Asynchronous ϵ-Greedy Bayesian Optimisation (Supplementary material)

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A INTRODUCTION

In this supplementary materials document we provide details of all additional experiments carried out in the course of this work. In Sections C, D and E we show the results of the synthetic problems, instance-based problems and the ablation study respectively. Section F details the ϵ-setting experiments and Section G contains the results of the experiments, ablation study and ϵ-setting experiments in the sequential setting (i.e. $q = 1$).

B EXPERIMENTAL DETAILS

Here we give additional details of the algorithms used.

B.1 NSGA-II

To find the approximate Pareto set we used NSGA-II [Deb et al., 2002] with a population size of $100d$, a mutation rate of d^{-1}, a crossover rate of 0.8, and mutation and distribution indices of $\eta_c = \eta_m = 20$.

C ADDITIONAL RESULTS: SYNTHETIC FUNCTIONS

In this section we show all convergence plots and results tables for the synthetic function experiments. Figures 1, 2, 3, and 4 show the convergence plots for each of the six methods on the fifteen benchmark problems for $q \in \{4, 8, 16\}$. Each plot shows the median log simple regret, with shading representing the interquartile range over 51 runs. Tables 1, 2 and 3 show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion. The method with the best (lowest) median regret is shown in dark grey, and those that are statistically equivalent to the best method according to a one-sided, paired Wilcoxon signed-rank test with Holm-Bonferroni correction [Holm, 1979] ($p \geq 0.05$) are shown in light grey.

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Table 1: Tabulated results for $q = 4$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.73×10^{-1}	2.02×10^{-1}	1.66×10^{2}	7.34×10^{1}	5.99	6.22	7.35×10^{-2}	6.39×10^{-2}	1.71×10^{-1}	1.07×10^{-1}
TS	4.39×10^{-3}	5.65×10^{-3}	6.51×10^{1}	2.06×10^{1}	3.81	5.60	2.60×10^{-4}	3.84×10^{-4}	1.08×10^{-2}	1.57×10^{-2}
KB	8.14×10^{-5}	1.16×10^{-4}	7.20×10^{1}	9.85×10^{1}	1.01	1.13	1.48×10^{-4}	1.43×10^{-4}	1.09×10^{-4}	1.29×10^{-4}
LP	1.24×10^{-4}	1.54×10^{-4}	7.32×10^{1}	9.70×10^{1}	1.49	1.42	2.05×10^{-4}	2.62×10^{-4}	1.06×10^{-4}	1.42×10^{-4}
PLAyBOOK	1.58×10^{-4}	1.96×10^{-4}	7.11×10^{1}	1.31×10^{1}	9.61×10^{-1}	1.04	2.92×10^{-4}	2.97×10^{-4}	1.25×10^{-4}	1.50×10^{-4}
AEGiS-RS	1.39×10^{-4}	1.96×10^{-4}	6.51×10^{1}	2.78×10^{1}	1.05	1.48	2.39×10^{-6}	2.79×10^{-6}	1.28×10^{-2}	1.61×10^{-2}
AEGiS	5.99×10^{-6}	6.90×10^{-6}	6.52×10^{1}	1.08×10^{1}	6.99×10^{-1}	7.67×10^{-1}	2.93×10^{-6}	3.31×10^{-6}	5.29×10^{-5}	5.59×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.62×10^{1}	1.71	2.19	2.81×10^{-1}	4.50×10^{1}	1.00×10^{1}	9.57×10^{-1}	3.60×10^{-1}	1.31×10^{4}	9.64×10^{3}
TS	3.70	8.13×10^{-1}	2.06	5.05×10^{-1}	1.45×10^{1}	2.05×10^{1}	2.78×10^{-3}	3.25×10^{-3}	3.00×10^{2}	2.54×10^{2}
KB	1.21×10^{1}	5.72	1.03	8.01×10^{-1}	1.46×10^{1}	4.55	7.51×10^{-3}	1.04×10^{-2}	5.42×10^{2}	3.00×10^{2}
LP	1.41×10^{1}	4.13	1.02	5.61×10^{-1}	1.47×10^{1}	5.64	4.34×10^{-3}	5.40×10^{-3}	5.86×10^{2}	3.31×10^{2}
PLAyBOOK	1.34×10^{1}	4.81	1.13	6.83×10^{-1}	1.46×10^{1}	1.24×10^{1}	4.32×10^{-3}	5.37×10^{-3}	4.45×10^{2}	3.00×10^{2}
AEGiS-RS	1.47×10^{1}	3.95	2.17	3.35×10^{-1}	1.45×10^{1}	1.21×10^{1}	7.62×10^{-3}	1.03×10^{-2}	5.37×10^{2}	3.14×10^{2}
AEGiS	2.81	1.19	1.54	4.77×10^{-1}	1.51×10^{1}	2.00×10^{1}	2.28×10^{-3}	3.09×10^{-3}	3.64×10^{2}	2.44×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	8.18×10^{1}	1.32×10^{1}	1.93×10^{1}	5.27×10^{-1}	5.97	4.53×10^{-1}	5.91×10^{4}	3.31×10^{4}	1.44×10^{2}	1.87×10^{1}
TS	8.14×10^{1}	3.55×10^{1}	1.10×10^{1}	1.97	6.21	4.07×10^{-1}	6.34×10^{2}	4.05×10^{2}	1.65×10^{2}	2.55×10^{1}
KB	4.27×10^{1}	1.29×10^{1}	1.67×10^{1}	1.55	5.18	6.20×10^{-1}	1.53×10^{3}	9.03×10^{2}	8.93×10^{1}	2.13×10^{1}
LP	4.54×10^{1}	1.61×10^{1}	1.59×10^{1}	2.48	5.20	5.34×10^{-1}	1.45×10^{3}	8.25×10^{2}	8.62×10^{1}	2.74×10^{1}
PLAyBOOK	4.20×10^{1}	1.29×10^{1}	1.62×10^{1}	2.26	5.08	7.27×10^{-1}	1.46×10^{3}	9.33×10^{2}	8.58×10^{1}	2.71×10^{1}
AEGiS-RS	3.22×10^{1}	1.60×10^{1}	1.84×10^{1}	7.77×10^{-1}	5.82	6.00×10^{-1}	8.69×10^{2}	5.11×10^{2}	6.85×10^{1}	2.54×10^{1}
AEGiS	3.10×10^{1}	1.79×10^{1}	1.38×10^{1}	2.30	5.57	7.16×10^{-1}	1.08×10^{3}	7.27×10^{2}	5.96×10^{1}	2.41×10^{1}

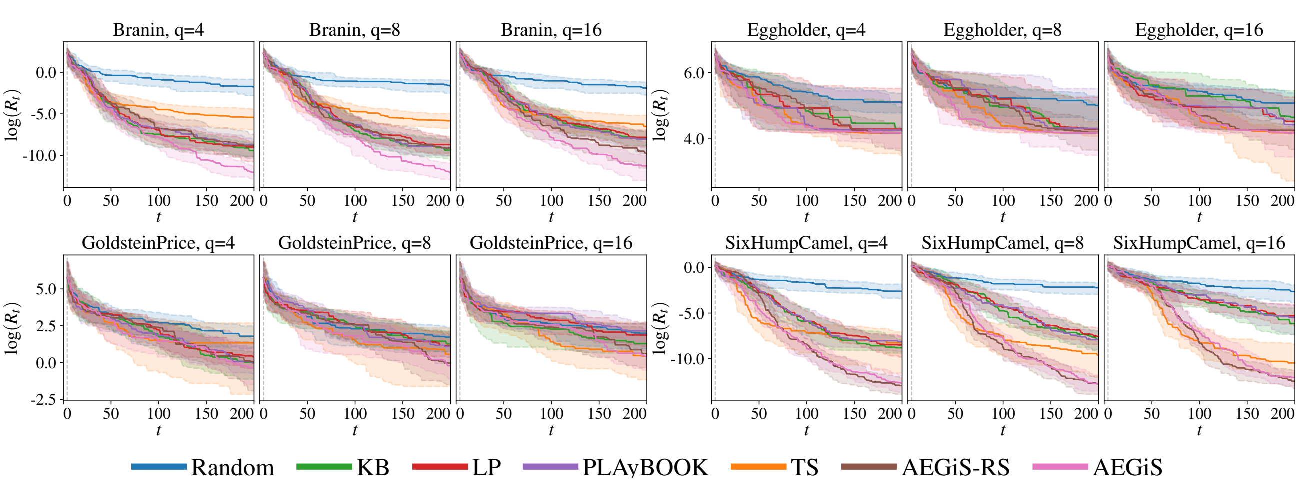

Figure 1: Convergence results for the synthetic test problems.

Table 2: Tabulated results for $q = 8$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.81×10^{-1}	2.21×10^{-1}	1.50×10^{2}	7.39×10^{1}	5.61	6.48	1.09×10^{-1}	8.72×10^{-2}	1.26×10^{-1}	8.14×10^{-2}
TS	2.91×10^{-3}	3.73×10^{-3}	6.51×10^{1}	9.41	1.76	2.60	6.73×10^{-5}	9.86×10^{-5}	7.89×10^{-3}	1.07×10^{-2}
KB	8.55×10^{-5}	1.14×10^{-4}	7.39×10^{1}	6.63×10^{1}	3.57	3.41	3.75×10^{-4}	4.68×10^{-4}	1.46×10^{-4}	1.69×10^{-4}
LP	1.66×10^{-4}	1.76×10^{-4}	7.37×10^{1}	8.40×10^{1}	3.18	3.19	5.13×10^{-4}	5.47×10^{-4}	1.26×10^{-4}	1.59×10^{-4}
PLAyBOOK	1.10×10^{-4}	1.27×10^{-4}	7.48×10^{1}	7.94×10^{1}	2.97	3.67	3.66×10^{-4}	4.38×10^{-4}	2.73×10^{-4}	3.28×10^{-4}
AEGiS-RS	1.09×10^{-4}	1.19×10^{-4}	6.64×10^{1}	4.31×10^{1}	9.66×10^{-1}	1.39	2.92×10^{-6}	3.27×10^{-6}	8.17×10^{-3}	1.16×10^{-2}
AEGiS	5.32×10^{-6}	6.51×10^{-6}	6.51×10^{1}	1.35×10^{1}	8.10×10^{-1}	8.82×10^{-1}	2.98×10^{-6}	3.83×10^{-6}	9.99×10^{-5}	1.03×10^{-4}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.59×10^{1}	2.07	2.30	2.86×10^{-1}	4.29×10^{1}	1.22×10^{1}	1.05	3.09×10^{-1}	1.33×10^{4}	7.34×10^{3}
TS	3.53	6.96×10^{-1}	2.14	4.66×10^{-1}	1.48×10^{1}	1.68×10^{1}	4.17×10^{-3}	4.99×10^{-3}	3.42×10^{2}	2.79×10^{2}
KB	1.31×10^{1}	5.66	1.12	6.19×10^{-1}	1.74×10^{1}	1.68×10^{1}	1.01×10^{-2}	1.37×10^{-2}	8.14×10^{2}	4.87×10^{2}
LP	1.37×10^{1}	3.02	1.23	4.53×10^{-1}	1.74×10^{1}	1.20×10^{1}	6.42×10^{-3}	8.18×10^{-3}	9.32×10^{2}	5.76×10^{2}
PLAyBOOK	1.38×10^{1}	4.38	1.22	4.05×10^{-1}	1.77×10^{1}	1.73×10^{1}	1.25×10^{-2}	1.78×10^{-2}	7.10×10^{2}	4.25×10^{2}
AEGiS-RS	1.44×10^{1}	3.48	2.11	2.98×10^{-1}	1.46×10^{1}	1.98×10^{1}	7.11×10^{-3}	8.85×10^{-3}	8.65×10^{2}	5.77×10^{2}
AEGiS	3.39	8.01×10^{-1}	1.53	5.62×10^{-1}	1.53×10^{1}	1.89×10^{1}	2.67×10^{-3}	3.16×10^{-3}	5.04×10^{2}	3.54×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	8.26×10^{1}	1.67×10^{1}	1.93×10^{1}	3.95×10^{-1}	6.13	4.68×10^{-1}	5.88×10^{4}	2.73×10^{4}	1.43×10^{2}	1.82×10^{1}
TS	8.65×10^{1}	2.52×10^{1}	1.01×10^{1}	2.46	6.02	4.71×10^{-1}	6.52×10^{2}	4.64×10^{2}	1.65×10^{2}	2.38×10^{1}
KB	4.96×10^{1}	1.70×10^{1}	1.66×10^{1}	1.28	5.19	8.12×10^{-1}	2.40×10^{3}	1.65×10^{3}	9.73×10^{1}	2.26×10^{1}
LP	4.83×10^{1}	1.88×10^{1}	1.62×10^{1}	1.75	5.16	7.35×10^{-1}	2.16×10^{3}	1.14×10^{3}	8.94×10^{1}	2.89×10^{1}
PLAyBOOK	5.27×10^{1}	2.19×10^{1}	1.60×10^{1}	1.66	5.29	5.77×10^{-1}	2.14×10^{3}	1.30×10^{3}	8.64×10^{1}	1.95×10^{1}
AEGiS-RS	4.25×10^{1}	1.91×10^{1}	1.82×10^{1}	1.38	5.94	4.83×10^{-1}	1.09×10^{3}	8.64×10^{2}	6.50×10^{1}	2.25×10^{1}
AEGiS	3.23×10^{1}	1.69×10^{1}	1.41×10^{1}	2.83	5.61	5.60×10^{-1}	9.97×10^{2}	5.96×10^{2}	6.75×10^{1}	2.31×10^{1}

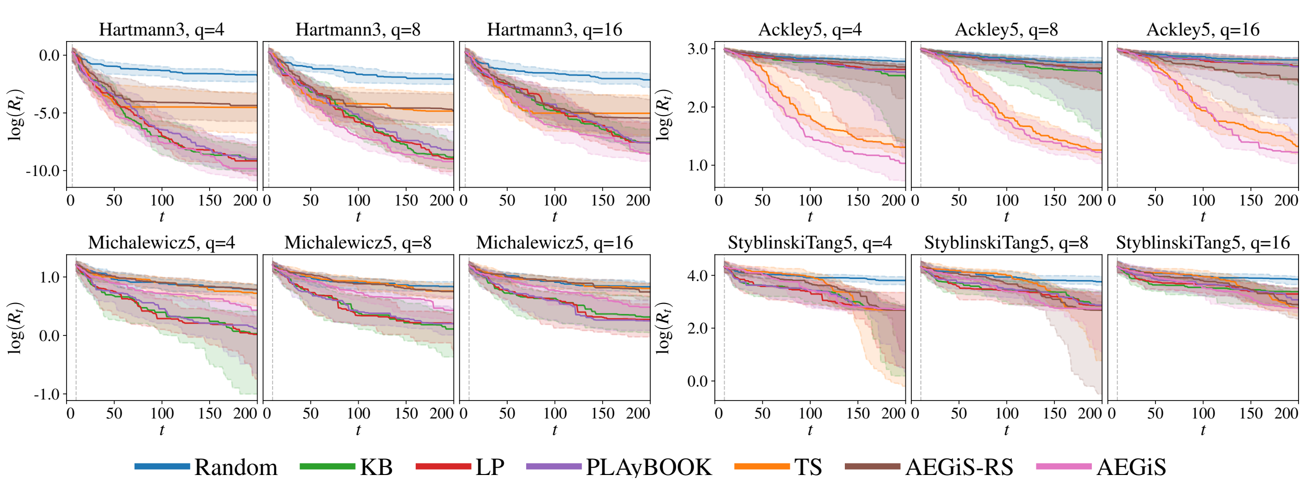

Figure 2: Convergence results for the synthetic test problems.

