Signal Processing 24 (1991) 1-10
Elsevier

Blind separation of sources, Part I: An adaptive algorithm based on neuromimetic architecture

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Received 2 April 1990
Revised 24 October 1990 and 21 February 1991

Abstract. The separation of independent sources from an array of sensors is a classical but difficult problem in signal processing. Based on some biological observations, an adaptive algorithm is proposed to separate simultaneously all the unknown independent sources. The adaptive rule, which constitutes an independence test using non-linear functions, is the main original point of this blind identification procedure. Moreover, a new concept, that of INdependent Components Analysis (INCA), more powerful than the classical Principal Components Analysis (in decision tasks) emerges from this work.

Zusammenfassung. Die Trennung unabhängiger Quellen stellt ein klassisches jedoch schwieriges Problem bei der Signalverarbeitung dar. Aufgrund neurobiologischer Beobachtungen stellen wir in diesem Artikel einen selbstanpassenden Algorithmus vor, der gleichzeitig alle unbekannten, unabhängigkeitstest unter Anwendung von nicht linearen Funktionen darstellt, ist der zentralste Punkt dieses blindend Identifikationsverfahrens. Ausserdem hebt sich ein neues Konzept, das der unabhängigen Komponenten-Analyse (INCA), leistungsfähiger in den Entscheidungsvorgängen als die Analyse der Hauptkomponenten, aus dieser Arbeit hervor.

Résumé. La séparation de sources indépendantes constitue un problème classique mais difficile de traitement du signal. D'après des observations neurobiologiques, nous proposons dans cet article un algorithme auto-adaptatif capable de séparer simultanément toutes les sources indépendantes inconnues. La règle d'adaptation, qui effectue un test d'indépendance grâce à l'utilisation de fonctions non-linéaires, est le point le plus central de cette méthode d'identification aveugle. De plus, un nouveau concept, celui d'analyse en composantes indépendantes (INCA), plus puissant dans les opérations de décision que celui d'analyse en composantes principales, émerge de ce travail.

Keywords. Separation of sources, high order moments, principal components, independent components, neural networks, linear recursive adaptive filter.

Introduction

In a large number of cases, the signal received by a sensor (antenna, microphone, etc.) is the sum (mixture) of elementary contributions that we can call *sources.* For instance, the signal received by an antenna is a superimposition of signals emitted by all the sources which are in its receptive field. Generally, sources as well as mixtures are unknown. In this case, without any knowledge on the sources (except independence assumption), this problem is called *blind separation of sources.*

Numerous approaches have been developed to solve the difficult problem of extracting the different sources from signals received by the sensors. However, it has been shown that the information contained in the spectral matrix is not sufficient to give a solution [14]. Most of the works related to this problem use the information contained in the received signals together with other information. For instance, the problem becomes simpler if we can assume that one source is a deterministic signal while all the others are gaussian noises [2]. Of course, in these cases, the problem is no more *blind* separation of sources.

Such a superimposition is also observed in biological systems. For instance, during a movement, nervous fibres Ia and II transmit to the central nervous system mixtures of information about joint stretch and stretch speed [9, 16]. However, it seems that the central nervous system is able to separate speed and stretch from these mixtures. Taking into account the large number of muscular fibers, a genetically wired solution is not realistic. Moreover, according to tracking abilities imposed by the possible time evolution of sensors (lesion, aging, etc.), an adaptive network constitutes the more consistent solution for this problem of source separation. We first addressed this problem in 1985 [8] and proposed a solution based on a neural approach. The architecture proposed in Section 1.1 (Fig. 1) is very common at various levels in the central nervous system: sensorial, cortical as well as cerebellar level. However, taking into account the linearity of 'neurons', the architecture can be viewed as a recursive linear adaptive filter.

Using a recursive fully interconnected neural network with learning abilities, we propose in this paper a blind (without a priori information) identification procedure, based on the use of high order moments. Other studies, related to the problem of blind source separation have been conducted recently using cumulants [3, 13] or high-order moments analysis [1, 5]. For a review of these methods, see [4].

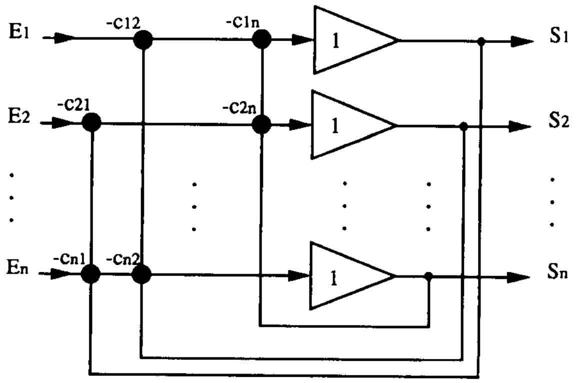

Fig. 1. Architecture of the neural network.

