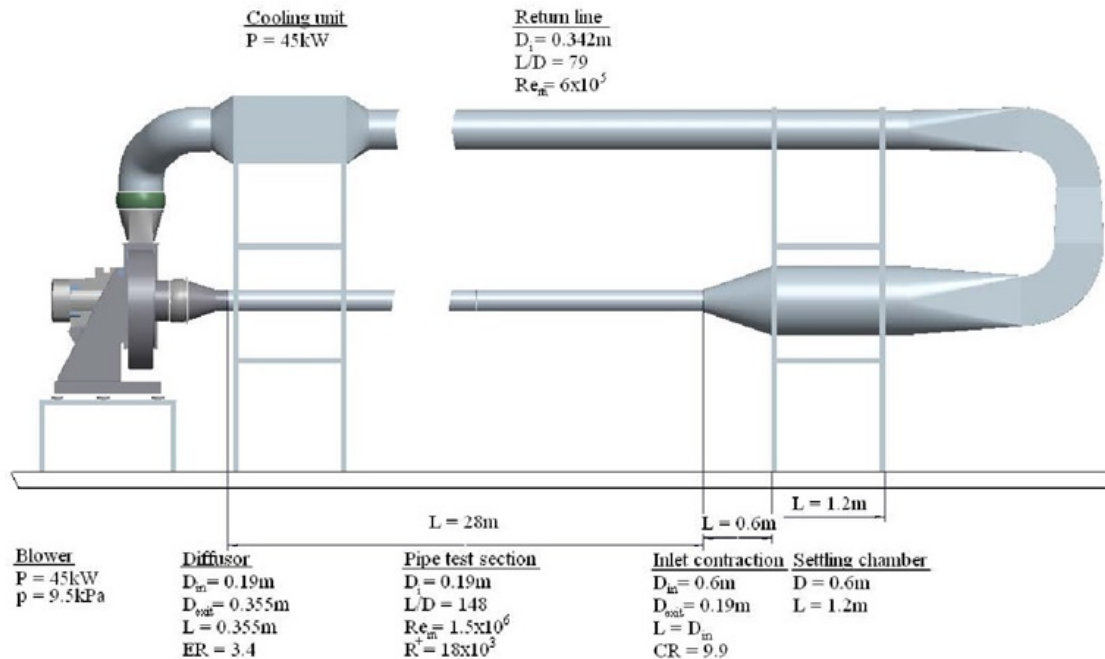


Figure 37. CoLa-Pipe Facility in BTU

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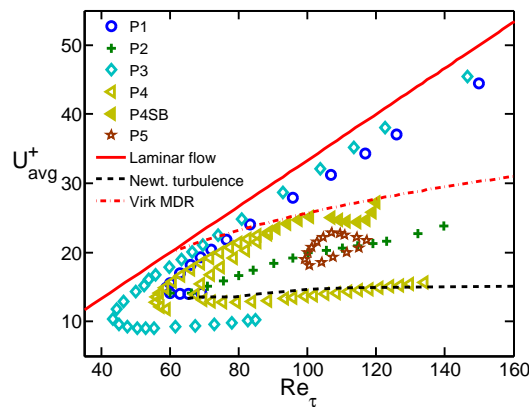
## NEW CLASSES OF EXACT COHERENT STATES IN PLANE POISEUILLE FLOW

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Five new families of nonlinear traveling wave solutions to the Navier-Stoke equations in plane Poiseuille flow are presented. A Prandtl-von Kármán plot is used to characterize the solutions as shown in Figure 55. We also compare our solutions to laminar, Newtonian turbulence and the so-called maximum drag reduction (MDR) asymptote. This is the asymptotic upper limit of turbulent drag reduction by polymer additives identified by Virk [1]. Two solution families, P1 and P3, display a possible connection to Waleffe's solution [2]. One solution family (P4) shows very intriguing behavior: its lower and upper branches appear to approach the MDR asymptote and the classical Newtonian profile, respectively, both in terms of bulk velocity and mean velocity profile. On the lower solution branch, a subharmonic bifurcation arises, giving rise to period doubling. Various symmetries and streak modes are found in the solutions, leading to different vortical structures. The spatial structures of streamwise velocity fluctuations are presented along with a critical layer, where the local streamwise velocity matches the wave speed. While some of the solutions clearly display critical layer dynamics, in others this connection is not as clear. Lastly, dynamical trajectories on the unstable manifold of the solutions are computed. Some solutions of two families are shown to lie on the basin boundary between laminar and turbulent flows and are thus edge states of the flow.



**Figure 38.** A Prandtl-von Kármán plot for traveling wave solutions in plane Poiseuille flow with curves for laminar, Newtonian turbulence and the maximum drag reduction. On the lower branch of P4, a subharmonic bifurcation (SB) arises around  $Re_\tau \approx 90$ , indicated by solid symbols.

### ACKNOWLEDGEMENTS

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## LAMINAR-TURBULENT TRANSITION IN THE FLOW THROUGH A GRADUAL EXPANSION IN A CIRCULAR PIPE

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We present the results of a combined experimental and numerical investigation of the transition to turbulence in a slowly diverging pipe, *i.e.* a gradual expansion in a circular pipe. The science of sudden expansion flow has been studied [1, 2] and indicate a very different transition scenario from circular pipe flow. Initially, the inlet velocity profile is parabolic along with the formation of axisymmetric recirculation region in the divergent section. There is a critical velocity for the existence of the recirculation. When the flow increases further, a subcritical transition for localised turbulence arises. The experiments describe the growth of the turbulent patch. These turbulent patches are reminiscent of turbulent puffs [2] in uniform pipe flow suggesting that the observed patches may contain solutions similar to those in pipe flow. In order to quantify further the localised turbulence regime, relaminarisation experiments were performed. The turbulent patches are generated, then the flow velocity is reduced and the decay to laminar flow is monitored. We observe a linear increase of the relaminarisation time. This will be discussed and reanalyzed. Moreover, a direct numerical simulation [3, 4] is carried out to study the onset of localised turbulence and the amplitude of perturbation required to trigger them.



**Figure 39.** Numerical simulation results of the streamwise vorticity at  $Re = 2000$ . The diverging angle is  $45^\circ$ .

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## NONLINEAR DELAY OPTOELECTRONIC OSCILLATOR EXHIBITING LAMINAR-TURBULENT TRANSITION

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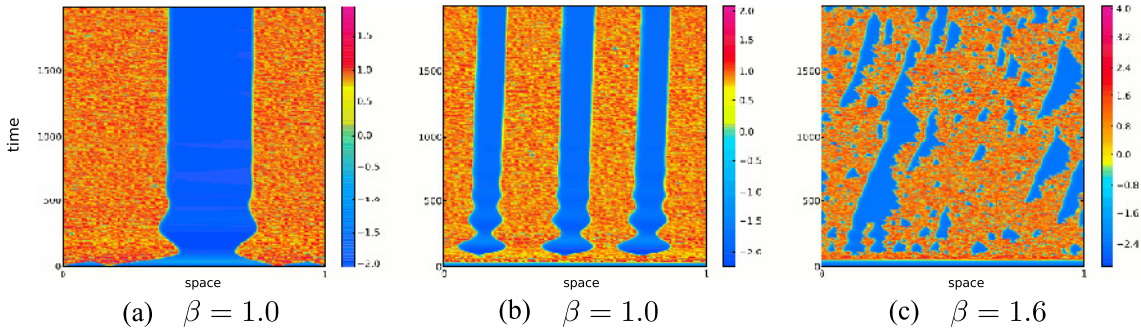
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Systems modeled by delay differential equations (DDE) may exhibit very complex behavior, because of their infinite dimensional phase space. Recently, a new class of nonlinear DDE involving a second order differential model, has attracted a growing interest. Their model is derived from physical optoelectronic setup [1, 2] and they also provide qualitatively new solutions compared to conventional scalar DDE (Ikeda or Mackey-Glass dynamics). An example of such solutions, so-called *virtual chimera states* discovered recently in [3], is emphasized in this contribution for a new optoelectronic setup. They are characterized by spontaneous symmetry breaking resembling laminar-turbulent flow when observed in a space-time representation of DDE [4]. The dynamical system of concern is governed by an Ikeda-like model comprising an integral term:

$$\varepsilon \dot{x} = -x - \delta \int_{t_0}^t x(\xi) d\xi + \beta F[x(t - \tau)], \quad (6)$$

where  $\varepsilon$  and  $\delta$  are typically small bifurcation parameters,  $\beta$  is the third one, and  $F$  is some function describing the nonlinear delayed feedback of the dynamics. In our optoelectronic setup,  $F$  is provided by the wavelength-to-intensity nonlinear transformation obtained from a Fabry-Pérot interferometer, i.e.  $F[x] = [1 + m \sin^2(x + \Phi_0)]^{-1}$ ,  $m = 8$ ,  $\Phi_0 = -0.4$ .

We report on the particular bifurcation scenario for the chimera states appearance as  $\beta$  increases, observed both numerically for the model (6) and experimentally for the setup, and compare the phenomenon with typical laminar-turbulent patterns in fluids. Figure 40 represents characteristic space-time plots of chimera state [(a) "one wall" and (b) "three



**Figure 40.** Bifurcation scenario for the model (6): from chimera states [(a) and (b) different initial conditions] to turbulent regime (c),  $\varepsilon = 5 \cdot 10^{-3}$ ,  $\delta = 1.59 \cdot 10^{-2}$ .

walls"] bifurcated into "turbulent flow" (c) as  $\beta$  increases. It is important to admit that chimeras, resembling laminar-turbulent patterns, coexist robustly with other space-time regimes. This kind of coexistence is typical for laminar-turbulent transition.