Table 3: Tabulated results for $q = 16$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.45×10^{-1}	1.47×10^{-1}	1.61×10^{2}	1.05×10^{2}	7.34	8.20	7.09×10^{-2}	7.83×10^{-2}	1.19×10^{-1}	1.01×10^{-1}
TS	1.47×10^{-3}	2.11×10^{-3}	6.51×10^{1}	4.59×10^{1}	1.56	2.29	2.86×10^{-5}	3.62×10^{-5}	6.53×10^{-3}	9.08×10^{-3}
KB	3.53×10^{-4}	3.72×10^{-4}	1.04×10^{2}	8.39×10^{1}	3.62	3.45	2.14×10^{-3}	3.07×10^{-3}	5.25×10^{-4}	6.34×10^{-4}
LP	3.99×10^{-4}	4.98×10^{-4}	9.27×10^{1}	8.75×10^{1}	6.52	6.35	4.97×10^{-3}	6.41×10^{-3}	4.96×10^{-4}	6.01×10^{-4}
PLAyBOOK	3.18×10^{-4}	4.11×10^{-4}	8.36×10^{1}	8.29×10^{1}	8.66	9.65	4.02×10^{-3}	5.46×10^{-3}	5.15×10^{-4}	6.17×10^{-4}
AEGiS-RS	5.47×10^{-5}	7.30×10^{-5}	7.07×10^{1}	2.74×10^{1}	1.77	2.32	3.85×10^{-6}	4.28×10^{-6}	4.33×10^{-3}	6.27×10^{-3}
AEGiS	1.28×10^{-5}	1.80×10^{-5}	6.53×10^{1}	1.27×10^{1}	1.56	1.84	5.82×10^{-6}	5.32×10^{-6}	1.93×10^{-4}	2.80×10^{-4}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	1.66×10^{1}	1.22	2.30	3.03×10^{-1}	4.67×10^{1}	9.10	9.78×10^{-1}	4.73×10^{-1}	1.58×10^{4}	1.03×10^{4}
TS	3.75	7.74×10^{-1}	2.23	3.20×10^{-1}	1.60×10^{1}	1.29×10^{1}	3.79×10^{-3}	4.91×10^{-3}	3.84×10^{2}	2.63×10^{2}
KB	1.49×10^{1}	3.57	1.37	6.89×10^{-1}	2.93×10^{1}	1.65×10^{1}	1.14×10^{-2}	1.53×10^{-2}	1.49×10^{3}	1.18×10^{3}
LP	1.49×10^{1}	4.20	1.31	5.50×10^{-1}	2.69×10^{1}	1.25×10^{1}	1.14×10^{-2}	1.48×10^{-2}	1.25×10^{3}	9.30×10^{2}
PLAyBOOK	1.48×10^{1}	3.31	1.29	5.21×10^{-1}	2.17×10^{1}	1.43×10^{1}	1.40×10^{-2}	1.93×10^{-2}	1.18×10^{3}	8.95×10^{2}
AEGiS-RS	1.13×10^{1}	6.40	2.10	4.70×10^{-1}	1.78×10^{1}	1.42×10^{1}	1.33×10^{-2}	1.70×10^{-2}	7.87×10^{2}	5.00×10^{2}
AEGiS	3.39	1.12	1.62	5.10×10^{-1}	1.56×10^{1}	1.09×10^{1}	3.21×10^{-3}	3.82×10^{-3}	7.38×10^{2}	5.13×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Random	8.46×10^{1}	1.09×10^{1}	1.93×10^{1}	5.99×10^{-1}	6.11	3.89×10^{-1}	5.93×10^{4}	2.66×10^{4}	1.51×10^{2}	1.79×10^{1}
TS	8.53×10^{1}	2.04×10^{1}	9.96	2.36	6.15	4.44×10^{-1}	6.55×10^{2}	3.56×10^{2}	1.61×10^{2}	2.42×10^{1}
KB	5.48×10^{1}	1.99×10^{1}	1.66×10^{1}	1.54	5.39	5.95×10^{-1}	2.87×10^{3}	1.44×10^{3}	8.68×10^{1}	2.69×10^{1}
LP	5.58×10^{1}	1.96×10^{1}	1.66×10^{1}	1.90	5.36	8.89×10^{-1}	3.13×10^{3}	1.86×10^{3}	9.27×10^{1}	2.58×10^{1}
PLAyBOOK	5.43×10^{1}	1.69×10^{1}	1.63×10^{1}	8.28×10^{-1}	5.26	5.96×10^{-1}	3.23×10^{3}	2.04×10^{3}	9.86×10^{1}	3.53×10^{1}
AEGiS-RS	4.63×10^{1}	1.70×10^{1}	1.86×10^{1}	6.95×10^{-1}	6.01	5.74×10^{-1}	1.56×10^{3}	6.77×10^{2}	7.84×10^{1}	3.30×10^{1}
AEGiS	4.32×10^{1}	1.80×10^{1}	1.46×10^{1}	2.22	5.74	5.74×10^{-1}	1.60×10^{3}	1.01×10^{3}	7.20×10^{1}	2.83×10^{1}

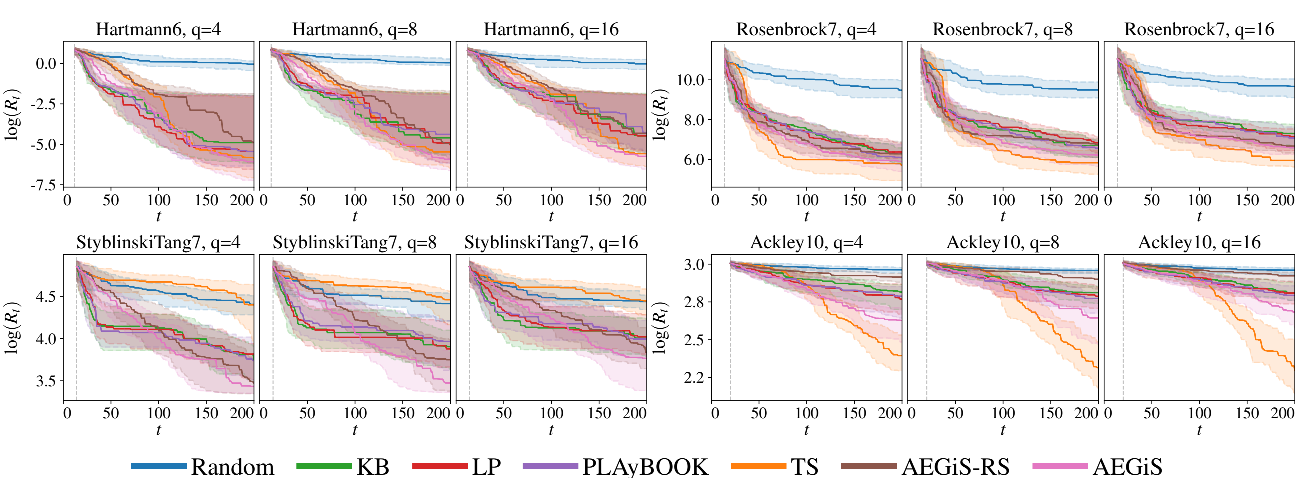

Figure 3: Convergence results for the synthetic test problems.

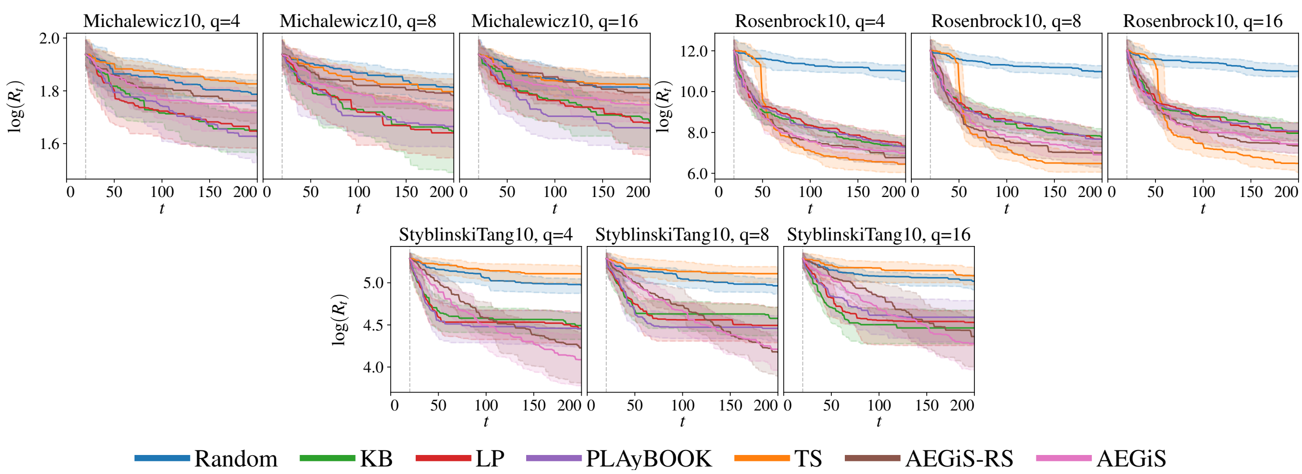

Figure 4: Convergence results for the synthetic test problems.

Table 4: Parameters tuned in the Profet benchmark.

Benchmark	Parameter Name	Range	Log scaled
SVM	C	$[e^{-10},\ e^{10}]$	Yes
	γ	$[e^{-10},\ e^{10}]$	Yes
FC-NET	Learning rate	$[10^{-6},\ 10^{-1}]$	Yes
	Batch size	$[2^3,\ 2^7]$	Yes
	Layer 1: Units	$[2^4,\ 2^9]$	Yes
	Layer 2: Units	$[2^4,\ 2^9]$	Yes
	Layer 1: dropout rate	$[0,\ 0.99]$	-
	Layer 2: dropout rate	$[0,\ 0.99]$	-
XGBoost	Learning rate	$[10^{-6},\ 10^{-1}]$	Yes
	γ	$[0, 2]$	-
	L1 regularisation	$[10^-5,\ 10^3]$	Yes
	L2 regularisation	$[10^-5,\ 10^3]$	Yes
	Number of estimators	$[10, 500]$	-
	Subsampling	$[0.1, 1]$	-
	Maximum depth	$[1, 15]$	-
	Minimum child weight	$[0, 20]$	-

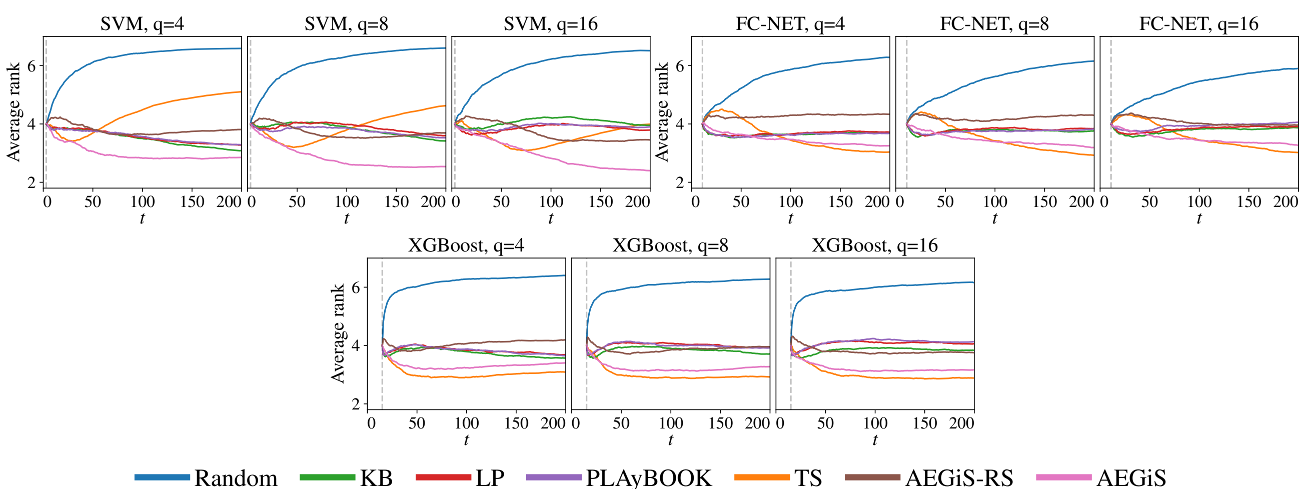

Figure 5: Average rank scores for the SVM, FC-NET and XGBoost hyperparameter optimisation problems.