Section 1 is devoted to theoretical aspects: modelling of the problem and the existence of a solution with a recursive neural network. The second section defines the learning rule. Some experiments are presented in the third section. In the conclusion, we discuss on the extension of the algorithm in more realistic situations: the number of sources is unknown and different from the number of sensors, and outputs of the sensors are non-linear combinations of sources or convolutions of sources.

1. Mixture model and theoretical solution

1.1. Model of the mixture

First, we assume that the number of sources n is equal to the number of sensors p, and we propose a simple linear model for the mixture. Letting $E_i(t)$ be the signal measured at the output of the ith sensor, we then assume

$$E_i(t) = \sum_k a_{ik} X_k(t), \tag{1}$$

where $X_k(t)$ is the unknown signal emitted by the kth source and a_{ik} is a real unknown scalar.

In order to separate the sources $X_k(t)$ from signals $E_i(t)$ simultaneously, we proposed in 1985 [8] a solution, derived from a neural approach, the schematic architecture of which is given in Fig. 1.

The recursive network is constituted by n linear 'neurons'. The output $S_i(t)$ of the ith operator is a weighted sum of input signal $E_i(t)$ and other outputs $S_k(t)$ $(k \neq i)$. In Fig. 1, each bold point represents a multiplier providing a partial product $-c_{ik}S_k(t)$. The horizontal line and the operator (triangle) performs the sum of the partial products and of the input $E_i(t)$:

$$S_i(t) = E_i(t) - \sum_{k \neq i} c_{ik} S_k(t), \quad 1 \leq i \leq n. \tag{2}$$

Let $X(t)$, $E(t)$ and $S(t)$ be the vectors for which the components are unknown sources $X_i(t)$, the signals $E_i(t)$ and the network outputs $S_i(t)$, respectively. Let A be the mixture matrix and C the matrix of the network coefficients. Note that the diagonal coefficients of matrix C are equal to zero. Equations (1) and (2) can be written using matrix and vector notations:

$$E(t) = AX(t), \tag{3}$$

$$S(t) = E(t) - CS(t). \tag{4}$$

In (3), the matrix A and the source $X(t)$ are unknown. Moreover, there is no extra information about signals or mixture. The sole hypothesis is the statistical independence of components of the vector $X(t)$. Assuming that the operators are very fast according to variations of signals $E(t)$, and assuming that the recursive network is stable, (4) becomes

$$S(t) = (I + C)^{-1}E(t). \tag{5}$$

1.2. Theoretical solution

Pertinent values of weights c_{ik} provide source separation, that is to say, each output $S_i(t)$ becomes proportional to an unknown source, say $X_k(t)$. Such a solution is then achieved with some undetermination (especially, a permutation P defining the pairs (i, k)), which is presented in details in Part II, Section 1.

However, we propose to examine the simple case $n = p = 2$. In this case, the network is shown in Fig. 2 and (1) and (5) can be written as

$$\begin{aligned} E_1(t) &= a_{11}X_1(t) + a_{12}X_2(t), \\ E_2(t) &= a_{21}X_1(t) + a_{22}X_2(t), \end{aligned} \tag{6}$$

$$\begin{aligned} S_1(t) &= \frac{E_1(t) - c_{12}E_2(t)}{1 - c_{12}c_{21}}, \\ S_2(t) &= \frac{E_2(t) - c_{21}S_1(t)}{1 - c_{12}c_{21}}. \end{aligned} \tag{7}$$

By substituting $E_1(t)$ and $E_2(t)$ from (6) into (7), we get

$$\begin{aligned} S_1(t) &= \frac{(a_{11} - c_{12}a_{21})X_1(t) + (a_{12} - c_{12}a_{22})X_2(t)}{1 - c_{12}c_{21}}, \\ S_2(t) &= \frac{(a_{21} - c_{21}a_{11})X_1(t) + (a_{22} - c_{21}a_{12})X_2(t)}{1 - c_{12}c_{21}}. \end{aligned} \tag{8}$$

From (8), two different pairs of coefficients c_{ik} lead clearly toward two correct solutions:

(i) If $c_{12} = a_{12}/a_{22}$ and $c_{21} = a_{21}/a_{11}$, then $S_1(t) = a_{11}X_1(t)$ and $S_2(t) = a_{22}X_2(t)$,

(ii) if $c_{12} = a_{11}/a_{21}$ and $c_{21} = a_{22}/a_{12}$, then $S_1(t) = a_{12}X_2(t)$ and $S_2(t) = a_{21}X_1(t)$.

