Second-Order Blind Separation of First- and Second-Order Cyclostationary Sources—Application to AM, FSK, CPFSK, and Deterministic Sources

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***Abstract*—Most of the second-order (SO) and higher order (HO) blind source separation (BSS) methods developed this last decade aim at blindly separating statistically independent sources that are assumed zero-mean, stationary, and ergodic. Nevertheless, in many situations of practical interest, such as in radiocommunications contexts, the sources are nonstationary and very often cyclostationary (digital modulations). The behavior of the current SO and fourth-order (FO) cumulant-based BSS methods in the presence of cyclostationary sources has been analyzed, recently, in a previous paper by Ferréol and Chevalier, assuming zero-mean sources. However, some cyclostationary sources used in practical situations are not zero-mean but have a first-order (FIO) cyclostationarity property, which is, in particular, the case for some amplitude modulated (AM) signals and for some nonlinearly modulated digital sources such as frequency shift keying (FSK) or some continuous phase frequency shift keying (CPFSK) sources. For such sources, the results presented in the previous paper by Ferréol and Chevalier no longer hold, and the purpose of this paper is to analyze the behavior and to propose adaptations of the current SO BSS methods for sources that are both FIO and SO cyclostationary and cyclo-ergodic. An extension for deterministic sources is also proposed in the paper.**

***Index Terms*—AM, blind, deterministic sources, first-order cyclostationary, FSK and CPFSK sources, second order, SOBI, source separation.**

I. Introduction

FOR more than a decade, blind source separation (BSS) methods exploiting either the second-order (SO) [3] or the higher order (HO) [16] or both the SO and HO [5], [13] statistics of the data have been strongly developed, as depicted in the overview presented in [14]. These methods aim at blindly separating several statistically independent sources that are assumed to be *zero-mean*, *stationary*, and *ergodic*. Nevertheless, in many applications, such as in the radiocommunications context, the sources are nonstationary and very often *cyclostationary* (digital modulations). Under these conditions, it becomes important to analyze the behavior of these current SO and HO blind methods that have been developed for zero-mean stationary sources in the presence of cyclostationary sources whose cyclostationarity property appears explicitly at the processing level as soon as the sources are oversampled. This is generally the case for numerous applications such as, for example, the passive listening context, where the different sources may have very different baud rate or bandwidth [17].

The behavior of the current SO and fourth-order (FO) cumulant-based blind source separation methods in the presence of cyclostationary sources has been analyzed in a recent paper [19], assuming *zero-mean* sources. Under this last assumption, which is valid in particular for linearly modulated digital sources, it has been shown in particular that under weak conditions of cyclo-ergodicity [4], the current SO blind methods are not affected by the cyclostationarity of the sources. On the contrary, the current FO cumulant-based blind methods, such as the JADE method [5], have been shown to be strongly affected, in some cases, by the cyclostationarity property of the sources and an adaptation of these FO methods, taking into account the SO cyclic frequencies of the sources has been proposed. A FO alternative approach aiming at blindly separating statistically independent zero-mean cyclostationary sources with no knowledge or estimation of the cyclic frequencies of the sources has been proposed recently in [24]. Finally, other approaches of blind spatial filtering or blind source separation of zero-mean cyclostationary sources, aiming, in this case, at recovering the sources signals directly from the cyclic statistics of the observations, have also been proposed in the literature at both the SO [1], [2], [25] and the HO [6], [18].

However, the cyclostationary sources used in practical applications are not necessarily zero-mean but may be *first-order (FIO) cyclostationary*, which is, in particular, the case for some amplitude modulated (AM) sources [21] and for some nonlinearly modulated digital sources such as frequency shift keying (FSK) sources [31] or some continuous phase frequency shift keying (CPFSK) sources, which belong to the more general family of the so-called continuous phase modulation (CPM) sources [23], [28], [31], [32]. For such sources, the analysis presented in [19] no longer applies, and for this reason, the purpose of this paper is to analyze the behavior and to propose adaptations of the current SO blind source separation methods in the presence of statistically independent sources that are both FIO and SO cyclostationary. An extension for polyperiodic deterministic sources is also proposed in the paper. The behavior analysis of the current HO blind methods in the same context is partially presented in [20].

The current SO BSS problem for zero-mean stationary independent narrowband (NB) sources together with the second-order blind identification (SOBI) algorithm [3], and the empirical estimator of the SO statistics of the data are recalled in Section II. Then, the problem of SO blind separation of FIO

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and SO cyclostationary sources together with examples of such sources (some AM, FSK, and some CPFSK sources) are presented in Section III, where, in particular, the limitations of the empirical estimator of the SO statistics for nonzero mean sources are pointed out. The situations for which these limitations may have bad consequences on the behavior of the current SO BSS methods, jointly with the behavior description of the latter, are presented in Section IV, where it is shown in particular that the performance of the SOBI method may be strongly affected by the FIO cyclostationary properties of the sources. To overcome this problem, an adaptation of the current SO blind methods taking into account the possible FIO cyclostationarity of the sources is proposed in Section V. Unfortunately, this adaptation does not allow the processing of deterministic sources, and to overcome this drawback, an extension of this adaptation, called second-order blind extraction of FIO cyclostationary sources (SOBEFOCYS), is described in Section VI. Most of the results presented in the paper are finally illustrated in Section VII by computer simulations. Note that the results presented in this paper have already been partially presented in [10] and [11].

II. SO BSS for Zero Mean Stationary Sources

A. Problem Formulation

In the classical SO BSS problem, a noisy mixture of P zero-mean, stationary, and narrowband (NB) independent sources is assumed to be received by an array of N sensors. Under this assumption, the vector $\boldsymbol{x}(t)$ of the complex envelopes of the signals present at time t at the output of the sensors can be written as

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_p(t)\boldsymbol{a}_p + \boldsymbol{b}(t) \triangleq A\boldsymbol{m}(t) + \boldsymbol{b}(t) \tag{1}$$

where $\boldsymbol{b}(t)$ is the noise vector, which is assumed zero-mean, stationary, ergodic, circular, and spatially white, $m_p(t)$ and $\boldsymbol{a}_p$ correspond to the complex envelope and the steering vector of the source p, respectively, $\boldsymbol{m}(t)$ is the vector whose components are the signals $m_p(t)$, and A is the $(N \times P)$ matrix whose columns are the vectors $\boldsymbol{a}_p$.

Under these assumptions, the classical SO BSS problem consists of finding, from the SO statistics of the observations, the $(N \times P)$ *linear* and *time-invariant (TI)* source separator W, whose $(P \times 1)$ output vector

$$\boldsymbol{y}(t) \triangleq W^{\dagger}\boldsymbol{x}(t) \tag{2}$$

corresponds, to within a diagonal matrix Λ and a permutation matrix Π, to the best estimate $\hat{\boldsymbol{m}}(t)$ of the vector $\boldsymbol{m}(t)$. Note that the symbol † means transpose and complex conjugate. The separator W is defined to within a diagonal and a permutation matrix since neither the value of each output power of the separator nor the order in which the outputs are arranged change the estimation quality of the sources.

B. SO Statistics of the Data

Under the previous assumptions, the SO statistics of the data are characterized by the correlation matrices $R_x(\tau)$, which also correspond to SO cumulant matrices, defined by

$$\begin{aligned} R_x(\tau) &\triangleq \mathrm{E}\left[\boldsymbol{x}(t)\boldsymbol{x}(t-\tau)^{\dagger}\right] \\ &= AR_m(\tau)A^{\dagger} + \eta_2(\tau)\mathrm{I} \\ &\triangleq R_s(\tau) + \eta_2(\tau)\mathrm{I} \end{aligned} \tag{3}$$

where $\eta_2(\tau)$ is the SO correlation function of the noise on each sensor, I is the identity matrix, the $R_m(\tau) \triangleq \mathrm{E}[\boldsymbol{m}(t)\boldsymbol{m}(t-\tau)^{\dagger}]$ diagonal under the previous hypotheses is the correlation matrix of the vector $\boldsymbol{m}(t)$, and $R_s(\tau) \triangleq AR_m(\tau)A^{\dagger}$ is the correlation matrix of the mixed sources.

C. Philosophy of the SO BSS Methods (SOBI)

Let us now briefly recall the philosophy of the SOBI [3] method, which can be considered currently, for zero-mean stationary sources, as the most powerful SO BSS method but requires that the sources have different spectral densities. This separator aims at separating the received sources from the blind identification of their steering vectors. These identified steering vectors may then be used to build and to apply to the data, for each source, a well-suited spatial filter such as the spatial matched filter or the optimal interference canceller [8], [9]. This blind identification requires the prewhitening of the data, by the pseudo-inverse, noted F, of a square root of the matrix $R_s(0)$, noted R_s in the following, computed from the $R_x \triangleq R_x(0)$ matrix and the knowledge of the noise correlation matrix. Usually, the matrix F is chosen to be equal to the $(P \times N)$ matrix $F = \Lambda_s^{-1/2}U_s^{\dagger}$, where the $(N \times P)$ matrix U_s and the $(P \times P)$ diagonal matrix Λ_s correspond to the matrices of the orthonormalized eigenvectors and associated nonzero eigenvalues of R_s, respectively. This prewhitening operation aims at orthonormalizing the sources steering vectors to search for the latter through a unitary matrix U, which is simpler to handle. If we note that $\boldsymbol{z}(t) \triangleq F\boldsymbol{x}(t)$ is the whitened observation vector, the matrix U is chosen to optimize an SO criterion, which is function of the elements of the correlation matrices $R_z(\tau)$ of the vector $z(t)$ for several nonzero values τ_q of τ. The matrix $R_z(\tau)$ can be easily computed from (3) and is given by

$$\begin{aligned} R_z(\tau) &\triangleq \mathrm{E}\left[\boldsymbol{z}(t)\boldsymbol{z}(t-\tau)^{\dagger}\right] \\ &= A'R_{m'}(\tau){A'}^{\dagger} + \eta_2(\tau)FF^{\dagger} \\ &\triangleq R_{s'}(\tau) + \eta_2(\tau)FF^{\dagger} \end{aligned} \tag{4}$$

where A' is the $(P \times P)$ unitary matrix of the whitened sources steering vectors $\boldsymbol{a}'_i \triangleq \pi_i^{1/2}F\boldsymbol{a}_i (1 \leq i \leq P)$, $\pi_i \triangleq E[|m_i(t)|^2]$ is the input power of the source i, $R_{m'}(\tau) \triangleq \mathrm{E}[\boldsymbol{m}'(t)\boldsymbol{m}'(t-\tau)^{\dagger}]$ corresponds to the correlation matrix of $\boldsymbol{m}'(t)$, which is the normalized vector $\boldsymbol{m}(t)$ such that each component has a unit power, and $R_{s'}(\tau) \triangleq A'R_{m'}(\tau)A'^{\dagger}$ is the correlation matrix of the whitened mixed sources.

Assuming that the sources have not the same spectral density and, for simplicity, that the coefficients $r'_i(\tau) \triangleq \mathrm{E}[m'_i(t)m'_i(t-\tau)^*](1 \leq i \leq P)$ are not zero for the considered value of τ, where $*$ means complex conjugate and $m'_i(t)$ is the normalized complex envelope $m_i(t)$, the SOBI method [3] is based on the fact that the P orthonormalized vectors $\boldsymbol{a}'_i (1 \leq i \leq P)$ are eigenvectors of the $R_{s'}(\tau)$ matrix associated with its P nonzero eigenvalues $r'_i(\tau)$, which also correspond to the P eigenvalues of $R_{s'}(\tau)$ having the greatest absolute value. Then, an arbitrary eigenvector $\boldsymbol{v}$ of $R_{s'}(\tau)$ associated with a nonzero eigenvalue is necessarily a linear combination of the P vectors $\boldsymbol{a}'_i$. Under these conditions, it is easy to verify [3] that the unitary matrix A' is, to within a permutation and a unitary diagonal matrix, the only one that jointly diagonalizes the set of Q matrices $R_{s'}(\tau_q)(1 \leq q \leq Q)$ provided that, for each couple (i, j) of sources, there is at least a τ_q such that $r'_i(\tau_q) \neq r'_j(\tau_q)$. In other words, the unitary matrix A' maximizes, with respect to the unitary matrix variable $U \triangleq (\boldsymbol{u}_1, \ldots, \boldsymbol{u}_P)$, the following joint diagonalization criterion [3]:

$$C(U) = \sum_{q=1}^{Q} \sum_{l=1}^{P} \left| \boldsymbol{u}_l{}^{\dagger} R_{s'}(\tau_q) \boldsymbol{u}_l \right|^2 \tag{5}$$

Nevertheless, as the matrix $R_{s'}(\tau_q)$ is not observable, it must theoretically be estimated from the observable matrix $R_z(\tau_q)$. Under a temporally white noise assumption, which is done in [3], the quantity $\eta_2(\tau)$ is zero for $\tau \neq 0$, and the matrix $R_{s'}(\tau_q)$ can be replaced by the matrix $R_z(\tau_q)$ in (5). However, in practical situations, the reception band is finite, and the noise can only be assumed temporally white within the reception band. Under these conditions, if the reception band is sufficiently high with respect to the bandwidth of the sources, it is possible to find τ_q such that $R_{s'}(\tau_q) \neq 0$ and $\eta_2(\tau_q) \approx 0$, which still allows the use of $R_z(\tau_q)$ instead of $R_{s'}(\tau_q)$ in (5), which is done in the following.

D. Implementation of the SO BSS

In situations of practical interests, the SO statistics of the data are not known *a priori* and have to be estimated from the data by temporal averaging operations using the SO ergodicity property of the data. Noting that T_e is the sample period and $\boldsymbol{x}(k)$ the kth sample of the observation vector $\boldsymbol{x}(t)$, the empirical estimator $\hat{R}_x(qT_e)(K)$ currently used to estimate the matrix $R_x(\tau)$, for $\tau = qT_e$, from K data snapshots is defined by

$$\hat{R}_x(qT_e)(K) \triangleq \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{x}(k) \boldsymbol{x}(k-q)^{\dagger}. \tag{6}$$

It is well known that for a stationary and SO ergodic vector $\boldsymbol{x}(t)$, the empirical estimator $\hat{R}_x(qT_e)(K)$ generates, as K becomes infinite, an unbiased and consistent estimate of $R_x(qT_e)$.

III. SO Blind Source Separation for FIO and SO Cyclostationary Sources

A. Problem Formulation

In many applications such as in radiocommunications or in passive listening contexts, the received sources are very often cyclostationary (digital modulations) with a potential carrier residu (passive listening). Under these conditions, the observation model (1) currently used in stationary contexts becomes too restrictive, and we must adopt, for the complex observation vector $\boldsymbol{x}(t)$, the following model [19]:

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_p(t) \mathrm{e}^{\mathrm{j}(2\pi \Delta f_p t + \phi_p)} \boldsymbol{a}_p + \boldsymbol{b}(t) \triangleq A \boldsymbol{m}_c(t) + \boldsymbol{b}(t) \tag{7}$$

where a noisy mixture of P first and SO cyclostationary, cyclo-ergodic, and NB independent sources is assumed to be received by the array of N sensors. In (7), the vector $\boldsymbol{b}(t)$ is the noise vector, which is assumed to be zero-mean, stationary, circular and spatially white, $m_p(t)$, Δf_p, ϕ_p, and $\boldsymbol{a}_p$ correspond to the complex envelope, NB, first and SO cyclostationary, the carrier residue, the phase and the steering vector of the source p, respectively, $\boldsymbol{m}_c(t)$ is the vector whose components are the signals $m_{pc}(t) \triangleq m_p(t) \exp[j(2\pi \Delta f_p t + \phi_p)]$, and A is the $(N \times P)$ matrix whose columns are the vectors $\boldsymbol{a}_p$. To simplify the developments, we limit the analysis to instantaneous mixtures of sources, which is typical of some applications such as the spatial telecommunications, some high data rate line-of-sight (LOS) contexts, or some airborne electronic warfare applications.