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## INFLUENCE OF A SHEAR-THINNING RHEOLOGY ON NONLINEAR WAVES IN PIPE FLOW

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In pipe flow of Newtonian fluids, [1, 2] discovered numerically ‘exact coherent structures’ that are nonlinear traveling waves. We focus here on the waves with a 3-fold rotational symmetry: if  $(r, \theta, z)$  are the cylindrical coordinates with  $z$  the axis of the pipe, the waves are invariant under  $\theta \mapsto \theta + 2\pi/3$ ; they are also invariant under  $z \mapsto z + 2\pi/q$  with  $q$  the axial wavenumber. The onset Reynolds number of these ‘FEWK waves’, at which they appear through a saddle-node bifurcation, is  $R = \overline{W}d/\nu = 1251$ , with  $\overline{W}$  the bulk velocity,  $d$  the pipe diameter,  $\nu$  the viscosity. This is a lower bound of the Reynolds numbers at which turbulence exists. For this reason and others, we see these waves as ‘precursors’ of turbulence. In non-Newtonian fluids, a delay for the onset of developed turbulence in pipes has been evidenced experimentally by several authors, e.g. [3]. Most non-Newtonian fluids are shear-thinning and viscoelastic. Here we focus on the influence of the shear-thinning effects, neglecting the elastic response of the fluid, which has been studied in the literature. We use the Carreau model for the viscosity

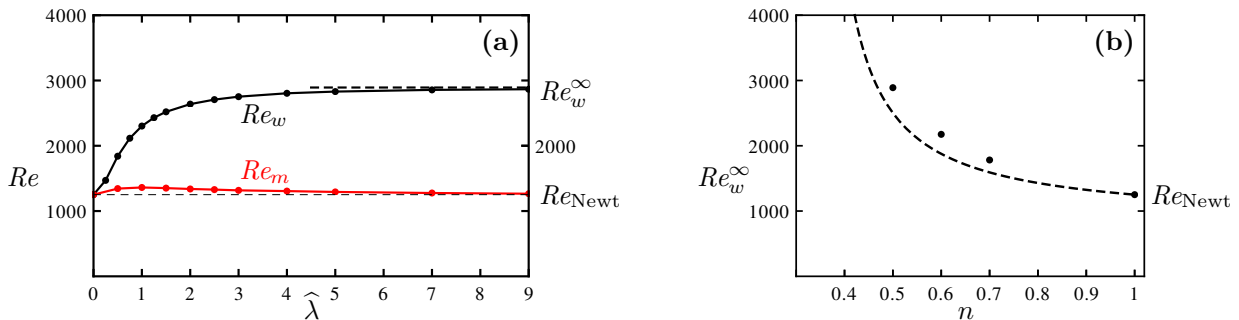
$$\nu = \nu_0 (1 + \lambda^2 D_2)^{(n-1)/2} \quad (7)$$

with  $\nu_0$  the viscosity at rest,  $\lambda$  the characteristic time of the fluid,  $n < 1$  the shear-thinning index,  $D_2$  the second invariant of the rate-of-strain tensor. This model is mathematically regular, but also approaches the simpler ‘power-law’ fluid model at large rate-of-strain or large values of  $\hat{\lambda} = \lambda/t_a$  with  $t_a = d/W_0$  the advection time,  $W_0$  being the centerline velocity of the laminar flow at the same mean pressure gradient. This limit  $\hat{\lambda} \rightarrow +\infty$  is the experimentally relevant one. Using the code introduced in [4], we study the evolution of the FEWK waves starting from the Newtonian case  $\hat{\lambda} = 0$  and increasing  $\hat{\lambda}$ , with continuation methods. The axial wavenumber  $q$  is always optimized. The onset Reynolds number  $Re_w$  of the waves, constructed, as in the experimental studies, using the wall viscosity  $\nu_w$ , increases with  $\hat{\lambda}$ , and converges to a limit  $Re_w^\infty$  as  $\hat{\lambda} \rightarrow +\infty$  (Fig. 1a). The Reynolds number  $Re_m$  constructed using the mean viscosity  $\nu_m$  of the critical wave is almost constant, which shows the relevance of this viscosity (Fig. 1a). By estimating  $\nu_w$  and  $\nu_m$  with the power-law fluid model solved for the laminar flow, we obtain an analytical formula for the onset Reynolds number at large  $\hat{\lambda}$ ,

$$Re_w^\infty = (\nu_m^\infty / \nu_w^\infty) Re_m^\infty \simeq (\nu_m^{pl} / \nu_w^{pl}) Re_{Newt} = 830 n (n - 1/3)^{-1}. \quad (8)$$

This law (the dashed line in Fig. 1b) turns out to be relevant for  $n = 0.5$  but also for larger values of  $n$ , as shown by the disks in Fig. 1b.

In conclusion, a delay of emergence of the FEWK waves has been found and characterized in shear-thinning fluids. This confirms the view that in non-Newtonian fluids coherent structures tend to be delayed, and is in line with some experimental results. Surprisingly, the delay can be estimated using simple laminar estimations of the viscosities.



**Figure 41. a :** For a shear-thinning index  $n = 0.5$ , onset Reynolds numbers of the waves, constructed on the wall (‘w’) or mean (‘m’) viscosity. **b :** For various values of  $n$ , wall-viscosity onset Reynolds number in the limit  $\hat{\lambda} \rightarrow +\infty$ .

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## SUBCRITICAL TRANSITIONS IN FLOWS AND ELSEWHERE: FROM REYNOLDS TO CATASTROPHE THEORY

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In the classical paper where the Reynolds number is introduced, the author reported experiments of transition to turbulence in a pipe flow and gave a hint that it is not due to a linear instability, something more or less forgotten afterwards. Nevertheless the study of linear stability of parallel flows was undertaken afterwards. In 1924 Werner Heisenberg did show in his PhD thesis - a masterpiece of WKB analysis - that plane Poiseuille is linearly unstable. Later on, in 1966 Iordanskii and Kulikovskii showed that this flow is convectively, not absolutely unstable. However this did not help much to understand the phenomenology of the transition in parallel flow. The coexistence of laminar and turbulent domains (turbulent flashes in the words of Reynolds) was left unexplained, although this coexistence can be stationary, in Taylor-Couette for instance. In a paper presented at a Conference at Los Alamos in 1985 [1], I showed that, in extended structures like parallel flows, subcritical transitions lead to the coexistence of domains with a different behaviour, turbulent and laminar for example, a bit like two different thermodynamic phases of the same substance, liquid and vapour can coexist. In this paper I developed various ideas connected to that. I did explain that in a finite range of parameters fronts separating two different "phases" are pinned on the underlying lattice if one of the phase is a steady periodic pattern and the other the laminar state, as observed in Benard-Marangoni thermal convection with hexagons. The transition from laminar to turbulent belongs to the class of directed percolation at the onset. If time permits I'll say a few words on the extension of this kind of idea to the theory of catastrophe and how they can be (sometimes) predicted. Catastrophes are fast transitions of finite amplitude in physical systems, with examples like supernova explosion and earthquakes.

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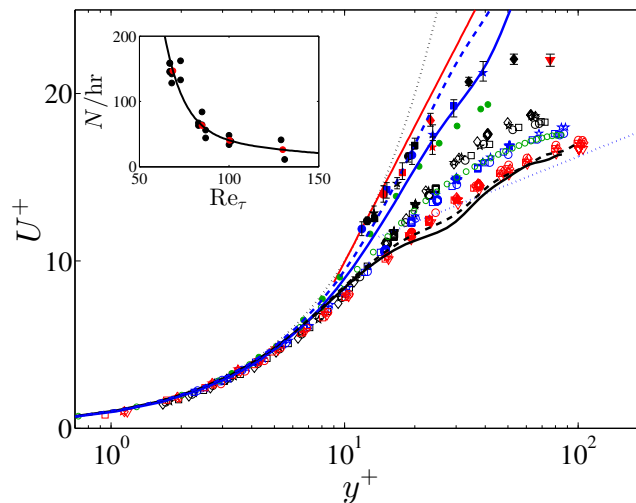
## HIBERNATING TURBULENCE IN NEWTONIAN FLUIDS AND DILUTE POLYMER SOLUTIONS

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The asymptotic upper limit of turbulent drag reduction with polymer additives – often termed Virk’s Maximum Drag Reduction (MDR) asymptote [1] – is a well-known phenomenon in the turbulent flow of complex fluids. One of the most intriguing features of MDR is its universality. Recent direct numerical simulations [2][2] have identified time intervals showing key features of MDR. These intervals have been termed “hibernating turbulence”. They are a weak turbulence state which is characterised by low wall-shear stress and extremely weak vortical flow structures. Here we report the results of an experimental investigation which shows that the streamwise velocity of a turbulent channel flow collapses to the MDR asymptote, even in the absence of a polymer additive, during intervals of hibernating turbulence. Our experiments are conducted in a fully developed turbulent channel flow of Newtonian fluids and semi-dilute polymer solutions, 300 ppm of a very high molecular weight polyacrylamide, at a Reynolds number of  $Re = uh/\nu = 85$ , where  $u$  is the friction velocity,  $h$  is channel half-height and  $\nu$  is kinematic viscosity. We measure the instantaneous wall-shear stress with a flush-mounted hot-film probe, which we use as an indicator for hibernating turbulence, whilst simultaneously sampling the streamwise and wall-normal velocity components with Laser Doppler Anemometry. We show that hibernating turbulence, Figure 1 showing Newtonian data only, which is strongly related to MDR, is a Newtonian phenomenon and could be interpreted as the weakest turbulence state for self-sustaining wall-bounded turbulence. The addition of a polymer additive entices the turbulence to enter a state of hibernation more frequently [1]. On a time-averaged point of view, this yields a large reduction in turbulent skin-friction drag, and causes the collapse of the polymer solution’s time-averaged streamwise velocity profile onto the MDR asymptote.



**Figure 42.** Mean streamwise velocity scaled with mean wall-shear stress (open symbols) and conditionally sampled ensemble-averaged streamwise velocity (hibernating turbulence data: closed symbols) scaled with conditionally sampled ensemble-averaged ( $0.7 < t^* < 2.8$ ) hibernating (low) wall-shear stress. Upper left inset shows the number of hibernating events detected each hour with increasing  $Re$ : individual experimental runs (black), average number of events at  $Re = 70, 85, 100$  and  $130$  (red) and a (black) line to guide the eye. Open symbols are mean data and closed symbols are conditionally sampled ensemble-averaged data:  $Re = 70$  (black),  $Re = 85$  (blue),  $Re = 100$  (red), and DNS at  $Re = 80$  (green). Nonlinear travelling-wave solutions at  $Re = 85$ : lower branch (blue solid line) and upper branch (black solid line). Thin black dotted line is viscous sublayer:  $U^+ = y^+$ , thick dashed black dotted line is the Von Karman log-law:  $U^+ = 2.44\ln y^+ + 5$ , thick dashed red line is MDR asymptote:  $U^+ = 11.7\ln y^+ - 17$ , thick red line is best fit to experimental data:  $U^+ = 8.553\ln y^+ - 10.46$ .