In fact, for the sake of stability, the loop gain of the recursive network must be less than one: $c_{12}c_{21} < 1$, and it is easy to verify that only one of the two previous solutions is possible.

Theoretically, the architecture of the network allows to separate the unknown sources. Nevertheless, the theoretical values of c_{ik} predicted above cannot be computed directly: they are functions of unknown coefficients a_{ik}. Therefore, we propose an adaptive algorithm to estimate the matrix C.

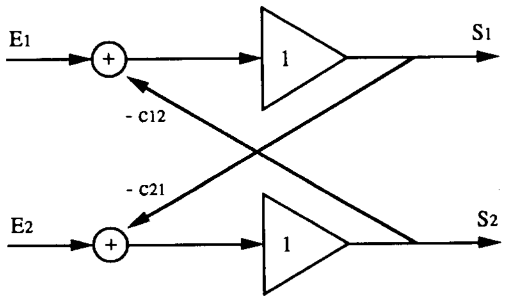

Fig. 2. The 2-operator network: 2 neurons and 2 coefficients only.

2. Adaptive computation of coefficients c_{ik}

2.1. Error term

For simplicity, we suppose here that $P = I$, that is to say, that the solution is defined by: $S_k(t)$ is proportional to $X_k(t)$, for all k. This assumption does not change the generality of the solution because the index of a source is arbitrary. Assume that the network is very close to a solution: $n - 1$

outputs $S_k(t)$ are already proportional to $X_k(t)$ $(1 \leqslant k \leqslant n-1)$. According to (2), the last output $S_n(t)$ can be written as

$$S_n(t) = E_n(t) - \sum_{k \neq n} c_{nk} S_k(t). \tag{9}$$

Using (1), we have

$$S_n(t) = \sum_{k \neq n} (a_{nk} - c_{nk} a_{kk}) X_k(t) + a_{nn} X_n(t). \tag{10}$$

Let $s_n(t) = S_n(t) - \langle S_n(t) \rangle$ and $x_n(t) = X_n(t) - \langle X_n(t) \rangle$. While signals $X_k(t)$ are assumed statistically independent, (10) leads to

$$\langle s_n^2(t) \rangle = \sum_{k \neq n} (a_{nk} - c_{nk} a_{kk})^2 \langle x_k^2(t) \rangle + a_{nn}^2 \langle x_n^2(t) \rangle. \tag{11}$$

When the nth output becomes proportional to $X_n(t)$, i.e., when all the coefficients $(a_{nk} - c_{nk} a_{kk})$ are equal to zero, the mean square term $\langle s_n^2(t) \rangle$ is minimum. So, for the ith operator, the term $s_i^2(t)$ can be considered as an error term to which we can apply the gradient method.

2.2. Gradient method

Let us compute the partial derivative of $s_i^2(t)$ with respect to c_{ik}:

$$\partial s_i^2(t) / \partial c_{ik} = 2 s_i(t)\, \partial s_i(t) / \partial c_{ik}. \tag{12}$$

With zero mean vectorial signals $s(t)$ and $e(t)$, (5) leads to

$$s(t) = (I + C)^{-1} e(t).$$

After taking the partial derivative, we obtain

$$\partial s(t) / \partial c_{ik} = -(\mathrm{I} + \mathrm{C})^{-1}\, \partial (I + C) / \partial c_{ik} \times (I + C)^{-1} e(t),$$

which can be written as

$$\partial s(t) / \partial c_{ik} = -(I + C)^{-1}\, \partial C / \partial c_{ik}\, s(t). \tag{13}$$

In the last equation, $\partial C / \partial c_{ik}$ is a square matrix with only one non-zero element: the one located on row i and column k. That element is equal to one. Therefore, the partial derivative of the component $s_m(t)$ is

$$\partial s_m(t) / \partial c_{ik} = -q_{mi} s_k(t), \tag{14}$$

where q_{mi} is the term with index mi of the matrix $(I + C)^{-1}$.

Referring to (12), it is found that

$$\partial s_m^2(t) / \partial c_{ik} = -2 q_{mi} s_m(t) s_k(t), \tag{15}$$

and for $m = i$ we obtain

$$\partial s_i^2(t) / \partial c_{ik} = -2 q_{ii} s_i(t) s_k(t). \tag{16}$$

By applying the gradient method, we then find the following adaptation rule:

$$\mathrm{d} c_{ik} / \mathrm{d} t = a q_{ii} s_i(t) s_k(t) \quad \forall i \neq k, \tag{17}$$

where a is a positive adaptation gain.