Under the previous assumptions, although in the presence of cyclostationary sources it may be useful to use *polyperiodic* (PP) [12] and, for noncircular sources [29], *widely linear* [7], [30] structures of separation, one may still prefer to try to recover the sources through a *linear* and *TI* structure of separation, which is easier to handle. Under these conditions, the problem is to find, from the SO statistics of the data, the $(N \times P)$ *linear* and *TI* source separator W, whose output vector (2) aims at corresponding, to within a diagonal matrix Λ and a permutation matrix Π, to the best estimate $\hat{\boldsymbol{m}}_c(t)$ of the vector $\boldsymbol{m}_c(t)$.

B. FIO and SO Statistics of the Data

1) FIO Statistics: In the presence of FIO cyclostationary sources, the FIO statistic of the vector $\boldsymbol{x}(t)$, which is given by (7), can be written as

$$\boldsymbol{e}_x(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)] = \sum_{p=1}^{P} e_p(t) \mathrm{e}^{\mathrm{j}(2\pi \Delta f_p t + \phi_p)} \boldsymbol{a}_p$$
$$\triangleq \sum_{p=1}^{P} e_{pc}(t) \boldsymbol{a}_p \triangleq A \boldsymbol{e}_{mc}(t) \tag{8}$$

where $e_p(t)$, $e_{pc}(t)$, and $\boldsymbol{e}_{mc}(t)$ are the expected values of $m_p(t)$, $m_{pc}(t) = m_p(t) \mathrm{e}^{\mathrm{j}(2\pi \Delta f_p t + \phi_p)}$, and $\boldsymbol{m}_c(t)$, respectively. The quantities $e_p(t)$ and $e_{pc}(t)$ have a Fourier serial expansion, and we obtain

$$e_p(t) \triangleq \mathrm{E}[m_p(t)] = \sum_{\gamma_p \in \Gamma_p} e_p^{\gamma_p} \mathrm{e}^{\mathrm{j}2\pi \gamma_p t} \tag{9}$$

$$e_{pc}(t) \triangleq \mathrm{E}[m_{pc}(t)] = \sum_{\gamma_{pc} \in \Gamma_{pc}} e_{pc}^{\gamma_{pc}} \mathrm{e}^{\mathrm{j}2\pi \gamma_{pc} t}$$
$$= \sum_{\gamma_p \in \Gamma_p} e_p^{\gamma_p} \mathrm{e}^{\mathrm{j}[2\pi(\Delta f_p + \gamma_p)t + \phi_p]} \tag{10}$$

where $\Gamma_p = \{\gamma_p\}$ and $\Gamma_{pc} = \{\gamma_{pc} = \gamma_p + \Delta f_p\}$ are the set of cyclic frequencies γ_p and γ_{pc} of $e_p(t)$ and $e_{pc}(t)$, respectively, and $e_p^{\gamma_p}$ and $e_{pc}^{\gamma_{pc}}$ are what we call the cyclic mean of $m_p(t)$

and $m_{pc}(t)$, respectively, for the cyclic frequencies γ_p and γ_{pc}, respectively, which are defined by

$$e_p^{\gamma_p} = \left\langle e_p(t)\mathrm{e}^{-\mathrm{j}2\pi\gamma_p t}\right\rangle_c \tag{11}$$

$$e_{pc}^{\gamma_{pc}} \triangleq \left\langle e_{pc}(t)\mathrm{e}^{-\mathrm{j}2\pi\gamma_{pc} t}\right\rangle_c = e_p^{\gamma_{pc}-\Delta f_p}\mathrm{e}^{\mathrm{j}\phi_p} \tag{12}$$

where the symbol $\langle f(t)\rangle_c \triangleq \lim_{T\to\infty}(1/T)\int_{-T/2}^{T/2} f(t)dt$ corresponds to the continuous-time temporal mean operation of $f(t)$ over an infinite observation interval. Note that for a zero mean source p, the quantities $e_p^{\gamma_p}$ and $e_{pc}^{\gamma_{pc}}$ are zero for all the cyclic frequencies γ_p and γ_{pc}, respectively. Besides, for a stationary source p that is not zero mean, only the quantity $e_{pc}^{\gamma_{pc}}$ for $\gamma_{pc}=0$ is not zero. A consequence of the previous results is that the vectors $\boldsymbol{e}_{mc}(t)$ and $\boldsymbol{e}_x(t)$ also have a Fourier serial expansion, and using (10) into (8), we obtain

$$\boldsymbol{e}_{mc}(t) \triangleq \sum_{\gamma\in\Gamma} \boldsymbol{e}_{mc}^{\gamma}\mathrm{e}^{\mathrm{j}2\pi\gamma t} \tag{13}$$

$$\begin{aligned}\boldsymbol{e}_x(t) &\triangleq \sum_{\gamma\in\Gamma} \boldsymbol{e}_x^{\gamma}\mathrm{e}^{\mathrm{j}2\pi\gamma t} = \sum_{\gamma\in\Gamma} A\boldsymbol{e}_{mc}^{\gamma}\mathrm{e}^{\mathrm{j}2\pi\gamma t}\\ &= \sum_{p=1}^{P}\sum_{\gamma_{pc}\in\Gamma_{pc}} e_{pc}^{\gamma_{pc}}\mathrm{e}^{\mathrm{j}2\pi\gamma_{pc}t}\boldsymbol{a}_p\end{aligned} \tag{14}$$

where $\Gamma = \cup_{1\le p\le \mathrm{P}}\{\Gamma_{pc}\}$ is the set of cyclic frequencies γ of $\boldsymbol{e}_{mc}(t)$ and $\boldsymbol{e}_x(t)$, the vectors $\boldsymbol{e}_{mc}^{\gamma}$ and $\boldsymbol{e}_x^{\gamma}$ are the cyclic mean of $\boldsymbol{m}_c(t)$ and $\boldsymbol{x}(t)$, respectively, defined by

$$\boldsymbol{e}_{mc}^{\gamma} = \left\langle \boldsymbol{e}_{mc}(t)\mathrm{e}^{-\mathrm{j}2\pi\gamma t}\right\rangle_c \tag{15}$$

$$\boldsymbol{e}_{x}^{\gamma} = \left\langle \boldsymbol{e}_{x}(t)\mathrm{e}^{-\mathrm{j}2\pi\gamma t}\right\rangle_c. \tag{16}$$

Under these assumptions, the first and SO cyclostationary vector $\boldsymbol{x}(t)$ can be decomposed as the sum of a deterministic (quasi)-periodic part $\boldsymbol{e}_x(t)$ and a zero-mean (quasi)-cyclostationary random part $\Delta\boldsymbol{x}(t)$ such that

$$\Delta\boldsymbol{x}(t) \triangleq \boldsymbol{x}(t) - \boldsymbol{e}_x(t) = A\Delta\boldsymbol{m}_c(t) + \boldsymbol{b}(t) \tag{17}$$

where $\boldsymbol{\Delta m}_c(t) \triangleq \boldsymbol{m}_c(t) - \boldsymbol{e}_{mc}(t)$ is the zero-mean vector of the source signals, with components $\Delta m_{pc}(t) \triangleq \Delta m_p(t)\mathrm{e}^{\mathrm{j}(2\pi\Delta f_p t+\phi_p)}$, where $\Delta m_p(t) \triangleq m_p(t) - e_p(t)$.

2) SO Statistics: Under the previous assumptions, using (7) and (17), the first correlation matrix $R_x(t,\tau) \triangleq \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t-\tau)^\dagger]$ of the data, which is time dependent, can be written as

$$\begin{aligned}R_x(t,\tau) &= AR_{mc}(t,\tau)A^\dagger + \eta_2(\tau)\mathrm{I}\\ &= R_{\Delta x}(t,\tau) + \boldsymbol{e}_x(t)\boldsymbol{e}_x(t-\tau)^\dagger\end{aligned} \tag{18}$$

where $\eta_2(\tau)$ is the SO correlation function of the noise on each sensor, I is the identity matrix, $R_{mc}(t,\tau) \triangleq \mathrm{E}[\boldsymbol{m}_c(t)\boldsymbol{m}_c(t-\tau)^\dagger]$ is the first correlation matrix of the vector $\boldsymbol{m}_c(t)$, and $R_{\Delta x}(t,\tau)$ is a cumulant matrix corresponding to the covariance matrix of $\boldsymbol{x}(t)$, which is defined by

$$R_{\Delta x}(t,\tau) \triangleq \mathrm{E}\left[\Delta\boldsymbol{x}(t)\Delta\boldsymbol{x}(t-\tau)^\dagger\right] = AR_{\Delta mc}(t,\tau)A^\dagger + \eta_2(\tau)\mathrm{I} \tag{19}$$

where $R_{\Delta mc}(t,\tau) \triangleq \mathrm{E}[\boldsymbol{\Delta m}_c(t)\boldsymbol{\Delta m}_c(t-\tau)^\dagger]$ is the covariance matrix of $\boldsymbol{m}_c(t)$. Using (8) and (19) into (18), we finally obtain

$$R_x(t,\tau) = A\left[R_{\Delta mc}(t,\tau) + \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t-\tau)^\dagger\right]A^\dagger + \eta_2(\tau)\mathrm{I}. \tag{20}$$

The SO cyclostationary property of the sources implies that the matrices $R_{mc}(t,\tau)$ and $R_{\Delta mc}(t,\tau)$ and, thus, the matrices $R_x(t,\tau)$ and $R_{\Delta x}(t,\tau)$, have Fourier serial expansions introducing the SO cyclic frequencies and statistics of $\boldsymbol{m}_c(t)$, $\boldsymbol{\Delta m}_c(t)$, $\boldsymbol{x}(t)$, and $\boldsymbol{\Delta x}(t)$, respectively. In particular, the first cyclic correlation matrix of $\boldsymbol{x}(t)$ for the zero cyclic frequency corresponds to the temporal mean of $R_x(t,\tau)$, which can be written, from (18), as

$$R_x(\tau) \triangleq \langle R_x(t,\tau)\rangle_c = AR_{mc}(\tau)A^\dagger + \eta_2(\tau)\mathrm{I} \triangleq R_s(\tau) + \eta_2(\tau)\mathrm{I} \tag{21}$$

where $R_s(\tau) \triangleq AR_{mc}(\tau)A^\dagger$, and where, from (20), $R_{mc}(\tau) \triangleq \langle R_{mc}(t,\tau)\rangle_c$ is given by

$$\begin{aligned}R_{mc}(\tau) &\triangleq \langle R_{mc}(t,\tau)\rangle_c\\ &= R_{\Delta mc}(\tau) + \left\langle \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t-\tau)^\dagger\right\rangle_c\\ &= R_{\Delta mc}(\tau) + E_{mc}(\tau)\end{aligned} \tag{22}$$

where $R_{\Delta mc}(\tau) \triangleq \langle R_{\Delta mc}(t,\tau)\rangle_c$, and $E_{mc}(\tau) \triangleq \langle \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t-\tau)^\dagger\rangle_c$.

For observations $\boldsymbol{x}(nT_e)$, which are sampled at the sample period T_e, the matrices $R_x(t,\tau)$ and $R_x(\tau)$ are defined only for the time instants $t = nT_e$ and $\tau = kT_e$, which are multiple of this sample period. Under these conditions, it is possible to show [19], [27] that, for bandlimited sources, the matrice $R_x(kT_e)$ defined by (21) for $\tau = kT_e$ can be computed only from the sampled matrices $R_x(nT_e, kT_e)$ instead of $R_x(t,\tau)$, provided that the data are sufficiently oversampled. In other words, for sufficiently oversampled data, it is possible to replace, in (21), the continuous-time temporal mean operation $\langle f(t)\rangle_c$ by a discrete one over an infinite number of samples $\langle f(nT_e)\rangle_d \triangleq \lim_{K\to\infty}(1/K)\sum_{k=1}^{K} f(k)$.

C. Two Examples of FIO Cyclostationary Sources: AM and FSK Sources

1) AM Sources: The AM sources [21] are amplitude modulated analog sources used in particular in old radio communications systems. If the source p is an AM source, its complex envelope can be written as

$$m_p(t) = \beta_p\left(1 + \mu_p l_p(t)\right) \tag{23}$$

where β_p is a scalar, $\mu_p (0\le\mu_p\le 1)$ is the modulation indice, and $l_p(t)$, such that $0\le|l_p(t)|\le 1$, is, in the general case, a nonzero mean nonstationary random signal. Defining $el_p(t) \triangleq \mathrm{E}[l_p(t)]$, the statistical mean of $m_p(t)$ is then given by

$$e_p(t) = \beta_p\left(1 + \mu_p el_p(t)\right) \tag{24}$$

which is, in the general case, a nonzero time-dependent function. The signal $m_p(t)$ becomes FIO cyclostationary for the particular case of FIO cyclostationary signals $l_p(t)$. In this latter case, $e_p(t)$ has a Fourier serial expansion (9), and we can easily verify that the cyclic mean e_p^{γ} is given by

$$e_p^{\gamma} = \beta_p\left(\delta(\gamma) + \mu_p el_p^{\gamma}\right) \tag{25}$$

where $\delta(.)$ is the Kronecker symbol and where $el_p^{\gamma} \triangleq \langle el_p(t)e^{-\mathrm{j}2\pi\gamma t}\rangle_c$. Thus, the FIO cyclic frequencies of an AM source p correspond to the zero cyclic frequency and to the FIO cyclic frequencies of $l_p(t)$.

2) FSK Sources: The FSK sources [31] are nonlinearly modulated digital sources used, in particular, for low data rate radio communications in the HF band. If the source p is an FSK source, its complex envelope can be written as

$$m_p(t) = {\pi_p}^{\frac{1}{2}} \sum_n \exp\left\{\mathrm{j}\left[\theta_{pn} + 2\pi f_{dp} a_n^p (t - nT_p)\right]\right\} \times \mathrm{Rect}_p(t - nT_p) \tag{26}$$

where $\pi_p \triangleq \langle \mathrm{E}[|m_p(t)|^2]\rangle_c$ is the input power of the source p, T_p is the symbol duration, the a_n^p are the transmitted M_p-ary symbols that are assumed i.i.d and taking their values in the alphabet $\pm 1, \pm 3, \ldots, \pm(M_p - 1)$, where M_p is generally a power of two, f_{dp} is the peak frequency deviation, $\mathrm{Rect}_p(t)$ is the rectangular pulse of amplitude 1 and of duration T_p, and θ_{pn} is the phase of the symbol n. Note that for M_p-ary symbols, the associated FSK source p is called an M_p-FSK source p.

When the signal $m_p(t)$ is built from only one local oscillator that is hopping from one frequency to another, at every symbol period T_p, the phases θ_{pn} may be considered to be i.i.d random variables that are statistically independent of the symbols a_n^p and uniformly distributed between 0 and 2π. In this case, it is easy to verify that $m_p(t)$ is zero mean. However, when the signal $m_p(t)$ is built from M_p local oscillators, one oscillator per symbol value, among which one oscillator is switched at every symbol period T_p, the phase θ_{pn} of the symbol n corresponds to the phase of the switched oscillator for this symbol n and becomes a function $\theta_{pn}(a_n^p)$ of the symbol n value, taking its values in the alphabet $\{\theta_{-p(Mp-1)}, \ldots, \theta_{-p1}, \theta_{p1}, \theta_{p3}, \ldots, \theta_{p(Mp-1)}\}$. In this case, the statistical mean of $m_p(t)$ is given by (27), shown at the bottom of the page, which corresponds to a periodic function of t with a period T_p. In other words, the associated M_p-FSK source p is an FIO cyclostationary source p with FIO cyclic frequencies γ_p's multiple of $1/T_p$. Under these conditions, $e_p(t)$ has a Fourier serial expansion (9), and the cyclic mean e_p^γ is shown in Appendix A to be given by

$$e_p^\gamma = \sum_{\mathrm{i}} e_p^{\gamma_{pi}} \delta\left(\gamma - \frac{i}{T_p}\right) \tag{28}$$

where $e_p^{\gamma_{pi}}$, which is the cyclic mean for the cyclic frequency $\gamma_{pi} = i/T_p$, is given by (29), shown at the bottom of the page.