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## VISUALIZATIONS AND PIV MEASUREMENTS OF THE TURBULENT SPOTS AND SPIRALS IN THE TAYLOR-COUETTE SYSTEM

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In the Taylor-Couette flow, the laminar-turbulent coexistence regimes which accompany the subcritical transition to turbulence have been studied since the 1960s [1, 2, 2] and reproduced in numerical simulations since 2009 [3, 4]. Nevertheless the detailed mechanism which causes and sustains turbulent spots or turbulent spirals remains unknown.

In this study we will focus on the laminar-turbulence transition with the intermittency and spiral turbulence regimes in the Taylor-Couette flow. The intermittency is characterized by turbulent spots, appearing and disappearing erratically in an otherwise laminar flow. The spiral turbulence regime is characterized by a turbulent spiral rotating in an otherwise laminar flow. The intermittency regime has been studied numerically by Coughlin and Marcus [6], who proposed a mechanism for spot formation in which the spiral vortex flow confined in the centrifugally unstable region near the inner cylinder triggers the finite amplitude unstable shear flow in the outer region. Colovas and Andereck [7]] and by Goharzadeh and Mutabazi [8] also studied the intermittency regime experimentally through flow visualization.

Our objective is to provide new experimental information on the organization of the flow. The experiments are conducted in a rather small aspect ratio Taylor-Couette flow with  $\eta = 0.8$  and  $\Gamma = 46$ . Visualizations using Kalliroscope [9] and Particle Image Velocimetry (PIV) measurements of the turbulent spots and turbulent spirals are realized and discussed in the context of the theoretical models as those proposed by Barkley [10] and Manneville [11]. These experiments are also intended to provide data for statistical analysis of the transitions which are difficult to obtain numerically for large aspect ratio systems and timescales.

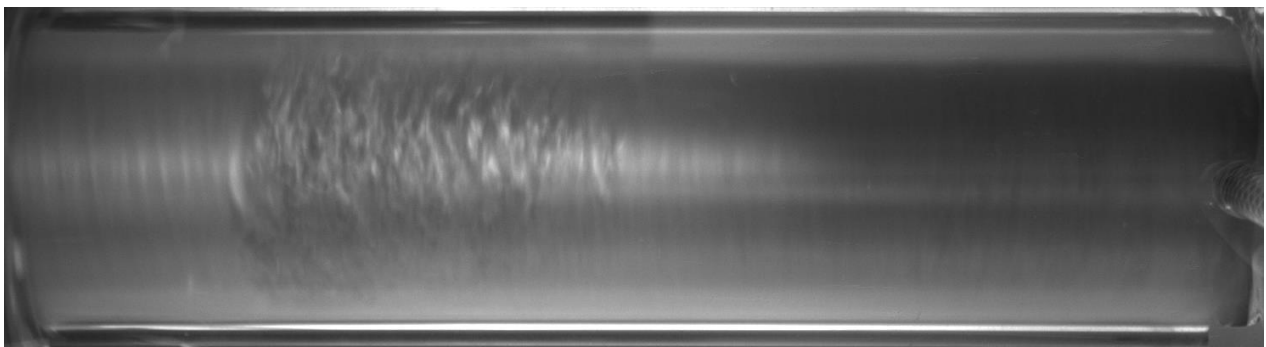


Figure 43. Snapshot of the spiral turbulence for  $R_o = -1092$  and  $R_i = 200$ .

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## A MODEL FOR SUBCRITICAL TURBULENCE IN CHANNELFLOW

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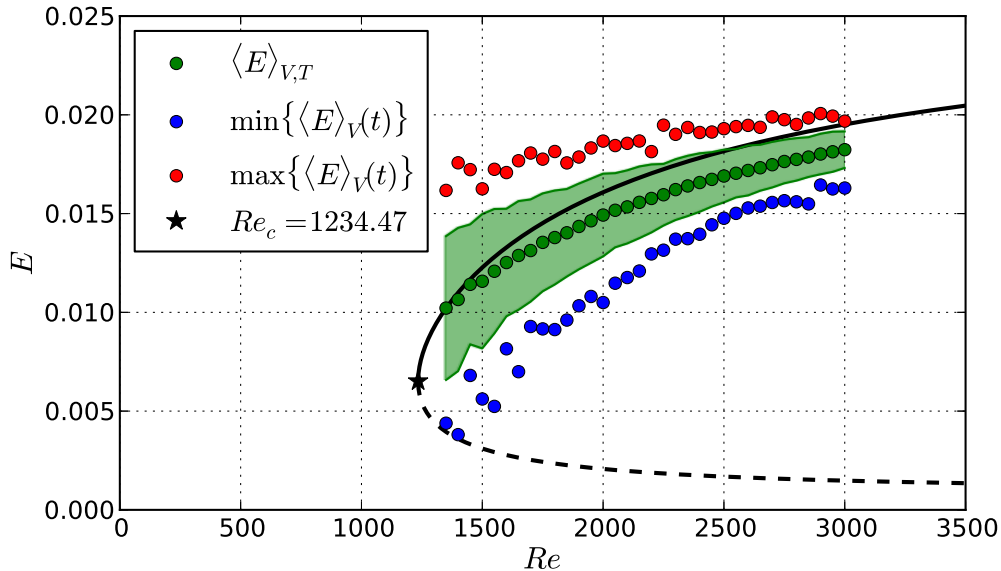
### Abstract

We present a model based on the incompressible Navier-Stokes equation that gives some insight into the transitional dynamics of shear flows. Using the example of plane channel flow, the pressure driven flow between two infinitely extended plates, we run direct numerical simulations of the incompressible Navier-Stokes equation and extract our model from the calculated data. The pipe flow model of Dwight Barkley [1] was the inspiration to start this work, because, despite being not too complicated from a mathematical point of view, it contains astonishingly many transitional phenomena. The key idea is to interpret transitional shear flow as an excitable, bistable medium, with a stable laminar solution always present and, depending on a certain threshold, a second excitable or stable, turbulent solution.

Starting with the incompressible Navier-Stokes equation, we decompose the velocity field into base flow  $\mathbf{u}_0$  and perturbation  $\mathbf{u}'$ . Projecting the Navier-Stokes equation for the perturbation onto the streamwise  $\mathbf{u}'_{\parallel}$  and transversal  $\mathbf{u}'_{\perp}$  components, we arrive at a set of two coupled equations. Averaging these over planes perpendicular to the flow direction leaves the downstream direction as a variable, and the advection and diffusion terms contribute partial derivatives. Then we can express the remaining terms as functions of the two variables  $\pi := \langle \mathbf{u}'_{\parallel}{}^2 \rangle / 2 = \langle u_x'^2 \rangle / 2$  and  $\sigma := \langle \mathbf{u}'_{\perp}{}^2 \rangle / 2 = \langle u_y'^2 + u_z'^2 \rangle / 2$  by parameter estimation from DNS data on fully turbulent flows, obtained using channelflow [2]. In the homogeneous case, this set of equations then has as fixed points the laminar state,  $\pi = \sigma = 0$ , and, as the Reynolds number increases, two new states are created in a saddle node bifurcation at  $Re = 1234.47$  (cf. FIG. 44):

$$\begin{aligned} \partial_t \pi + a_{adv}^{\parallel} \partial_x \pi + \tilde{P}(Re, \pi, \sigma, \partial_x) &= T_{\pi}(Re, \pi, \sigma, \partial_x, \partial_{xx}) + \tilde{\epsilon}^{\parallel}(Re, \pi) + \frac{1}{Re} \partial_{xx} \pi \\ \partial_t \sigma + a_{adv}^{\perp} \partial_x \sigma + T_{\sigma}(Re, \pi, \sigma, \partial_x, \partial_{xx}) &= \tilde{\epsilon}^{\perp}(Re, \sigma) + \frac{1}{Re} \partial_{xx} \sigma \end{aligned}$$

whereas  $\tilde{P}$ : turbulence production rate,  $T_i$ : intercomponent energy transfer rate and  $\tilde{\epsilon}^i$ : dissipation rates. The precise form of the spatial derivatives is crucial for the spatio-temporal behavior of the flow and will be investigated.



**Figure 44.** Bifurcation for the total energy  $E = \pi + \sigma$ , DNS-data and model (black line: model, not a direct fit, shaded green: 2 standard deviations)

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## SUBCRITICAL MAGNETOROTATIONAL DYNAMO ACTION IN KEPLERIAN SHEAR FLOW

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Magnetorotational (MRI) dynamo action in Keplerian shear flow is a three-dimensional, nonlinear magnetohydrodynamic mechanism whose study is relevant to dynamo theory and to the understanding of accretion processes in astrophysics. Transition to this form of dynamo action is subcritical and shares many of the characteristics of subcritical transition to turbulence in hydrodynamic shear flows. In the first part of this presentation, I will present numerical evidence that the emergence of three-dimensional chaos and transient magnetohydrodynamic turbulence in this dynamo problem is primarily associated with global homoclinic and heteroclinic bifurcations involving the stable and unstable manifolds of nonlinear MRI dynamo cycles born out of saddle node bifurcations. I will then show how the detailed analysis of cyclic dynamics makes it possible to gain new physical insights into the regimes of excitation of instability-driven dynamos and turbulent angular momentum transport processes in astrophysical accretion flows and perhaps also laboratory experiments.

## FLUCTUATION CRISIS AT THE DISAPPEARANCE OF OBLIQUE LAMINAR-TURBULENT BANDS OF PLANE COUETTE FLOW

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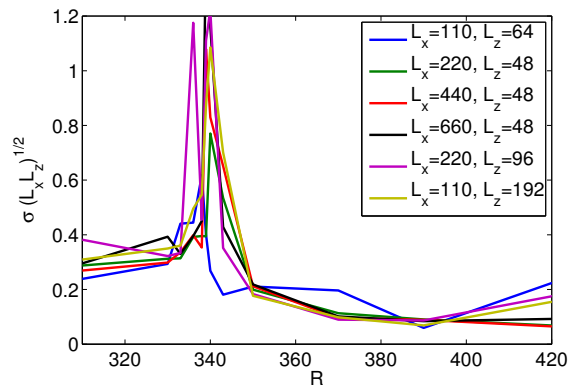
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The coexistence of laminar and turbulent flow is the key feature of the subcritical transition to turbulence in wall bounded flows. In plane Couette flow, this coexistence takes the form of oblique bands in the range of Reynolds numbers  $[R_g; R_t] \simeq [325; 415]$  [1, 2]. Although they are much more sedate and regular than the puffs of pipe flow, the bands are not entirely stationary. Indeed, the turbulent noise originating from the small scale temporal chaos perturbs the bands and can cause orientation reversals in small systems [2] and coexistence of both orientations in large systems [1]. The amplitude of the modulation of turbulence is measured with a pair of order parameters  $m_{\pm}$  either using the Hilbert transform of the measured light intensity [1] or the Fourier transform [2] of the velocity field. The time fluctuations of the bands reflect themselves in the fluctuations of the modulus of the order parameter by causing a maximum near  $R_t$  [2].