D INSTANCE-BASED PROBLEMS

In this section we give further details of the hyperparameter optimisation benchmark problems and the robot pushing problems, as well as providing the average rank score plots for each problem.

D.1 HYPERPARAMETER OPTIMISATION BENCHMARK

The SVM problem was built using a set of SVM classification models trained on 16 OpenML tasks, with 2 input parameters corresponding to the SVM's hyperparameters. The FC-NET problem was built using a set of feed-forward neural networks trained on the same OpenML tasks, with 6 input parameters corresponding to the network hyperparameters. XGBoost was built using a set of XGBoost regression models trained on 11 UCI datasets and has 8 input parameters, corresponding to the XGBoost hyperparameters. Table 4 shows the hyperparameters optimised in the three Profet benchmark problems [Klein et al., 2019], their search spaces, and whether or not they were log scaled. Like the other optimisation runs performed in this work, the inputs are rescaled to reside in $[0, 1]$. Figure 5 shows the average rank score plots for the three functions.

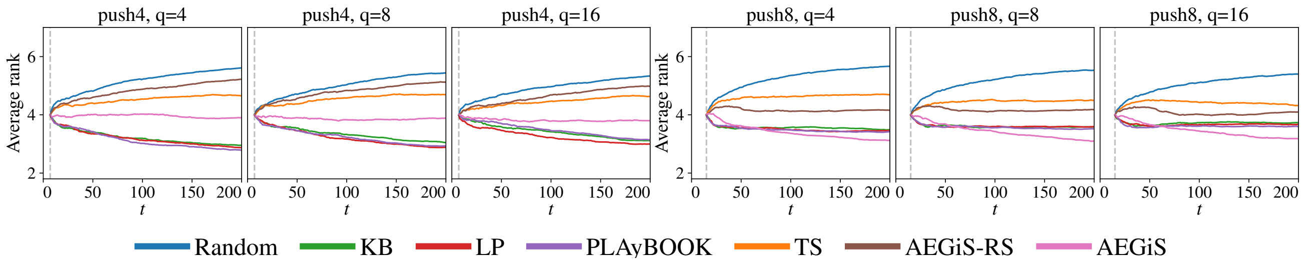

Figure 6: Average rank scores for the push4 and push8 problems.

D.2 ROBOT PUSHING PROBLEMS

Here, we optimised the control parameters for two active learning robot pushing problems. In the first problem (push4), a robot hand is given the task of pushing an object towards an unknown target location. Once the robot has finished pushing the object, it receives feedback in the form of the object-target distance. The adjustable parameters in this problem are the robot's starting coordinates on the $2d$ plain, the orientation of its hand and how long it pushes for. Therefore, this problem can be casts as a minimisation in which we optimise the four parameters in order to minimise the object-target distance. The object's initial location in push4 is always the centre of the domain, and for each problem instance the target location is randomly generated. Note that these instances are shared between methods.

The second problem (push8) is similar to the first except there are two robots moving in the same arena, both having to push their own objects to their respective targets. The final object-target distance from both robots are summed to give an objective function to minimise, resulting in an 8-dimensional problem. Problem instances for push8 were generated such that the minimum distance between the targets was sufficient that each of the objects could be placed on the targets without overlapping. However, this does not mean that each problem instance can be successfully optimised because the targets may be positioned such that the robots block each others path *en route* to the target location.

Figure 6 shows the average rank score plots for push4 and push8.

E ABLATION STUDY

In this section we show all convergence plots and results tables for the ablation study on AEGiS using the synthetic functions. Figure 7 shows the convergence plots for each of the six methods on the fifteen benchmark problems for $q \in \{4, 8, 16\}$. Each plot shows the median log simple regret, with shading representing the interquartile range over 51 runs. Tables 5, 6 and 7 show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion. The method with the best (lowest) median regret is shown in dark grey, and those that are statistically equivalent to the best method according to a one-sided, paired Wilcoxon signed-rank test with Holm-Bonferroni correction [Holm, 1979] ($p \geq 0.05$) are shown in light grey.

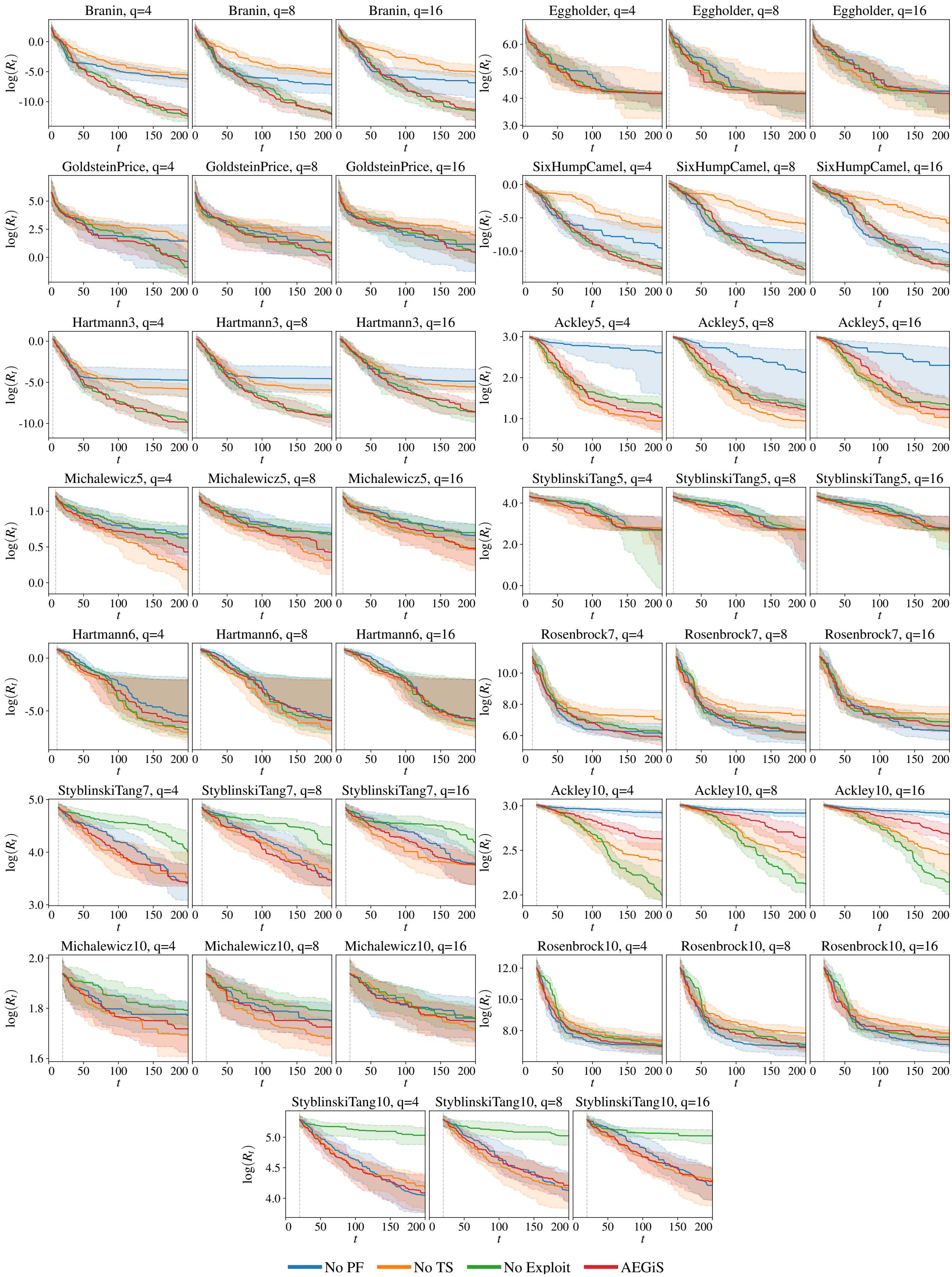

Figure 7: Convergence results for the ablation study.

Table 5: Tabulated results for $q = 4$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	1.87×10^{-3}	2.58×10^{-3}	6.51×10^{1}	1.80×10^{1}	4.24	6.26	7.00×10^{-5}	1.03×10^{-4}	8.84×10^{-3}	1.22×10^{-2}
No TS	3.78×10^{-3}	4.51×10^{-3}	6.63×10^{1}	8.79×10^{1}	3.64	3.73	1.50×10^{-3}	1.77×10^{-3}	2.97×10^{-3}	2.78×10^{-3}
No Exploit	4.20×10^{-6}	4.73×10^{-6}	6.51×10^{1}	1.74×10^{1}	4.08×10^{-1}	5.43×10^{-1}	3.22×10^{-6}	4.12×10^{-6}	5.30×10^{-5}	6.79×10^{-5}
AEGiS	5.99×10^{-6}	6.90×10^{-6}	6.52×10^{1}	1.08×10^{1}	6.99×10^{-1}	7.67×10^{-1}	2.93×10^{-6}	3.31×10^{-6}	5.29×10^{-5}	5.59×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	1.36×10^{1}	4.78	1.98	4.16×10^{-1}	1.47×10^{1}	1.47×10^{1}	4.20×10^{-3}	5.19×10^{-3}	4.76×10^{2}	3.30×10^{2}
No TS	2.56	7.91×10^{-1}	1.19	3.18×10^{-1}	1.63×10^{1}	1.13×10^{1}	8.95×10^{-4}	1.13×10^{-3}	1.12×10^{3}	7.51×10^{2}
No Exploit	3.61	9.86×10^{-1}	1.87	4.42×10^{-1}	1.45×10^{1}	1.73×10^{1}	1.21×10^{-3}	1.50×10^{-3}	4.50×10^{2}	2.59×10^{2}
AEGiS	2.81	1.19	1.54	4.77×10^{-1}	1.51×10^{1}	2.00×10^{1}	2.28×10^{-3}	3.09×10^{-3}	3.64×10^{2}	2.44×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	3.02×10^{1}	1.95×10^{1}	1.85×10^{1}	1.09	5.89	5.40×10^{-1}	1.15×10^{3}	9.62×10^{2}	5.74×10^{1}	2.10×10^{1}
No TS	3.36×10^{1}	2.25×10^{1}	1.08×10^{1}	3.93	5.43	6.51×10^{-1}	1.53×10^{3}	8.32×10^{2}	6.39×10^{1}	2.64×10^{1}
No Exploit	5.46×10^{1}	2.71×10^{1}	7.38	1.39	6.01	4.58×10^{-1}	1.18×10^{3}	7.14×10^{2}	1.53×10^{2}	3.35×10^{1}
AEGiS	3.10×10^{1}	1.79×10^{1}	1.38×10^{1}	2.30	5.57	7.16×10^{-1}	1.08×10^{3}	7.27×10^{2}	5.96×10^{1}	2.41×10^{1}

Table 6: Tabulated results for $q = 8$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	7.73×10^{-4}	1.14×10^{-3}	6.51×10^{1}	3.39×10^{1}	3.78	5.46	1.55×10^{-4}	2.26×10^{-4}	1.05×10^{-2}	1.44×10^{-2}
No TS	4.90×10^{-3}	5.69×10^{-3}	6.54×10^{1}	7.27×10^{1}	3.26	3.61	2.44×10^{-3}	2.86×10^{-3}	2.60×10^{-3}	2.39×10^{-3}
No Exploit	5.28×10^{-6}	6.77×10^{-6}	6.60×10^{1}	2.61×10^{1}	1.56	1.95	2.94×10^{-6}	3.49×10^{-6}	1.26×10^{-4}	1.13×10^{-4}
AEGiS	5.32×10^{-6}	6.51×10^{-6}	6.51×10^{1}	1.35×10^{1}	8.10×10^{-1}	8.82×10^{-1}	2.98×10^{-6}	3.83×10^{-6}	9.99×10^{-5}	1.03×10^{-4}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	8.41	8.03	2.00	4.95×10^{-1}	1.44×10^{1}	3.17	3.54×10^{-3}	4.78×10^{-3}	4.85×10^{2}	4.39×10^{2}
No TS	2.58	7.86×10^{-1}	1.37	4.58×10^{-1}	1.44×10^{1}	1.23×10^{1}	1.22×10^{-3}	1.60×10^{-3}	1.46×10^{3}	9.07×10^{2}
No Exploit	3.68	7.93×10^{-1}	1.96	3.87×10^{-1}	1.51×10^{1}	1.31×10^{1}	2.83×10^{-3}	3.96×10^{-3}	4.79×10^{2}	3.09×10^{2}
AEGiS	3.39	8.01×10^{-1}	1.53	5.62×10^{-1}	1.53×10^{1}	1.89×10^{1}	2.67×10^{-3}	3.16×10^{-3}	5.04×10^{2}	3.54×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	3.20×10^{1}	1.80×10^{1}	1.85×10^{1}	1.09	5.77	5.89×10^{-1}	1.09×10^{3}	7.33×10^{2}	6.22×10^{1}	2.28×10^{1}
No TS	3.70×10^{1}	2.21×10^{1}	1.12×10^{1}	2.31	5.37	5.73×10^{-1}	2.55×10^{3}	1.58×10^{3}	6.46×10^{1}	2.72×10^{1}
No Exploit	6.21×10^{1}	3.51×10^{1}	8.39	1.34	5.99	3.30×10^{-1}	1.22×10^{3}	6.31×10^{2}	1.52×10^{2}	2.54×10^{1}
AEGiS	3.23×10^{1}	1.69×10^{1}	1.41×10^{1}	2.83	5.61	5.60×10^{-1}	9.97×10^{2}	5.96×10^{2}	6.75×10^{1}	2.31×10^{1}

