Learning Probabilistic Sentential Decision Diagrams Under Logic Constraints by Sampling and Averaging Supplementary Material

Renato Lui Geh[1] **Denis Deratani Mauá**[1]

[1]Department of Computer Science, Institute of Mathematics and Statistics, University of São Paulo, Brazil

A FAST IMPLEMENTATION OF SAMPLEPSDD

In order to produce valid (partial) partitions, SAMPLEPSDD requires that the FORGET operation be called for every sub lest the scope of the formula contradicts the respective scope in vtree. Despite FORGET taking polynomial time in the size of the BDD, we can make sampling more efficient by directly "forgetting" a variable when returning a PSDD structure. The algorithm below modifies Algorithm 2 to handle variables not appearing in the formula ϕ:

Algorithm 3 SAMPLEPSDD

Input BDD ϕ, vtree node v, number of primes k
Output A sampled PSDD structure
1: **if** v is a leaf **then**
2:     **if** $v \in \phi$ **then**
3:         **if** ϕ is a literal **then return** ϕ as a literal node
4:         **if** $\phi|_v \equiv \top$ **then return** v as a literal node
5:         **if** $\phi|_{\neg v} \equiv \top$ **then return** $\neg v$ as a literal node
6:     **return** a Bernoulli over v
7: **else if** $\phi \equiv \top$ **then**
8:     **return** a fully factorized circuit over $\mathrm{Sc}(v)$
9: $\mathbf{E} \leftarrow$ SAMPLEPARTIALPARTITION$(\phi, \mathrm{Sc}(v^{\leftarrow}), k)$
10: Create an OR gate S
11: Randomly compress elements in $\mathbf{E}$ with equal subs
12: Randomly merge elements in $\mathbf{E}$ with equal subs
13: **for** each element $(p, s) \in \mathbf{E}$ **do**
14:     $l \leftarrow$ SAMPLEEXACTPSDD$(p, v^{\leftarrow}, k)$
15:     $r \leftarrow$ SAMPLEPSDD$(s, v^{\rightarrow}, k)$
16:     Add an AND gate with inputs l and r as a child of S
17: **return** S

Lines 2 – 7 of the algorithm above ensure that the formula correctly accounts for the forgetting of variables not in the scope of the vtree. Hence, we can omit the FORGET operation in Algorithm 1, resulting in Algorithm 4.

Algorithm 4 SAMPLEPARTIALPARTITION

Input BDD ϕ, vtree node v, number of primes k
Output A set of sampled elements
Define $\mathbf{E}$ as an empty collection of sampled elements
Sample an ordering $X_1, \ldots, X_m$ of $\mathrm{Sc}(v^{\leftarrow}) \cap \mathrm{Sc}(\phi)$
Let $\mathbf{Q}$ be a queue initially containing $(\phi, 1, \{\})$
$j \leftarrow 1$ ▷ Counter of sampled elements
while $|\mathbf{E}| < k$ **do**
    Pop top item (ψ, i, p) from $\mathbf{Q}$
    if $j \geq k$ or $i > m$ or $\psi \equiv \top$ **then**
        Add $(p, \phi|_p)$ to $\mathbf{E}$
        continue
    $\alpha \leftarrow \psi|_{X_i}, \beta \leftarrow \psi|_{\neg X_i}$
    if $\alpha \equiv \beta$ **then** enqueue $(\psi, i+1, p)$ in $\mathbf{Q}$
    else
        if $\alpha \not\equiv \bot$ **then** push $(\alpha, i+1, p \wedge X_i)$ to $\mathbf{Q}$
        if $\beta \not\equiv \bot$ **then** push $(\beta, i+1, p \wedge \neg X_i)$ to $\mathbf{Q}$
        $j \leftarrow j + 1$
return $\mathbf{E}$

Since the restricion $\psi|X$ is linearithmic in the size of the BDD, and constructing a conjunction of literals α is linear in $|\mathrm{Sc}(\alpha)|$, the algorithm is highly efficient.

B ADDITIONAL EXPERIMENTS

We repeat the accuracy vs. sample size plots including the results of running STRUDEL and MIXSTRUDEL for 1000 iterations, as used in the original paper. Figure 1 shows all results with the added Strudel and MixStrudel with 1000 iterations curves.

C TABLES WITH ALL RESULTS

Tables 1 through 5 show all log-likelihood values for all learned circuits mentioned in the article.