2.3. Evaluation of this adaptation rule

In fact, the convergence is achieved when the time averages $\langle \mathrm{d} c_{ik} / \mathrm{d} t \rangle$ are equal to zero, i.e. if $\langle s_i(t) s_k(t) \rangle = 0$ $\forall i, k \neq i$. These conditions mean that the two outputs $S_i(t)$ and $S_k(t)$ are not correlated. Remember that we assume the statistical independence of the unknown sources $X_i(t)$. Therefore, when a good solution of the problem of source separation is obtained, any pair of outputs $S_i(t)$ and $S_k(t)$ must be independent and not only non-correlated. So, (17) should be improved towards an independence test of $s_i(t)$ and $s_k(t)$.

Moreover, the error term we proposed in the last section is not exact. It was computed assuming that the system was very close to a solution. If one coefficient, say c_{ik}, varies, the corresponding output $S_i(t)$ is no longer proportional to one source, but becomes a mixture of sources. Variations of output $S_i(t)$ then affect all the outputs of the network, through the $n-1$ connections c_{mi} coming from output $S_i(t)$.

It seems that the adaptation rule cannot be associated with a Lyapunov function. More precisely, we cannot find an error function, whose global minimization provides the solution.

2.4. Adaptation rule

In the adaptation rule (17), q_{ii} is the diagonal term with index i of the matrix $(I+C)^{-1}$. Diagonal coefficients of matrix C being zero, so long as other coefficients c_{ik} $(i \neq k)$ remain small, first order expansion leads to

$$q_{ii} = 1 - c_{ii} = 1 \quad \forall i. \tag{18}$$

As a result of (18), we have $\mathrm{d}c_{ik}/\mathrm{d}t = \mathrm{d}c_{ki}/\mathrm{d}t$. This relation even holds in the case of $n = p = 2$. In the following, we consider $q_{ii} = 1$.

We will see in the last section that the rule (17) or its simplified version with $q_{ii} = 1$, leads only to a zero-covariance test. To achieve an independence test, we propose to use two non-linear and different odd functions:

$$\mathrm{d}c_{ik}/\mathrm{d}t = af(s_i(t))g(s_k(t)). \tag{19}$$

Of course, these functions must be different, otherwise we always have $\mathrm{d}c_{ik}/\mathrm{d}t = \mathrm{d}c_{ki}/\mathrm{d}t$, which implies symetric modifications of the matrix C. These two functions introduce high-order moments. In fact, let us assume that these functions can be expanded in a Taylor series. They contain only odd power terms:

$$f(x) = \sum_j f_{2j+1}x^{2j+1}, \qquad g(x) = \sum_m g_{2m+1}x^{2m+1}.$$

Referring to these expansions, (19) can be expressed as

$$\frac{\mathrm{d}c_{ik}}{\mathrm{d}t} = a \sum_j \sum_m f_{2j+1}g_{2m+1}s_i^{2j+1}(t)s_k^{2m+1}(t). \tag{20}$$

The convergence of the algorithm corresponds now to the following condition:

$$\left\langle \frac{\mathrm{d}c_{ik}}{\mathrm{d}t} \right\rangle = a \sum_j \sum_m f_{2j+1}g_{2m+1}\langle s_i^{2j+1}(t)s_k^{2m+1}(t)\rangle = 0. \tag{21}$$

Convergence is then achieved if all the high-order moments $\langle s_i^{2j+1}(t)s_k^{2m+1}(t)\rangle$ are equal to zero. Statistical independence between $s_i(t)$ and $s_k(t)$ implies $\langle s_i^{2j+1}(t)s_k^{2m+1}(t)\rangle = \langle s_i^{2j+1}(t)\rangle\langle s_k^{2m+1}(t)\rangle$. If the probability density of signal $s_i(t)$ is an even function, it is easy to verify that all the odd moments of $s_i(t)$ are null: $\langle s_i^{2j+1}(t)\rangle = 0$. In practice, for audiofrequency signals (speech, music, etc.), the even probability density function is not a restrictive condition. The adaptation rule (19) is then an approximation of an independence test.

Finally, note that signal $S_i(t)$ is not a zero-mean signal, a priori. Therefore, the zero-mean signal $s_i(t)$ must be estimated by the relation

$$s_i(t) = S_i(t) - \langle S_i(t)\rangle, \tag{22}$$

where $\langle S_i(t)\rangle$ is an estimation of the time average of signal $S_i(t)$. In practice, a simple first-order low pass filter is a good enough estimation of the average:

$$\langle S_i(t)\rangle = S_i(t) * \exp(-t/T), \tag{23}$$

where $*$ denotes a convolution product and T is the time constant of the filter.