In particular, we deduce from (29) that an M_p-FSK source such that the product $f_{dp}T_p$ is an integer has exactly M_p equal power FIO cyclic frequencies $\gamma_p = \pm(2k+1)f_{dp}$, $0 \le k \le (M_p - 2)/2$, such that

$$e_p^\gamma = {\pi_p}^{\frac{1}{2}} \frac{1}{M_p} \sum_{m=0}^{\frac{(M_p-2)}{2}} \left[\exp\left\{\mathrm{j}\theta_{p(2m+1)}\right\} \delta\left(\gamma - (2m+1)f_{dp}\right) + \exp\left\{\mathrm{j}\theta_{-p(2m+1)}\right\} \delta\left(\gamma + (2m+1)f_{dp}\right)\right]. \tag{30}$$

D. Third Example of FIO Cyclostationary Sources: Some CPFSK Sources

1) CPFSK Sources as a Particular Case of CPM Sources: The CPM sources [28], [31] are nonlinearly modulated digital sources used in many applications of practical interest such as the mobile cellular radio communications (GSM) or the spatial telecommunications. One characteristic of CPM sources is that their spectral efficiency is much better than that of nonlinear modulations without a continuous phase. Another property is that their complex envelope has a constant amplitude, which allows the use of cheap amplifiers working at a saturation level without any distorsion on the transmitted information. If the source p is a CPM source, its complex envelope can be written as

$$m_p(t) = {\pi_p}^{\frac{1}{2}} \exp\left\{\mathrm{j}2\pi\left[\sum_n a_n^p h_n^p v_p(t - nT_p)\right]\right\} \tag{31}$$

where $\pi_p \triangleq \langle \mathrm{E}[|m_p(t)|^2]\rangle_c$ is the input power of the source p, T_p is the symbol duration, the a_n^p are the transmitted M_p-ary symbols that are assumed i.i.d and taking their values in the alphabet $\pm 1, \pm 3, \ldots, \pm(M_p - 1)$, where M_p is generally a power of two, $\{h_n^p\}$ is a sequence of modulation indices, and $v_p(t)$ is the waveform shape that is represented as the integral of a pulse $g_p(t)$ that is nonzero and bounded on the interval [0, L_pT_p], where L_p is a nonzero integer, and such that

$$v_p(t) = \int_{-\infty}^{t} g_p(u)\mathrm{d}u = \left\{\begin{matrix} \frac{1}{2}(t \ge L_pT_p) \\ 0\ (t < 0) \end{matrix}\right\}. \tag{32}$$

When $h_n^p = h_p$ for all n, the modulation is said to be mono-indice; otherwise, the modulation is qualified by multi-indices. When $L_p = 1$, the CPM source p is called *full response CPM*; otherwise, it is called *partial response CPM*. To each choice of the pulse function $g_p(t)$, it corresponds a family of CPM source

$$e_p(t) = {\pi_p}^{\frac{1}{2}} \frac{1}{M_p} \sum_n \sum_{m=0}^{\frac{(M_p-2)}{2}} \left(\exp\left\{\mathrm{j}\left[\theta_{p(2m+1)} + 2\pi f_{dp}(2m+1)(t - nT_p)\right]\right\} + \exp\left\{\mathrm{j}\left[\theta_{-p(2m+1)} - 2\pi f_{dp}(2m+1)(t - nT_p)\right]\right\}\right) \mathrm{Rect}_p(t - nT_p) \tag{27}$$

$$e_p^{\gamma_{pi}} = {\pi_p}^{\frac{1}{2}} \frac{1}{M_p} \sum_{m=0}^{\frac{(M_p-2)}{2}} \int_0^1 \left(\exp\left\{\mathrm{j}\left[\theta_{p(2m+1)} + 2\pi w\left(f_{dp}T_p(2m+1) - i\right)\right]\right\} + \exp\left\{\mathrm{j}\left[\theta_{-p(2m+1)} - 2\pi w\left(f_{dp}T_p(2m+1) + i\right)\right]\right\}\right) \mathrm{d}w \tag{29}$$

p. The GMSK modulation, which is the modulation of the GSM standard and for which the pulse function $g_p(t)$ has only an approximated finite duration, belongs to one of these families.

The CPFSK source p is a particular case of the mono-indice full response CPM source p for which the pulse $g_p(t)$ is a rectangular pulse of amplitude $1/2T_p$ and of duration T_p. For such sources p, it is possible to show, after easy computations, that $m_p(t)$ can be written as (26), where $f_{dp} \triangleq h_p/2T_p$ is the peak frequency deviation, and θ_{pn}, which represents the accumulation (memory) of all symbols up to $(n-1)T_p$, is defined by

$$\theta_{pn} \triangleq 2\pi f_{dp} T_p \sum_{k=-\infty}^{n-1} a_k^p. \tag{33}$$

For M_p-ary symbols, the associated CPFSK source p is called an M_p-CPFSK source p. Note that a binary CPFSK source $p(M_p=2)$ with a modulation index $h_p = 1/2$ is called a minimum shift keying (MSK) source p.

2) FIO Statistics of CPFSK Sources: It is shown in Appendix B that, under the previous assumptions, the statistical mean of $m_p(t)$ is, for an M_p-CPFSK source p, given by

$$e_p(t) = \pi_p{}^{\frac{1}{2}} \frac{2}{M_p} K_p \sum_n (\rho_p)^n u_p(t - nT_p) \tag{34}$$

where the quantities K_p, $\rho_p (0 \leq \rho_p \leq 1)$, and $u_p(t)$ are defined by

$$K_p \triangleq \lim_{l \to \infty} (\rho_p)^l \tag{35}$$

$$\rho_p \triangleq \frac{2}{M_p} \sum_{m=0}^{\frac{(M_p-2)}{2}} \cos\left[(2m+1)\pi h_p\right] \tag{36}$$

$$u_p(t) \triangleq \sum_{m=0}^{\frac{(M_p-2)}{2}} \cos\left[(2m+1)\left(\frac{\pi h_p}{T_p}\right) t\right] \mathrm{Rect}_p(t). \tag{37}$$

From the previous expressions, two cases have to be considered depending on the value of h_p. These cases correspond to the cases where h_p is an integer or not, respectively.

a) h_p Is not an Integer: If h_p is not an integer, it is obvious that $|\rho_p| < 1$, which implies the nullity of both K_p and $e_p(t)$. In other words, an M_p-CPFSK source p whose modulation indice is not an integer is a zero-mean source p.

b) h_p Is an Integer: If h_p is an integer, it is obvious that $\rho_p = 1$ if h_p is even and $\rho_p = -1$ if h_p is odd, which implies that $K_p = 1$ in the first case and that expression (35) has no limit $(K_p = \pm 1)$, whereas $|K_p| = 1$ in the second case. In this latter case, note that if we assume that the number of past symbols is finite, then $K_p = \pm 1$. A consequence of the previous results is that $e_p(t)$ reduces to

$$e_p(t) = \pi_p{}^{\frac{1}{2}} \frac{2}{M_p} K_p \sum_n (-1)^{nh_p} u_p(t - nT_p) \tag{38}$$

which corresponds to a periodic function of t with a period T_p if h_p is even and with a period $2T_p$ if h_p is odd. In other words, the associated M_p-CPFSK source p is an FIO cyclostationary source p with FIO cyclic frequencies γ_p's multiple of $1/T_p$ in the first case and multiple of $1/2T_p$ in the second case. Under these conditions, $e_p(t)$ has a Fourier serial expansion (9), and the cyclic mean e_p^γ is shown in Appendix B to be given by

$$e_p^\gamma = \pi_p{}^{\frac{1}{2}} \frac{1}{M_p} K_p \times \sum_{m=0}^{\frac{(M_p-2)}{2}} \left[\delta\left(\gamma - (2m+1) f_{dp}\right) + \delta\left(\gamma + (2m+1) f_{dp}\right)\right] \tag{39}$$

where $\delta(.)$ is the Kronecker symbol. Thus, a M_p-CPFSK source p whose modulation indice is an integer has exactly M_p equal power FIO cyclic frequencies γ_p such that

$$e_p^\gamma = \pi_p{}^{\frac{1}{2}} \frac{1}{M_p} K_p, \quad \text{for}$$
$$\gamma \in \Gamma_p = \left\{ \gamma_p = \pm(2k+1) f_{dp}, 0 \leq k \leq \frac{(M_p-2)}{2} \right\}. \tag{40}$$

Note that in this case, $\langle e_p(t) \rangle_c = 0$, despite the fact that $e_p(t) \neq 0$.

E. Problem Addressed in This Paper

For FIO and SO cyclostationary and bandlimited vectors $\boldsymbol{x}(t)$ having an SO cyclo-ergodicity property [4] and for sufficiently oversampled data, the empirical estimator $\hat{R}_x(lT_e)(K)$, which is defined by (6) with l instead of q, gives an asymptotically unbiased and consistent estimate of $R_x(lT_e)$, which is defined by (21) with $\tau = lT_e$, by definition of the cyclo-ergodicity property. In other words, in cyclostationary contexts, the SO BSS methods such as the SOBI method exploit, asymptotically or in the steady state, the information contained in several time averaged correlation matrices $R_x(\tau_q) (1 \leq q \leq Q)$, which are defined by (21).

However, while these matrices correspond to cumulant matrices for zero-mean stationary sources and to time-averaged cumulant matrices for zero-mean cyclostationary sources [19], it is no longer the case for FIO and SO cyclostationary sources for which $\boldsymbol{e}_{mc}(t) \neq \mathbf{0}$, $R_{mc}(\tau) \neq R_{\Delta mc}(\tau)$, and $R_x(\tau) \neq R_{\Delta x}(\tau)$, as shown by (8), (18), and (22). As a consequence, while, for zero-mean statistically independent sources, $R_{mc}(\tau)$ and $R_{\Delta mc}(\tau)$, appearing in (22), coincide and are diagonal, only the $R_{\Delta mc}(\tau)$ matrix keeps in all cases a diagonal structure for nonzero mean sources by definition of the statistical independence of the sources, whereas the matrix $R_{mc}(\tau)$ may loose its diagonal character. In this latter case, if the element $[i, j]$, $R_{mc}(\tau)[i,j]$, of the matrix $R_{mc}(\tau)$, with $i \neq j$, is not zero, we will say that the FIO cyclostationarity of the statistically independent sources i and j creates an *apparent SO correlation* of the sources in the $R_{mc}(\tau)$ and $R_x(\tau)$ matrices. This apparent SO correlation is directly related to the so-called impure SO cycle frequencies of the SO statistics of the sources discussed in [22].

In this context, we must first must identify the conditions that two statistically independent FIO and SO cyclostationary sources have to be verified to create an apparent SO correlation in the $R_x(\tau)$ matrices. Then, we must analyze the consequences of such an apparent SO correlation between two sources on the output performances of the current SO BSS methods such as the SOBI one. These two questions are adressed in Section IV.

IV. Behavior Analysis of Current SO BSS Methods for FIOand SO Cyclostationary Sources

A. Structure Analysis of the $E_{mc}(\tau)$ Matrix

The matrix $R_{mc}(\tau)$ is not diagonal if and only if the matrix $E_{mc}(\tau)$ introduced in (22) is not diagonal. Thus, two sources i and j become apparently SO correlated in the $R_x(\tau)$ matrix if and only if the element $[i,j]$, $E_{mc}(\tau)[i,j]$ of the matrix $E_{mc}(\tau)$ is not zero, which is a situation that is analyzed in this section.

1) General Case: Using (13) and (22), the $E_{mc}(\tau)$ matrix can be written as

$$\begin{aligned} E_{mc}(\tau) &\triangleq \left\langle \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t-\tau)^{\dagger}\right\rangle_c \\ &= \sum_{\gamma\in\Gamma}\sum_{\omega\in\Gamma} \boldsymbol{e}_{mc}^{\gamma}\boldsymbol{e}_{mc}^{\omega\,\dagger}\mathrm{e}^{\mathrm{j}2\pi\tau\omega}\left\langle \mathrm{e}^{\mathrm{j}2\pi(\gamma-\omega)t}\right\rangle_c \end{aligned} \tag{41}$$

and using the fact that $\langle \mathrm{e}^{\mathrm{j}2\pi\alpha t}\rangle_c = \delta(\alpha)$, we obtain

$$E_{mc}(\tau) = \sum_{\gamma\in\Gamma} \boldsymbol{e}_{mc}^{\gamma}\boldsymbol{e}_{mc}^{\gamma\,\dagger}\mathrm{e}^{\mathrm{j}2\pi\tau\gamma}. \tag{42}$$

The element $[i,j]$ of the matrix $E_{mc}(\tau)$ is thus given by

$$E_{mc}(\tau)[i,j] = \sum_{\gamma_{ij}\in\Gamma_{ij}} e_{ic}^{\gamma_{ij}} e_{jc}^{\gamma_{ij}\,*}\mathrm{e}^{\mathrm{j}2\pi\tau\gamma_{ij}} \tag{43}$$

where $\Gamma_{ij} \triangleq \Gamma_{ic}\cap\Gamma_{jc}$ is the set of cyclic frequencies γ_{ij} belonging to both Γ_{ic} and Γ_{jc}, which are defined in (10), and e_{ic}^{γ} and e_{jc}^{γ} are defined by (12) with γ instead of γ_{pc}. Expression (43) shows that $E_{mc}(\tau)[i,j]$ is generally not zero, i.e., the two sources i and j become *apparently SO correlated* in the matrice $R_x(\tau)$ if condition C1) is verified, where C1) is defined by the following.

C1) The two sources i and j share at least one FIO cyclic frequency, i.e., $e_{ic}(t)$ and $e_{jc}(t)$ share at least one cyclic frequency.

2) Application to FSK Sources: In the particular case of a M_i-FSK source i and a M_j-FSK source j built from M_i and M_j oscillators, respectively, such that $f_{di}T_i$ and $f_{dj}T_j$ are integer and such that $\theta_{i(2m+1)} = \theta_{-i(2m+1)} = \theta_i(0\le m\le (M_i-2)/2)$ and $\theta_{j(2m+1)} = \theta_{-j(2m+1)} = \theta_j(0\le m\le (M_j-2)/2)$, (43) becomes

$$\begin{aligned} E_{mc}(\tau)[i,j] &= \mathrm{e}^{\mathrm{j}(\phi_i-\phi_j)}\pi_i^{\frac{1}{2}}\pi_j^{\frac{1}{2}}\frac{1}{M_iM_j} \\ &\quad\times\exp\{\mathrm{j}(\theta_i-\theta_j)\}\sum_{\gamma_{ij}\in\Gamma_{ij}}\mathrm{e}^{\mathrm{j}2\pi\tau\gamma_{ij}} \end{aligned} \tag{44}$$

which is not zero if Γ_{ij} is not empty, which is the case if it exists at least one value of $m(0\le m\le (M_i-2)/2)$ and one value of $n(0\le n\le (M_j-2)/2)$ such that

$$\Delta f_i \pm (2m+1)f_{di} = \Delta f_j \pm (2n+1)f_{dj}. \tag{45}$$

3) Application to CPFSK Sources: In the particular case of a M_i-CPFSK source i and a M_j-CPFSK source j, using the results of Section III, a necessary condition to obtain $E_{mc}(\tau)[i,j] \neq 0$ is that the two sources have integer modulation indices h_i and h_j, respectively. Under these conditions, using (40) and (12), (43) becomes

$$E_{mc}(\tau)[i,j] = \mathrm{e}^{\mathrm{j}(\phi_i-\phi_j)}\pi_i^{\frac{1}{2}}\pi_j^{\frac{1}{2}}\frac{1}{M_iM_j}K_iK_j\sum_{\gamma_{ij}\in\Gamma_{ij}}\mathrm{e}^{\mathrm{j}2\pi\tau\gamma_{ij}} \tag{46}$$

which is not zero if Γ_{ij} is not empty, which is the case if there exists at least one value of $m(0\le m\le (M_i-2)/2)$ and one value of $n(0\le n\le (M_j-2)/2)$ such that (45) is verified.