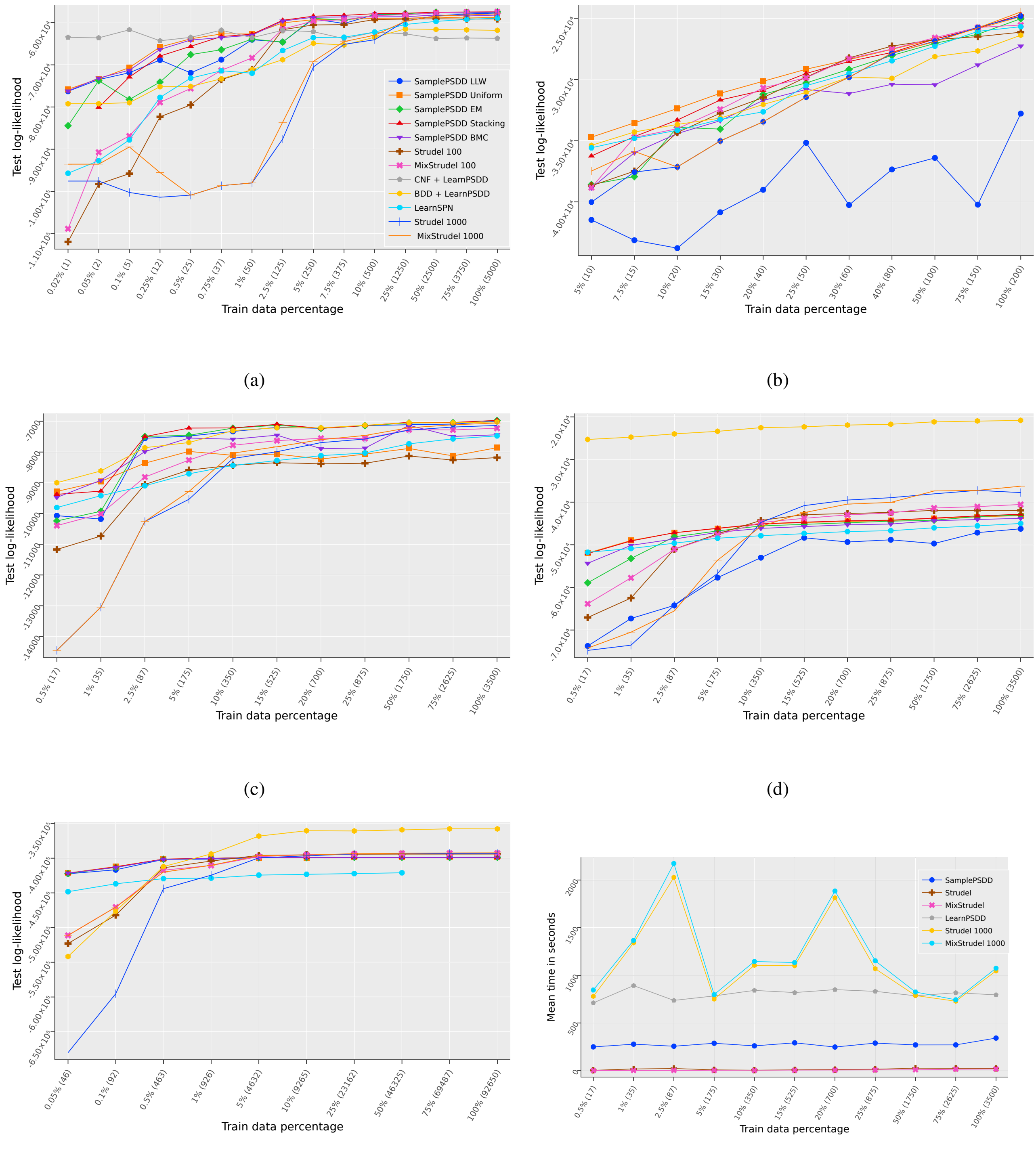


Figure 1: (a) Log-likelihoods for the unpixelized `led`, (b) `led-pixels`, (c) `sushi` 10-choose-5, (d) `sushi` ranking, and (e) `dota` datasets. (f) Mean average in seconds of each PSDD learning algorithm.