As a result of the reasoning put forward in this section, we propose the following adaptation rule:

$$\mathrm{d}c_{ik}/\mathrm{d}t = af(s_i(t))g(s_k(t)), \tag{24}$$

which, on the average, is equivalent to

$$\mathrm{d}c_{ik}/\mathrm{d}t = af(S_i(t))g(s_k(t)). \tag{25}$$

As can be experimentally verified, these rules are very robust. In particular, a large class of odd functions f and g is acceptable; in most of the simulations presented in the following section, we have used $f(x) = x^3$ and $g(y) = \tan^{-1}(y)$, but $g(y) = y$ or $g(y) = \mathrm{sign}(y)$ lead also to source separation. Limitations related to the density probability distribution of the signals are proved in Parts II and III. These limitations seem to be very severe, but in many practical situations, for audiofrequency signals for instance (speech, music, etc.), it can be shown that these limitations do not apply, which has been experimentally confirmed.

Relations (24) and (25) are very close to mathematic formalizations [6] of rules for synaptic plasticity proposed as by Hebb [7] or Rauschecker and Singer [15]. These relations are local rules: the adaptation increment of synaptic weight c_{ik}, from neuron k to neuron i, depends on the weight c_{ik} and on outputs $S_i(t)$ and $S_k(t)$ of neurons i and

k. Moreover, non-linear functions (logarithmic, sigmoïd, etc.) are very common in neurosciences: the metabolism of the central nervous system is associated with the competition of chemical processes which typically induce non-linearities.

3. Experimental results

In this section, we propose some simulation results as illustrations of the theory developed in the previous parts.

3.1. Signal extractions

Consider now a set of two sensors receiving an unknown mixture of two unknown independent signals. We assume nothing else: the nature of these signals (deterministic or stochastic, with large or narrow bandwidth, etc.) has no effect on the algorithm.

In this experiment, there is a mixture of a deterministic signal $X_1(t)$ and a random noise $X_2(t)$ with uniform probability density. The energy of the noise $X_2(t)$ is about 50 times greater than that of signal $X_1(t)$. We use here a single 2-neuron network (Fig. 2), with the initial conditions $c_{12} = c_{21} = 0$. Before the beginning of learning, from (4), we deduce $S_i(t) = E_i(t)$. In Fig. 3, we can see the outputs $S_1(t)$ and $S_2(t)$: before learning ($t < 0$, the beginning of the learning is indicated by an arrow), outputs are very similar, because the signal $X_1(t)$ is drowned by the noise $X_2(t)$. After the beginning of learning, the network provides an adaptive estimation of the mixture, and after about 300 learning steps, both signals $X_1(t)$ and $X_2(t)$ are separated on the outputs of the network, with a residual crosstalk of about -25 dB.

Depending on the initial conditions ($c_{12}(t = 0)$ and $c_{21}(t = 0)$), the algorithm may converge rapidly or slowly, or may even diverge (Fig. 4).

If the mixture of signals is time dependent, the algorithm is able to track the solution if the rate of mixture evolution is less than the rate of learning.

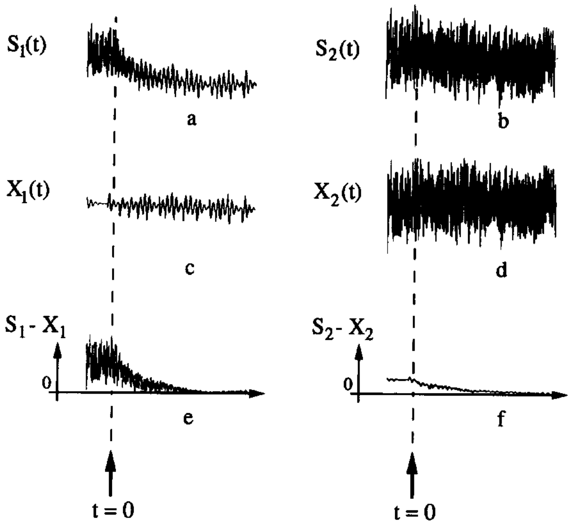

Fig. 3. Separation of two signals with a 2-neuron network. Before $t = 0$, coefficients of the matrix C are equal to zero, which implies $S_i(t) = E_i(t)$, $t < 0$. After $t = 0$, the learning process becomes active, and after some hundred learning steps, each output $S_i(t)$ extracts an unknown signal $X_i(t)$. (a), (b) outputs of the networks, (c), (d) unknown sources to be extracted, (e), (f) errors on the outputs.

3.2. Image preprocessing

Now consider Fig. 5(a). With a random sampling of points (x, y) from this image, we obtain a time series of points $(x(t), y(t))$ that we may consider as a sampled 2-dimensional signal. This signal was processed with a 2-neuron network and the algorithm converged after 300 to 500 learning steps.