In other words, for a M_i-CPFSK source i and a M_j-CPFSK source j, condition C1) becomes C1′), which is defined by the following.

$C1'$): a) The two sources i and j have an integer modulation indice.

b) There exists at least one $m(0\le m\le (M_i-2)/2)$ and one $n(0\le n\le (M_j-2)/2)$ such that $\Delta f_i \pm (2m+1)f_{di} = \Delta f_j \pm (2n+1)f_{dj}$

We must now wonder how the presence of *apparently SO correlated* sources in the matrices $R_x(\tau)$ may modify the behavior of the current SO BSS method and the SOBI method in particular. These questions are addressed in Sections IV-B to D.

B. Prewhitening of the Data

In this section, we evaluate the consequences of *an apparent SO correlation of the sources* on the prewhitening operation of the observations.

1) Apparent SO Correlation Coefficient of Two Sources: To characterize the degree of apparent SO correlation of two sources i and j in the matrix $R_x \triangleq R_x(0)$, we introduce the apparent SO correlation coefficient of these sources $\rho_{ij}(0\le|\rho_{ij}|\le 1)$ defined by

$$\rho_{ij} \triangleq \rho_{ij}(0) \triangleq \frac{R_{mc}(0)[i,j]}{(R_{mc}(0)[i,i]R_{mc}(0)[j,j])^{\frac{1}{2}}} \tag{47}$$

where $R_{mc}(0)[i,j] \triangleq \langle \mathrm{E}[m_{ic}(t)m_{jc}(t)^*]\rangle_c$. In particular, from (46) and using the fact that $R_{mc}(0)[i,j] = E_{mc}(0)[i,j]$ for $i\neq j$, the apparent SO correlation coefficient of a M_i-CPFSK source i and a M_j-CPFSK source j, with integer modulation indices, is given by

$$\rho_{ij} = \mathrm{e}^{\mathrm{j}(\phi_i-\phi_j)}\frac{1}{M_iM_j}K_iK_j\,\mathrm{card}(\Gamma_{ij}),\quad i\neq j \tag{48}$$

where $\mathrm{card}(\Gamma_{ij})$ is the number of elements of Γ_{ij}. This expression shows in particular that for given values of M_i and M_j, $|\rho_{ij}|$ increases with the number of couples (m,n) verifying (45). For example, if $M_i = M_j = M$, $\Delta f_i = \Delta f_j$, and $f_{di} = f_{dj}$, we obtain $|\rho_{ij}| = 1/M$.

2) Eigenstructure of $R_x(0)$: To simplify the developments, we limit, in the following, the analysis to the two FIO cyclostationary source cases, and we assume that the source matrix R_s, which is defined from (21) for $\tau = 0$, is not rank deficient, i.e., that the sources are not apparently SO coherent ($|\rho_{12}|\neq 1$). As the eigenstructure of R_x is directly deduced from that of R_s (same eigenvectors and eigenvalues obtained by the addition of

η_2 to the eigenvalues of R_s), we limit the analysis to the eigendecomposition of R_s. Under these assumptions, it is possible to show, after easy but tedious algebraic manipulations, that the two nonzero eigenvalues of R_s, λ_{s+}, and λ_{s-} are given by

$$\lambda_{s\pm} = \left(\frac{1}{2}\right)\left[B_4 \pm \left({B_4}^2 - 4B_5\right)^{\frac{1}{2}}\right] \tag{49}$$

where B_4 and B_5 are scalar quantities defined by

$$\begin{aligned} B_4 \triangleq\, & \pi_1 {\boldsymbol{a}_1}^\dagger \boldsymbol{a}_1 + \pi_2 {\boldsymbol{a}_2}^\dagger \boldsymbol{a}_2 \\ & + 2\left[\pi_1\pi_2\left({\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1\right)\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\right]^{\frac{1}{2}} \operatorname{Re}(\rho_{12}\alpha_{21}) \end{aligned} \tag{50}$$

$$B_5 \triangleq \pi_1\pi_2\left({\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1\right)\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\left(1-|\rho_{12}|^2\right)\left(1-|\alpha_{12}|^2\right) \tag{51}$$

where $\pi_i \triangleq \langle \mathrm{E}[|m_{ic}(t)|^2]\rangle_c (1 \le i \le 2)$, and $\alpha_{12} = \alpha_{21}^* (0 \le |\alpha_{12}| \le 1)$ is the spatial correlation coefficient of the sources 1 and 2 defined by

$$\alpha_{12} \triangleq \frac{{\boldsymbol{a}_1}^\dagger \boldsymbol{a}_2}{\left[\left({\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1\right)\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\right]^{\frac{1}{2}}}. \tag{52}$$

The associated orthonormalized eigenvectors $\boldsymbol{u}_+$ and $\boldsymbol{u}_-$ are defined by

$$\boldsymbol{u}_{s\pm} \triangleq \exp(\mathrm{j}\varphi_\pm)\left(\frac{1}{\|B_6\boldsymbol{a}_1 - C_\pm\boldsymbol{a}_2\|}\right)[B_6\boldsymbol{a}_1 - C_\pm\boldsymbol{a}_2] \tag{53}$$

where $\phi_\pm$ is an arbitrary phase value, and B_6 and $C_\pm$ are scalar quantities defined by

$$\begin{aligned} B_6 \triangleq\, & \pi_1\left[\left({\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1\right)\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\right]^{\frac{1}{2}}\alpha_{12} \\ & + (\pi_1\pi_2)^{\frac{1}{2}}\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\rho_{12} \end{aligned} \tag{54}$$

$$\begin{aligned} C_\pm \triangleq\, & \pi_1 {\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1 + \left[\pi_1\pi_2\left({\boldsymbol{a}_1}^\dagger\boldsymbol{a}_1\right)\left({\boldsymbol{a}_2}^\dagger\boldsymbol{a}_2\right)\right]^{\frac{1}{2}} \\ & \times \rho_{12}\alpha_{21} - \lambda_{s\pm}. \end{aligned} \tag{55}$$

3) Whitened Observations: From the previous expressions, it is possible to build the $(2 \times N)$ whitening matrix $F \triangleq \Lambda_s^{-1/2}U_s^\dagger$, where $\Lambda_s \triangleq \mathrm{Diag}(\lambda_{s+}, \lambda_{s-})$ and $U_s \triangleq [\boldsymbol{u}_{s+}, \boldsymbol{u}_{s-}]$. The whitened observation vector $\boldsymbol{z}(t) \triangleq F\boldsymbol{x}(t)$ is then given by

$$\boldsymbol{z}(t) \triangleq F\boldsymbol{x}(t) = \sum_{p=1}^{P} {m_{pc}}'(t){\boldsymbol{a}_p}' + F\boldsymbol{b}(t) \triangleq A'{\boldsymbol{m}_c}'(t) + F\boldsymbol{b}(t) \tag{56}$$

where $\boldsymbol{m}_c'(t)$ is the (2×1) vector of the normalized complex envelopes $m_{pc}'(t)$ of $m_{pc}(t) (0 \le p \le 2)$, such that $\langle \mathrm{E}[|m_{pc}'(t)|^2]\rangle_c = 1$, A' is the (2×2) matrix of the whitened source steering vectors $\boldsymbol{a}_p' (0 \le p \le 2)$, such that the whitened steering vector $\boldsymbol{a}_p'$ is defined by

$$ {\boldsymbol{a}_p}' \triangleq {\pi_p}^{\frac{1}{2}} F\boldsymbol{a}_p = {\pi_p}^{\frac{1}{2}}\begin{Bmatrix} {\lambda_{s+}}^{-\frac{1}{2}} {\boldsymbol{u}_{s+}}^\dagger \boldsymbol{a}_p \\ {\lambda_{s-}}^{-\frac{1}{2}} {\boldsymbol{u}_{s-}}^\dagger \boldsymbol{a}_p \end{Bmatrix}. \tag{57}$$

Meanwhile, for apparently SO uncorrelated sources ($|\rho_{12}| = 0$), which is, in particular, the case for zero-mean sources, the whitened source steering vectors $\boldsymbol{a}_p'$, $(0 \le p \le 2)$ are orthonormalized vectors, and the matrix A' is an unitary matrix, and it is no longer the case for apparently SO correlated sources ($|\rho_{12}| \ne 0$), for which the vectors $\boldsymbol{a}_p'$, $(0 \le p \le 2)$ are neither normalized nor orthogonal.

Proof: To show the previous result, let us first assume that the matrix A' is orthogonal. Under this assumption, as the matrix $R_{s'} \triangleq A'R_{mc'}A'^\dagger$ corresponds to the identity matrix, the matrix $A'^\dagger R_{s'}A' = A'^\dagger A'$ is diagonal and equal to $A'^\dagger A'R_{mc'}A'^\dagger A'$, implying that the matrix $R_{mc'}$ is diagonal, which is not the case for apparently SO correlated sources.

Let us now assume that the columns of A' are normalized. In this case, as $A'^\dagger R_{s'}A' = A'^\dagger A' = A'^\dagger A'R_{mc'}A'^\dagger A'$, we obtain that $R_{mc'}A'^\dagger A' = A'^\dagger A'R_{mc'} = I$, which means that $R_{mc'}$ is the inverse of $A'^\dagger A'$, which is the case for ($|\rho_{12}| = 0$) since the two matrices are the identity matrix but which is generally not the case for ($|\rho_{12}| \ne 0$) since $R_{mc'}$ not diagonal does not depend on the spatial properties of the sources.

To illustrate the previous result, assume that we can simplify the computations that the sensors are omnidirectional ($\boldsymbol{a}_1^\dagger\boldsymbol{a}_1 = \boldsymbol{a}_2^\dagger\boldsymbol{a}_2 = N$) and that the two sources 1 and 2 are orthogonal ($\alpha_{12} = 0$). Under these conditions, it is possible to show, after tedious computations, that

$$ {{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_1}' = {{\boldsymbol{a}_2}'}^\dagger{\boldsymbol{a}_1}' = \frac{1}{(1 - |\rho_{12}|^2)} \tag{58}$$

$$ {{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_2}' = \frac{-\rho_{12}}{(1 - |\rho_{12}|^2)} \tag{59}$$

which shows that the modulus of the spatial correlation coefficient α_{12}' of the whitened sources 1 and 2, which is defined by the normalized inner product of ${\boldsymbol{a}_1}'$ and ${\boldsymbol{a}_2}'$, is equal to $|\alpha_{12}'| = |\rho_{12}|$ and increases with $|\rho_{12}|$.

C. Blind Identification From Matrices $R_z(\tau)$ by the SOBI Method

After the whitening operation of the observations, the temporal mean $R_z(\tau)$ of the correlation matrix of the whitened observation vector $\boldsymbol{z}(t)$ is given by

$$\begin{aligned} R_z(\tau) & \triangleq \langle R_z(t,\tau)\rangle_c = A'R_{mc'}(\tau){A'}^\dagger + \eta_2(\tau)FF^\dagger \\ & \triangleq R_{s'}(\tau) + \eta_2(\tau)FF^\dagger \end{aligned} \tag{60}$$

where $R_{mc'}(\tau) \triangleq \langle \mathrm{E}[{\boldsymbol{m}_c}'(t){\boldsymbol{m}_c}'(t-\tau)^\dagger]\rangle_c$, and ${\boldsymbol{m}_c}'(t)$ is the normalized vector $\boldsymbol{m}_c(t)$ with components ${m_{pc}}'(t)(1 \le p \le 2)$. Choosing Q parameters $\tau_q (1 \le q \le Q)$, such that $\eta_2(\tau_q) = 0$, the process of joint diagonalization of the Q matrices $R_z(\tau_q)$ gives a (2×2) unitary matrix U, maximizing the criterion (5) with $P = 2$. Meanwhile, for apparently SO uncorrelated sources ($|\rho_{12}| = 0$), A' is a unitary matrix that jointly diagonalizes the set of Q matrices $R_z(\tau_q)$, and it is no longer the case for apparently SO correlated sources ($|\rho_{12}| \ne 0$) since A' is neither a unitary matrix nor an orthogonal matrix. Under these conditions, even for sources with different spectrum, the two orthonormalized vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ corresponding to the two columns of U become necessarily linear combinations of the whitened steering vectors ${\boldsymbol{a}_1}'$ and ${\boldsymbol{a}_2}'$, which are given by

$$\boldsymbol{u}_i = \alpha_i{\boldsymbol{a}_1}' + \beta_i{\boldsymbol{a}_2}', \quad i = 1, 2 \tag{61}$$

where the coefficients α_i and $\beta_i (i = 1, 2)$ are dependent on the SO properties of the sources and are such that

$$\begin{aligned} & |\alpha_i|^2{{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_1}' + |\beta_i|^2{{\boldsymbol{a}_2}'}^\dagger{\boldsymbol{a}_2}' \\ & \quad + 2\mathrm{Re}\left[{\alpha_i}^*\beta_i{{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_2}'\right] = 1, \quad i = 1, 2 \end{aligned} \tag{62}$$

$$\begin{aligned} & {\alpha_1}^*\alpha_2{{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_1}' + {\beta_1}^*\beta_2{{\boldsymbol{a}_2}'}^\dagger{\boldsymbol{a}_2}' + {\alpha_1}^*\beta_2{{\boldsymbol{a}_1}'}^\dagger{\boldsymbol{a}_2}' \\ & \quad + {\beta_1}^*\alpha_2{{\boldsymbol{a}_2}'}^\dagger{\boldsymbol{a}_1}' = 0. \end{aligned} \tag{63}$$

Consequently, the blind identification stage of the SOBI method is perturbed by the apparent SO correlation of the sources, and the behavior of the SOBI method is modified in the steady state. This nonideal behavior of the blind identification of the whitened source steering vector generates a degradation of the source separation process, as it is shown in Section IV-D.

D. Blind Source Separation by the SOBI Method

1) Performance Criterion and Spatial Filter Choice: Following the description of the SOBI method in Section II-C, from the blindly identified vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$, which are considered to be estimates of $\boldsymbol{a}_1{}'$ and $\boldsymbol{a}_2{}'$, it is possible to obtain, to within a scalar factor, an estimate of the true steering vectors of the sources defined by $\hat{a}_{\boldsymbol{i}} = \boldsymbol{F}^{\#}\boldsymbol{u}_i (i = 1, 2)$, where # is the pseudo-inverse operation. Using (61) and the fact that $\boldsymbol{a}_i{}' \triangleq \pi_i^{1/2} F\boldsymbol{a}_i (1 \le i \le 2)$, the vectors $\hat{\boldsymbol{a}}_i$ can be written as

$$\hat{\boldsymbol{a}}_i = \alpha_i\sqrt{\pi_1}\boldsymbol{a}_1 + \beta_i\sqrt{\pi_2}\boldsymbol{a}_2, \quad i = 1, 2. \tag{64}$$

Under these conditions, both the optimal linear and TI spatial filters associated with the vectors $\hat{\boldsymbol{a}}_i (1 \le i \le 2)$ and the performance of the associated source separator can be computed.