In that matter, plane Couette flow resembles the highly turbulent Von Kármán flow which displays a divergence of susceptibility to symmetry breaking  $\chi$  (square root of the volume times the fluctuations  $\sigma$ ) [3]. A formal analogy with critical phenomena was proposed to describe concomitance of a symmetry breaking and the divergence of the susceptibility. This approach seems all the more adapted to the appearance of the bands at  $R_t$  that the Stochastic Ginzburg–Landau models that describe the band are the same that describe equilibrium phase transitions [4]. The main difference between Von Kármán flow and Couette flow is that the former has a finite extension and therefore, no thermodynamic limit.

In order to address this question, we use the classical approach of the numerical study of equilibrium phase transition: Finite Size Analysis. The range of Reynolds number around  $R_t$  is simulated in systems of increasing sizes in order to examine the convergence toward the thermodynamic limit. This has the advantages of exhibiting the finite size scalings of the system and facilitating the computation of critical exponents if the transition is of second order type. We use low order modeling for our numerical procedure. Indeed, very long time series are necessary for statistical convergence and fine graining in  $R$  around  $R_t$  is necessary for a precise determination of the maximum of the susceptibility.



**Figure 45.** Susceptibility of the order parameter, computed numerically for six system sizes, as a function of Reynolds number

We give the example of the susceptibility (Fig. 55). For a critical phenomenon, the temperature of the maximum of  $\chi$ ,  $T_m$ , in a system of finite size  $L$  verifies  $T_m = T_c \pm L^{-1/\nu}$  and said maximum diverges with the size of the system like  $\chi_m \propto L^{\gamma/\nu}$ , with  $T_c$  the critical temperature,  $\gamma$  the exponent of the response function and  $\nu$  that of the correlation length. We performed numerical simulations in domains of various sizes  $L_x \times L_z$ . One can see that there is indeed an increase of the value of the maximum of  $\chi = \sqrt{L_x L_z} \sigma$  with the size. Finer data in  $R$  and larger domains will be necessary to determine precisely the behaviour of the maximum with the size. A simple fit of  $\chi$  for  $R > R_t$  gives  $\gamma \simeq -0.5$  and  $R_c \simeq 333$ . Similar size effects are found in the order parameter, the turbulent fraction *etc.*

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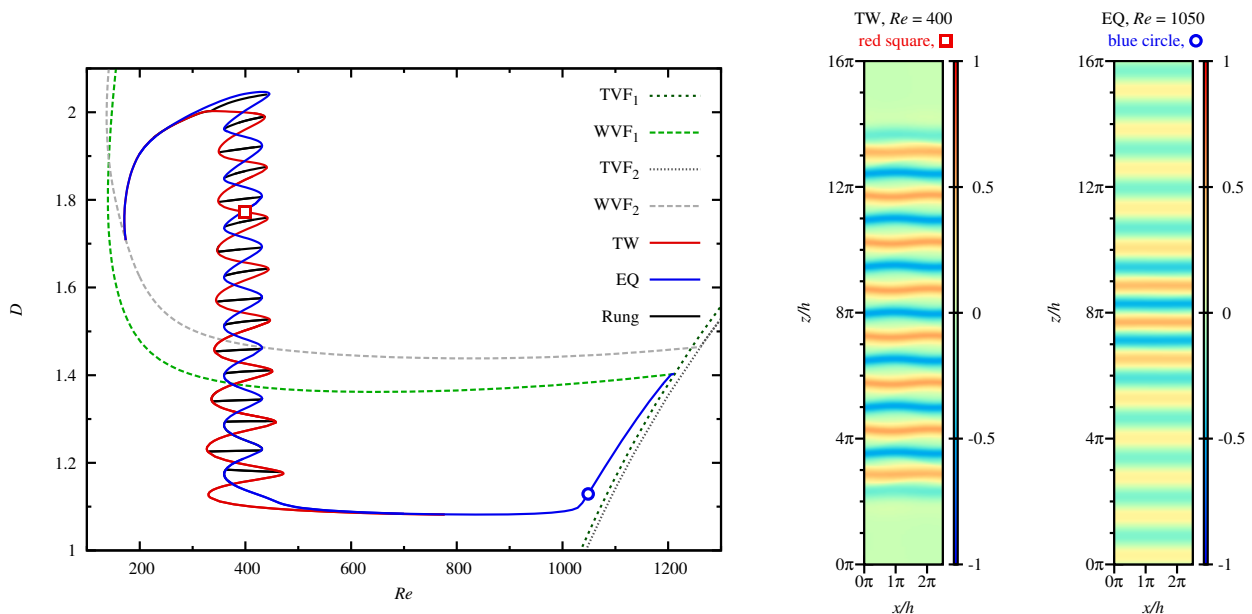
## LOCALIZED SOLUTIONS IN ROTATING PLANE COUETTE FLOW

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We investigate localized solutions of plane Couette flow (PCF) subject to rotation. Anticipating the scenario studied by Knobloch and others (for a comprehensive introduction and further references, see [1]), particularly that of the Swift-Hohenberg equation, we continue localized solutions of plane Couette flow [2] in rotation to their origins; this follows the reverse procedure used by Nagata [2] to find finite-amplitude solutions in PCF. The localized solutions, which occur as equilibrium (EQ) and traveling wave (TW) solutions, survive into nonzero rotation and preserve the characteristic snakes-and-ladders bifurcation structure, as seen below. Continuing in either Reynolds number,  $Re$ , or rotation number,  $Ro$ , we find that the localized solutions originate from the secondary flow, known as wavy vortex flow (WVF) [2]. This secondary flow occurs as a subcritical bifurcation from the streamwise-independent Taylor vortex flow (TVF). When connecting to their origin, the localized solutions cross the critical point for the TVF-instability, becoming embedded in a patterned rather than the laminar background.



**Figure 46.** Bifurcation diagram (*left*) showing dissipation,  $D$ , against Reynolds number,  $Re$ . This plot shows the full snakes-and-ladders structure with Traveling Wave (TW), Equilibrium (EQ), and Rung solutions. The TW and EQ solutions both originate from the wavy vortex solution,  $WVF_1$  and terminate on the wavy vortex solution  $WVF_2$ . The wavy vortex solutions  $WVF_n$  emerge subcritically as secondary flows from the Taylor vortex flows,  $TVF_n$ . Plots (*right*) of the midplane streamwise velocity field,  $u(x, 0, z)$ , show a localized TW within the snaking region at  $Re = 400$ , as well as a localized EQ at  $Re = 1050$  which is embedded in a TVF<sub>2</sub>-background.

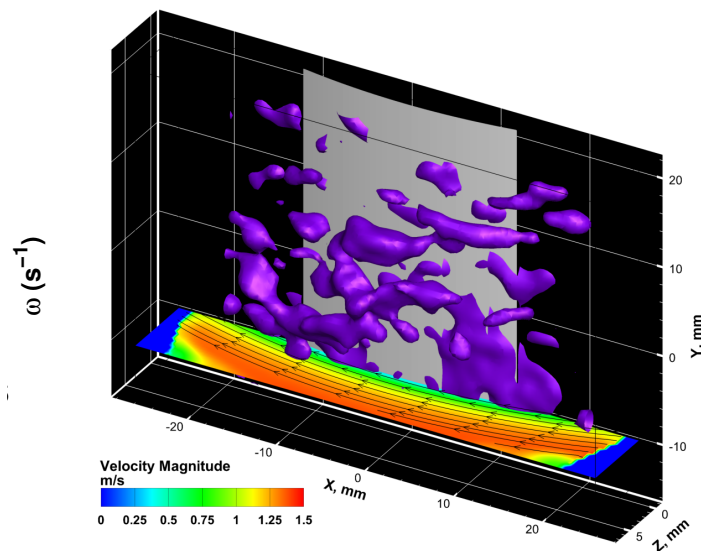
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## SUBCRITICAL TURBULENT TRANSITION IN CIRCULAR COUETTE FLOW EXPERIMENTS

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The flow between concentric rotating cylinders can undergo a direct transition to turbulence in a manner similar to that found in laminar pipe and plane channel flows. In experiments carried out in a centrifugally-stable regime (with the inner cylinder at rest), turbulent spots are observed to grow from localized, finite-amplitude perturbations applied by injecting/withdrawing fluid through an axial array of jets located on the inner cylinder. Velocity fields in fluid volumes are obtained using tomographic PIV; by rapidly sampling the flow, we measured 3D velocity field time series that are time-resolved in the turbulent regime (Figure 55). The long time behavior of the moving turbulent spots can be reconstructed by repeated experiments (at fixed Reynolds number and perturbation parameters) where the interrogation volume is positioned at different azimuthal locations (relative to the location of the jets). In other experiments with independent counter-rotation of both cylinders, flow visualization is used to characterize qualitatively the hysteretic transition between laminar and turbulent flows. The experiments seek to connect transitional and turbulent flows to dynamically important unstable Navier-Stokes solutions known as Exact Coherent Structures and, thereby, to advance the development of deterministic models of shear-flow turbulence.



**Figure 47.** Q-criterion-visualization of a turbulent spot in experiments on circular Couette flow at  $R_{\text{outer}} = 7750$ ,  $R_{\text{inner}} = 0$  with a radius ratio of 0.905 and axial aspect ratio of 5.26.

This work is partially supported by the National Science Foundation (CBET-0853691)



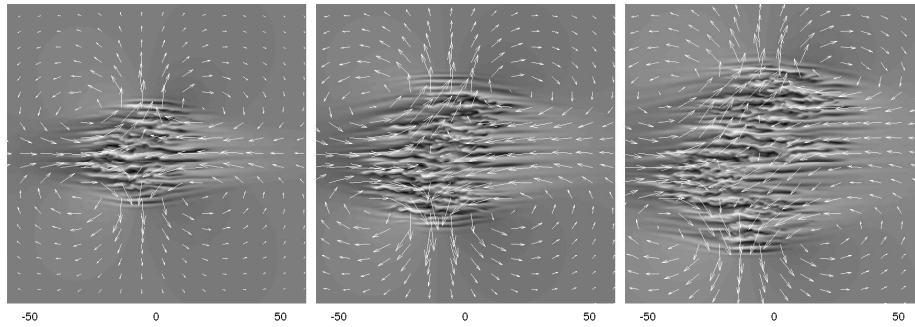
## OBLIQUE LAMINAR-TURBULENT INTERFACES IN TRANSITIONAL SHEAR FLOWS

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The onset of transition to turbulence in subcritical wall-bounded flows is usually characterised by large-scale localised structures such as turbulent spots or turbulent stripes. Interestingly, the laminar-turbulent interfaces associated with these structures always display obliqueness with respect to the mean direction of the flow [1]. We will attempt to explain this phenomenon using an assumption of scale separation between large and small scales, and we can show analytically why the corresponding laminar-turbulent interfaces are always oblique with respect to the mean direction of the flow in the case of plane Couette flow. This mechanism can be easily extended to other flows such as Plane Poiseuille flow, Ekman boundary layers or Taylor-Couette flow.