Train %	LLW	UNIFORM	EM	STACKING	BMC	STRUDEL	MIXSTRUDEL	CNF	BDD	LEARNSPN	STRUDEL 1000	MIXSTRUDEL 1000
0.02	-76270.12	-75787.23	-84437.88	-Inf	-76188.67	-111934.70	-108848.94	-63491.43	-79214.25	-95691.57	-97544.40	-93539.76
0.05	-73497.69	-73346.26	-73749.15	-80090.00	-73134.93	-98263.88	-90720.27	-63601.97	-79214.25	-92723.15	-97544.40	-93539.76
0.10	-71874.12	-70645.43	-78218.27	-72887.26	-71279.73	-95780.74	-86851.08	-61695.59	-78972.30	-87784.83	-100241.28	-89465.54
0.25	-68850.67	-65765.90	-74079.05	-67914.35	-66355.85	-82308.32	-78892.63	-64301.48	-75181.47	-77728.44	-101371.88	-95533.46
0.50	-71908.34	-63836.00	-67574.64	-65669.59	-64123.61	-79503.19	-75601.22	-63489.18	-75120.22	-73147.62	-100880.62	-100880.62
0.75	-68764.98	-62672.24	-66423.16	-63291.72	-63591.08	-73509.03	-71312.21	-61786.42	-73306.98	-71405.55	-98628.08	-98628.08
1.00	-64046.13	-62690.14	-63900.15	-62744.10	-62962.83	-71170.68	-68344.86	-63567.95	-71121.15	-71979.82	-97986.81	-97986.81
2.50	-64606.12	-60058.62	-64606.01	-59435.88	-59623.46	-61670.14	-61527.77	-61817.41	-68781.20	-66602.53	-87585.31	-83696.90
5.00	-59126.68	-59295.69	-58950.44	-58466.21	-58723.25	-60542.42	-59623.12	-62142.91	-64901.41	-63540.15	-70519.08	-69183.37
7.50	-60206.00	-59243.14	-59451.17	-58280.74	-58667.93	-60474.21	-59302.01	-63779.07	-65295.23	-63461.16	-65143.38	-64492.63
10.00	-58082.24	-58990.91	-58218.28	-57876.12	-58603.25	-59258.70	-58335.49	-62272.04	-63449.87	-62230.84	-64059.13	-62980.20
25.00	-58127.23	-58977.09	-57827.87	-57777.27	-58589.02	-59116.05	-58116.16	-62649.53	-61490.03	-60435.37	-59619.32	-59663.81
50.00	-57683.16	-58687.88	-57526.56	-57512.01	-58189.97	-59084.32	-57654.38	-63760.85	-61614.24	-59703.44	-58395.85	-58447.46
75.00	-57731.25	-58900.85	-57564.91	-57514.10	-58416.90	-59160.69	-57480.32	-63656.23	-61716.55	-59227.08	-57947.51	-58162.58
100.00	-57533.82	-58777.85	-57444.14	-57432.12	-58217.43	-59140.02	-57421.31	-63717.25	-61823.57	-58923.36	-57727.95	-57921.16

Table 1: All results for the `led` dataset.

Train %	LLW	UNIFORM	EM	STACKING	BMC	STRUDEL	MIXSTRUDEL	CNF	BDD	LEARNSPN	STRUDEL 1000	MIXSTRUDEL 1000
0.05	-422502.57	-421390.69	-422148.83	-421743.46	-421503.71	-522994.12	-511269.34	–	-541716.99	-448310.30	-680217.64	-511269.34
0.10	-416764.85	-412215.39	-413252.45	-412497.30	-413309.05	-482048.24	-470524.47	–	-476355.45	-436929.97	-595314.50	-470524.47
0.50	-401852.30	-401428.03	-401651.98	-401342.48	-401639.60	-413893.71	-417290.26	–	-412120.93	-429491.77	-443999.07	-420096.18
1.00	-401399.54	-400217.55	-400304.56	-400241.36	-400415.36	-404177.25	-410592.47	–	-393893.11	-428727.36	-424539.00	-410274.56
5.00	-399210.71	-399209.46	-398999.00	-398995.65	-399170.53	-395999.62	-397754.60	–	-368214.89	-424401.59	-399534.98	-396268.43
10.00	-399308.46	-399114.38	-399111.18	-399101.08	-399139.16	-394996.70	-395111.24	–	-360499.20	-423240.08	-396642.23	-395462.40
25.00	-398759.03	-398967.54	-398809.05	-398789.22	-398884.48	-394338.42	-393943.77	–	-360817.30	-422176.41	-393840.34	-393563.40
50.00	-398818.59	-398947.09	-398781.72	-398803.82	-398825.88	-394185.68	-393086.71	–	-359123.74	-421069.29	-393395.77	-392753.17
75.00	-398935.65	-398983.57	-398955.77	-398956.10	-398958.45	-394144.51	-392858.52	–	-357708.97	0.00	-393130.99	-392476.99
100.00	-398814.38	-398946.54	-398847.74	-398851.61	-398903.84	-394104.98	-392767.17	–	-357820.05	0.00	-393080.77	-392305.31

Table 2: All results for the `dota` dataset.