Table 7: Tabulated results for $q = 16$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	1.06×10^{-3}	1.55×10^{-3}	7.07×10^{1}	4.58×10^{1}	3.19	4.63	3.56×10^{-5}	4.69×10^{-5}	7.75×10^{-3}	1.13×10^{-2}
No TS	6.67×10^{-3}	7.63×10^{-3}	6.51×10^{1}	5.92×10^{1}	7.21	8.09	3.09×10^{-3}	4.11×10^{-3}	3.86×10^{-3}	2.95×10^{-3}
No Exploit	1.01×10^{-5}	1.36×10^{-5}	6.51×10^{1}	3.15×10^{1}	1.65	2.05	4.54×10^{-6}	5.45×10^{-6}	1.83×10^{-4}	2.41×10^{-4}
AEGiS	1.28×10^{-5}	1.80×10^{-5}	6.53×10^{1}	1.27×10^{1}	1.56	1.84	5.82×10^{-6}	5.32×10^{-6}	1.93×10^{-4}	2.80×10^{-4}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	9.98	8.05	1.93	2.98×10^{-1}	1.63×10^{1}	1.82×10^{1}	3.16×10^{-3}	3.50×10^{-3}	5.44×10^{2}	4.00×10^{2}
No TS	2.80	9.83×10^{-1}	1.59	4.78×10^{-1}	1.47×10^{1}	1.23×10^{1}	1.66×10^{-3}	1.90×10^{-3}	1.61×10^{3}	1.13×10^{3}
No Exploit	3.80	7.76×10^{-1}	1.99	4.63×10^{-1}	1.51×10^{1}	6.02	2.62×10^{-3}	3.46×10^{-3}	9.71×10^{2}	6.76×10^{2}
AEGiS	3.39	1.12	1.62	5.10×10^{-1}	1.56×10^{1}	1.09×10^{1}	3.21×10^{-3}	3.82×10^{-3}	7.38×10^{2}	5.13×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	4.35×10^{1}	2.08×10^{1}	1.83×10^{1}	9.93×10^{-1}	5.82	7.68×10^{-1}	1.20×10^{3}	8.13×10^{2}	6.75×10^{1}	2.87×10^{1}
No TS	4.29×10^{1}	1.89×10^{1}	1.17×10^{1}	2.85	5.58	5.76×10^{-1}	2.53×10^{3}	1.45×10^{3}	7.29×10^{1}	3.20×10^{1}
No Exploit	6.58×10^{1}	2.60×10^{1}	8.50	1.54	5.81	4.38×10^{-1}	1.95×10^{3}	1.27×10^{3}	1.52×10^{2}	2.42×10^{1}
AEGiS	4.32×10^{1}	1.80×10^{1}	1.46×10^{1}	2.22	5.74	5.74×10^{-1}	1.60×10^{3}	1.01×10^{3}	7.20×10^{1}	2.83×10^{1}

F SELECTING ϵ

In this section we show all convergence plots and results table for choosing a suitable value of ϵ ($\epsilon_T = \epsilon_P = \epsilon/2$). Here, AEGiS corresponds to the value used throughout the rest of the paper, $\epsilon = \min(2/\sqrt{d}, 1)$. The labels *faster* and *slower* correspond to using values of $\epsilon = \min(2/(d-2), 1)$ and $\epsilon = \min(2/\log(d+3), 1)$ respectively, where *faster* and *slower* refer to the increased and decrease rate of decay of ϵ with respect to the problem dimensionality d. Therefore, *faster* exploits more than AEGiS, and *slower* explores more than AEGiS for a given problem dimensionality. Figure 8 shows the three ϵ decay curves. Figure 9 shows the convergence plots for AEGiS with the three ϵ decay rates evaluated on the 15 synthetic test functions for $q \in \{4, 8, 16\}$. Tables 8, 9, and 10 show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion.

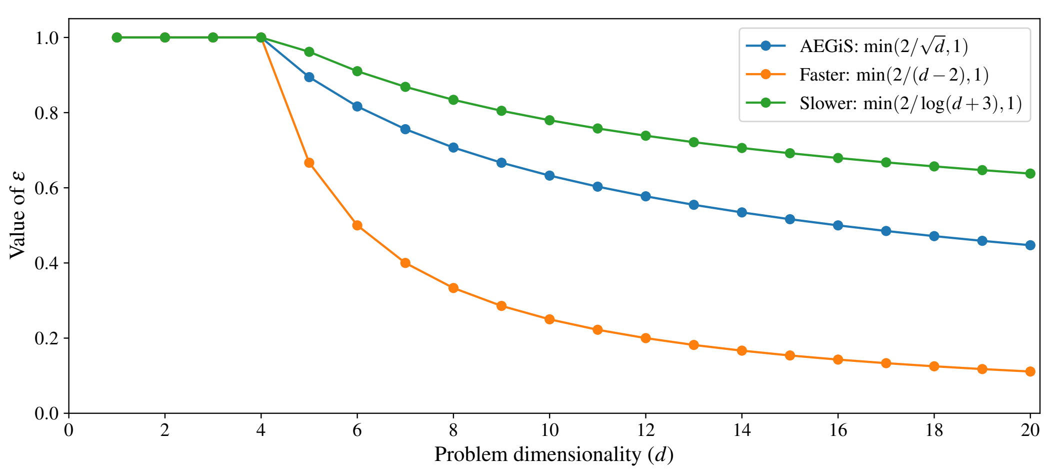

Figure 8: Epsilon decay rate for the three evaluated heuristics. Here, *faster* refers to a higher rate of decay and thus more exploitation, whereas *slower* refers to a lower rate of decay and therefore more exploration.

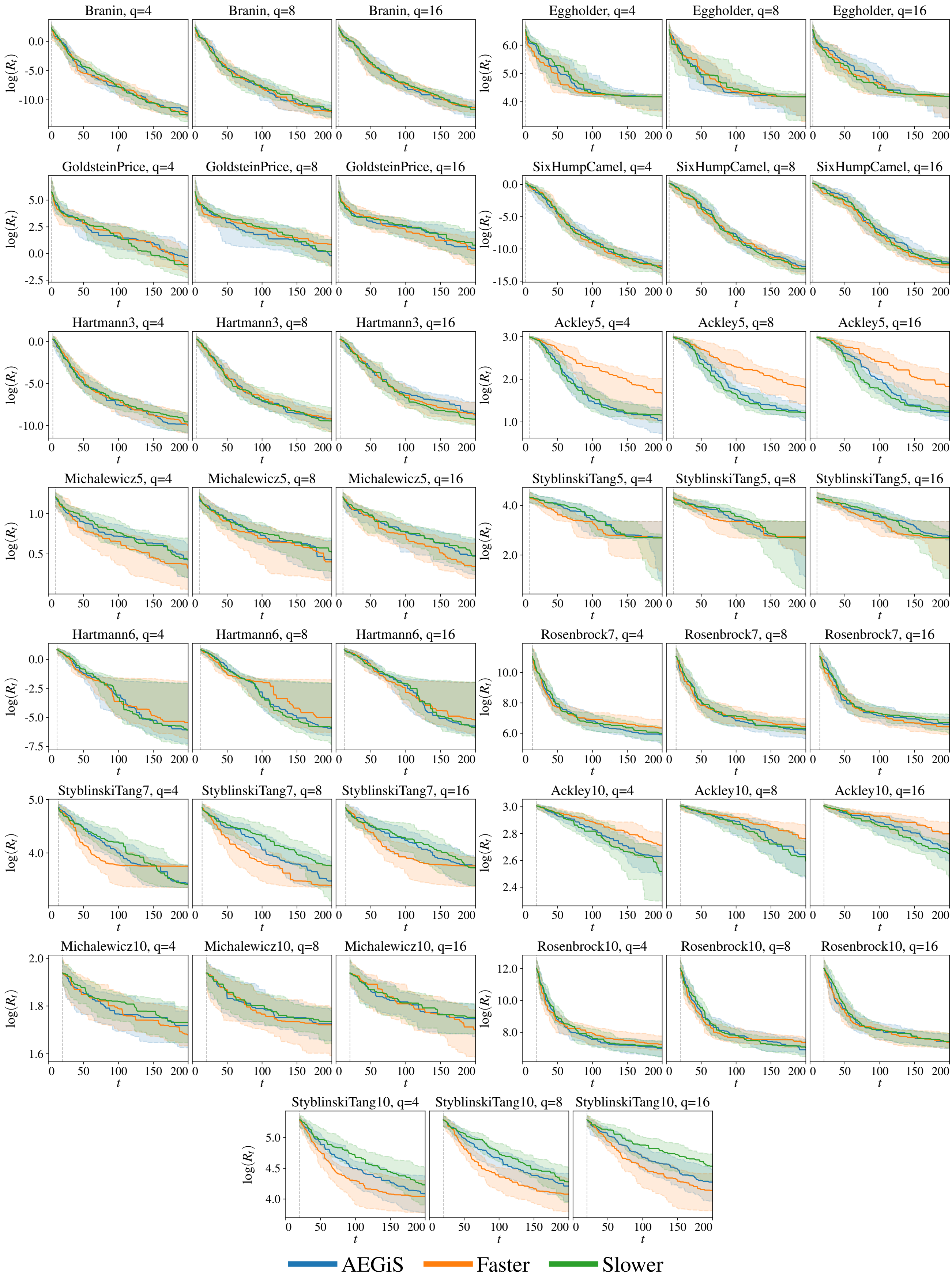

Figure 9: Convergence results for the ϵ value experiments.

Table 8: Tabulated results for $q = 4$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	5.99×10^{-6}	6.90×10^{-6}	6.52×10^{1}	1.08×10^{1}	6.99×10^{-1}	7.67×10^{-1}	2.93×10^{-6}	3.31×10^{-6}	5.29×10^{-5}	5.59×10^{-5}
Faster	4.81×10^{-6}	5.56×10^{-6}	6.51×10^{1}	1.10×10^{1}	2.84×10^{-1}	3.57×10^{-1}	2.83×10^{-6}	3.17×10^{-6}	4.76×10^{-5}	5.89×10^{-5}
Slower	3.47×10^{-6}	4.18×10^{-6}	6.51×10^{1}	1.61×10^{1}	3.60×10^{-1}	4.71×10^{-1}	2.25×10^{-6}	2.87×10^{-6}	7.01×10^{-5}	8.80×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	2.81	1.19	1.54	4.77×10^{-1}	1.51×10^{1}	2.00×10^{1}	2.28×10^{-3}	3.09×10^{-3}	3.64×10^{2}	2.44×10^{2}
Faster	5.34	3.53	1.38	4.77×10^{-1}	1.50×10^{1}	1.99×10^{1}	4.41×10^{-3}	6.05×10^{-3}	5.72×10^{2}	3.43×10^{2}
Slower	3.20	6.71×10^{-1}	1.53	6.13×10^{-1}	1.47×10^{1}	1.32×10^{1}	2.28×10^{-3}	3.07×10^{-3}	4.04×10^{2}	3.08×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	3.10×10^{1}	1.79×10^{1}	1.38×10^{1}	2.30	5.57	7.16×10^{-1}	1.08×10^{3}	7.27×10^{2}	5.96×10^{1}	2.41×10^{1}
Faster	4.25×10^{1}	2.04×10^{1}	1.51×10^{1}	2.52	5.36	7.35×10^{-1}	1.40×10^{3}	9.75×10^{2}	5.70×10^{1}	2.25×10^{1}
Slower	3.03×10^{1}	1.93×10^{1}	1.24×10^{1}	3.37	5.65	6.11×10^{-1}	1.13×10^{3}	8.73×10^{2}	6.87×10^{1}	3.00×10^{1}

Table 9: Tabulated results for $q = 8$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	5.32×10^{-6}	6.51×10^{-6}	6.51×10^{1}	1.35×10^{1}	8.10×10^{-1}	8.82×10^{-1}	2.98×10^{-6}	3.83×10^{-6}	9.99×10^{-5}	1.03×10^{-4}
Faster	5.96×10^{-6}	6.64×10^{-6}	6.51×10^{1}	1.42×10^{1}	2.36	3.10	2.09×10^{-6}	2.07×10^{-6}	9.89×10^{-5}	1.16×10^{-4}
Slower	7.57×10^{-6}	1.02×10^{-5}	6.51×10^{1}	2.30×10^{1}	1.16	1.62	2.02×10^{-6}	2.63×10^{-6}	7.94×10^{-5}	9.45×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	3.39	8.01×10^{-1}	1.53	5.62×10^{-1}	1.53×10^{1}	1.89×10^{1}	2.67×10^{-3}	3.16×10^{-3}	5.04×10^{2}	3.54×10^{2}
Faster	6.09	2.54	1.49	5.13×10^{-1}	1.53×10^{1}	1.94×10^{1}	6.73×10^{-3}	9.57×10^{-3}	6.20×10^{2}	4.57×10^{2}
Slower	3.37	6.10×10^{-1}	1.69	5.22×10^{-1}	1.47×10^{1}	2.03×10^{1}	2.91×10^{-3}	4.06×10^{-3}	5.45×10^{2}	2.68×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	3.23×10^{1}	1.69×10^{1}	1.41×10^{1}	2.83	5.61	5.60×10^{-1}	9.97×10^{2}	5.96×10^{2}	6.75×10^{1}	2.31×10^{1}
Faster	2.96×10^{1}	1.94×10^{1}	1.58×10^{1}	2.13	5.59	8.40×10^{-1}	1.57×10^{3}	6.94×10^{2}	5.92×10^{1}	2.36×10^{1}
Slower	4.29×10^{1}	1.85×10^{1}	1.34×10^{1}	2.84	5.67	4.79×10^{-1}	1.14×10^{3}	8.00×10^{2}	7.23×10^{1}	2.85×10^{1}

Table 10: Tabulated results for $q = 16$ asynchronous workers, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	1.28×10^{-5}	1.80×10^{-5}	6.53×10^{1}	1.27×10^{1}	1.56	1.84	5.82×10^{-6}	5.32×10^{-6}	1.93×10^{-4}	2.80×10^{-4}
Faster	1.26×10^{-5}	1.55×10^{-5}	6.54×10^{1}	1.32×10^{1}	1.36	1.92	3.71×10^{-6}	4.40×10^{-6}	1.61×10^{-4}	1.92×10^{-4}
Slower	9.08×10^{-6}	9.72×10^{-6}	6.54×10^{1}	1.73×10^{1}	2.20	3.04	4.51×10^{-6}	5.24×10^{-6}	9.79×10^{-5}	1.17×10^{-4}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	3.39	1.12	1.62	5.10×10^{-1}	1.56×10^{1}	1.09×10^{1}	3.21×10^{-3}	3.82×10^{-3}	7.38×10^{2}	5.13×10^{2}
Faster	6.26	3.70	1.41	5.64×10^{-1}	1.46×10^{1}	2.03×10^{1}	5.44×10^{-3}	7.27×10^{-3}	6.18×10^{2}	5.03×10^{2}
Slower	3.51	6.31×10^{-1}	1.60	4.95×10^{-1}	1.49×10^{1}	1.74×10^{1}	2.82×10^{-3}	3.14×10^{-3}	8.27×10^{2}	7.18×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
AEGiS	4.32×10^{1}	1.80×10^{1}	1.46×10^{1}	2.22	5.74	5.74×10^{-1}	1.60×10^{3}	1.01×10^{3}	7.20×10^{1}	2.83×10^{1}
Faster	4.28×10^{1}	2.04×10^{1}	1.64×10^{1}	2.21	5.48	7.44×10^{-1}	1.61×10^{3}	1.12×10^{3}	6.30×10^{1}	2.76×10^{1}
Slower	4.12×10^{1}	1.62×10^{1}	1.41×10^{1}	2.44	5.77	5.26×10^{-1}	1.67×10^{3}	1.11×10^{3}	9.33×10^{1}	2.98×10^{1}

G SEQUENTIAL BAYESIAN OPTIMISATION ($q = 1$)

In this section we evaluate AEGiS in the sequential BO setting, i.e. with $q = 1$ workers and compare it to the other asynchronous methods in Section G.1, carry out an ablation study in Section G.2 and investigate faster and slower ϵ decay rates in Section G.3.