For statistically independent zero-mean sources, the concepts of both source separator performance and optimal linear and TI source separator have been clearly defined in [8]. In particular, an estimate of the latter consists to implement, for each source i, the estimated spatial matched filter $\hat{\boldsymbol{w}}_{ix}$ defined by $\hat{\boldsymbol{w}}_{ix} \triangleq R_x^{-1}\hat{\boldsymbol{a}}_i$ [8]. However, for statistically independent sources that are FIO cyclostationary, due to the potential apparent SO correlation of the latter, the concepts of source separator performance together with that of optimal source separator have to be redefined. Indeed, the power of the output $y(t) = \boldsymbol{w}^\dagger\boldsymbol{x}(t)$ of a linear and TI spatial filter $\boldsymbol{w}$, whose input vector is given by (7) with $P = 2$ apparently SO correlated statistically independent FIO cyclostationary sources, is given by

$$\begin{aligned}\pi_y &\triangleq \left\langle \mathrm{E}\left[|y(t)|^2\right]\right\rangle_c \\ &= \boldsymbol{w}^\dagger R_x \boldsymbol{w} \\ &= \pi_1|\boldsymbol{w}^\dagger\boldsymbol{a}_1|^2 + \pi_2|\boldsymbol{w}^\dagger\boldsymbol{a}_2|^2 \\ &\quad + 2\mathrm{Re}\left[(\pi_1\pi_2)^{\frac{1}{2}}(\boldsymbol{w}^\dagger\boldsymbol{a}_1)(\boldsymbol{w}^\dagger\boldsymbol{a}_2)^*\rho_{12}\right] + \eta_2\boldsymbol{w}^\dagger\boldsymbol{w}.\end{aligned} \tag{65}$$

Then, for each source $i (i = 1, 2)$, do we have to consider the term $2\mathrm{Re}[(\pi_1\pi_2)^{1/2}(\boldsymbol{w}^\dagger\boldsymbol{a}_1)(\boldsymbol{w}^\dagger\boldsymbol{a}_2)^*\rho_{12}]$ as a useful term for the source i, as an interference term for the source i, or as a term that is a combination of a useful and an interference part for the source i? These questions have no easy answers, and their analysis is beyond the scope of this paper. Nevertheless, they have to be clarified to introduce the concepts of both source separator performance and optimal linear and TI source separator in the presence of FIO cyclostationary sources that are apparently SO correlated.

In the following, to simplify the problem, we still use the concept of source separator performances introduced in [8]. In other words, for each source $k (k = 1, 2)$, the signal-to-interference-plus-noise ratio for the source k at the output of a spatial filter $\boldsymbol{w}_i$ is defined by

$$\mathrm{SINR}k[\boldsymbol{w}_i] \triangleq \pi_k \frac{\left|\boldsymbol{w}_i{}^\dagger\boldsymbol{a}_k\right|^2}{\boldsymbol{w}_i{}^\dagger R_{bk}\boldsymbol{w}_i} \tag{66}$$

where R_{bk} is the total noise correlation matrix for the source k, corresponding to the R_x matrix in the absence of the source k. For example, $R_{b1} \triangleq \pi_2\boldsymbol{a}_2\boldsymbol{a}_2^\dagger + \eta_2\mathrm{I}$. Under these conditions, the restitution's quality of the source k at the output of the separator W, whose columns are the $\boldsymbol{w}_i$, can be evaluated by the maximum value of $\mathrm{SINR}k[\boldsymbol{w}_i]$ when i varies from 1 to 2, which is noted $\mathrm{SINRM}k[W]$. It is well known that for the previous performance criterion, the optimal source separator is the one that implements, for each source i, the spatial matched filter $\boldsymbol{w}_i$ that is defined, to within a scalar, by $w_i \triangleq R_{bi}^{-1}\boldsymbol{a}_i$, which requires the knowledge of the nonobservable matrix R_{bi}. However, while the filter $\boldsymbol{w}_i$ is colinear to the filter $\boldsymbol{w}_{ix} \triangleq R_x^{-1}\boldsymbol{a}_i$ for zero mean sources, it is no longer the case for FIO cyclostationary sources that are apparently SO correlated. For this reason, for each source i, we prefer to implement, in the following, an estimate of the optimal interference canceller (OIC) for the source i, whose performance is very close to that of the optimal filter in most cases [8]. The OIC for the source i is defined [8] by

$$\boldsymbol{w}_{i,\mathrm{OIC}} \triangleq P_{bi}\boldsymbol{a}_i = \left[\mathrm{I} - \left(\boldsymbol{a}_j{}^\dagger\boldsymbol{a}_j\right)^{-1}\boldsymbol{a}_j\boldsymbol{a}_j{}^\dagger\right]\boldsymbol{a}_i, \quad i, j = 1, 2,\ j \ne i \tag{67}$$

where μ is a scalar, and P_{bi} is the operator of orthogonal projection on the space orthogonal to the steering vectors of the interference for the source i. Then, an estimate of the filter (67) ($\hat{\boldsymbol{w}}_{i,\mathrm{OIC}}$) can be obtained by replacing in (67) the true steering vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ by their blind estimates $\hat{\boldsymbol{a}}_1$ and $\hat{\boldsymbol{a}}_2$, which are generated by the SOBI method, which finally gives

$$\hat{\boldsymbol{w}}_{i,\mathrm{OIC}} \triangleq \left[\mathrm{I} - \left(\hat{\boldsymbol{a}}_j{}^\dagger\hat{\boldsymbol{a}}_j\right)^{-1}\hat{\boldsymbol{a}}_j\hat{\boldsymbol{a}}_j{}^\dagger\right]\hat{\boldsymbol{a}}_i, \quad i, j = 1, 2,\ j \ne i. \tag{68}$$

Considering $\hat{\boldsymbol{w}}_{i,\mathrm{OIC}}$ as the column i of the $(N \times 2)$ matrix $\hat{W}_{\mathrm{OIC}}$, the associated source separator can be written as [8]

$$\hat{W}_{\mathrm{OIC}} = \hat{A}[\hat{A}^\dagger\hat{A}]^{-1} \tag{69}$$

where $\hat{A} \triangleq [\hat{\boldsymbol{a}}_1, \hat{\boldsymbol{a}}_2]$.

2) Performance Computation: To simplify the performance computation, we assume in this section that the sources are orthogonal (i.e., $\boldsymbol{a}_1^\dagger\boldsymbol{a}_2 = 0$) and strong ($\varepsilon_j \triangleq \boldsymbol{a}_j^\dagger\boldsymbol{a}_j\pi_j/\eta_2 \gg 1$, $j = 1, 2$) and that the sensors are omnidirectional ($\boldsymbol{a}_i^\dagger\boldsymbol{a}_i = N$, $i = 1, 2$). Under these assumptions, for each source $k (k = 1, 2)$, the $\mathrm{SINRM}k$ at the output of the source separator $\hat{W}_{\mathrm{OIC}}$, deduced from the SOBI method, can be computed using (64), and after tedious elementary algebraic manipulations, we obtain (in the steady state)

$$\mathrm{SINRM}k[\hat{W}_{\mathrm{OIC}}] \approx \varepsilon_k\left[1 - \frac{\varepsilon_k\nu_k}{1 + \varepsilon_k\nu_k}\right] \tag{70}$$

where the quantities $\nu_k (k = 1, 2)$ are defined by

$$\nu_1 \triangleq \mathrm{Min}\left[\frac{|\alpha_1|^2}{|\beta_1|^2}, \frac{|\alpha_2|^2}{|\beta_2|^2}\right] = \mathrm{Min}\left[\frac{|\rho_{12}{}^*\zeta - 1|^2}{|\zeta - \rho_{12}|^2}, |\zeta|^2\right] \tag{71}$$

$$\nu_2 \triangleq \mathrm{Min}\left[\frac{|\beta_1|^2}{|\alpha_1|^2}, \frac{|\beta_2|^2}{|\alpha_2|^2}\right] = \mathrm{Min}\left[\frac{|\zeta - \rho_{12}|^2}{|\rho_{12}{}^*\zeta - 1|^2}, \frac{1}{|\zeta|^2}\right] \tag{72}$$

where $\zeta = \zeta(\rho_{12}) \triangleq \alpha_2/\beta_2$, and where the α_i and $\beta_i (i = 1, 2)$ verify (62) and (63), where (58) and (59) are valid for orthogonal sources.

Expression (70) shows that SINRMk does not depend on the input signal-to-noise ratio (SNR) of the source different of the source k and is a decreasing function of ν_k. The performance of the SOBI method is optimal, and the SINRMk is maximum and equal to ε_k when $\nu_k = 0$, i.e., when the blind identification of the two source whitened steering vectors is perfect. This situation always occurs for zero-mean sources having different spectrum but has no reason to occur for FIO cyclostationary sources that are apparently SO correlated, as shown in the previous sections. In the latter case, (70)–(72) show that the performance at the output of the SOBI method degrades. In this case, the quantities ν_1 and ν_2, and thus the SINRM1 and the SINRM2, are related to each other and become a function of both ζ and the apparent SO correlation coefficient of the two sources ρ_{12}, which are themselves directly related to the SO statistics of the FIO cyclostationary sources. In particular, while for two equal power sources, SINRM1 corresponds to SINRM2 when $\rho_{12} = 0$, it is no longer the case when $\rho_{12} \neq 0$, as shown by (71) and (72), which shows that the apparent SO correlation of the sources introduces a difference in the restitution's quality of the sources. Some situations for which one of the parameters ν_1 and ν_2 is sufficiently high to generate strong performance degradation of the SOBI method are described in Section VII.

These results show that to prevent strong performance degradation or, in the worst case, a very poor source separation at the output of the SOBI method and, more generally, at the output of the current SO cumulant-based BSS methods in FIO (quasi)-cyclostationary contexts, the SO statistics of the data from which the blind identification of the source steering vectors is performed have to correspond to time-averaged SO cumulants of the data and have to take into account the potential FIO cyclostationarity of the sources. Such an estimator of the time-averaged SO cumulants of the observations is presented in Section V.

V. Adapted SO Blind Source Separation for FIO and SO Cyclostationary Sources: Covariance Methods

A. *Adapted SO BSS Philosophy for FIO Cyclostationary Sources: Covariance Philosophy*

It has been shown in the previous sections that the potential performance degradation of the current SO BSS methods is directly related to the potential nondiagonal character of the source correlation matrix temporal mean $R_{mc}(\tau)$ defined by (22), which appears in the expression of the observation correlation matrix temporal mean $R_x(\tau) \triangleq \langle R_x(t,\tau)\rangle_c$ given by (21). Moreover, the potential nondiagonal character of $R_{mc}(\tau)$ is directly related to the potential nondiagonal character of $E_{mc}(\tau)$, whereas the matrix source covariance matrix temporal mean $R_{\Delta mc}(\tau)$ is always diagonal, regardless of the FIO characteristic of the sources, by definition of the statistical independence of the latter. Consequently, to prevent poor performance of SO BSS methods for FIO cyclostationary sources, it is necessary to exploit the information contained in the $R_{\Delta mc}(\tau)$ matrix instead of $R_{mc}(\tau)$. This can be done by exploiting the information contained in the temporal mean of the observation cumulant matrix $R_{\Delta x}(\tau)$ given, using (18) and (19), by

$$R_{\Delta x}(\tau) = AR_{\Delta mc}(\tau)A^{\dagger} + \eta_2(\tau)\mathrm{I} = R_x(\tau) - E_x(\tau) \quad (73)$$

where, using (14), $E_x(\tau)$ is given by

$$E_x(\tau) \triangleq \left\langle \boldsymbol{e}_x(t)\boldsymbol{e}_x(t-\tau)^{\dagger}\right\rangle_c = \sum_{\gamma\in\Gamma} \boldsymbol{e}_x^{\gamma}\boldsymbol{e}_x^{\gamma\dagger}\mathrm{e}^{\mathrm{j}2\pi\tau\gamma} \quad (74)$$

and where it is recalled that Γ is the set of the FIO cyclic frequencies γ of $\boldsymbol{x}(t)$, and e_x^{γ} is the cyclic mean of $\boldsymbol{x}(t)$ for the cyclic frequency γ, which is defined by (16). Note from (73) and (74) that in the general case of FIO cyclostationary sources and contrary to the stationary sources case, the $R_{\Delta x}(\tau)$ matrix cannot be obtained by substracting, from the $R_x(\tau)$ matrix, only the part of $E_x(\tau)$ associated with the zero cyclic frequency and given by $\boldsymbol{e}_x^0\boldsymbol{e}_x^0{}^{\dagger} \triangleq \langle \boldsymbol{e}_x(t)\rangle_c \langle \boldsymbol{e}_x(t)\rangle_c^{\dagger}$ but has to take into account all the FIO cyclic frequencies of the observation vector $\boldsymbol{x}(t)$, which then requires a preliminary step of FIO cyclic frequency estimation of the data.

Such a SO philosophy, which is called covariance philosophy, prevents generation of apparently SO correlated FIO cyclostationary sources when the latter are statistically independent and thus allows good source steering vector blind identification performances and then good separation performances to within the spatial filtering process limits, regardless of the FIO characteristic of the sources, provided they do not have the same spectrum. Moreover, for zero-mean cyclostationary sources (containing in particular the zero mean stationary ones), this philosophy corresponds, in the steady state, to the classical one. For this reason, the proposed philosophy can be considered to be an extension of the classical one, allowing also the processing of FIO cyclostationary sources (also containing the nonzero-mean stationary sources).

Nevertheless, the new proposed philosophy of SO BSS is unable to process cyclostationary deterministic sources, such as sinusoid sources or, more generally, poly-periodic sources, whose contribution in the $R_{\Delta x}(\tau)$ matrix disappears. For this reason, an extension of the covariance method, allowing the processing of FIO cyclostationary sources jointly with deterministic sources, is presented in Section VI.

B. *Covariance Philosophy Implementation*

In situations of practical interest, the cyclic frequencies and the statistics of the observations are unknown *a priori* and have to be estimated from the data, by temporal averaging operations, using the FIO and SO cyclo-ergodicity property of the data [26].

For this purpose, if K data snapshots of the observation vector $\boldsymbol{x}(t)$ are available and provided that the data are sufficiently oversampled, we introduce the estimates $\hat{e}_x^{\gamma}(K)$ and $\hat{\pi}_x(K)$, of e_x^{γ} and $\pi_x \triangleq \langle \mathrm{E}[\boldsymbol{x}(t)^{\dagger}\boldsymbol{x}(t)]\rangle_c$, respectively, defined by

$$\hat{e}_x^{\gamma}(K) \triangleq \frac{1}{K}\sum_{m=1}^{K} \boldsymbol{x}(m)\mathrm{e}^{-\mathrm{j}2\pi\gamma m T_e} \quad (75)$$

$$\hat{\pi}_x(K) \triangleq \frac{1}{K}\sum_{m=1}^{K} \boldsymbol{x}(m)^{\dagger}\boldsymbol{x}(m). \quad (76)$$

Under these conditions, an FIO cyclic frequency detector of the observations can be implemented by selecting the cyclic frequencies γ, which makes the criterion $V(\gamma)(K)$ greater than a threshold, whose value has to be chosen to maximize the detection probability of high power cyclic frequency γ for a given false alarm rate, where $V(\gamma)(K)$ is defined by

$$V(\gamma)(K) \triangleq \frac{\hat{e}_x^{\gamma}(K)^{\dagger}\hat{e}_x^{\gamma}(K)}{\hat{\pi}_x(K)}. \tag{77}$$

Once the active FIO cyclic frequencies of the observations have been detected, the $R_{\Delta x}(\tau)$ matrix for $\tau = qT_e$, which is defined by (73) and (74), can be estimated, from the K data snapshots, by the quantity $\hat{R}_{\Delta x}(qT_e)(K)$ defined by

$$\hat{R}_{\Delta x}(qT_e)(K) = \hat{R}_x(qT_e)(K) - \sum_{\gamma\in\Gamma}\hat{e}_x^{\gamma}(K)\hat{e}_x^{\gamma}(K)^{\dagger}\mathrm{e}^{\mathrm{j}2\pi\gamma qT_e} \tag{78}$$

where $\hat{R}_x(qT_e)(K)$ is defined by (6).