**Figure 48.** Obliquely growing spot in plane Couette flow at  $R = 360$  (grey: streamwise velocity in the midplane) and associated  $y$ -integrated large-scale flow  $(\bar{U}_x, \bar{U}_z)(x, z)$  (arrows). From left to right:  $t = 200, 300$  and  $400$ . Simulations in a periodic domain with  $\Lambda = 500$  and  $1536 \times 33 \times 2048$  spectral modes. Only the subdomain  $[-60 : 60] \times [-60 : 60]$  is displayed here.

Especially the origin of the obliqueness of the associated interfaces has long remained mysterious. We assume Couette flow, the flow between two sliding plates that move with opposite velocities  $\pm U$  in the streamwise direction  $x$ . We also define the half-gap  $h$  between the plates in the  $y$  direction, the spanwise direction  $z$ , and the Reynolds number  $R$  as  $Uh/\nu$ , where  $\nu$  is the kinematic viscosity of the fluid, supposed Newtonian and incompressible. Lengths and velocities are non-dimensionalised by respectively  $h$  and  $U$ . Direct numerical simulations were carried out with the same pseudo-spectral code as in Ref. [2], in a periodic domain of size  $\Lambda \times 2 \times \Lambda$ , with  $\Lambda = 500$ .

We base our analysis on the existence of two distinct characteristic scales. The small scales correspond to the coherence of the turbulent fluctuations inside a turbulent patch while the large scales correspond to the diffusive tails of the streaks, so that the scale separation grows like  $O(R)$ . Using this hypothesis we separate the flow field  $\mathbf{u}$  into small scales  $\tilde{\mathbf{u}} = H\mathbf{u}$  and large scales  $\mathbf{U} = L\mathbf{u}$  using adequate filters, respectively  $H$  and  $L$ . Denoting  $y$ -averaging with a bar  $(\bar{\cdot})$ , we deduce from the incompressibility of the flow a two-dimensional divergence-free condition in the  $xz$ -plane for the large-scale flow,  $\partial_x \bar{U}_x + \partial_z \bar{U}_z = 0$ .

The growth of a turbulent patch is shown in Fig. 55 for  $Re = 360$  along with the corresponding large scale flow  $(\bar{U}_x, \bar{U}_z)$ . The streamwise ends of a such a turbulent patch are characterised by so-called overhang regions where locally turbulent flow on one wall faces nearly laminar flow near the other wall. Those regions correspond to a mismatch in the flow rates  $\bar{U}_x \neq 0$ , whereas  $\bar{U}_x = 0$  everywhere else. As a consequence  $\partial_x \bar{U}_x \neq 0$  in the overhang regions, hence  $\bar{U}_z \neq 0$ , and the large-scale flow is locally oblique with respect to the streamwise direction. We use the decomposition introduced earlier and apply successively the filters  $L$  and  $H$  to the wall-normal momentum equation. The scale separation hypothesis results in a simplified system, similar to a transport equation for the small scales by the large scales. This indicates that newly nucleated streaks at the tips of the spots will be advected by the large-scale flow, which we know has a non-zero angle with respect to the streamwise direction. As a consequence, the growth of the spots will be distorted by the presence of the large-scale flow and proceed obliquely as well [2].

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## PATTERNS AND INVARIANT SOLUTIONS IN TRANSITIONAL FLOWS

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When shear flows become turbulent spatio-temporal patterns emerge in the chaotically fluctuating flow. Those patterns such as localized turbulent spots or laminar-turbulent stripes appear to be captured by spatially localized exact invariant solutions of the 3D Navier-Stokes equations. Specific equilibrium and traveling wave solutions are organized in a snakes-and-ladders structure strikingly similar to that observed in simpler pattern-forming PDE systems, suggesting that well-developed theories of patterns in simpler PDE models carry over to transitional turbulent flows. We will present invariant solutions in plane Couette flow and discuss the situation when convective driving or rotation [cf. presentation by Matthew Salewski] destabilized the base flow. These studies shed light on the mechanisms underlying the emergence of homoclinic snaking in shear-driven flows.

## EXACT COHERENT STATES IN PURELY ELASTIC PARALLEL SHEAR FLOWS

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Using an analogy with the Newtonian self-sustaining process in parallel shear flows [1], we attempt to construct the purely elastic counterpart for plane Couette flow of polymer solutions. By introducing a forcing term to the coupled Stokes and Oldroyd-B equations, we observe the formation of purely elastic streaks and consider their linear stability. We find that there exists a previously unrecognised purely elastic analogue of the Kelvin-Helmholtz instability that gives rise to the streamwise waviness of Newtonian coherent structures. We discuss how this instability might close the cycle and lead to a sustained purely elastic coherent structure.

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## NUMERICAL INVESTIGATION OF TURBULENT PIPE FLOW STRUCTURES

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<sup>1</sup>*Department of Aerodynamics and Fluid Mechanics,  
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Since the formation of boundary layer concept, considerable attention has been given to the study of wall bounded turbulent flows. In this area turbulent pipe flow is known to be linearly stable which undergoes sub critical transition. The increasing interest in the physics of this flow type at high Reynolds numbers has motivated construction of experimental facilities, able to run at such turbulent regimes. The same motivation has triggered countless numerical studies in terms of Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) as a consequence of recent advances in high performance computing facilities.

During the past decades, considerable attention has been given to the study of coherent structures in various geometries and at different Reynolds numbers. This has resulted in new findings and has given rise to new questions concerning their identification and scaling methods. According to Marusic et al. [1], these organized structures are classified to three basic eddy motions. While near-wall streaks obey a near wall cycle and have a spanwise scale of about 100 viscous length scales, large-scale motions (LSMs) are described as packets of individual eddies (Adrian [2]) and have a scale of pipe radius. On the other hand very large-scale motions (VLSMs) are interpreted differently by various scientists as joint series of vorticity packets (Adrian [2]) or as meandering superstructures (Marusic et al. [1]) and have a scale of 10 pipe radii. At this stage many key questions concerning LSMs and VLSMs are still unanswered including a uniform scaling law for their identification. Differing views on the origin and definition of low wave number VLSMs exist which questions their dependence on geometry and outer layer variables.

In the present study DNS and LES has been performed for turbulent incompressible viscous flow in a pipe at friction Reynolds number of  $Re_\tau = 181$  and  $361$  ( $Re_b = 5300$  and  $11700$ ). Streamwise periodic length of  $25 R$  is selected for a pipe of radius  $R$ . To validate the results in lower ranges of Reynolds number, DNS results by El Khory et al. [2] have been used as a benchmark to compare the flow statistics. For higher ranges of Reynolds number, DNS and LES of pipe flow will be performed and the results will be compared against experimental data from Cottbus Large (CoLa) pipe test facility. Additionally, kinetic energy budget of the mean flow and turbulent flow will be investigated. Analysis of the effect of interactions between coherent structures in pipe flow, their effects on the mean flow and streamwise turbulent intensity and their comparison with experimental data from CoLa pipe are of the future goals of this study.

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## SUBCRITICAL TURBULENCE IN QUASI-KEPLERIAN ROTATING FLOWS?

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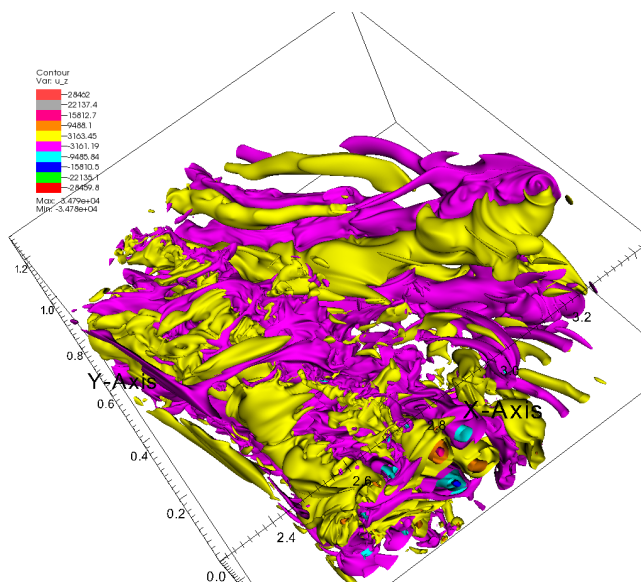
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Turbulence plays an important role in astrophysical accretion processes but its origin remains still unclear. The inviscid analysis tells that the Keplerian velocity profiles are linearly stable and therefore a different instability mechanism is required for turbulence to arise. While in hot discs turbulence can be triggered through magnetorotational instability, cooler discs lack sufficient ionization and it is unclear how turbulence sets in. In analogy to other linearly stable flows like pipe and Couette flow, subcritical transition to turbulence may be the mechanism. Recently, experimental studies [2, 3, 4] of Taylor-Couette flow in quasi-Keplerian regime have given conflicting results and numerical simulations [2] of above experimental flows showed that the top and bottom end-wall leads to strong deviations from the Keplerian velocity profile and drives turbulence. In order to clarify this, we perform direct numerical simulations of incompressible Taylor-Couette flow without end walls in the quasi Keplerian regime for  $Re$  up to  $2 \times 10^5$ . Secondary instability is observed and gives rise to strongly distorted motion, suggesting that for large enough  $Re$  this mechanism may lead to turbulence even for Keplerian flows.