Train %	LLW	UNIFORM	EM	STACKING	BMC	STRUDEL	MIXSTRUDEL	CNF	BDD	LEARNSPN	STRUDEL 1000	MIXSTRUDEL 1000
0.50	-10071.51	-9281.12	-10241.60	-9375.70	-9476.25	-67065.20	-63836.65	–	-25300.02	-51765.32	-14452.67	-14452.67
1.00	-10178.28	-8954.16	-9927.91	-9271.89	-8919.37	-62498.23	-57767.33	–	-24760.09	-50882.04	-13046.79	-13046.79
2.50	-7551.81	-8362.66	-7494.87	-7498.91	-7985.90	-51036.17	-51070.28	–	-23990.52	-49625.57	-10259.14	-10259.14
5.00	-7480.58	-7979.85	-7454.45	-7221.24	-7545.54	-47444.71	-47699.34	–	-23393.66	-48465.94	-9538.90	-9285.30
10.00	-7330.72	-8105.71	-7222.63	-7216.68	-7583.54	-44289.65	-44785.08	–	-22534.80	-47881.50	-8210.50	-8035.49
15.00	-7196.89	-8067.99	-7129.28	-7099.35	-7450.98	-42929.19	-43793.82	–	-22336.59	-47350.66	-7989.75	-7828.43
20.00	-7230.35	-8228.08	-7230.35	-7230.13	-7889.98	-42692.17	-42998.28	–	-21918.48	-46898.91	-7693.88	-7598.46
25.00	-7140.91	-8068.33	-7140.90	-7140.75	-7872.47	-42384.75	-42544.64	–	-21738.22	-46700.17	-7577.25	-7461.51
50.00	-7111.22	-7884.88	-7054.73	-7031.96	-7131.19	-41938.09	-41373.59	–	-21169.19	-46031.00	-7286.91	-7210.35
75.00	-7091.82	-8125.42	-7043.95	-7036.04	-7493.26	-41931.85	-41055.14	–	-20951.09	-45576.38	-7187.18	-7115.33
100.00	-6995.82	-7859.13	-6972.82	-6970.53	-7439.17	-41931.72	-40550.63	–	-20824.09	-44999.66	-7135.58	-7055.88

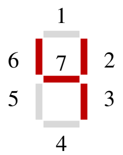
Table 3: All results for the `sushi-ranking` dataset.

Train %	LLW	UNIFORM	EM	STACKING	BMC	STRUDEL	MIXSTRUDEL	CNF	BDD	LEARNSPN	STRUDEL 1000	MIXSTRUDEL 1000
0.50	-73738.04	-51933.84	-58925.72	-52070.48	-54347.08	-11170.18	-10393.62	–	-8998.46	-9806.06	-74782.40	-74245.00
1.00	-67295.90	-48998.01	-53225.82	-49112.97	-50140.36	-10733.47	-10010.65	–	-8615.35	-9419.33	-73557.43	-70504.16
2.50	-64222.40	-47191.60	-48124.73	-47171.38	-48649.17	-9054.54	-8814.77	–	-7859.15	-9097.40	-64196.67	-65531.31
5.00	-57715.96	-46215.44	-46757.26	-46121.74	-47099.80	-8587.40	-8262.15	–	-7692.06	-8709.09	-56744.91	-53619.79
10.00	-53029.28	-45182.87	-45582.79	-45096.27	-46157.73	-8428.53	-7779.55	–	-7303.22	-8435.54	-44767.54	-45684.07
15.00	-48359.59	-44776.85	-45173.90	-44696.40	-45732.28	-8346.72	-7630.41	–	-7212.20	-8272.97	-40815.68	-42453.92
20.00	-49352.41	-44503.85	-44841.65	-44375.66	-45316.36	-8382.45	-7553.45	–	-7213.00	-8118.19	-39514.08	-40466.87
25.00	-48837.90	-44320.47	-44473.52	-44232.87	-45147.34	-8366.84	-7558.48	–	-7126.86	-8031.02	-38959.38	-40067.63
50.00	-49715.52	-43810.03	-44236.83	-43679.79	-44361.31	-8129.12	-7274.86	–	-7047.23	-7733.97	-38055.99	-37387.91
75.00	-47155.99	-43439.89	-43385.69	-43236.88	-44080.44	-8261.48	-7280.35	–	-7050.77	-7571.72	-37245.72	-37239.73
100.00	-46253.45	-43146.33	-43017.12	-42836.74	-43796.68	-8181.48	-7227.68	–	-7022.86	-7475.26	-37761.35	-36269.11

Table 4: All results for the `sushi-top5` dataset.