Under the assumption of FIO and SO (quasi)-cyclostationary and cyclo-ergodic bandlimited observations, and for sufficiently oversampled data, the estimator (78) is asymptotically unbiased and consistent, which means that it generates, in the steady state, the true matrix $R_{\Delta x}(qT_e)$, provided that the cyclic frequencies γ are exactly known [15]. Finally, the current SO BSS methods, qualified by correlation methods, can be implemented from the $\hat{R}_{\Delta x}(qT_e)(K)$ matrix instead of $\hat{R}_x(qT_e)(K)$, giving birth to covariance methods.

VI. SO Blind Extraction of Stochastic and Deterministic FIO and SO Cyclostationary Sources: SOBEFOCYS Method

A. General Philosophy

The so-called *correlation SOBI method* [3], which exploits the information contained in the $R_x(\tau)$ matrices, allows the processing of zero-mean statistically independent cyclostationary sources jointly with deterministic sources, provided the latter do not generate nondiagonal terms in the $E_{mc}(\tau)$ matrix. This requires that the spectrum of the deterministic sources have no common frequencies, i.e., that these sources are spectrally separable, which can be considered as the definition of independent deterministic sources. Otherwise, the deterministic sources become correlated and generate nonzero nondiagonal terms in the $E_{mc}(\tau)$ matrix, and the *Correlation SOBI method* is no longer adapted for this problem. In the same way, FIO cyclostationary sources sharing at least one FIO cyclic frequency still generate nonzero nondiagonal terms in the $E_{mc}(\tau)$ matrix and become no longer separable by the *correlation SOBI method*, as shown in Section IV.

On the contrary, the so-called *covariance SOBI method* (Section V), which exploits the information contained in the $R_{\Delta x}(\tau)$ matrices, allows the processing of both zero-mean and FIO cyclostationary statistically independent sources but is unable to process deterministic sources since their contribution disappears from the $R_{\Delta x}(\tau)$ matrices.

In this context, the purpose of this section is to propose an SO BSS scheme that allows the joint processing of both zero-mean and FIO cyclostationary statistically independent sources that are either stochastic or deterministic. This scheme, which is called SOBEFOCYS, implements a first step allowing the processing and the extraction of stochastic sources, zero-mean or not, from the covariance method proposed in Section V, and a second step allowing the processing and the extraction of deterministic sources from the results of the first step. Note that other schemes may be proposed, but their analysis is beyond the scope of the paper.

B. SOBEFOCYS Method

1) Observation Model: In the presence of P_1 stochastic and P_2 deterministic statistically independent sources such that $P_1 + P_2 = P$, the observation model (7) can be written as

$$\boldsymbol{x}(t) = A_1\boldsymbol{m}_{1c}(t) + A_2\boldsymbol{m}_{2c}(t) + \boldsymbol{b}(t) \tag{79}$$

where A_1 and A_2 are the $(N \times P_1)$ and $(N \times P_2)$ matrices of the steering vectors of the stochastic and deterministic sources, respectively, and $\boldsymbol{m}_{1c}(t)$ and $\boldsymbol{m}_{2c}(t)$ are the $(P_1 \times 1)$ and $(P_2 \times 1)$ vectors of the complex envelope (with potential carrier residues) of the stochastic and deterministic sources, respectively. Note that the stochastic sources are assumed to be FIO cyclostationary and may share some FIO cyclic frequencies, whereas the deterministic sources are assumed to be polyperiodic. The statistical independence of the stochastic sources means that the components of $\boldsymbol{m}_{1c}(t)$ are statistically independent. The statistical independence of the deterministic sources means that the spectrum of the latter share no frequencies. Finally, the statistical independence of stochastic and deterministic sources means, in this paper, that the vectors $\boldsymbol{m}_{1c}(t)$ and $\boldsymbol{m}_{2c}(t-\tau)$ are not correlated, regardless of the value of τ, i.e., that $\langle \mathrm{E}[\boldsymbol{m}_{1c}(t)\boldsymbol{m}_{2c}(t-\tau)^{\dagger}]\rangle_c = 0\ \forall(t,\tau)$.

2) FIO and SO Statistics of the Data: Under the previous assumptions, the FIO statistics of the vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{e}_x(t) = A_1\boldsymbol{e}_{1c}(t) + A_2\boldsymbol{e}_{2c}(t) \tag{80}$$

where $\boldsymbol{e}_{1c}(t) \triangleq \mathrm{E}[\boldsymbol{m}_{1c}(t)]$ and $\boldsymbol{e}_{2c}(t) \triangleq \mathrm{E}[\boldsymbol{m}_{2c}(t)]$ have a Fourier serial expansion. The deterministic character of $\boldsymbol{m}_{2c}(t)$ implies that the latter vector disappears from the temporal mean $R_{\Delta x}(\tau)$ of the covariance matrix $R_{\Delta x}(t,\tau)$, which is given by

$$R_{\Delta x}(\tau) = A_1R_{\Delta m1c}(\tau){A_1}^{\dagger} + \eta_2(\tau)\mathrm{I} \tag{81}$$

where $R_{\Delta m1c}(\tau) \triangleq \langle \mathrm{E}[\boldsymbol{\Delta m}_{1c}(t)\boldsymbol{\Delta m}_{1c}(t-\tau)^{\dagger}]\rangle_c$ is a diagonal matrix, and $\boldsymbol{\Delta m}_{1c}(t) \triangleq \boldsymbol{m}_{1c}(t) - \boldsymbol{e}_{1c}(t)$. Finally, under the assumptions of Section VI-B1, the temporal mean $R_x(\tau)$ of the correlation matrix $R_x(t,\tau)$ is given by

$$R_x(\tau) = A_1R_{m1c}(\tau){A_1}^{\dagger} + A_2R_{m2c}(\tau){A_2}^{\dagger} + \eta_2(\tau)\mathrm{I} \tag{82}$$

where $R_{mic}(\tau) \triangleq \langle \mathrm{E}[\boldsymbol{m}_{ic}(t)\boldsymbol{m}_{ic}(t-\tau)^{\dagger}]\rangle_c$, $1 \le i \le 2$, such that $R_{m2c}(\tau)$ is a diagonal matrix.

3) Detection of Deterministic Sources From P1 and P2 Estimation: The presence of deterministic sources can be detected from the estimation of the number of sources P_1 and P contained in the matrices (81) and (82) for $\tau = 0$, respectively. The different steps of this process are presented in the following:

- estimation $\hat{R}_x(K)$ of R_x from K data snapshots $\boldsymbol{x}(k)$, using the empirical estimator (6) with $q = 0$;
- estimation $\hat{P}$ of P from the eigendecomposition of $\hat{R}_x(K)$ using a classical eigenvalue test;
- estimation of the FIO cyclic frequencies γ of the observations using (75)–(77);

— estimation $\hat{R}_{\Delta x}(K)$ of $\hat{R}_{\Delta x}$ from $\hat{R}_x(K)$ and the estimated cyclic frequencies γ using (78) for $q = 0$;
— estimation $\hat{P}_1$ of P_1 from the eigendecomposition of $\hat{P}_x(K)$, using a classical eigenvalue test;
— estimation $\hat{P}_2$ of P_2 by $\hat{P}_2 = \hat{P} - \hat{P}_1$;
— deterministic sources are detected if $\hat{P}_2 \neq 0$.

4) First Step of the SOBEFOCYS Method: Extraction of the Stochastic Sources: The blind estimation $\hat{A}_1$ of the mixing matrix A_1 can be obtained, to within a $(\hat{P}_1 \times \hat{P}_1)$ permutation matrix Π_1 and a $(\hat{P}_1 \times \hat{P}_1)$ diagonal matrix Λ_1, by implementing the so-called covariance SOBI method, from $\hat{P}_1$, $\hat{R}_{\Delta x}(K)$ and several covariance matrices $\hat{R}_{\Delta x}(qT_e)(K)$ with $q \neq 0$, computed from (78), (6), and the cyclic frequencies γ. Then, from the obtained $\tilde{A}_1 \triangleq \hat{A}_1\Lambda_1\Pi_1$ matrix, an $(N \times \hat{P}_1)$ stochastic sources separator W_1 has to be built to allow the extraction of the stochastic sources, i.e., the estimation $\hat{\boldsymbol{m}}_{1c}(t)$ of the stochastic sources vector $\boldsymbol{m}_{1c}(t)$ to within a permutation and a diagonal matrix by

$$y_1(\mathrm{t}) = {W_1}^{\dagger}\boldsymbol{x}(t). \tag{83}$$

In the absence of deterministic sources ($\hat{P}_2 = 0$), as suggested in Section IV-D1, the separator W_1 is chosen to implement the OIC for each stochastic source, which is equivalent, for a spatially white noise, to implementing the least square separator [8], which is defined by

$$W_1 = \tilde{A}_1\left[{\tilde{A}_1}^{\dagger}\tilde{A}_1\right]^{-1} = \hat{A}_1\left[{\hat{A}_1}^{\dagger}\hat{A}_1\right]^{-1}{\Lambda_1}^{-\dagger}\Pi_1 \tag{84}$$

and which gives

$$y_1(\mathrm{t}) = {\Pi_1}^{\dagger}{\Lambda_1}^{-1}\left[{\hat{A}_1}^{\dagger}\hat{A}_1\right]^{-1}{\hat{A}_1}^{\dagger}\boldsymbol{x}(t) = {\Pi_1}^{\dagger}{\Lambda_1}^{-1}\hat{\boldsymbol{m}}_{1c}(t). \tag{85}$$

In the presence of deterministic sources ($\hat{P}_2 \neq 0$), the column i, $\boldsymbol{w}_{1i}$ of the separator W_1 is chosen to minimize the output power $\boldsymbol{w}_{1i}^{\dagger}\hat{R}_x(K)\boldsymbol{w}_{1i}$ under a first constraint ${\boldsymbol{w}_{1i}}^{\dagger}\tilde{\boldsymbol{a}}_{1i} = 1$ of zero distorsion in the direction of the source associated with the column i, $\tilde{\boldsymbol{a}}_{1i}$, of $\tilde{A}_1$, and under a second constraint of nulling all the other stochastic sources, i.e., $\boldsymbol{w}^{\dagger}\tilde{\boldsymbol{a}}_{1j} = 0$ for $j \neq i$. It is easy to verify that W_1 is written as

$$\begin{aligned} W_1 &= \hat{R}_x(K)^{-1}\tilde{A}_1\left[{\tilde{A}_1}^{\dagger}\hat{R}_x(K)^{-1}\tilde{A}_1\right]^{-1} \\ &= \hat{R}_x(K)^{-1}\hat{A}_1\left[{\hat{A}_1}^{\dagger}\hat{R}_x(K)^{-1}\hat{A}_1\right]^{-1}{\Lambda_1}^{-\dagger}\Pi_1 \end{aligned} \tag{86}$$

and gives

$$\begin{aligned} y_1(\mathrm{t}) &= {\Pi_1}^{\dagger}{\Lambda_1}^{-1}\left[{\hat{A}_1}^{\dagger}\hat{R}_x(K)^{-1}\hat{A}_1\right]^{-1}{\hat{A}_1}^{\dagger}\hat{R}_x(K)^{-1}\boldsymbol{x}(t) \\ &= {\Pi_1}^{\dagger}{\Lambda_1}^{-1}\hat{\boldsymbol{m}}_{1c}(t). \end{aligned} \tag{87}$$

Note that in the absence of deterministic sources, the separator (86) asymptotically (i.e., when K becomes infinite) corresponds to (84).

5) Second Step of the SOBEFOCYS Method: Extraction of the Deterministic Sources: Once the stochastic sources have been extracted, the deterministic sources can be processed if $\hat{P}_2 \neq 0$. To this aim, we first remove the stochastic sources from the observation vector $\boldsymbol{x}(t)$ by building the projection $\boldsymbol{v}(t)$ of the observation vector $\boldsymbol{x}(t)$ on the subspace orthogonal to the column of $\tilde{A}_1$ defined by

$$\boldsymbol{v}(t) \triangleq \mathrm{F}_1\boldsymbol{x}(t) \tag{88}$$

where $F_1 \triangleq I - \tilde{A}_1[\tilde{A}_1^{\dagger}\tilde{A}_1]^{-1}\tilde{A}_1^{\dagger}$. Note that for a perfect blind identification of the matrix A_1, we obtain $\hat{A}_1 = A_1$, and the vector $\boldsymbol{v}(t)$ takes the form

$$\boldsymbol{v}(t) = \mathrm{F}_1A_2\boldsymbol{m}_{2c}(t) + \mathrm{F}_1\boldsymbol{b}(t). \tag{89}$$

The blind estimation $\hat{A}_{\mathrm{F}2} \triangleq \mathrm{F}_1\hat{A}_2$ of the projected mixing matrix F_1A_2 can be obtained, to within a $(\hat{P}_2 \times \hat{P}_2)$ permutation matrix Π_2 and a $(\hat{P}_2 \times \hat{P}_2)$ diagonal matrix Λ_2, by implementing the so-called correlation SOBI method [3] from $\hat{P}_2$, $\hat{R}_v(K)$, and several correlation matrices $\hat{R}_v(qT_e)(K)$ with $q \neq 0$ computed from (6) with the indice x replaced by v. Then, from the obtained $\tilde{A}_{\mathrm{F}2} \triangleq \mathrm{F}_1\tilde{A}_2 \triangleq \hat{A}_{\mathrm{F}2}\Lambda_2\Pi_2$ matrix, an $(N \times \hat{P}_2)$ deterministic source separator W_2 has to be built to allow the extraction of the deterministic sources, i.e., the estimation $\hat{\boldsymbol{m}}_{2c}(t)$ of the deterministic source vector $\boldsymbol{m}_{2c}(t)$, to within a permutation and a diagonal matrix, by

$$y_2(t) = {W_2}^{\dagger}\boldsymbol{x}(t). \tag{90}$$

The separator W_2 is chosen to implement the least square separator for the deterministic sources once the stochastic sources have been removed from the observation vector. It is then defined by

$$W_2 = \mathrm{F}_1\tilde{A}_{\mathrm{F}2}\left[{\tilde{A}_{\mathrm{F}2}}^{\dagger}\tilde{A}_{\mathrm{F}2}\right]^{-1} = \mathrm{F}_1\hat{A}_2\left[{\hat{A}_2}^{\dagger}\mathrm{F}_1\hat{A}_2\right]^{-1}{\Lambda_2}^{-\dagger}\Pi_2 \tag{91}$$

which gives

$$\begin{aligned} y_2(\mathrm{t}) &= {\Pi_2}^{\dagger}{\Lambda_2}^{-1}\left[{\hat{A}_2}^{\dagger}\mathrm{F}_1\hat{A}_2\right]^{-1}{\hat{A}_2}^{\dagger}\mathrm{F}_1\boldsymbol{x}(t) \\ &= {\Pi_2}^{\dagger}{\Lambda_2}^{-1}\hat{\boldsymbol{m}}_{2c}(t). \end{aligned} \tag{92}$$

VII. SIMULATIONS

The results presented in Sections I–V are illustrated in Figs. 1–4, where two statistically independent binary CPFSK sources are assumed to be received by a circular array of five uniformly spaced sensors with a radius $r = 0.55\ \lambda$ (λ is the wavelenght). The two sources are assumed to be orthogonal to each other (${\boldsymbol{a}_1}^{\dagger}\boldsymbol{a}_2 = 0$), which is, in particular, the case when their angle of arrival is such that $\theta_1 = 50°$ and $\theta_2 = 91°$. They have the same input signal-to-noise ratio (SNR) of 10 dB and are synchronized. Their symbol durations T_i and their modulation indices $h_i (i = 1, 2)$ are such that $h_1/T_1 = h_2/T_2 = 1/4T_e$ for $h_1 = 2$ and $h_2 = 4$. Besides, the considered SOBI method aims at diagonalizing an estimation of only one correlation or one covariance matrix temporal mean of the whitened observation matrix for $\tau = 4T_e$. In the first case, the SOBI method (which is the current one) is called SOBI_COR, whereas in the second case, the SOBI method (which is the new one) is called either SOBI_COV when all the FIO cyclic frequencies belonging to Γ are taken into account in (78) or SOBI_ACOV (for approximated covariance) when only the zero cyclic frequency is taken into account in (78). Finally, the SINRMk ($k = 1, 2$) at the output of the SOBI methods, computed in these figures, are averaged over 200 realizations.