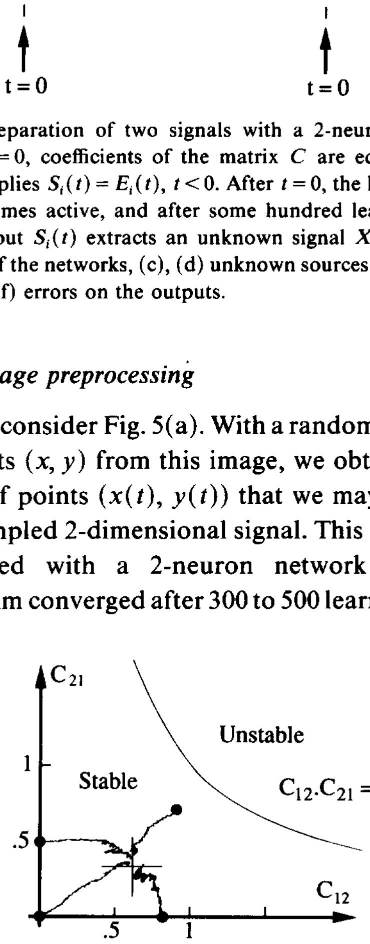

Fig. 4. Evolution of coefficients of matrix C during the learning. The cross corresponds to the theoretical solution. The bold points are the starting points for 4 different convergence trajectories. For the sake of stability, the recursive 2-neuron network must satisfy the condition $C_{12}C_{21} < 1$. The first quadrant is thus separated into two regions (stable or unstable) by the hyperbola $C_{12}C_{21} = 1$.

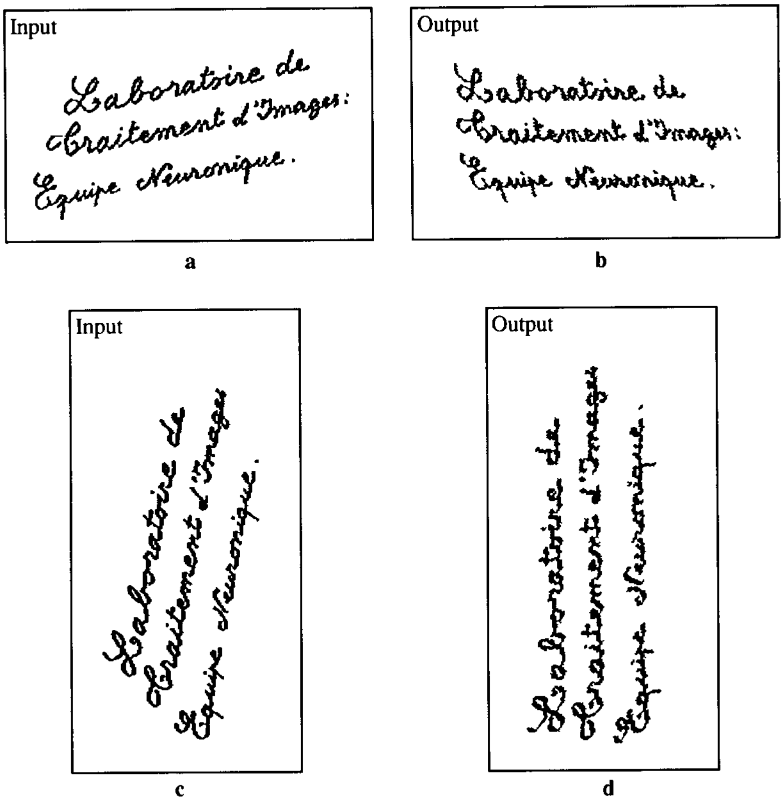

Fig. 5. Application to image preprocessing. A 2-neuron network can eliminate dependence between coordinates of the points of a sloped text. After convergence, the initial images in Figs. 5(a) and 5(c) are transformed into the images in Figs. 5(b) and 5(d), respectively. Lines of transformed images are horizontal or vertical, according to the slope of the lines in the initial images.

Then, the initial image was transformed by the network into the image in Fig. 5(b), where lines and characters, sloped in the initial image, are straightened. This result is due to the fact that a sloped line introduces dependence between coordinates x and y. The algorithm minimizes this dependency, which is reached if lines are either horizontal or vertical.

In fact, if the slope of the lines of the initial text is closer to a vertical line than to a horizontal line (Fig. 5(c)), after convergence, the lines of the initial image are vertical in the transformed image (Fig. 5(d)).

3.3. INdependent Components Analysis (INCA)

A new concept in data analysis, that of INdependent Components Analysis (INCA) emerges from this algorithm. This new concept may be compared to the classical concept of Principal Components Analysis (PCA). In fact, emergence of independent signals and not only non-correlated signals is due only to non-linear functions in the learning rule which introduce high-order statistical moments and not only second-order moments as in most classical methods.