**Figure 49.** Turbulent motion induced by secondary instabilities. The axial velocity is plotted.

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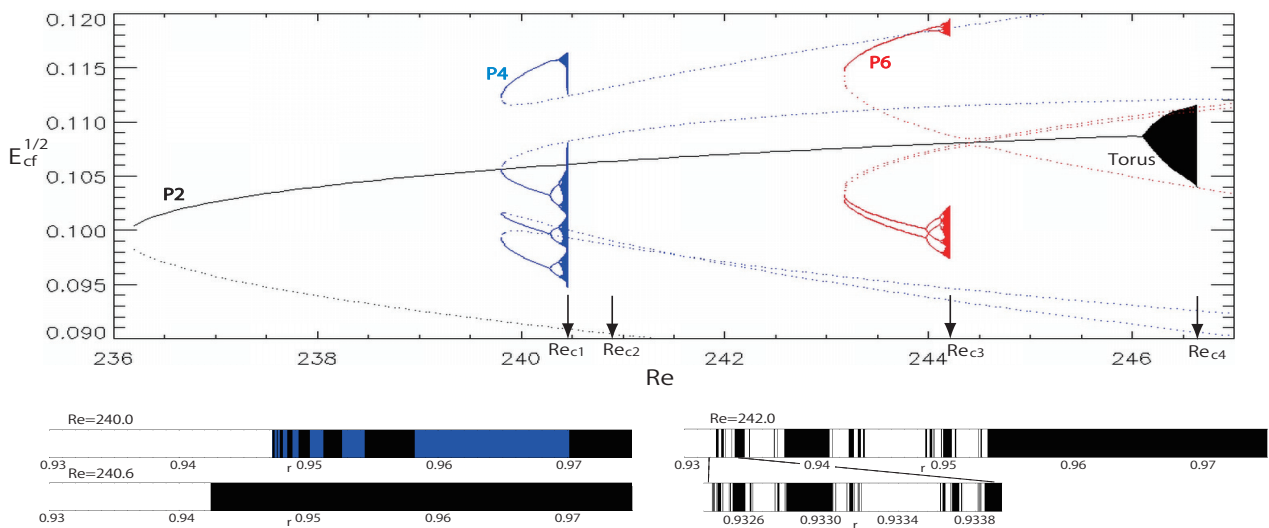
## ROUTE TO CHAOS IN MINIMAL PLANE COUETTE FLOW

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Here we consider incompressible Newtonian flow in minimal plane Couette system with the domain size  $1.755\pi * 2 * 1.2\pi$ . (for example, see [3]) No additional symmetry is imposed besides the periodic boundary conditions (unlike in [1] and [2]). We integrate numerically this system using spectral-Galerkin method and investigate final flow state for  $236 \leq Re \leq 247$ . Top figure shows the bifurcation of this system. Laminar flow (LF), which corresponds to  $E_{cf} = 0$ , is linear stable. For  $236.1 \leq Re \leq 246.6$ , "several attractors coexist with LF". This coexistence makes it easy to find global changes in phase space. The onset of the nonlinear solutions is the pair of periodic orbits (P2) caused by saddle-node bifurcation [3], and each of them has two local maxima per cycle but these are the same value. (This is also the case for P6.) The upper-branch of P2 ( $UB_{P2}$ ) loses stability at  $Re = 246.1$  [3] by supercritical Neimark-Sacker bifurcation resulting in the stable torus. P4 and P6 also appear from saddle-node bifurcation and each of them leads to the chaotic attractor via period-doubling cascade.

Some important global bifurcation points are discussed below. At  $Re_{c1} = 240.4$  the boundary crisis occurs between  $UB_{P2}$  and the chaotic attractor, and the chaotic trajectory can approach to any neighborhood of the  $LB_{P4}$ . Above this  $Re$  the chaotic attractor is replaced by the chaotic saddle and all trajectories in its neighborhood are attracted to  $UB_{P2}$ . This chaotic set also touches  $LB_{P2}$  at  $Re_{c2} = 240.9$ , and "the fractal basin boundary" appears between  $UB_{P2}$  and LF (bottom figure). Above  $Re_{c3}$  the chaotic attractor originating from P6 disappears and trajectories starting from in its neighborhood decay to LF. At  $Re_{c3}$  "the chaotic orbit touches the fractal basin boundary". Finally at  $Re_{c4}$  the collapse of the torus occurs through  $LB_{P6}$ , and there is no attractor except for LF. At the workshop we will discuss more detail about the formation/destruction of the fractal basin boundaries caused by the boundary crises, and the heteroclinic connections among the periodic orbits. We are grateful to Dr. Paul Manneville for his insightful comments and suggestions.



Top: Bifurcation diagram of minimal plane Couette flow for  $236 \leq Re \leq 247$ . Lines and filled areas represent attractors and dotted lines are saddles.  $E_{cf}$  is local-maximum of the cross-flow energy, which is the same quantity used in Kreilos and Eckhardt[1]. At  $Re = 236.1, 239.8$  and  $243.2$  periodic orbits are created by saddle-node bifurcation. During one cycle these have two, four and six local maxima of  $E_{cf}$  respectively and have stable upper branch (P2, P4 and P6). Major global bifurcation points are represented by allows at  $Re_c = 240.4, 240.9, 244.2$  and  $246.6$ .

Bottom: Basin of attractors along lines in phase space. White, Black and Blue represent basin of the origin (laminar flow), P2 and P4 respectively. The horizontal axis  $r$  denotes the distance from the origin.

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## RELATIVE PERIODIC ORBITS IN TURBULENT PIPE FLOW

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The transition to turbulence for incompressible pipe flow is accompanied by the appearance of spatially and temporally unstable recurring flows, including traveling waves (TWs) and cyclic regenerative flows. In the argot of dynamical systems, these flows are relative equilibria and relative periodic orbits (RPOs), exact solutions to the Navier-Stokes equations which strongly influence turbulent trajectories and give shape to the state space manifold on which turbulent dynamics plays out.

We present the latest results of our searches for RPOs in two disparate computational domains. Using a simple symmetry reduction scheme to account for down-stream drifts of the flow, we are able to identify a set of RPOs, recurrent flows thought to constitute most of the natural measure of turbulent flow [1]. Visualizations of the state space dynamics reveal that these RPOs explore its turbulent reaches, suggesting that transport characteristics of turbulent flow may be ascertained from the measured properties of the RPOs embedded within it.

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## A PASSIVE CONTROL THAT RELAMINARISES FULLY TURBULENT PIPE FLOW

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Turbulent motion is accompanied by rapid dissipation of kinetic energy and causes intense fluctuations of velocities and stresses. At sufficiently high Reynolds numbers, shear flow turbulence is self-sustained and will persist unless its self-sustaining process [1, 2, 3] can be intercepted. Great reduction in the friction drag and detrimental effects resulting from turbulence will be achieved if turbulence can be relaminarised. However, current control measures are usually costly and only affect the flow locally. In light of the fact that the mean shear is the energy source of the near-wall turbulence, Hof [4] proposed a method that redistributes the mean shear at the upstream edge of a localized turbulence and eliminated it completely. The Barkley model [5] also indicates the turbulence intensity decreases as the mean shear decreases in the turbulence core region, supporting the idea that even fully turbulent flow can be relaminarised by sufficiently reducing the mean shear. We here show for the example of pipe flow that a very simple modification of the velocity profile leads to a total collapse of turbulence and complete relaminarisation in direct numerical simulations and laboratory experiments.

In simulations, we introduce a force term which forces the velocity to increase near the wall and decrease near the pipe center while keeping the mass flux fixed, so that redistributes the shear in the flow. Simulations at high Reynolds numbers up to 25000 indicates that fully turbulent flow indeed can be relaminarised with this control mechanism. In contrast to present theories neither velocity scales nor shear rates explain the collapse of turbulence, instead the non-normality of the velocity profile appears to set a threshold dividing flows that can sustain turbulence from those that relaminarise. The linear transient growth is calculated for the forced flow and surprisingly a critical level is found under which turbulence fails to regenerate and finally relaminarises.

Under sufficiently strong forces, turbulence intensity decays quickly and the velocity profile approaches to a parabola. The fast decay suggests a localized implementation of the forcing in a more realistic manner and a simulation at  $Re = 8000$  in a 180 diameter-long pipe shows a 40 diameter-long forcing suffices to tear up turbulence and open up a laminar region down stream of the control area, where flow remains laminar thanks to the linear stability. This indicates that turbulent flow is possible to be relaminarised in laboratory experiment with a localized control method that modifies the velocity profile properly. In our experimental implementation, a control technique with a carefully designed obstacle, mounted inside the pipe, can successfully deform the velocity profile and relaminarise turbulence up to  $Re \simeq 7000$ . Great friction drag reduction is achieved downstream of the control area.

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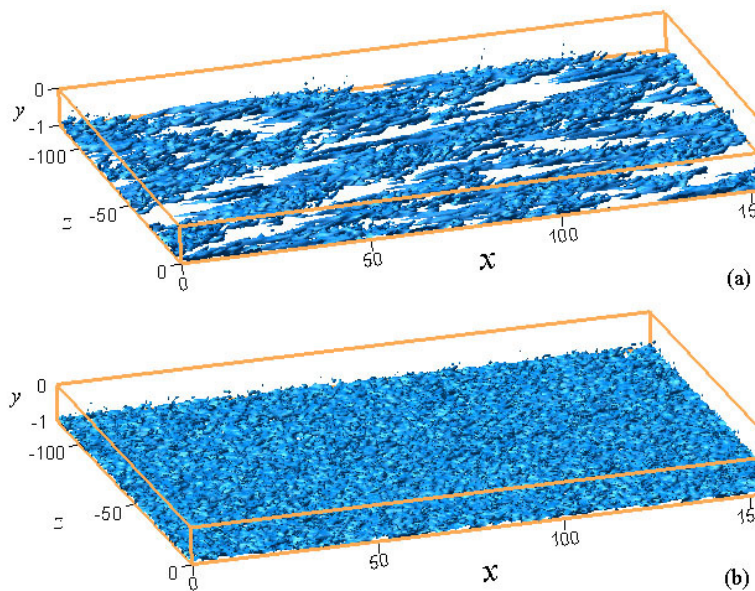


## THE TRANSITION STAGES OF PLANE-POISEUILLE FLOW

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The pressure-driven Hagen-Poiseuille flow, plane-Poiseuille flow and the shear-driven plane-Couette flow are three typical models to study the subcritical transition. Recent experiments and numerical simulations have revealed that there are several stages between the steady laminar flows and the fully developed turbulence [1, 2, 3, 4]. We simulated plane-Poiseuille flow (PPF) in a large computational domain. Based on statistical analyses, the thresholds of different transition stages, i.e. the transient localized turbulence, the temporally persistent turbulence and the uniform turbulence, are identified. It is illustrated that the typical flow structures at moderate Reynolds numbers are not spots but isolated turbulent bands, which are the counterparts of puffs in pipe flows and grow transiently by oblique extension. We show that the thresholds of PPF are consistent with the corresponding values of Hagen-Poiseuille flow and plane-Couette flow in terms of a locally defined Reynolds number [5].