G.1 EVALUATION

We first compare compare AEGiS to the other asynchronous methods in the sequential setting. Note that KB, LP and PLAyBOOK are equivalent because each select the first location to be evaluated using EI. Therefore, we compare AEGiS and AEGiS-RS to EI, TS and Latin hypercube sampling (i.e. Random) on the 15 synthetic benchmark functions. Figure 10 shows the convergence plots for the benchmark functions, with solid lines showing the median log simple regret and shading showing the interquartile range. Table 11, show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion. The method with the best (lowest) median regret is shown in dark grey, and those that are statistically equivalent to the best method according to a one-sided, paired Wilcoxon signed-rank test with Holm-Bonferroni correction [Holm, 1979] ($p \geq 0.05$) are shown in light grey. Figure 11 summarises the tabulated results and shows the number of times each method is best or statistically equal to the best performing method.

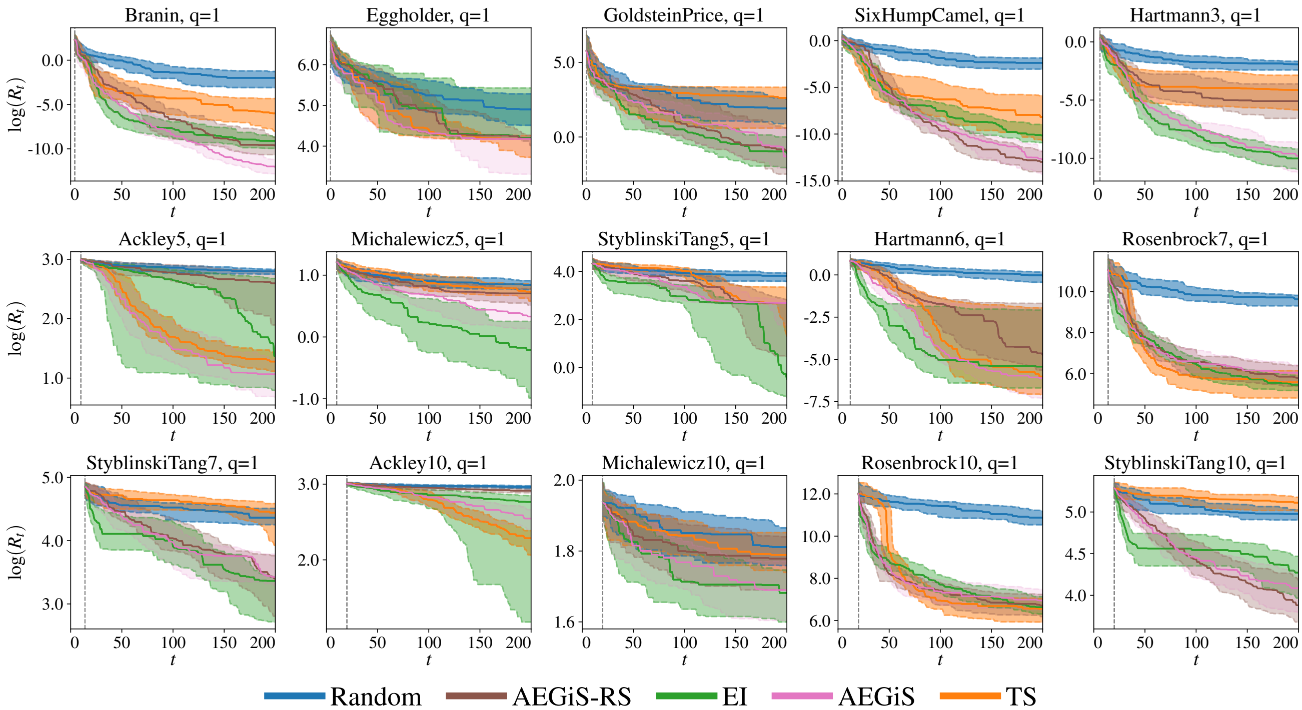

Figure 10: Convergence results for the sequential BO experiments.

Table 11: Tabulated results for the sequential ($q = 1$) BO optimisation runs, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)** Median	MAD	**Eggholder (2)** Median	MAD	**GoldsteinPrice (2)** Median	MAD	**SixHumpCamel (2)** Median	MAD	**Hartmann3 (3)** Median	MAD
Random	1.37×10^{-1}	1.41×10^{-1}	1.35×10^{2}	7.08×10^{1}	6.75	7.57	9.30×10^{-2}	7.41×10^{-2}	1.46×10^{-1}	7.08×10^{-2}
TS	2.51×10^{-3}	3.58×10^{-3}	6.51×10^{1}	1.13×10^{1}	1.32×10^{1}	1.91×10^{1}	2.81×10^{-4}	4.14×10^{-4}	1.59×10^{-2}	2.15×10^{-2}
EI	1.11×10^{-4}	9.39×10^{-5}	6.71×10^{1}	9.25	3.46×10^{-1}	4.25×10^{-1}	3.87×10^{-5}	4.74×10^{-5}	4.42×10^{-5}	4.74×10^{-5}
AEGiS-RS	6.76×10^{-5}	9.41×10^{-5}	6.58×10^{1}	7.47	3.96×10^{-1}	5.23×10^{-1}	2.30×10^{-6}	2.64×10^{-6}	6.16×10^{-3}	8.59×10^{-3}
AEGiS	5.69×10^{-6}	4.87×10^{-6}	6.51×10^{1}	2.09×10^{1}	2.71×10^{-1}	3.67×10^{-1}	3.08×10^{-6}	3.94×10^{-6}	5.62×10^{-5}	6.94×10^{-5}

Method	**Ackley5 (5)** Median	MAD	**Michalewicz5 (5)** Median	MAD	**StyblinskiTang5 (5)** Median	MAD	**Hartmann6 (6)** Median	MAD	**Rosenbrock7 (7)** Median	MAD
Random	1.64×10^{1}	1.01	2.32	2.49×10^{-1}	4.56×10^{1}	1.32×10^{1}	9.51×10^{-1}	4.03×10^{-1}	1.51×10^{4}	4.90×10^{3}
TS	3.59	8.21×10^{-1}	2.07	4.32×10^{-1}	1.45×10^{1}	1.94×10^{1}	2.44×10^{-3}	2.89×10^{-3}	2.68×10^{2}	2.35×10^{2}
EI	3.94	4.60	8.09×10^{-1}	6.85×10^{-1}	6.13×10^{-1}	7.75×10^{-1}	4.40×10^{-3}	5.64×10^{-3}	2.33×10^{2}	1.19×10^{2}
AEGiS-RS	1.34×10^{1}	6.18	2.02	3.77×10^{-1}	1.46×10^{1}	1.09×10^{1}	8.86×10^{-3}	1.16×10^{-2}	3.24×10^{2}	2.10×10^{2}
AEGiS	2.90	1.44	1.39	5.38×10^{-1}	1.46×10^{1}	2.95	2.21×10^{-3}	2.77×10^{-3}	4.20×10^{2}	2.33×10^{2}

Method	**StyblinskiTang7 (7)** Median	MAD	**Ackley10 (10)** Median	MAD	**Michalewicz10 (10)** Median	MAD	**Rosenbrock10 (10)** Median	MAD	**StyblinskiTang10 (10)** Median	MAD
Random	7.85×10^{1}	1.81×10^{1}	1.93×10^{1}	4.99×10^{-1}	6.11	4.75×10^{-1}	5.32×10^{4}	2.30×10^{4}	1.45×10^{2}	1.46×10^{1}
TS	8.48×10^{1}	2.92×10^{1}	9.82	2.18	5.98	5.30×10^{-1}	6.98×10^{2}	5.45×10^{2}	1.66×10^{2}	2.04×10^{1}
EI	2.89×10^{1}	1.74×10^{1}	1.59×10^{1}	4.32	5.37	5.65×10^{-1}	7.59×10^{2}	3.91×10^{2}	7.18×10^{1}	2.66×10^{1}
AEGiS-RS	3.00×10^{1}	1.93×10^{1}	1.84×10^{1}	6.36×10^{-1}	5.92	6.22×10^{-1}	8.23×10^{2}	5.14×10^{2}	4.85×10^{1}	1.96×10^{1}
AEGiS	3.07×10^{1}	1.89×10^{1}	1.27×10^{1}	2.38	5.42	6.94×10^{-1}	1.03×10^{3}	7.26×10^{2}	5.95×10^{1}	2.10×10^{1}

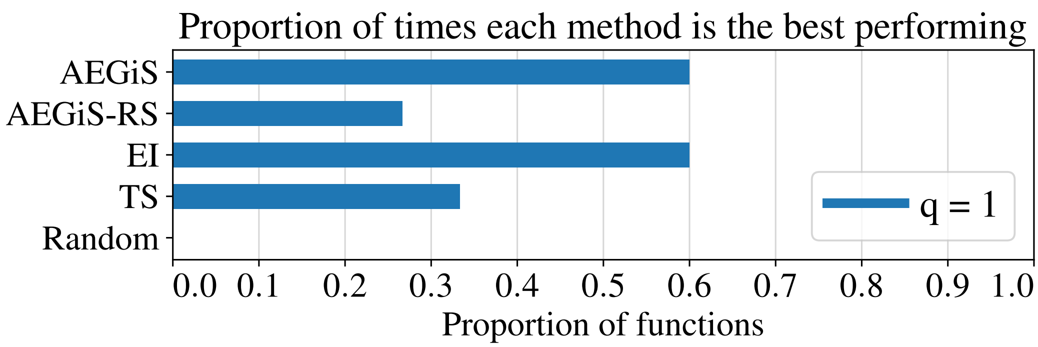

Figure 11: Synthetic function optimisation summary. Bar lengths correspond to the proportion of times that a method is best or statistically equivalent to the best method across the 15 synthetic functions.

G.2 ABLATION STUDY

Next, we perform an ablation study as before in the main paper. Figure 12 shows the convergence plots for the benchmark functions, with solid lines showing the median log simple regret and shading showing the interquartile range. Table 12, show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion. The method with the best (lowest) median regret is shown in dark grey, and those that are statistically equivalent to the best method according to a one-sided, paired Wilcoxon signed-rank test with Holm-Bonferroni correction [Holm, 1979] ($p \geq 0.05$) are shown in light grey. Figure 13 summarises the tabulated results and shows the number of times each method is best or statistically equal to the best performing method.

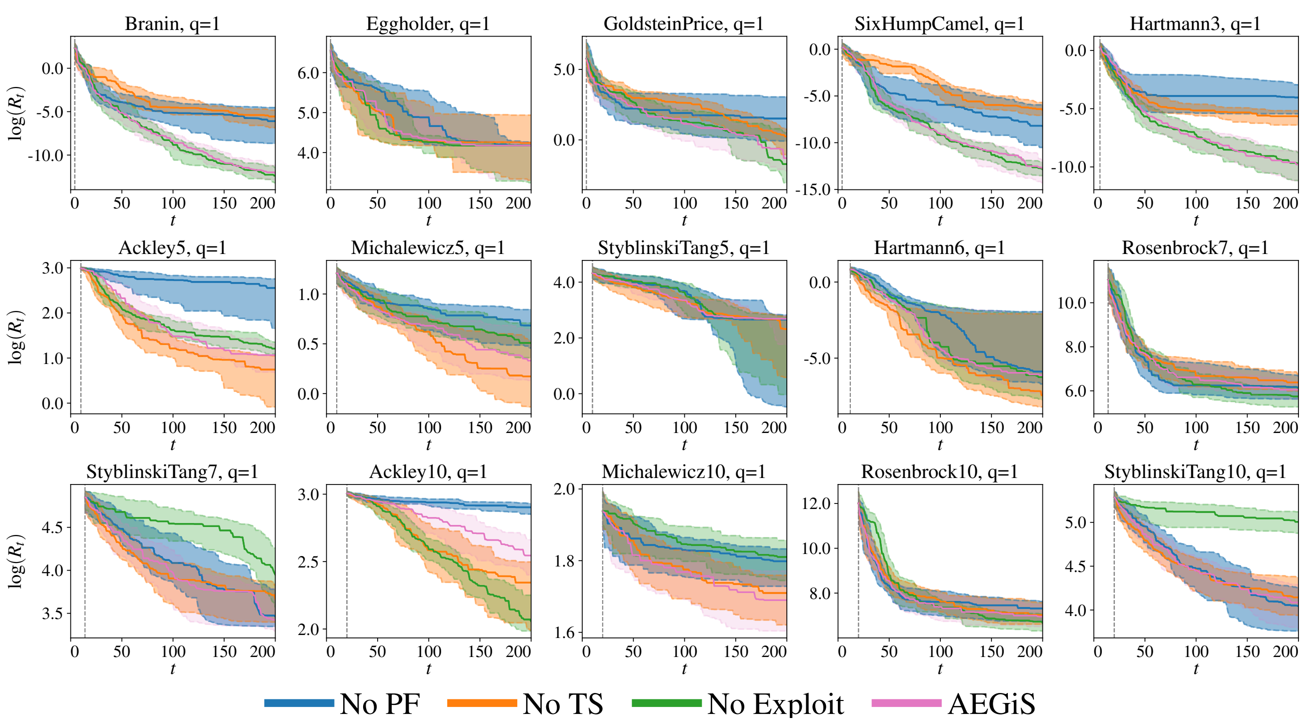

Figure 12: Convergence results for the ablation study experiments.