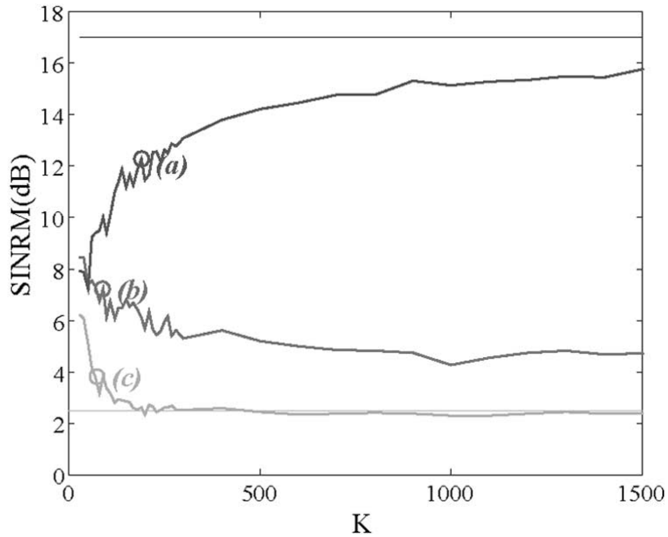

Fig. 1. SINRM1 at the output of SOBI_COR, SOBI_ACOV, and SOBI_COV as a function of K, $N = 5$, $P = 2$ 2-CPFSK sources, $\theta_1 = 50°$, $\theta_2 = 91°$, SNR $= 10$ dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, and $\Delta f_1 = \Delta f_2 = h_1/2T_1$. (a) SOBI_COV. (b) SOBI_ACOV. (c) SOBI_COR.

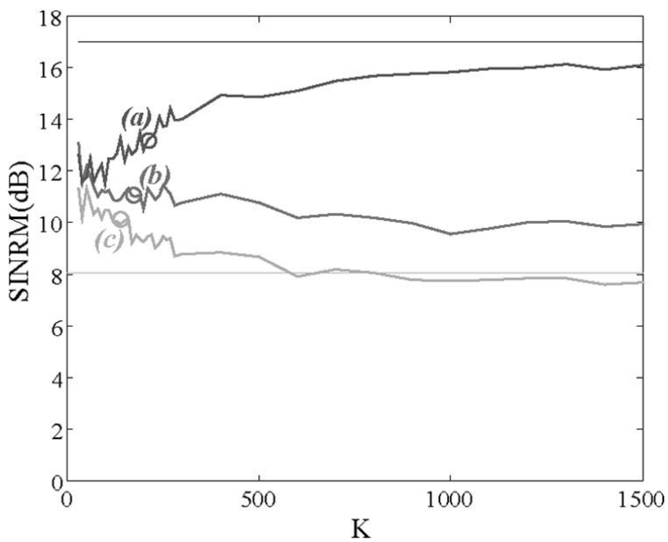

Fig. 2. SINRM2 at the output of SOBI_COR, SOBI_ACOV, and SOBI_COV as a function of K, $N = 5$, $P = 2$ 2-CPFSK sources, $\theta_1 = 50°$, $\theta_2 = 91°$, SNR $= 10$ dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, and $\Delta f_1 = \Delta f_2 = h_1/2T_1$. (a) SOBI_COV. (b) SOBI_ACOV. (c) SOBI_COR.

Under the previous assumptions and assuming that the two sources have a carrier residue such that $\Delta f_1 = \Delta f_2 = h_1/2T_1$, Figs. 1 and 2 show the variations of the SINRM1 and the SINRM2, respectively, at the output of the SOBI_COR, the SOBI_ACOV, and the SOBI_COV separators, as a function of K. As the two sources share the FIO cyclic frequencies $\gamma = 0$ and $\gamma = h_1/T_1$, they become apparently SO correlated in the $\hat{R}_x(4T_e)(K)$ matrix, with a coefficient ρ_{12} equal to 0.5. In this case, it can be shown that $\zeta = \sqrt{2}/(2+\sqrt{3})^{1/2}$ or $\zeta = -\sqrt{2}/(2-\sqrt{3})^{1/2}$, $\nu_1 = 2/(2+\sqrt{3})$, and $\nu_2 = (2-\sqrt{3})/2$, which explains the high performance degradation (the SINRM1 and the SINRM2 converge toward 2.5 and 8 dB, respectively, instead of 17 dB) and the poor separation of the sources at the output of the current SOBI (SOBI_COR) method. On the contrary, the implementation of the SOBI method from the $\hat{R}_{\Delta x}(4T_e)(K)$ matrix, given by (78) for $q = 4$, where the two cyclic frequencies $\gamma = 0$ and $\gamma = h_1/T_1$ have been used (SOBI_COV), shows performances approaching the optimality as the number of snapshots increases. Nevertheless, the use of (78) with only one ($\gamma = 0$) of the two common FIO cyclic frequencies of the sources (SOBI_ACOV) is not sufficient to obtain optimal performances.

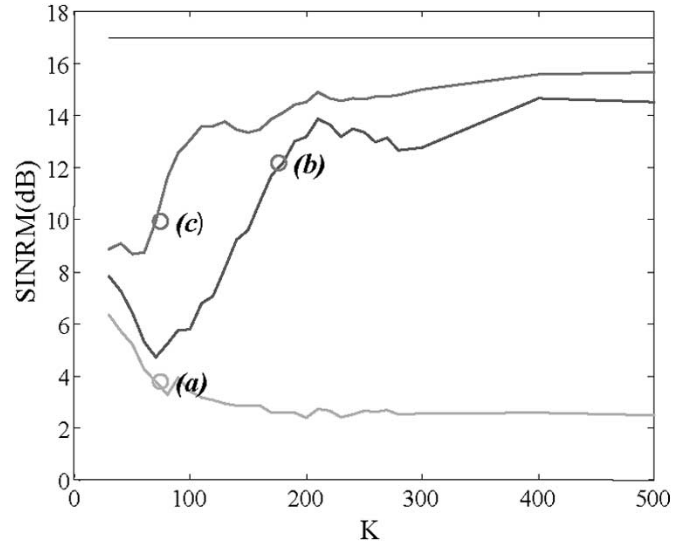

Fig. 3. SINRM1 at the output of SOBI_COR as a function of K, $N = 5$, $P = 2$ 2-CPFSK sources, $\theta_1 = 50°$, $\theta_2 = 91°$, SNR $= 10$ dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, and $\Delta \mathrm{f}_1 = \mathrm{h}_1/2\mathrm{T}_1$. $(\Delta \mathrm{f}_1 - \Delta \mathrm{f}_2)\mathrm{xT_e} =$ (a) 0, (b) 0.005, and (c) 0.01.

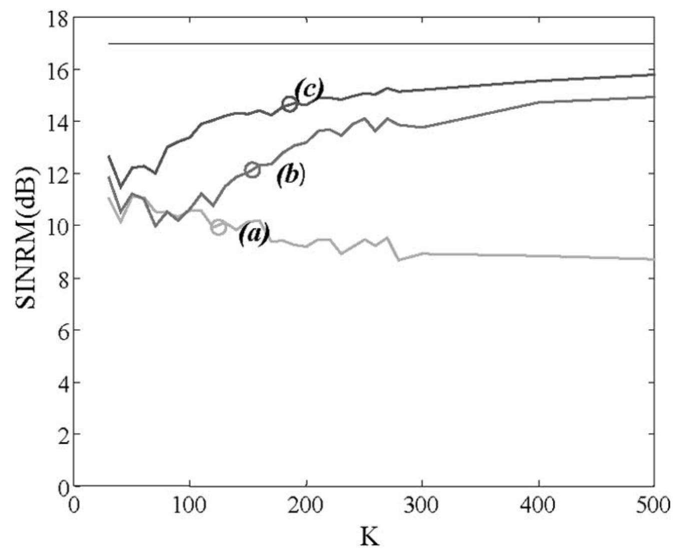

Fig. 4. SINRM2 at the output of SOBI_COR as a function of K, $N = 5$, $P = 2$ 2-CPFSK sources, $\theta_1 = 50°$, $\theta_2 = 91°$, SNR $= 10$ dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, and $\Delta \mathrm{f}_1 = \mathrm{h}_1/2\mathrm{T}_1$. $(\Delta \mathrm{f}_1 - \Delta \mathrm{f}_2)\mathrm{xT_e} =$ (a) 0, (b) 0.005, and (c) 0.01.

Figs. 3 and 4 show, for several values of Δf_2, the variations of the SINRM1 and the SINRM2, respectively, at the output of the SOBI_COR separator, as a function of the number of snapshots K, for several values 0, 0.005, and 0.01 of the differential carrier residue $(\Delta f_1 - \Delta f_2)\mathrm{x}T_e$. Note the poor separation of the two sources when $\Delta f_1 = \Delta f_2$, even in the steady-state (SINRM1 $=$ 2.7 dB, SINRM2 $= 8$ dB), and the decreasing convergence

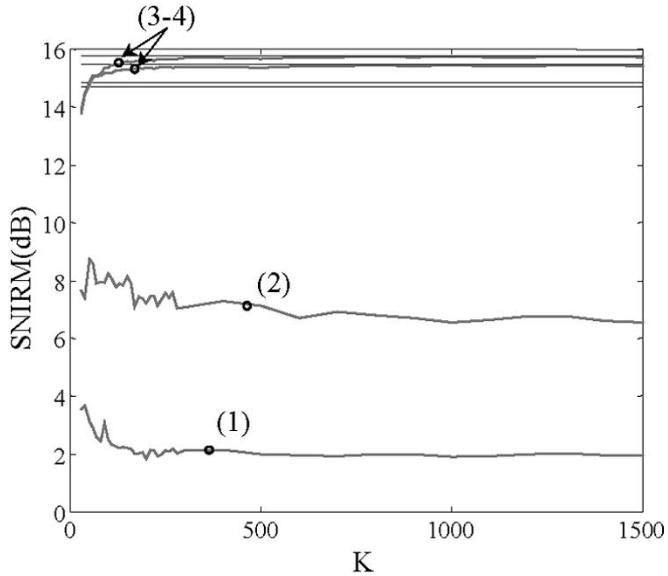

Fig. 5. SINRM$i(1 \leq i \leq 4)$ at the output of SOBI_COR as a function of K, $N = 5$, $P = 4$: 2 2-CPFSK sources and two sinusoids $\theta_1 = 50°$, $\theta_2 = -179°$, $\theta_3 = 125°$, $\theta_4 = 93°$, SNR = 10 dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, $\Delta f_1 = \Delta f_2 = h_1/2T_1$, $\Delta f_3 = 1/3T_e$, and $\Delta f_4 = 1/5T_e$.

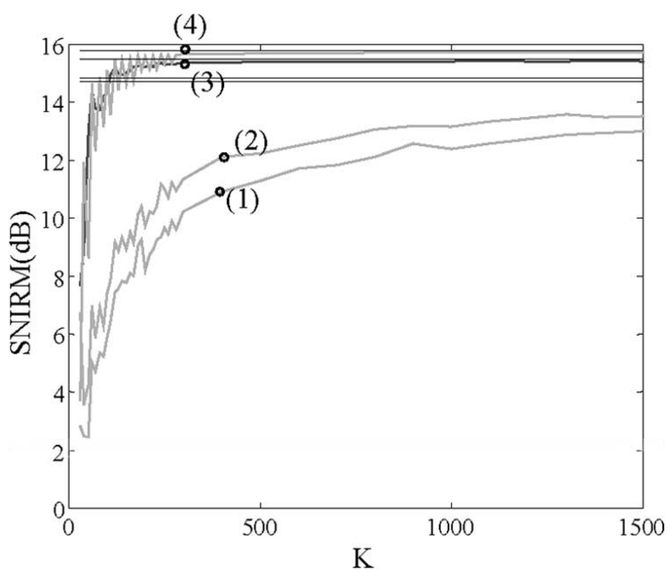

Fig. 6. SINRM$i(1 \leq i \leq 4)$ at the output of the SOBEFOCYS method as a function of K, $N = 5$, $P = 4$: two 2-CPFSK sources and two sinusoids $\theta_1 = 50°$, $\theta_2 = -179°$, $\theta_3 = 125°$, $\theta_4 = 93°$, SNR = 10 dB, $h_1/T_1 = h_2/T_2 = 1/4T_e$, $h_1 = 2$, $h_2 = 4$, $\tau = 4T_e$, $\Delta f_1 = \Delta f_2 = h_1/2T_1$, $\Delta f_3 = 1/3T_e$, and $\Delta f_4 = 1/5T_e$.

speed of the SOBI_COR separator as $(\Delta f_1 - \Delta f_2)$ decreases. In this latter case, the steady-state output performance are not affected by the use of the empirical SO statistics estimator since the source correlation matrix is diagonal due to the fact that the sources do not share any FIO cyclic frequencies. Nevertheless, when the FIO cyclic frequencies of the sources are close to each other, the output performance degradation obtained from a short-time observation is a decreasing function of the difference between the FIO cyclic frequencies of the sources.

The results presented in Section VI are illustrated in Figs. 5 and 6, where the context is the same as the one depicted for Figs. 1–4, to within the DOA of the source 2, which is equal to $\theta_2 = -179°$, but where two independent deterministic sources, corresponding to two sinusoïds, have been added in the observation vector. These deterministic sources have an SNR of 10 dB, come from the directions $\theta_3 = 125°$ and $\theta_4 = 93°$, respectively, and are such that $\Delta f_3 = 1/3T_e$ and $\Delta f_4 = 1/5T_e$. In this context, Figs. 5 and 6 show the variations of the SINRM$i(1 \leq i \leq 4)$ at the output of the SOBI_COR and the SOBEFOCYS method, respectively, as a function of K. Note that the SOBI_COR method separates the deterministic sources but has some difficulties to separate the FO cyclostationary stochastic sources as the latter share the FIO cyclic frequencies $\gamma = 0$ and $\gamma = h_1/T_1$. In the same context, the SOBEFOCYS method allows good separation of all the sources, deterministic or not, despite the FIO cyclostationarity of the latter and the fact that sources 1 and 2 share some FIO cyclic frequencies.

VIII. Conclusion

In this paper, the behavior of the current SO cumulant-based BSS methods, such as the SOBI method, initially developed for zero mean, stationary, and ergodic sources, has been analyzed for cyclostationary and cyclo-ergodic sources that are assumed FIO cyclostationary. Examples of such sources correspond to CPFSK sources having an integer modulation indice some FSK and AM sources.

It has been shown in the paper that when two sources share at least one FIO cyclic frequency, they become apparently SO correlated in the temporal mean of the data correlation matrix, and the performance of the current SO BSS methods may be strongly affected by such sources, despite the fact that they are statistically independent.

To solve this problem, it has been proposed in the paper to implement the current SO BSS method from the temporal mean of the data covariance matrix instead of the correlation matrix, which generates covariance methods instead of correlation ones. For this purpose, an asymptotically unbiased and consistent estimator of the data covariance matrix temporal mean has been proposed for FIO and SO cyclostationary and cyclo-ergodic sources. However, the use of this estimator requires the knowledge or the *a priori* estimation of all the FIO cyclic frequencies of the observations.

The so-called covariance BSS philosophy proposed in this paper allows the separation of both stationary and cyclostationary statistically independent sources that are either zero mean or not (FIO cyclostationary), provided they do not have the same spectrum. In that sense, it extends the applicability of the current correlation BSS methods [3] that have been developed for stationary sources to FIO and SO cyclostationary sources.