To demonstrate the advantage of this new concept, we compare in the following experiment these 2 methods: INCA and PCA. We consider 3 unknown noisy digital signals, and assume they are transmitted along 3 wires with crosstalk. So, at the end of the wires, we observe 3 signals ($E_1(t)$, $E_2(t)$ and $E_3(t)$) which are unknown mixtures of unknown digital signals (Fig. 6(a)). These signals may also be represented by their distribution in a 3-dimensional space (Figs. 6(b) and 7(a)). In these

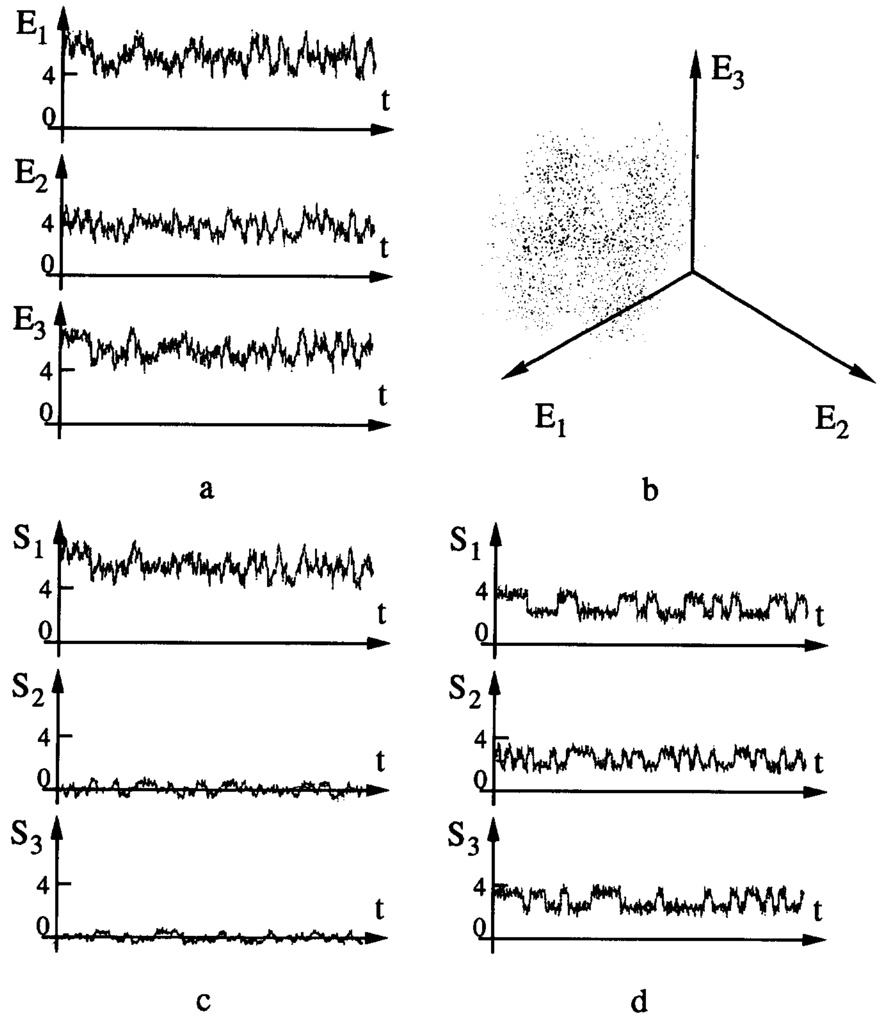

Fig. 6. INdependent Components Analysis (INCA) versus Principal Components Analysis (PCA). (a) Inputs are unknown mixtures of 3 unknown noisy digital sources. In this figure, the ratio S/B of each source is equal to 10 dB. (b) Isometric representation of input distribution in the plane (E_1, E_2, E_3). (c) Signals transformed by PCA: digital sources are not found. (d) Signals transformed by INCA after convergence of the algorithm: each output extracts a noisy digital source. Noise can easily be eliminated by a simple threshold element.

figures, we can see 8 clusters, associated to the 8 possible configurations of 3 binary variables. The diameter of each cluster depends on the noise energy.

By computing the covariance matrix of this distribution, and after diagonalization of this matrix, we obtain the Karhunen-Loève Transform (or PCA) by choosing eigen vectors as the new base. Figure 6(c) shows the 3 signals observed in this new base, and Fig. 7(b) the transformed distribution of Fig. 7(a).

By using our algorithm, after convergence (about 3000 learning steps), the network provides a transform of the original signals and distributions (Figs. 6(d) and 7(c)). In Fig. 6(d), it is clear that INCA extracts 3 noisy binary signals from mixtures $E_1(t)$, $E_2(t)$ and $E_3(t)$ of Fig. 6(a).