Table 12: Tabulated results for the ablation study of the sequential ($q = 1$) BO optimisation runs, showing the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	2.24×10^{-3}	3.28×10^{-3}	6.51×10^{1}	8.41	4.54	6.51	2.73×10^{-4}	4.01×10^{-4}	1.75×10^{-2}	2.20×10^{-2}
No TS	3.85×10^{-3}	4.64×10^{-3}	6.96×10^{1}	8.70×10^{1}	1.28	1.36	1.47×10^{-3}	1.41×10^{-3}	3.43×10^{-3}	2.87×10^{-3}
No Exploit	4.20×10^{-6}	4.25×10^{-6}	6.51×10^{1}	1.32×10^{1}	1.79×10^{-1}	2.50×10^{-1}	2.59×10^{-6}	2.57×10^{-6}	6.05×10^{-5}	7.30×10^{-5}
AEGiS	5.69×10^{-6}	4.87×10^{-6}	6.51×10^{1}	2.09×10^{1}	2.71×10^{-1}	3.67×10^{-1}	3.08×10^{-6}	3.94×10^{-6}	5.62×10^{-5}	6.94×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	1.29×10^{1}	5.68	1.98	5.14×10^{-1}	1.43×10^{1}	2.00×10^{1}	2.78×10^{-3}	3.38×10^{-3}	4.73×10^{2}	3.39×10^{2}
No TS	2.09	1.24	1.19	5.80×10^{-1}	1.02×10^{1}	1.05×10^{1}	5.86×10^{-4}	6.70×10^{-4}	5.93×10^{2}	3.03×10^{2}
No Exploit	3.35	7.60×10^{-1}	1.66	4.23×10^{-1}	1.46×10^{1}	1.03×10^{1}	1.93×10^{-3}	2.66×10^{-3}	3.12×10^{2}	1.88×10^{2}
AEGiS	2.90	1.44	1.39	5.38×10^{-1}	1.46×10^{1}	2.95	2.21×10^{-3}	2.77×10^{-3}	4.20×10^{2}	2.33×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
No PF	3.20×10^{1}	1.67×10^{1}	1.82×10^{1}	1.00	6.04	4.16×10^{-1}	1.51×10^{3}	9.78×10^{2}	5.73×10^{1}	2.04×10^{1}
No TS	3.92×10^{1}	1.32×10^{1}	1.04×10^{1}	3.56	5.53	5.38×10^{-1}	1.12×10^{3}	7.36×10^{2}	6.31×10^{1}	2.16×10^{1}
No Exploit	5.19×10^{1}	2.51×10^{1}	7.82	1.52	6.11	5.01×10^{-1}	7.89×10^{2}	4.23×10^{2}	1.49×10^{2}	2.73×10^{1}
AEGiS	3.07×10^{1}	1.89×10^{1}	1.27×10^{1}	2.38	5.42	6.94×10^{-1}	1.03×10^{3}	7.26×10^{2}	5.95×10^{1}	2.10×10^{1}

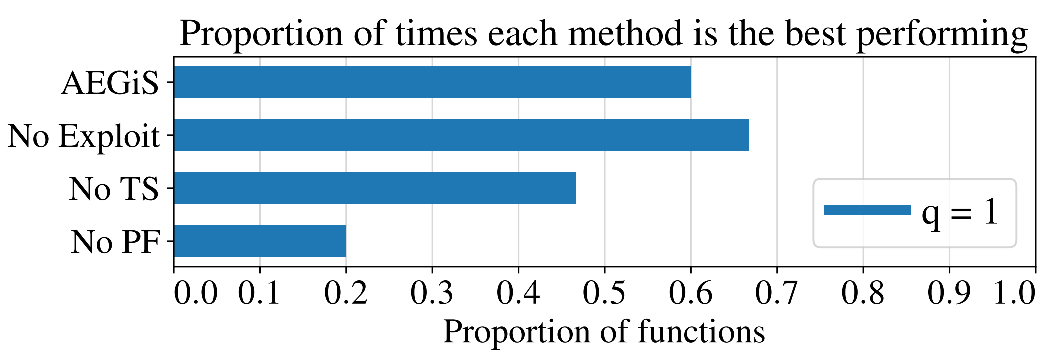

Figure 13: Ablation study summary. Bar lengths correspond to the proportion of times that a method is best or statistically equivalent to the best method across the 15 synthetic functions.

G.3 SETTING ϵ

Lastly, we compare different rates of ϵ decay ($\epsilon_T = \epsilon_P = \epsilon/2$). Here, *faster* corresponds to a quicker rate of decay and thus an increase in exploitation, and *slower* corresponds to a reduced rate, leading to less exploitation. Figure 14 shows the convergence plots for the benchmark functions, with solid lines showing the median log simple regret and shading showing the interquartile range. Table 13, show the median log simple regret as well as the median absolute deviation from the median (MAD), a robust measure of dispersion. The method with the best (lowest) median regret is shown in dark grey, and those that are statistically equivalent to the best method according to a one-sided, paired Wilcoxon signed-rank test with Holm-Bonferroni correction [Holm, 1979] ($p \geq 0.05$) are shown in light grey. Figure 15 summarises the tabulated results and shows the number of times each method is best or statistically equal to the best performing method.

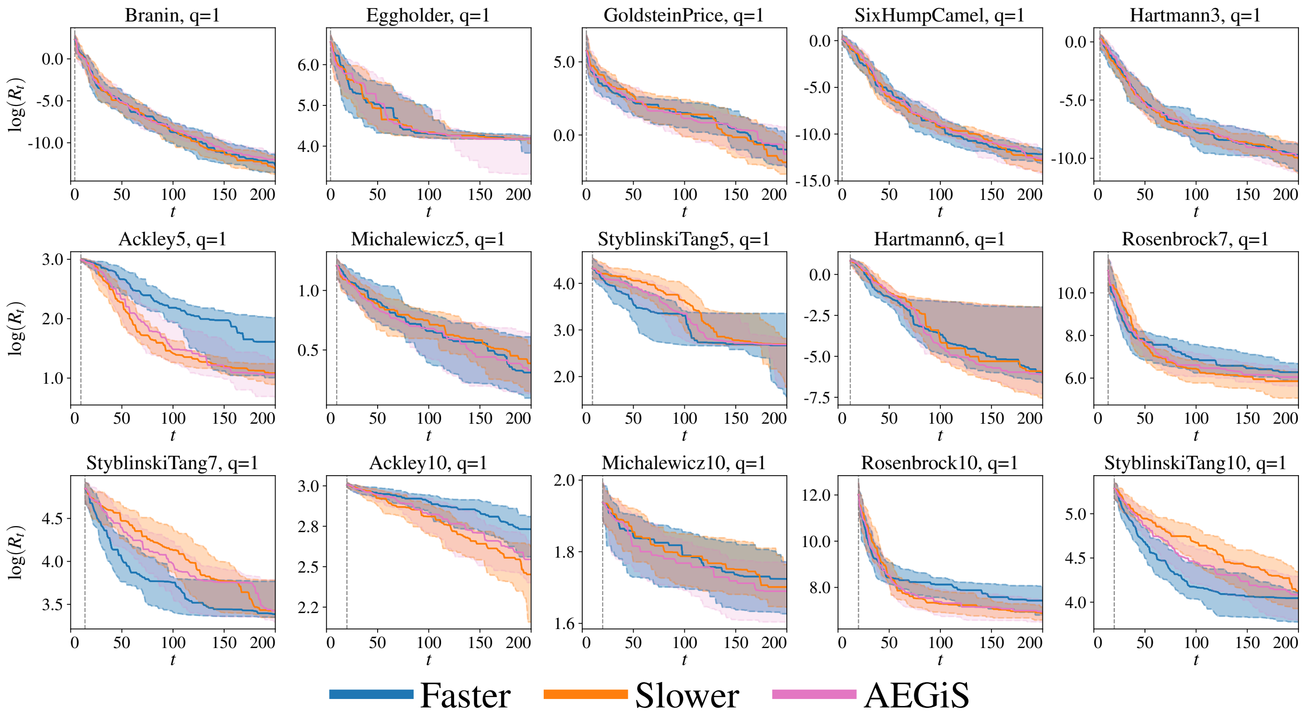

Figure 14: Convergence results for the setting ϵ experiments.

Table 13: Tabulated results for the setting ϵ experiments in the sequential setting. The table shows the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Faster	3.16×10^{-6}	4.21×10^{-6}	6.51×10^{1}	8.39	3.68×10^{-1}	4.81×10^{-1}	5.28×10^{-6}	5.65×10^{-6}	6.15×10^{-5}	7.50×10^{-5}
Slower	2.23×10^{-6}	2.69×10^{-6}	6.51×10^{1}	4.03	1.55×10^{-1}	1.84×10^{-1}	2.75×10^{-6}	3.13×10^{-6}	4.44×10^{-5}	6.19×10^{-5}
AEGiS	5.69×10^{-6}	4.87×10^{-6}	6.51×10^{1}	2.09×10^{1}	2.71×10^{-1}	3.67×10^{-1}	3.08×10^{-6}	3.94×10^{-6}	5.62×10^{-5}	6.94×10^{-5}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Faster	5.00	3.62	1.36	5.55×10^{-1}	1.45×10^{1}	2.03×10^{1}	2.59×10^{-3}	2.83×10^{-3}	5.34×10^{2}	3.07×10^{2}
Slower	2.95	7.71×10^{-1}	1.47	4.00×10^{-1}	1.47×10^{1}	3.93	2.45×10^{-3}	3.39×10^{-3}	3.49×10^{2}	2.82×10^{2}
AEGiS	2.90	1.44	1.39	5.38×10^{-1}	1.46×10^{1}	2.95	2.21×10^{-3}	2.77×10^{-3}	4.20×10^{2}	2.33×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
Faster	2.95×10^{1}	1.98×10^{1}	1.54×10^{1}	3.28	5.61	7.34×10^{-1}	1.69×10^{3}	1.44×10^{3}	5.71×10^{1}	2.00×10^{1}
Slower	3.08×10^{1}	1.87×10^{1}	1.16×10^{1}	3.73	5.48	5.56×10^{-1}	9.86×10^{2}	3.87×10^{2}	6.20×10^{1}	2.15×10^{1}
AEGiS	3.07×10^{1}	1.89×10^{1}	1.27×10^{1}	2.38	5.42	6.94×10^{-1}	1.03×10^{3}	7.26×10^{2}	5.95×10^{1}	2.10×10^{1}

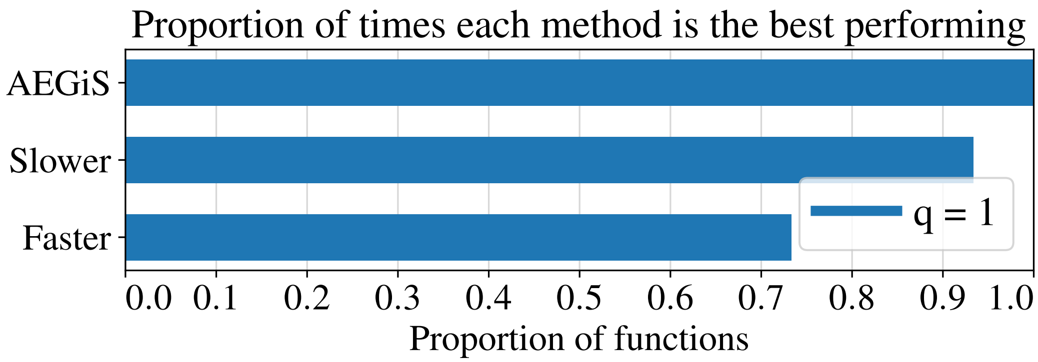

Figure 15: Setting ϵ summary. Bar lengths correspond to the proportion of times that a method is best or statistically equivalent to the best method across the 15 synthetic functions.

H PROPORTION OF TS TO PARETO SET SELECTION

In this section we show all convergence plots and results tables for the investigation into the optimal selection ratio between TS and Pareto set selection on the synthetic benchmark functions for $q \in \{4, 8, 16\}$. Specifically we evaluate AEGiS on the synthetic benchmark functions for $q \in \{4, 8, 16\}$ with the split between exploitation, TS and Pareto set selection being $1 - \epsilon$, $\epsilon_T = \gamma\epsilon$ and $\epsilon_P = (1 - \gamma)\epsilon$ respectively and $\gamma \in \{0, 0.1, \dots, 1\}$. Note that we use the default value $\epsilon = \min(2/\sqrt{d}, 1)$. Figure 16 shows the convergence plots for the 15 synthetic benchmark problems and and Tables 14, 15 and 16 show median log simple regret as well as the median absolute deviation from the median.