However, the main limitation of the proposed covariance philosophy is that it is unable to separate cyclostationary deterministic sources such as sinusoids or polyperiodic sources. For this reason, an SO BSS scheme, called the SOBEFOCYS method, allowing the joint processing of arbitrary modulated (stochastic or deterministic, zero-mean or not) statistically independent cyclostationary sources, has finally been proposed in the paper. After the estimation of the number of deterministic sources, this scheme implements the proposed covariance method in a first step, allowing the processing and the extraction of stochastic sources, and then implements a second step allowing the processing and the extraction of deterministic sources from the results of the first step. Note that to our knowledge, the SOBEFOCYS scheme is the first to allow the joint SO BSS of arbitrarily modulated (stochastic or deterministic, zero-mean or not) cyclostationary sources.

Appendix A

In this Appendix, we compute the FIO cyclic statistics of a M_p-FSK source p built from M_p local oscillators.

The cyclic mean $e_p^{\gamma_p}$ has to be computed for cyclic frequencies γ_p multiple of $(1/T_p)$. For the cyclic frequency $\gamma_{pi} = i/T_p$, using the fact that the function $e_p(t)\exp[-\mathrm{j}2\pi\gamma_{pi}t]$ is periodic with a period equal to T_p, we deduce from (27) that the cyclic mean $e_p^{\gamma_{pi}}$, which is defined by (11) with $\gamma_p = \gamma_{pi}$, is given by (A1), shown at the bottom of the page. Using the fact that $\mathrm{Rect}_p(t)$ is zero outside the interval [0, T_p] and making in (A1) the change of variables $v = t - nT_p$, $w = v/T_p$, we obtain, after some elementary manipulations, (29).

Appendix B

In this Appendix, we compute the FIO statistics of a M_p-CPFSK source p.

A. Computation of $e_p(t)$

From (26) and (33), using the statistical independance of the symbols a_n^p, we obtain

$$e_p(t) = \pi_p{}^{\frac{1}{2}} \sum_n \lim_{l\to\infty} \prod_{k=-l}^{n-1} \mathrm{E}\left\{\exp\left[\mathrm{j}\pi h_p a_k^p\right]\right\} \times \mathrm{E}\left\{\exp\left[\mathrm{j}\pi a_n^p\left(\frac{h_p}{T_p}\right)(t-nT_p)\right]\right\}\mathrm{Rect}_p(t-nT_p). \quad \text{(B1)}$$

On the other hand, for equiprobable symbols a_n^p, with probability $(1/M_p)$, it is easy to verify that for an arbitrary real value β, we obtain

$$\mathrm{E}\left\{\exp\left[\mathrm{j}\beta a_k^p\right]\right\} = \frac{2}{M_p}\sum_{m=0}^{\frac{(M_p-2)}{2}} \cos\left[(2m+1)\beta\right]. \quad \text{(B2)}$$

Applying (B2) into (B1), (34)–(37) follow immediately.

B. Computation of $\boldsymbol{e}_p^{\gamma_p}$ for $\rho_p = 1$

When $\rho_p = 1$, the cyclic mean $\boldsymbol{e}_p^{\gamma_p}$ has to be computed for the cyclic frequencies γ_p multiple of $(1/T_p)$. For the cyclic frequency $\gamma_{pi} = i/T_p$, using the fact that the function $e_p(t)\exp[-\mathrm{j}2\pi\gamma_{pi}t]$ is periodic with a period equal to T_p, we deduce from (38) that the cyclic mean $e_p^{\gamma_{pi}}$, which is defined by (11) with $\gamma_p = \gamma_{pi}$, is given by

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}}\frac{2}{M_p}\frac{1}{T_p}\int_0^{T_p}\sum_n u_p(t-nT_p)\exp[-\mathrm{j}2\pi\gamma_{pi}t]\mathrm{d}t. \quad \text{(B3)}$$

Using the fact that $u_p(t)$ is zero outside the interval [0, T_p], making in (B3) the change of variables $v = t - nT_p$, $w = v/T_p$ and using (37) in (B3) with $h_p = 2q$, where q is an integer, we obtain, after some elementary manipulations that

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}}\frac{2}{M_p}\sum_{m=0}^{\frac{(M_p-2)}{2}}\int_0^1 \cos[(2m+1)2q\pi w]\exp[-\mathrm{j}2\pi i w]\mathrm{d}w \quad \text{(B4)}$$

which, after elementary trigonometric manipulations, can also be written as

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}}\frac{1}{M_p}\sum_{m=0}^{\frac{(M_p-2)}{2}}\int_0^1 \left\{\cos\left[2\pi w\left((2m+1)q+i\right)\right] + \cos\left[2\pi w\left((2m+1)q-i\right)\right]\right\}\mathrm{d}w. \quad \text{(B5)}$$

For each value of m, $0 \le m \le (M_p-2)/2$, if $i \ne \pm(2m+1)q$, it is easy to verify that the expression (B5) gives $e_p^{\gamma_{pi}} = 0$. However, if $i = \pm(2m+1)q$, we deduce from (B5) that $e_p^{\gamma_{pi}} = \pi_p^{1/2}/M_p$. Recalling that $q = h_p/2 = f_{dp}T_p$, we deduce that $e_p^{\gamma_{pi}}$ is not zero for cyclic frequencies $\beta_{pi} = i/T_p = \pm(2m+1)q/T_p = \pm(2m+1)f_{dp}$, and the associated cyclic mean is $\pi_p^{1/2}/M_p$; this result is expressed by (39).

C. Computation of $e_p^{\gamma_p}$ for $\rho_p = -1$

When $\rho_p = -1$, the cyclic mean $e_p^{\gamma_p}$ has to be computed for the cyclic frequencies γ_p multiple of $(1/2T_p)$. For the cyclic frequency $\gamma_{pi} = i/2T_p$, using the fact that the function $e_p(t)\exp[-\mathrm{j}2\pi\gamma_{pi}t]$ is periodic with a period equal to $2T_p$, we deduce from (38) that the cyclic mean $e_p^{\gamma_{pi}}$ defined by (11) with $\gamma_p = \gamma_{pi}$ is given by

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}}K_p\frac{2}{M_p}\frac{1}{2T_p} \times \int_0^{2T_p}\sum_n(-1)^n u_p(t-nT_p)\exp[-\mathrm{j}2\pi\gamma_{pi}t]\mathrm{d}t. \quad \text{(B6)}$$

Using the fact that $u_p(t)$ is zero outside the interval [0, T_p], making in (B6) the change of variables $v = t - nT_p$, $w = v/T_p$,

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}}\frac{1}{M_p}\frac{1}{T_p}\int_0^{T_p}\sum_n\sum_{m=0}^{\frac{(M_p-2)}{2}}\Big(\exp\left\{\mathrm{j}\left[\theta_{p(2m+1)} + 2\pi f_{dp}(2m+1)(t-nT_p)\right]\right\} + \exp\left\{\mathrm{j}\left[\theta_{-p(2m+1)} - 2\pi f_{dp}(2m+1)(t-nT_p)\right]\right\}\Big)\mathrm{Rect}_p(t-nT_p)\exp[-\mathrm{j}2\pi\gamma_{pi}t]\mathrm{d}t \quad \text{(A1)}$$

and using (37) in (B6) with $h_p = (2q+1)$, where q is an integer, we obtain, after some elementary manipulations that

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}} K_p \frac{1}{M_p}(1 - \mathrm{e}^{-ji\pi}) \times \sum_{m=0}^{\frac{(M_p-2)}{2}} \int_0^1 \cos\left[(2m+1)(2q+1)\pi w\right] \exp[-\mathrm{j}\pi i w] \mathrm{d}w. \quad \text{(B7)}$$

If i is even, $\mathrm{e}^{-ji\pi} = 1$, and $e_p^{\gamma_{pi}} = 0$. However, if i is odd, i.e., if $i = (2s+1)$, where s in an integer, it is easy to verify, after some elementary trigonometric manipulations, that

$$e_p^{\gamma_{pi}} = \pi_p{}^{\frac{1}{2}} K_p \frac{1}{M_p} \times \sum_{m=0}^{\frac{(M_p-2)}{2}} \int_0^1 \left\{\cos\left[\pi w\left((2m+1)(2q+1) + (2s+1)\right)\right] + \cos\left[\pi w\left((2m+1)(2q+1) - (2s+1)\right)\right]\right\} \mathrm{d}w. \quad \text{(B8)}$$

For each value of m, $0 \le m \le (M_p-2)/2$, if $i = (2s+1) \neq \pm(2m+1)(2q+1)$, it is easy to verify that (B8) gives $e_p^{\gamma_{pi}} = 0$. However, if $i = (2s+1) = \pm(2m+1)(2q+1)$, we deduce from (B8) that $e_p^{\gamma_{pi}} = \pi_p^{1/2} K_p / M_p$. Recalling that $h_p = (2q+1) = 2f_{dp}T_p$, we deduce that $e_p^{\gamma_{pi}}$ is not zero for cyclic frequencies $\gamma_{pi} = i/2T_p = \pm(2m+1)(2q+1)/2T_p = \pm(2m+1)f_{dp}$, and the associated cyclic mean is $\pi_p{}^{1/2} K_p / M_p$; this result is expressed by (39).

References

[1] K. Abed-Meraim, Y. Xiang, J. H. Manton, and Y. Hua, "Blind source separation using second-order cyclostationary statistics," *IEEE Trans. Signal Processing*, vol. 49, pp. 694–701, Apr. 2001.

[2] B. G. Agee, S. V. Schell, and W. A. Gardner, "Spectral self-coherence restoral: a new approach to blind adaptive signal extraction using antenna array," *Proc. IEEE*, vol. 78, pp. 753–767, Apr. 1990.

[3] A. Belouchrani, K. Abed-Meraim, J. F. Cardoso, and E. Moulines, "A blind source separation technique using second-order statistics," *IEEE Trans. Signal Processing*, vol. 45, pp. 434–444, Feb. 1997.

[4] R. A. Boyles and W. A. Gardner, "Cycloergodic properties of discrete-parameter non stationary stochastic processes," *IEEE Trans. Inform. Theory*, vol. IT-39, pp. 105–114, Jan. 1983.

[5] J. F. Cardoso and A. Souloumiac, "Blind beamforming for nonGaussian signals," *Proc. Inst. Elect. Eng. F*, vol. 140, no. 6, pp. 362–370, Dec. 1993.

[6] L. Castedo and A. R. Figueiras-Vidal, "An adaptive beamforming technique based on cyclostationary signal properties," *IEEE Trans. Signal Processing*, vol. 43, pp. 1637–1650, July 1995.

[7] P. Chevalier, "Optimal time invariant and widely linear spatial filtering for radiocommunications," in *Proc. EUSIPCO*, Triestre, Italy, Sept. 1996, pp. 559–562.

[8] ——, "Optimal separation of independent narrow-band sources—concept and performance," *Signal Process.: Special Issue on Blind Source Separation and Multichannel Deconvolution*, vol. 73, no. 1–2, pp. 27–47, Feb. 1999.

[9] ——, "On the blind implementation of the ultimate source separators for arbitrary noisy mixtures," *Signal Process.*, vol. 78, no. 3, pp. 277–287, Nov. 1999.

[10] P. Chevalier, A. Ferreol, and L. Albera, "On the behavior of current second order blind source separation methods for first and second order cyclostationary sources—application to CPFSK sources," in *Proc ICASSP*, Orlando, FL, May 2002.

[11] ——, "Méthodologie générale pour la séparation aveugle de sources cyclostationnaires arbitraires—application à l'écoute passive des radiocommunications," in *Proc GRETSI*, Paris, France, Sept. 2003.

[12] P. Chevalier and A. Maurice, "Blind and informed cyclic array processing for cyclostationary signals," in *Proc EUSIPCO*, Rhodes, Greece, Sept. 1998, pp. 1645–1648.

[13] P. Comon, "Independent component analysis—a new concept?," *Signal Process.: Special Issue on Higher Order Statistics*, vol. 36, no. 3, pp. 287–314, Apr. 1994.

[14] P. Comon and P. Chevalier, "Blind source separation: Models, concepts, algorithms and performance," in *Unsupervised Adaptive Filtering—Tome 1—Blind Source Separation*, S. Haykin, Ed. New York: Wiley, 2000, ch. 5, pp. 191–235.

[15] A. V. Dandawate and G. B. Giannakis, "Asymptotic theory of mixed time averages and Kth-order cyclic-moment and cumulant statistics," *IEEE Trans. Inform. Theory*, vol. 41, pp. 216–232, Jan. 1995.

[16] L. DeLathauwer, B. DeMoor, and J. Vandewalle, "Independent component analysis based on higher-order statistics only," in *Proc IEEE SP Workshop Statistical Signal Array Process.*, Corfu, Greece, June 1996, pp. 356–359.

[17] C. Demeure and P. Chevalier, "The smart antennas at Thomson-CSF communications: concepts, implementations, performances and applications," *Ann. Telecommun.*, vol. 53, no. 11–12, pp. 466–482, 1998.

[18] A. Ferreol and P. Chevalier, "Higher order blind source separation using the cyclostationarity property of the signals," in *Proc. ICASSP*, Munich, Germany, Apr. 1997, pp. 4061–4064.

[19] ——, "On the behavior of current second and higher order blind source separation methods for cyclostationary sources," *IEEE Trans. Signal Processing*, vol. 48, pp. 1712–1725, June 2000. Erratum: vol 50, pp. 990, Apr. 2002.

[20] A. Ferreol, P. Chevalier, and L. Albera, "Higher order blind separation of non zero-mean cyclostationary sources," in *Proc. EUSIPCO*, Toulouse, France, Sept. 2002.

[21] W. A. Gardner, "Spectral correlation of modulated signals: Part I—Analog modulation," *IEEE Trans. Commun.*, vol. COM-35, pp. 584–594, June 1987.

[22] W. A. Gardner and C. M. Spooner, "The cumulant theory of cyclostationary time series, part I: foundation," *IEEE Trans. Signal Processing*, vol. 42, pp. 3387–3408, Dec. 1994.

[23] P. Gournay and P. Viraveau, "Corrélation spectrale théorique des modulations CPM—partie I: résultats analytiques pour les modulations CPFSK à 2 états (1-REC)," *Ann. Télécommun.*, vol. 53, no. 7–8, pp. 267–278, July/Aug. 1998.

[24] S. Houcke, A. Chevreuil, and P. Loubaton, "Séparation autodidacte d'un mélange de sources à débits multiples," in *Proc. GRETSI*, Toulouse, France, Sept. 2001.

[25] Y. C. Liang, A. R. Leyman, and B. H. Soong, "Blind source separation using second-order cyclic statistics," in *Proc. SPAWC*, Paris, France, 1997, pp. 57–60.

[26] L. Mazet and P. Loubaton, "Estimation de la période symbole d'un signal modulé linéairement basée sur des cyclo-corrélations," in *Proc. GRETSI*, Vannes, France, Sept. 1999.

[27] A. Napolitano, "Cyclic higher order statistics: input/output relations for discrete- and continuous-time MIMO linear almost-periodically time variant systems," *Signal Process.*, vol. 42, pp. 147–166, 1995.

[28] A. Napolitano and C. M. Spooner, "Cyclic spectral analysis of continuous-phase modulated signals," *IEEE Trans. Signal Processing*, vol. 49, pp. 30–44, Jan 2001.

[29] B. Picinbono, "On circularity," *IEEE Trans. Signal Processing*, vol. 42, pp. 3473–3482, Dec. 1994.

[30] B. Picinbono and P. Chevalier, "Widely linear estimation with complex data," *IEEE Trans. Signal Processing*, vol. 43, pp. 2030–2033, Aug. 1995.

[31] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[32] P. Viraveau and P. Gournay, "Corrélation spectrale théorique des modulations CPM—Partie II: Méthode de calcul générale et analyse," *Ann. Télécommun.*, vol. 53, no. 7–8, pp. 279–288, July/Aug. 1998.

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