Comparing Figs. 6(c) and 6(d), it is clear that INCA is very advantageous for the extraction of independent features. The 3 unknown digital signals are effectively separated by this algorithm, and the residual noise can be easily eliminated,

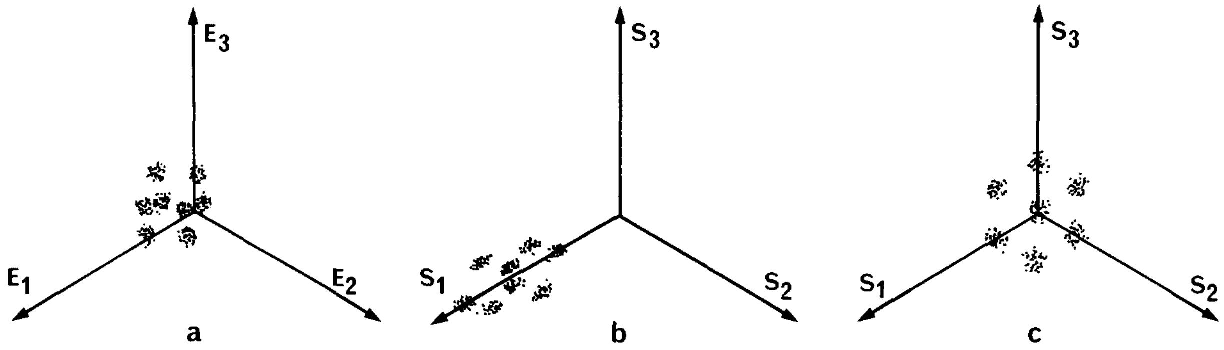

Fig. 7. INdependent Components Analysis (INCA) versus Principal Components Analysis (PCA). (a) Input distribution of 3 unknown mixtures of 3 unknown noisy digital signals. In this figure, the ratio S/B of each source is equal to 23 dB. (b) Output distribution after PCA. On the principal axis (S_1), there is still a mixture of sources. (c) Output distribution after INCA. Each axis now corresponds to an unknown source.

for instance with a simple threshold element. With PCA, the signal associated with the first eigen vector has the greatest energy, but it is always a mixture of the 3 binary sources. As a conclusion, we can say that PCA is an effective method for data compression, while INCA is an effective method for the extraction of independent features. Efficient interpretation or classification needs this type of pre-processing.

4. Discussion

In this paper, we have presented a new adaptive algorithm for the separation of sources. The main characteristic of this algorithm is the use of high order statistical moments (with non-linear functions) to perform an independence test. Although the examples given in this paper are very simple, generalization to more realistic situations is easy. Non-linear mixtures, degenerated mixtures (the number of sensors n is not equal to the number of sources p) and convolutive mixtures ('cocktail party' problem, for instance) have already been studied with success [10, 12] and will be published in a forthcoming paper.

In the case of non-linear mixtures, there is no longer an exact solution with a linear network. However, experimental and theoretical results show that the solution proposed by the algorithm corresponds to that related to the first order linear approximation of the non-linear mixture [10].

In case numbers of sources and sensors are different, we can mention the following experimental consistent results:

- if there are more sources (n) then sensors (p), there is no exact solution. The algorithm separates the p most energetical signals and the remaining signals induce noise, which decreases quality of the separation.
- if there are less sources (n) than sensors (p), n outputs (from p) of the networks provide the n sources, and the $p-n$ other outputs are on the average equal to zero.

The main drawback of this adaptive algorithm is the unknown number of learning steps necessary before convergence. Statistical explanations on how this algorithm works and the design of a non-adaptive algorithm has been studied by Comon [3]. Statistical explanations are given in the second part of this paper.

Finally, the stability of the algorithm has not been dealt with here. This difficult problem, complicated by non-linear functions, has been studied by E. Sorouchyari and constitutes the third part of this paper on a method of sources separation.

Several hardware configurations have already been implemented to run this algorithm. Since 1985, a discrete analog 2-neuton network [11], designed by the authors has been running in the

TIRF laboratory of the INP Grenoble. Another configuration, using the Digital Signal Processor 68930, with the support of Thomson Semiconductors Inc., was implemented in 1987. In 1989, a CMOS analog integrated circuit of a 6-neuron network was designed by Arreguit and Vittoz [17]. This circuit can separate three mixed audio-frequency signals, as speech or music, in about 5 ms.

Acknowledgments

This work was partially funded by the ESPRIT II-BRA project no. 3049 entitled NERVES for 'NEurocomputing Research and VLSI for Enhanced Systems'. The authors would like to express their gratitude to X. Arreguit, P. Comon, E. Sorouchyari and E. Vittoz for very fruitful discussions.

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