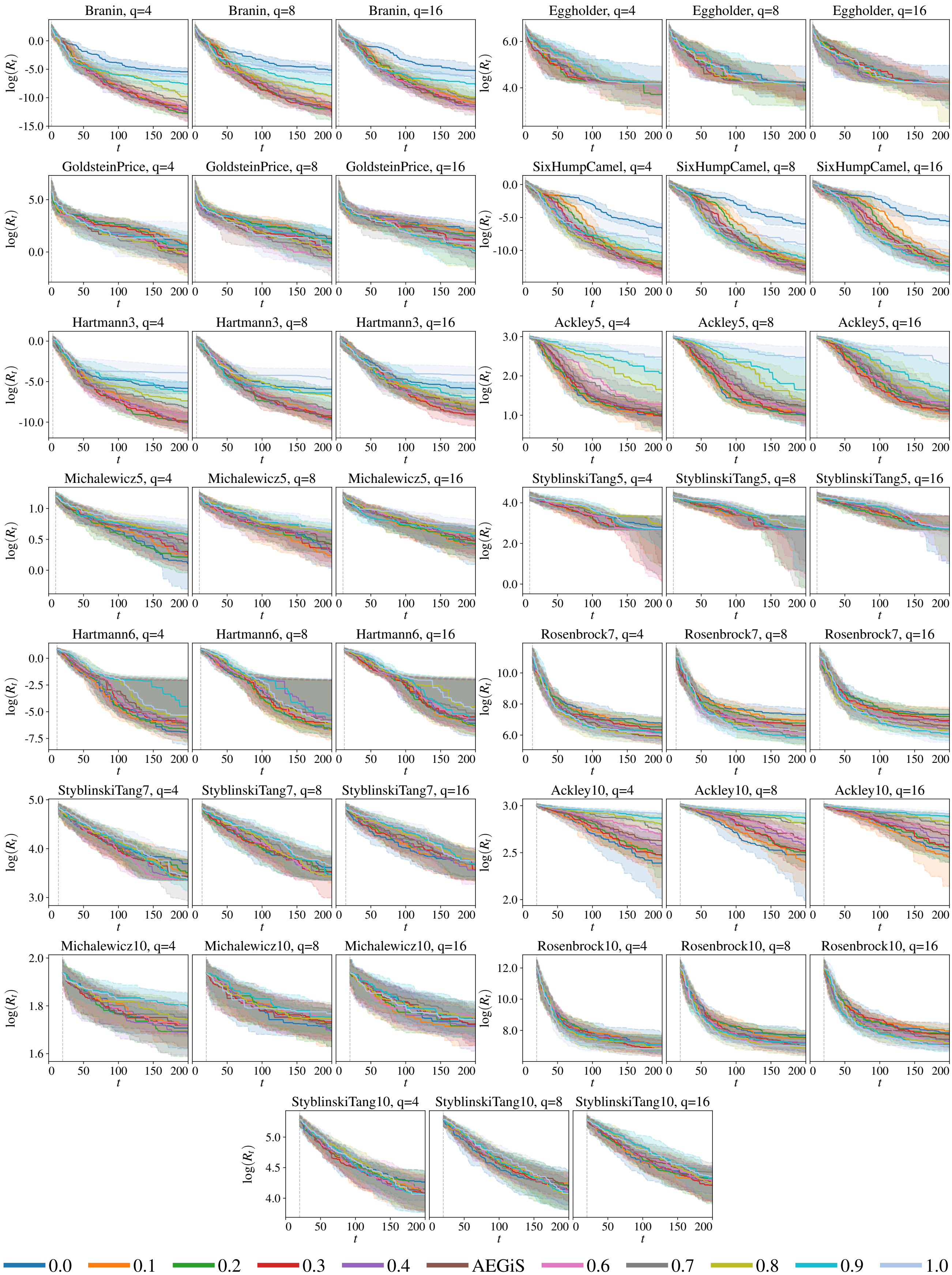

Figure 16: Convergence results for AEGiS with $\gamma \in \{0, 0.1, \ldots, 1\}$.

Table 14: Tabulated results for $q = 4$ workers for $\gamma \in \{0, 0.1, \ldots, 1\}$. The table shows the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	4.31×10^{-3}	4.14×10^{-3}	6.57×10^{1}	8.01×10^{1}	2.06	2.23	1.25×10^{-3}	1.45×10^{-3}	2.95×10^{-3}	3.03×10^{-3}
0.1	8.79×10^{-6}	9.91×10^{-6}	6.51×10^{1}	5.55×10^{1}	1.88	2.24	5.71×10^{-6}	6.51×10^{-6}	4.81×10^{-5}	5.92×10^{-5}
0.2	3.01×10^{-6}	3.01×10^{-6}	4.09×10^{1}	3.61×10^{1}	1.23	1.10	2.75×10^{-6}	2.61×10^{-6}	4.70×10^{-5}	5.47×10^{-5}
0.3	3.93×10^{-6}	4.77×10^{-6}	6.51×10^{1}	1.15×10^{1}	8.79×10^{-1}	1.08	2.41×10^{-6}	2.67×10^{-6}	4.15×10^{-5}	5.23×10^{-5}
0.4	3.27×10^{-6}	4.19×10^{-6}	6.51×10^{1}	8.93	6.32×10^{-1}	7.85×10^{-1}	2.02×10^{-6}	2.58×10^{-6}	4.12×10^{-5}	5.61×10^{-5}
AEGiS	5.99×10^{-6}	6.90×10^{-6}	6.52×10^{1}	1.08×10^{1}	6.99×10^{-1}	7.67×10^{-1}	2.93×10^{-6}	3.31×10^{-6}	5.29×10^{-5}	5.59×10^{-5}
0.6	6.94×10^{-6}	8.01×10^{-6}	4.78×10^{1}	3.05×10^{1}	7.51×10^{-1}	9.80×10^{-1}	2.08×10^{-6}	2.38×10^{-6}	9.57×10^{-5}	1.20×10^{-4}
0.7	1.22×10^{-5}	1.66×10^{-5}	6.51×10^{1}	1.43×10^{1}	6.47×10^{-1}	8.93×10^{-1}	8.43×10^{-6}	1.03×10^{-5}	2.67×10^{-4}	3.28×10^{-4}
0.8	5.61×10^{-5}	8.25×10^{-5}	6.51×10^{1}	1.01×10^{1}	8.99×10^{-1}	1.32	5.63×10^{-6}	7.55×10^{-6}	6.34×10^{-4}	7.93×10^{-4}
0.9	5.15×10^{-4}	6.74×10^{-4}	6.51×10^{1}	1.50×10^{1}	1.58	2.31	3.28×10^{-5}	4.52×10^{-5}	1.92×10^{-3}	2.74×10^{-3}
1.0	2.59×10^{-3}	3.43×10^{-3}	6.51×10^{1}	1.42×10^{1}	3.83	5.62	9.05×10^{-5}	1.34×10^{-4}	1.97×10^{-2}	2.72×10^{-2}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	2.66	9.48×10^{-1}	1.11	5.75×10^{-1}	1.61×10^{1}	9.85	1.00×10^{-3}	1.28×10^{-3}	8.69×10^{2}	6.58×10^{2}
0.1	2.69	1.09	1.27	5.23×10^{-1}	1.47×10^{1}	1.48×10^{1}	1.10×10^{-3}	1.57×10^{-3}	7.26×10^{2}	5.25×10^{2}
0.2	2.76	1.00	1.24	4.11×10^{-1}	1.46×10^{1}	1.88×10^{1}	1.10×10^{-3}	1.26×10^{-3}	6.88×10^{2}	4.17×10^{2}
0.3	2.69	1.08	1.28	5.30×10^{-1}	1.46×10^{1}	1.99×10^{1}	2.17×10^{-3}	2.91×10^{-3}	5.65×10^{2}	4.25×10^{2}
0.4	2.87	1.19	1.29	5.00×10^{-1}	1.49×10^{1}	2.04	1.77×10^{-3}	2.33×10^{-3}	4.69×10^{2}	4.00×10^{2}
AEGiS	2.81	1.19	1.54	4.77×10^{-1}	1.51×10^{1}	2.00×10^{1}	2.28×10^{-3}	3.09×10^{-3}	3.64×10^{2}	2.44×10^{2}
0.6	3.52	1.28	1.67	5.18×10^{-1}	1.45×10^{1}	1.96×10^{1}	2.62×10^{-3}	3.34×10^{-3}	4.15×10^{2}	3.00×10^{2}
0.7	3.01	1.61	1.56	5.82×10^{-1}	1.50×10^{1}	2.00×10^{1}	2.20×10^{-3}	2.87×10^{-3}	4.59×10^{2}	2.91×10^{2}
0.8	5.23	3.34	1.77	4.13×10^{-1}	1.87×10^{1}	1.47×10^{1}	4.43×10^{-3}	6.11×10^{-3}	3.39×10^{2}	1.86×10^{2}
0.9	7.92	5.46	1.80	5.88×10^{-1}	1.49×10^{1}	1.87×10^{1}	1.12×10^{-2}	1.64×10^{-2}	4.86×10^{2}	3.78×10^{2}
1.0	1.19×10^{1}	6.42	1.96	5.73×10^{-1}	1.53×10^{1}	1.93×10^{1}	4.90×10^{-3}	6.64×10^{-3}	4.17×10^{2}	3.55×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	4.01×10^{1}	1.40×10^{1}	1.09×10^{1}	3.36	5.51	5.36×10^{-1}	1.29×10^{3}	9.35×10^{2}	7.12×10^{1}	2.36×10^{1}
0.1	3.27×10^{1}	1.21×10^{1}	1.14×10^{1}	2.53	5.59	5.48×10^{-1}	1.28×10^{3}	7.31×10^{2}	6.23×10^{1}	2.75×10^{1}
0.2	3.34×10^{1}	1.42×10^{1}	1.18×10^{1}	3.20	5.42	6.76×10^{-1}	1.12×10^{3}	8.05×10^{2}	6.34×10^{1}	2.92×10^{1}
0.3	3.01×10^{1}	1.80×10^{1}	1.19×10^{1}	2.00	5.59	4.29×10^{-1}	9.57×10^{2}	6.96×10^{2}	5.95×10^{1}	2.24×10^{1}
0.4	3.03×10^{1}	1.87×10^{1}	1.32×10^{1}	2.75	5.38	7.18×10^{-1}	1.08×10^{3}	7.91×10^{2}	5.79×10^{1}	2.31×10^{1}
AEGiS	3.10×10^{1}	1.79×10^{1}	1.38×10^{1}	2.30	5.57	7.16×10^{-1}	1.08×10^{3}	7.27×10^{2}	5.96×10^{1}	2.41×10^{1}
0.6	2.94×10^{1}	9.29	1.39×10^{1}	2.91	5.59	5.22×10^{-1}	1.09×10^{3}	7.03×10^{2}	6.23×10^{1}	2.17×10^{1}
0.7	3.02×10^{1}	2.03×10^{1}	1.53×10^{1}	2.87	5.74	6.28×10^{-1}	1.17×10^{3}	9.73×10^{2}	5.81×10^{1}	2.21×10^{1}
0.8	2.92×10^{1}	2.00×10^{1}	1.65×10^{1}	2.47	5.73	5.23×10^{-1}	9.63×10^{2}	6.63×10^{2}	5.79×10^{1}	2.11×10^{1}
0.9	2.92×10^{1}	1.98×10^{1}	1.76×10^{1}	1.66	6.04	5.92×10^{-1}	1.20×10^{3}	8.13×10^{2}	5.79×10^{1}	2.18×10^{1}
1.0	2.93×10^{1}	1.96×10^{1}	1.84×10^{1}	8.55×10^{-1}	5.99	6.75×10^{-1}	9.18×10^{2}	6.00×10^{2}	5.83×10^{1}	2.25×10^{1}