**Figure 50.** Iso-surface of vorticity ( $|\omega| = 9$ ) in one eighth of the computational domain at (a)  $Re=1300$  and (b)  $1800$ , respectively.

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## DAMPING FILTER METHOD FOR A SPATIALLY LOCALIZED SOLUTION IN TWO-DIMENSIONAL CHANNEL FLOW

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Spatially localized structures are key components of turbulence. In order to investigate them from a dynamical systems viewpoint, it is desirable to obtain corresponding exact solutions. The dynamical systems viewpoint, however, has not yet successfully captured the full nonlinear spatio-temporal dynamics of turbulence. One major limitation is that there is no general framework for obtaining solutions corresponding to spatially localized structures in turbulence. For example, in channel flows there exist near-wall structures and large scale motion (LSM) that occupies the outer layer above the near-wall layer. In order to elucidate their intrinsic dynamics and interactions among them from the dynamical systems viewpoint, it is desirable to obtain corresponding exact solutions separately. For this purpose, we introduced a damping filter method in [1]. This method aims to provide a general way to obtain spatially localized solutions in various systems. Since this method does not use any property of edge states, it will be able to find spatially localized solutions which do not belong to the edge.

The damping filter method consists of three steps as follows:

1. To introduce a damping filter term into the equation
2. To obtain a solution to the filtered equation
3. To execute a continuation with the amplitude of the damping filter until it disappears

This method is being adopted to two-dimensional channel flow to obtain a stream-wisely localized traveling-wave solution. There are two issues to obtain a spatially localized solution:

1. Is there localized sustainable mechanism in two-dimensional channel flow?
2. How to determine the propagation speed?

We will talk about these issues.

The determination of the propagation speed is an issue discussed in [1] related with a criticality. In [1], it was found that the damping term acts as a singular perturbation in some cases. It breaks the translational and Galilean invariances which exist in various systems. The result of the continuation will contain important information which may cast a new light on the relationship between localized solutions in flows and theories of one-dimensional localized solutions.



**Figure 51.** A snapshot of a simulation of the filtered Navier-Stokes equation. We use the plane Poiseuille flow with fixed flow rate. This is a spatially localized chaotic state, and we use this as a guess for a traveling-wave solution.

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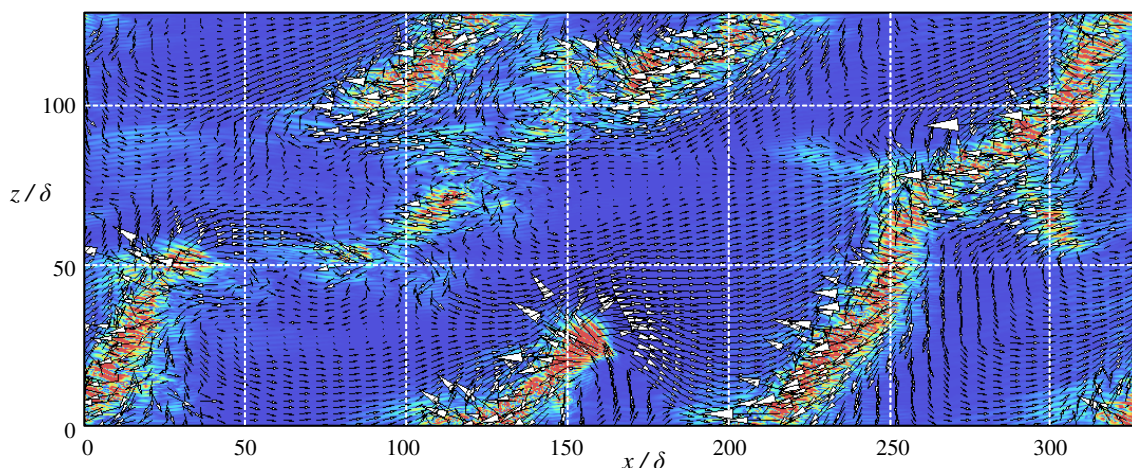
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## THE LOWER BOUND OF SUBCRITICAL TRANSITION IN PLANE POISEUILLE FLOW

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Although the transition in a pressure-driven channel flow, one of canonical wall-bounded flows, has attracted interests of many researchers, its equilibrium transitional flow state in the subcritical regime has been the subject of ongoing debate. According to the literature, the lower bound of the subcritical transitional regime was suggested to be about  $Re_c = 1000$  [1, 2], where  $Re_c$  is based on the channel centerline velocity and the channel half width  $\delta$ . It has been believed that, below this lower critical Reynolds number, any self-sustained turbulence cannot exist, while the laminar flow cannot be linearly stable above the upper critical value of  $Re_c = 5772$ . However, the lower critical value is still an unsolved issue and not yet proven to be unique, because it must be necessary to consider three-dimensional nonlinear phenomena and spatio-temporal intermittent behaviors characterized by large-scale turbulent patches (i.e., the turbulent spot, puff, and band). In this study, we conducted a series of DNS (direct numerical simulation) of the pressure-driven channel flow, i.e., the plane Poiseuille flow, considering the subcritical regime with the use of a high aspect-ratio computational domain, whose size is  $L_x \times L_y \times L_z = 327.68\delta \times 2\delta \times 128\delta$  in the streamwise, wall-normal, and spanwise directions, respectively). We performed also large-scale DNSs of the developing process of a turbulent spot triggered in the laminar flows.



**Figure 52.** Instantaneous velocity field in the channel central plane for the friction Reynolds number of  $Re_\tau = 44$ , corresponding to  $Re_c = 840$ : the color contour shows the wall-normal velocity fluctuation (red,  $|v'| > u_\tau$ ; blue,  $|v'| \approx 0$ ), and the vector represents the fluctuating velocity component ( $u'$ ,  $w'$ ) parallel to the channel walls. The mean flow goes from left to right.

As shown in Figure 52, we found that localized turbulence taking in the form of one-way oblique bands maintained themselves during a long time period, even at a very low friction Reynolds number of  $Re_\tau = 44$ , based on the friction velocity  $u_\tau$  and  $\delta$ . This flow field was obtained by stepwisely, but gradually, decreasing  $Re_\tau$  from the fully turbulent state. From our long-term simulation for a time period of  $730u_\tau/\delta$ , we have not observed any sign of disappearance of turbulent bands so far. As can be clearly seen from the figure, large-scale flows with non-zero spanwise components occur along the boundaries of the turbulent bands. A mechanism regarding the relationship between the large-scale flow and the turbulent band may be same with the scenario proposed by Duguet & Schlatter [3] for a subcritical plane Couette flow. We confirmed that the same intermittent flow with turbulent bands under laminar background was achieved as the terminal state of a single spot at the same Reynolds numbers (figure not shown). This Reynolds number is lower than the minimum  $Re_c$  for sustaining turbulence with hairpin vortices [4]. Once the turbulent spot was elongated in an oblique direction with similar angles of the bands with respect to the main flow direction, the region of localized turbulence was continuously expanded until the stripe pattern almost covered the domain like given in Figure 52. From this result, we may draw conclusions that the lower critical Reynolds number would be less than, at least,  $Re_c = 840$  and that the oblique-band formation should be responsible for the sustainability of localized turbulence.

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## THE SELF-SUSTAINING PROCESS FOR TAYLOR-VORTEX FLOW

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The Self-Sustaining Process (SSP) proposed by Waleffe [1] as the fundamental element of turbulence in low Reynolds number turbulence in wall-bounded shear flows consists of three phases. (i) Streamwise vortices bend the streamwise velocity contours via advection. (ii) The undulating streamwise velocity leads to wavy streamwise vortices. (iii) The nonlinear interaction of the wavy streamwise vortices promotes streamwise vortices. We explore the SSP for Taylor-vortex flow, for which streamwise (azimuthal) and wavy vortices are genuine steady states resulting from linear instabilities with well-defined thresholds. In particular, we determine the circumstances under which wavy vortices reinforce Taylor vortices.

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## PHASE TRANSITION TO SUSTAINED TURBULENCE IN PIPE AND PLANE-COUETTE FLOW

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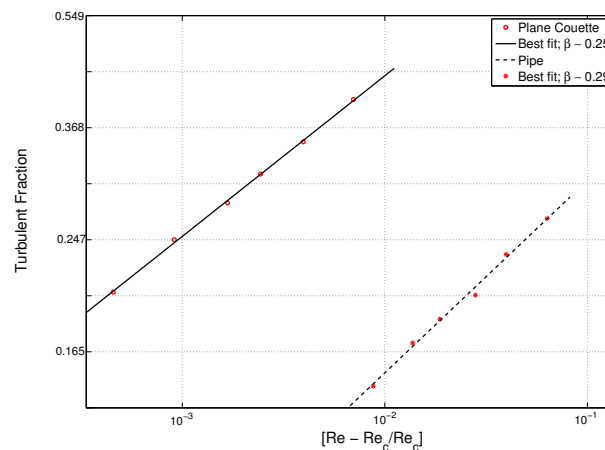
Many spatially extended systems exhibit a transition to chaos via spatio-temporal intermittency, with localized disordered states that fluctuate in both space and time. This occurs, for example, in the transition to turbulence in wall bounded shear flows such as Taylor-Couette, Plane-Couette or Poiseuille (pipe) flow. This is analogous to stochastic models of contamination processes such as fires or epidemics, which exhibit a second order phase transition in the universality class of directed percolation [4].

Experiments and model studies in such systems have however yielded a variety of exponents, with even first order transitions being observed (e.g [2, 3]). Moreover, in spite of its theoretical importance, there has only been one convincing experimental realization of directed percolation criticality ([5]). Here, we examine some of these issues via experimental studies of pipe flow and direct numerical simulations of Plane Couette flow (confined in two directions).

In the case of pipe flow, the long time scales involved ( $10^8$  advective units [1]) pose a challenge in studying the transition. We show that it is possible to exploit memoryless nature of the turbulence proliferation and decay processes close to the transition point, to construct a quasi-periodic pipe. Together with an accurate long-term control on the Reynolds number (better than 0.1%), it is possible for the first time to follow the intermittent dynamics for arbitrarily long times.