Table 15: Tabulated results for $q = 8$ workers for $\gamma \in \{0, 0.1, \ldots, 1\}$. The table shows the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	5.86×10^{-3}	6.80×10^{-3}	6.89×10^{1}	8.42×10^{1}	3.64	3.10	2.48×10^{-3}	2.67×10^{-3}	2.61×10^{-3}	1.75×10^{-3}
0.1	8.73×10^{-6}	9.79×10^{-6}	6.62×10^{1}	4.71×10^{1}	2.58	2.69	6.97×10^{-6}	7.29×10^{-6}	5.90×10^{-5}	7.27×10^{-5}
0.2	5.62×10^{-6}	7.18×10^{-6}	4.98×10^{1}	3.54×10^{1}	2.21	2.34	5.62×10^{-6}	6.64×10^{-6}	6.91×10^{-5}	8.71×10^{-5}
0.3	4.26×10^{-6}	5.78×10^{-6}	6.51×10^{1}	8.78	1.57	1.78	4.29×10^{-6}	4.97×10^{-6}	7.20×10^{-5}	6.30×10^{-5}
0.4	5.50×10^{-6}	5.91×10^{-6}	5.97×10^{1}	2.15×10^{1}	1.61	1.92	2.49×10^{-6}	2.32×10^{-6}	5.31×10^{-5}	6.85×10^{-5}
AEGiS	5.32×10^{-6}	6.51×10^{-6}	6.51×10^{1}	1.35×10^{1}	8.10×10^{-1}	8.82×10^{-1}	2.98×10^{-6}	3.83×10^{-6}	9.99×10^{-5}	1.03×10^{-4}
0.6	1.44×10^{-5}	2.03×10^{-5}	6.51×10^{1}	2.41×10^{1}	4.63×10^{-1}	6.19×10^{-1}	4.02×10^{-6}	4.17×10^{-6}	1.48×10^{-4}	2.06×10^{-4}
0.7	1.82×10^{-5}	2.48×10^{-5}	6.51×10^{1}	2.09×10^{1}	5.56×10^{-1}	7.69×10^{-1}	5.56×10^{-6}	6.83×10^{-6}	2.17×10^{-4}	2.81×10^{-4}
0.8	6.26×10^{-5}	9.11×10^{-5}	6.51×10^{1}	1.76×10^{1}	7.84×10^{-1}	1.10	7.66×10^{-6}	1.03×10^{-5}	1.05×10^{-3}	1.41×10^{-3}
0.9	4.51×10^{-4}	6.51×10^{-4}	6.55×10^{1}	9.19	2.66	3.84	1.41×10^{-5}	1.94×10^{-5}	1.65×10^{-3}	2.13×10^{-3}
1.0	3.87×10^{-3}	5.73×10^{-3}	6.51×10^{1}	1.15×10^{1}	1.56	2.28	1.16×10^{-4}	1.70×10^{-4}	9.29×10^{-3}	1.33×10^{-2}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	2.81	8.15×10^{-1}	1.40	5.86×10^{-1}	1.55×10^{1}	1.33×10^{1}	1.36×10^{-3}	1.71×10^{-3}	1.50×10^{3}	8.73×10^{2}
0.1	2.85	6.58×10^{-1}	1.27	4.24×10^{-1}	1.46×10^{1}	1.05×10^{1}	1.23×10^{-3}	1.43×10^{-3}	1.01×10^{3}	6.21×10^{2}
0.2	2.71	9.03×10^{-1}	1.52	4.98×10^{-1}	1.46×10^{1}	2.04×10^{1}	1.40×10^{-3}	1.66×10^{-3}	8.75×10^{2}	6.10×10^{2}
0.3	2.89	8.64×10^{-1}	1.33	6.12×10^{-1}	1.45×10^{1}	1.65×10^{1}	2.37×10^{-3}	3.09×10^{-3}	7.45×10^{2}	4.88×10^{2}
0.4	2.89	1.23	1.34	6.47×10^{-1}	1.50×10^{1}	5.53	3.15×10^{-3}	4.57×10^{-3}	5.45×10^{2}	3.73×10^{2}
AEGiS	3.39	8.01×10^{-1}	1.53	5.62×10^{-1}	1.53×10^{1}	1.89×10^{1}	2.67×10^{-3}	3.16×10^{-3}	5.04×10^{2}	3.54×10^{2}
0.6	2.93	1.33	1.74	4.11×10^{-1}	1.45×10^{1}	1.77×10^{1}	2.60×10^{-3}	3.15×10^{-3}	4.21×10^{2}	3.17×10^{2}
0.7	3.39	1.55	1.61	4.48×10^{-1}	1.47×10^{1}	2.05×10^{1}	3.17×10^{-3}	4.11×10^{-3}	3.40×10^{2}	2.27×10^{2}
0.8	3.99	2.12	1.87	5.16×10^{-1}	1.57×10^{1}	1.87×10^{1}	2.24×10^{-3}	2.93×10^{-3}	5.02×10^{2}	4.53×10^{2}
0.9	5.19	3.88	1.94	4.09×10^{-1}	1.47×10^{1}	2.02×10^{1}	4.91×10^{-3}	6.56×10^{-3}	3.48×10^{2}	2.50×10^{2}
1.0	1.18×10^{1}	7.29	1.96	3.60×10^{-1}	1.54×10^{1}	1.95×10^{1}	2.52×10^{-3}	3.13×10^{-3}	5.11×10^{2}	4.00×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	3.69×10^{1}	1.36×10^{1}	1.19×10^{1}	2.80	5.46	5.94×10^{-1}	2.18×10^{3}	1.21×10^{3}	6.28×10^{1}	2.31×10^{1}
0.1	3.89×10^{1}	1.75×10^{1}	1.11×10^{1}	3.11	5.65	6.58×10^{-1}	1.77×10^{3}	1.07×10^{3}	7.01×10^{1}	2.16×10^{1}
0.2	3.39×10^{1}	1.50×10^{1}	1.22×10^{1}	1.80	5.52	6.62×10^{-1}	1.90×10^{3}	1.25×10^{3}	6.55×10^{1}	3.04×10^{1}
0.3	3.39×10^{1}	1.75×10^{1}	1.24×10^{1}	3.06	5.66	5.56×10^{-1}	1.31×10^{3}	1.09×10^{3}	6.26×10^{1}	1.79×10^{1}
0.4	3.08×10^{1}	1.72×10^{1}	1.32×10^{1}	1.68	5.43	8.41×10^{-1}	1.35×10^{3}	8.19×10^{2}	6.26×10^{1}	2.20×10^{1}
AEGiS	3.23×10^{1}	1.69×10^{1}	1.41×10^{1}	2.83	5.61	5.60×10^{-1}	9.97×10^{2}	5.96×10^{2}	6.75×10^{1}	2.31×10^{1}
0.6	3.10×10^{1}	1.73×10^{1}	1.41×10^{1}	2.07	5.73	4.28×10^{-1}	1.01×10^{3}	4.93×10^{2}	6.06×10^{1}	2.81×10^{1}
0.7	3.11×10^{1}	1.81×10^{1}	1.66×10^{1}	2.23	5.57	4.39×10^{-1}	1.10×10^{3}	7.36×10^{2}	5.98×10^{1}	2.15×10^{1}
0.8	3.11×10^{1}	1.77×10^{1}	1.68×10^{1}	2.08	5.74	5.77×10^{-1}	9.57×10^{2}	7.16×10^{2}	5.83×10^{1}	2.18×10^{1}
0.9	3.22×10^{1}	1.77×10^{1}	1.75×10^{1}	1.41	5.77	5.40×10^{-1}	1.29×10^{3}	9.01×10^{2}	5.94×10^{1}	2.15×10^{1}
1.0	3.84×10^{1}	1.30×10^{1}	1.86×10^{1}	6.40×10^{-1}	5.85	4.42×10^{-1}	1.07×10^{3}	7.34×10^{2}	5.90×10^{1}	2.27×10^{1}

Table 16: Tabulated results for $q = 16$ workers for $\gamma \in \{0, 0.1, \ldots, 1\}$. The table shows the median log simple regret (*left*) and median absolute deviation from the median (MAD, *right*) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance are shown in light grey.

Method	**Branin (2)**		**Eggholder (2)**		**GoldsteinPrice (2)**		**SixHumpCamel (2)**		**Hartmann3 (3)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	5.57×10^{-3}	5.89×10^{-3}	6.70×10^{1}	8.02×10^{1}	7.14	8.27	3.80×10^{-3}	3.63×10^{-3}	2.66×10^{-3}	2.61×10^{-3}
0.1	2.01×10^{-5}	2.23×10^{-5}	6.81×10^{1}	1.13×10^{1}	5.78	5.54	1.67×10^{-5}	2.07×10^{-5}	1.49×10^{-4}	1.61×10^{-4}
0.2	1.20×10^{-5}	1.13×10^{-5}	6.51×10^{1}	3.07×10^{1}	4.92	4.59	7.57×10^{-6}	9.49×10^{-6}	1.55×10^{-4}	1.71×10^{-4}
0.3	1.10×10^{-5}	1.29×10^{-5}	6.53×10^{1}	3.61×10^{1}	3.20	3.73	5.35×10^{-6}	6.14×10^{-6}	1.15×10^{-4}	1.44×10^{-4}
0.4	9.80×10^{-6}	9.11×10^{-6}	6.51×10^{1}	1.97×10^{1}	3.38	3.58	3.89×10^{-6}	4.71×10^{-6}	1.74×10^{-4}	1.91×10^{-4}
AEGiS	1.28×10^{-5}	1.80×10^{-5}	6.53×10^{1}	1.27×10^{1}	1.56	1.84	5.82×10^{-6}	5.32×10^{-6}	1.93×10^{-4}	2.80×10^{-4}
0.6	1.10×10^{-5}	1.39×10^{-5}	6.60×10^{1}	1.50×10^{1}	1.66	1.98	6.89×10^{-6}	8.79×10^{-6}	1.35×10^{-4}	1.75×10^{-4}
0.7	3.74×10^{-5}	5.05×10^{-5}	6.51×10^{1}	4.12×10^{1}	8.82×10^{-1}	1.27	5.54×10^{-6}	6.69×10^{-6}	2.07×10^{-4}	2.59×10^{-4}
0.8	4.98×10^{-5}	6.76×10^{-5}	6.51×10^{1}	3.23×10^{1}	1.50	2.08	5.95×10^{-6}	7.22×10^{-6}	4.03×10^{-4}	5.34×10^{-4}
0.9	4.40×10^{-4}	6.46×10^{-4}	6.52×10^{1}	9.10	1.15	1.66	4.92×10^{-6}	6.51×10^{-6}	9.59×10^{-4}	1.32×10^{-3}
1.0	1.43×10^{-3}	2.04×10^{-3}	6.52×10^{1}	2.47×10^{1}	2.45	3.46	2.98×10^{-5}	4.25×10^{-5}	1.47×10^{-2}	2.10×10^{-2}

Method	**Ackley5 (5)**		**Michalewicz5 (5)**		**StyblinskiTang5 (5)**		**Hartmann6 (6)**		**Rosenbrock7 (7)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	2.97	8.93×10^{-1}	1.50	6.17×10^{-1}	1.58×10^{1}	1.96×10^{1}	2.30×10^{-3}	2.74×10^{-3}	1.53×10^{3}	8.24×10^{2}
0.1	2.84	8.76×10^{-1}	1.49	5.01×10^{-1}	1.60×10^{1}	1.90×10^{1}	1.87×10^{-3}	2.23×10^{-3}	1.35×10^{3}	9.36×10^{2}
0.2	3.16	1.03	1.54	5.19×10^{-1}	1.55×10^{1}	7.75	1.89×10^{-3}	2.28×10^{-3}	1.35×10^{3}	8.08×10^{2}
0.3	3.06	8.66×10^{-1}	1.60	4.88×10^{-1}	1.49×10^{1}	3.01	4.20×10^{-3}	5.80×10^{-3}	1.02×10^{3}	5.18×10^{2}
0.4	3.47	9.78×10^{-1}	1.51	4.74×10^{-1}	1.47×10^{1}	1.84×10^{1}	2.96×10^{-3}	3.84×10^{-3}	8.65×10^{2}	6.87×10^{2}
AEGiS	3.39	1.12	1.62	5.10×10^{-1}	1.56×10^{1}	1.09×10^{1}	3.21×10^{-3}	3.82×10^{-3}	7.38×10^{2}	5.13×10^{2}
0.6	3.43	1.34	1.66	4.10×10^{-1}	1.64×10^{1}	1.79×10^{1}	3.57×10^{-3}	4.54×10^{-3}	5.17×10^{2}	2.91×10^{2}
0.7	3.38	1.47	1.63	5.68×10^{-1}	1.52×10^{1}	1.76×10^{1}	4.13×10^{-3}	5.56×10^{-3}	5.78×10^{2}	3.51×10^{2}
0.8	3.99	2.05	1.83	4.57×10^{-1}	1.51×10^{1}	1.77×10^{1}	1.07×10^{-2}	1.51×10^{-2}	5.00×10^{2}	2.98×10^{2}
0.9	5.07	2.74	1.75	4.60×10^{-1}	1.52×10^{1}	1.64	3.70×10^{-3}	4.74×10^{-3}	4.28×10^{2}	2.70×10^{2}
1.0	1.02×10^{1}	7.76	2.05	3.46×10^{-1}	1.60×10^{1}	1.33×10^{1}	8.71×10^{-3}	1.21×10^{-2}	5.02×10^{2}	4.58×10^{2}

Method	**StyblinskiTang7 (7)**		**Ackley10 (10)**		**Michalewicz10 (10)**		**Rosenbrock10 (10)**		**StyblinskiTang10 (10)**	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
0.0	4.27×10^{1}	1.87×10^{1}	1.24×10^{1}	1.73	5.59	5.43×10^{-1}	2.54×10^{3}	1.89×10^{3}	7.53×10^{1}	2.26×10^{1}
0.1	3.67×10^{1}	1.36×10^{1}	1.16×10^{1}	2.69	5.54	4.19×10^{-1}	2.87×10^{3}	1.62×10^{3}	7.16×10^{1}	2.51×10^{1}
0.2	4.31×10^{1}	1.74×10^{1}	1.29×10^{1}	2.29	5.56	5.68×10^{-1}	2.38×10^{3}	1.22×10^{3}	7.49×10^{1}	2.53×10^{1}
0.3	3.55×10^{1}	1.44×10^{1}	1.29×10^{1}	3.09	5.61	5.26×10^{-1}	2.24×10^{3}	1.21×10^{3}	6.76×10^{1}	3.26×10^{1}
0.4	3.48×10^{1}	1.55×10^{1}	1.36×10^{1}	2.14	5.55	7.22×10^{-1}	1.48×10^{3}	8.46×10^{2}	7.11×10^{1}	2.29×10^{1}
AEGiS	4.32×10^{1}	1.80×10^{1}	1.46×10^{1}	2.22	5.74	5.74×10^{-1}	1.60×10^{3}	1.01×10^{3}	7.20×10^{1}	2.83×10^{1}
0.6	4.37×10^{1}	1.98×10^{1}	1.54×10^{1}	1.85	5.70	6.21×10^{-1}	1.33×10^{3}	6.92×10^{2}	7.11×10^{1}	1.95×10^{1}
0.7	3.95×10^{1}	1.75×10^{1}	1.63×10^{1}	1.99	5.77	5.68×10^{-1}	1.21×10^{3}	7.68×10^{2}	8.37×10^{1}	3.66×10^{1}
0.8	4.37×10^{1}	2.09×10^{1}	1.68×10^{1}	1.74	5.77	4.51×10^{-1}	1.33×10^{3}	8.95×10^{2}	7.06×10^{1}	2.13×10^{1}
0.9	4.01×10^{1}	1.62×10^{1}	1.79×10^{1}	1.09	5.70	6.43×10^{-1}	1.15×10^{3}	7.28×10^{2}	7.51×10^{1}	3.21×10^{1}
1.0	4.26×10^{1}	1.41×10^{1}	1.84×10^{1}	1.13	5.70	5.35×10^{-1}	1.13×10^{3}	5.77×10^{2}	7.81×10^{1}	2.74×10^{1}

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