In the Plane-Couette system, direct numerical simulations were carried out in a long, narrow domain, unconfined in the direction normal to the turbulent stripes. We find that turbulence becomes sustained at a distinct critical point once the spatial proliferation of the turbulent stripes outweighs their decay.

The scaling of the turbulent fraction (the order parameter for the transition) for the pipe and Plane-Couette flow is shown in Fig 1, with exponents being 0.29 and 0.25 respectively; the corresponding exponent for  $1 + 1D$  directed percolation is 0.276. Improved estimates of this and other exponents will be presented and compared with those of the directed percolation class.



**Figure 53.** Scaling of the order parameter (turbulent fraction) in Plane Couette (DNS) and pipe (experiments) flows.

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## LARGE-SCALE FLOW ASSOCIATED WITH TURBULENT SPOTS IN CHANNEL AND COMPARISON WITH LOW ORDER MODEL

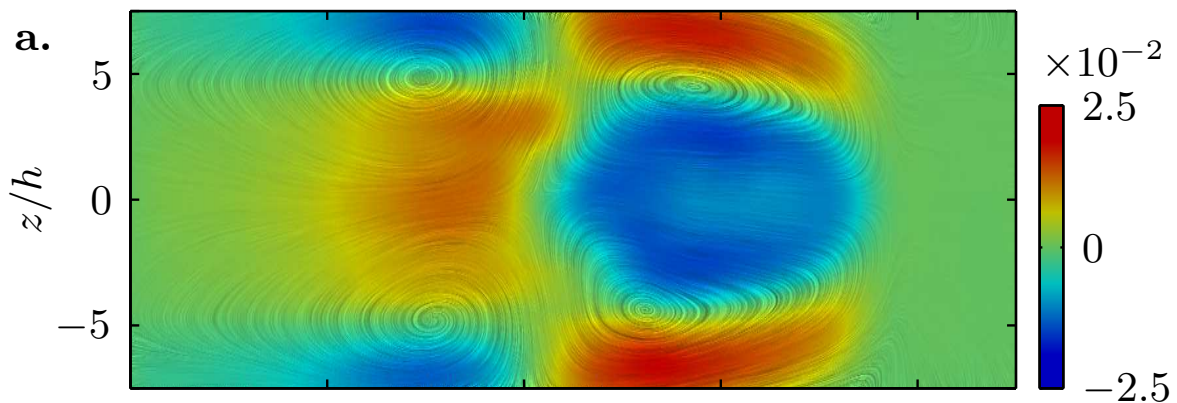
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Over the past years, many minimal or phenomenological models have been suggested to capture the main features of the transition to turbulence in wall bounded shear flows, such as their subcritical character with finite amplitude perturbations and the existence of localized domains.

We present new experimental results on the development of turbulent spots in channel flow. We measure the internal structure of a turbulent spot with Time Resolved Stereoscopic Particle Image Velocimetry and reconstruct the instantaneous three-dimensional velocity field. Special attention is paid to the large-scale flow surrounding the spot[1]. We show that this large-scale flow is an asymmetric quadrupole centred on the spot. We discuss the relation which exists between this modification of the mean flow and the evolution of the turbulent fluctuations.



**Figure 54.** Streamwise component of the large scale flow.

Finally, we compare our experimental data with the low-order model developed for pipe flow by D. Barkley[1]. We fit our data to identify the relevant parameters in the case of plane Poiseuille flow. We also extend the model in 2 dimensions in order to capture the richness of pattern formation allowed by the spanwise direction.

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## USING SYMMETRY REDUCTION TO CAPTURE RELATIVE PERIODIC ORBITS IN TURBULENT PIPE FLOW

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Over the last decade a new theoretical framework has emerged for the study of turbulence, where chaotic turbulent flow is viewed as a trajectory embedded in a high-dimensional space, with transitory visits to neighborhoods of invariant solutions. The visited solutions are weakly unstable relative equilibria, known as travelling waves (TWs), and relative periodic orbits (RPOs), patterns that repeat in time, but after one period are translated in space.

TWs have been shown to be important in the transition to turbulence, where they often lie in the laminar-turbulent boundary [1]. To capture the dynamics of persistent turbulence, we turn to RPOs, which are expected to form the skeleton underpinning the natural measure of turbulent flow. Continuous symmetries (translations and rotations), however, obscure the relationships between the invariant solutions, apparently knotting trajectories, as seen in projected visualizations of the flow. Symmetry reduction is thus a crucial step in untangling trajectories in state space, and for revealing the true relationship between influential solutions.

In ref. [2] we described the ‘method of slices’, which systematically removes shifts along symmetry directions, closing RPOs into POs, and providing a means for untangling state space trajectories. We recap the method, which is global and reduces symmetry, and comment on the coordinate transformation to the local ‘comoving frame’, which is a geometrically natural way of separating symmetry drifts from the ‘shape-changing’ dynamics, but is not a symmetry reduction scheme. Our recent refinements of the ‘method of slices’, applied to two different computational domains with conceptually and physically distinct slices, have enabled us to determine of a set of new RPOs. Implementation of the method does, however, require care. We discuss the origin of difficulties in reducing continuous symmetries, and approaches that alleviate them.

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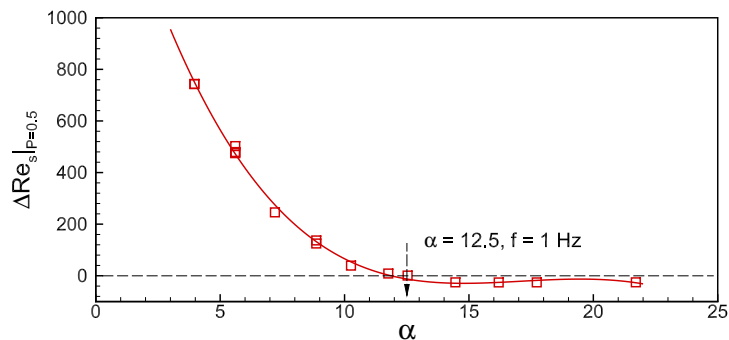
## TRANSITION TO TURBULENCE IN PULSATILE PIPE FLOW

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We report an experimental investigation of the transition to turbulence in a pulsatile pipe flow, which is a prototype of various pulsatile flows in nature and engineering, e.g., in the cardiovascular system where the onset of fluctuations and turbulence is often possibly related to various diseases like the formation of aneurysms. The experiments are carried out in a straight rigid pipe using a Newtonian fluid (water) with a sinusoidal modulation of the flow rate. The flow is governed by the Reynolds and the Womersley number ( $\alpha$ , dimensionless pulsation frequency) ranging to cover the flow characteristics in larger arteries. To characterize the transition to turbulence we measure lifetimes of individual puffs for a range of pulsation frequencies. Figure 55 shows the shift in  $Re$  required to obtain a puff half lifetime in the pulsatile pipe flow with the pulsation amplitude of 40% identical to that in the steady case. At high frequencies ( $\alpha > 12$ ) puff lifetimes are entirely unaffected by flow pulsations. At low frequencies on the other hand substantially higher Reynolds numbers are required to obtain puffs with equal lifetimes and hence transition is delayed. We also report on the effect of pulsation amplitude and structural changes in the flow.



**Figure 55.** Reynolds number shift (relative to steady pipe flow) against Womersley number at the puff surviving probability of 50% with the pulsation amplitude of 40%.



## CRISIS BIFURCATIONS AND THE ORIGIN OF SUBCRITICAL TURBULENCE IN PLANE POISEUILLE FLOW

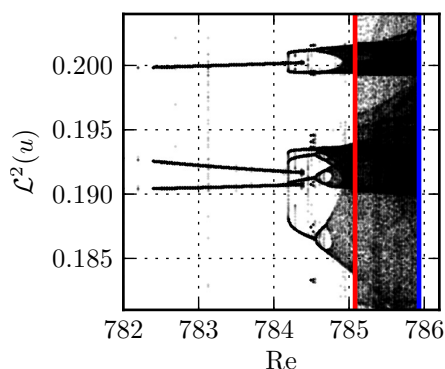
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In plane Poiseuille flow (PPF) sustaining turbulence can be observed for Reynolds numbers that are below the critical one  $Re_{crit} = 5772$  where the laminar profile becomes linearly unstable [1]. This is a feature that is also observed in other flows a plane Couette or pipe flow.

For the case of plane Couette flow it was shown that the origin of the turbulence can be related to bifurcations of exact solutions of the Navier stokes equation [2]. At a certain  $Re$  a pair of exact solutions is created in a saddle-node bifurcation. With increasing  $Re$  the more energetic upper branch solution undergoes various bifurcations leading to the formation of a chaotic attractor. The stable manifold of the lower branch solution forms the boundary of the basin of attraction of this chaotic attractor. At a certain Reynolds number the attractor is destroyed in a boundary crisis bifurcation. After this bifurcation the dynamics shows transient chaotic motion together with exponential distributed lifetimes that are also observed in experiments for this flow. A similar scenario was found for the case of pipe flow [3].

For the case of PPF we will show that the origin of the subcritical turbulences is quite similar. We study PPF in a small computational domain with a streamwise and spanwise extent of  $2\pi$  and  $1\pi$ , respectively. The flow is restricted to a subspace that is symmetric to reflections at the midplane at to spanwise reflections at the center of the domain. We for all our simulations the *channelflow*-code [4] is used. Using the technique of edge tracking [5], we identified the so called edge state of the system. This edge state is a lower branch travelling wave ( $TW_{Eyz}$ ). Using continuation methods it is possible to continue this travelling wave to other  $Re$  and to identify the upper branch of the solution. Slightly above the bifurcation point at  $Re \approx 640$  the upper branch solution is a stable attracting state. With increasing  $Re$  the upper branch undergoes further bifurcations. First a Hopf-bifurcation creates a stable relative periodic orbit. This orbit undergoes further bifurcations resulting in a chaotic attractor. At  $Re \approx 785.93$  the attractor is destroyed in a boundary crisis bifurcation[7]. Above this Reynolds number the lifetimes of the initial conditions show an exponential scaling. In figure 56 the bifurcation diagram close to the two crises is shown. For the bifurcation diagram we plot the minima of the energy-norm of the trajectories.

In addition to our detailed study of the small domains with restricted symmetry, we investigate the origin of the turbulence for PPF without symmetry restriction. For the system without symmetry restriction we show that the two observable transition scenarios, the bypass- and the TS-transition, can be related to two different travelling wave edge states.



**Figure 56.** Bifurcation diagram close to the boundary and interior crisis. The red line marks the position of the interior crisis and the blue line the position of the boundary crisis.

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