Train %	LLW	UNIFORM	EM	STACKING	BMC	STRUDEL	MIXSTRUDEL	CNF	BDD	LEARNSPN	STRUDEL 1000	MIXSTRUDEL 1000
5.00	-41467.80	-34709.51	-38580.12	-36250.22	-38865.10	-38681.10	-38833.61	–	-35379.58	-35577.04	-40016.81	-37480.79
7.50	-43131.74	-33541.11	-37933.14	-34661.19	-35984.88	-37486.80	-34728.47	–	-34287.35	-34802.33	-37560.84	-35873.42
10.00	-43778.51	-32372.82	-33907.77	-33312.05	-34414.20	-34325.42	-34019.61	–	-33669.67	-34144.75	-37146.60	-37146.60
15.00	-40846.42	-31125.19	-34049.79	-31674.03	-33368.23	-32755.51	-32434.84	–	-33155.99	-33261.28	-35019.44	-35019.44
20.00	-39009.42	-30139.82	-31209.07	-30876.98	-31708.73	-31474.09	-30675.72	–	-32038.90	-32636.76	-33453.36	-33453.36
25.00	-35170.26	-29158.72	-30278.57	-29505.30	-30811.56	-29808.79	-29856.29	–	-31056.47	-30501.52	-31443.56	-31443.56
30.00	-40254.35	-28374.13	-29166.40	-28524.61	-31150.31	-28229.82	-28293.39	–	-29796.42	-29515.66	-29824.86	-29824.86
40.00	-37346.83	-27532.89	-28033.83	-27784.87	-30386.84	-27267.30	-27504.01	–	-29904.46	-28468.36	-27865.87	-27815.05
50.00	-36404.20	-26711.99	-27047.65	-26693.77	-30428.74	-26734.57	-26579.84	–	-28127.06	-27274.67	-26863.06	-26913.21
75.00	-40223.88	-25846.50	-26300.40	-25801.80	-28806.88	-26494.04	-25744.18	–	-27665.63	-26064.91	-25733.69	-25780.46
100.00	-32780.47	-24858.39	-25031.07	-24687.48	-27260.98	-26133.39	-25549.14	–	-26379.98	-25684.45	-24818.43	-24478.19

Table 5: All results for the `led-pixels` dataset.



$d_4 = \neg X_1 \wedge X_2 \wedge X_3 \wedge \neg X_4 \wedge \neg X_5 \wedge X_6 \wedge X_7$

$\phi = \bigvee_{i=0}^{9} d_i$

Figure 2: LED segment numbering (left), and the corresponding formula for that digit (right).

D LOGIC CONSTRAINTS

Here we show all the logic constraints used for each dataset.

D.1 LED

Let $Y_1, Y_2, \dots, Y_7$ be the observable segments of a 7-segment LED display, with each Y_i representing whether the i-th segment (read from the top segment clockwise with the middle segment last) is observably on (true/1) or off (false/0). We assign a latent variable for each segment i and call it X_i. The latent variable indicates the true intent of the segment (i.e. whether it was supposed to be on or off regardless of technical problems). For each digit d_i, we add a positive literal if it is supposedly on, and a negative literal if it is supposedly off. Observable variables are free variables with no constraints. The final formula is given by a disjunction over all digits, as shown below.

$$\begin{aligned}
\phi = \; & X_1 \wedge X_2 \wedge X_3 \wedge X_4 \wedge X_5 \wedge X_6 \wedge \neg X_7 \vee \\
& \neg X_1 \wedge X_2 \wedge X_3 \wedge \neg X_4 \wedge \neg X_5 \wedge \neg X_6 \wedge \neg X_7 \vee \\
& X_1 \wedge X_2 \wedge \neg X_3 \wedge X_4 \wedge X_5 \wedge \neg X_6 \wedge X_7 \vee \\
& X_1 \wedge X_2 \wedge X_3 \wedge X_4 \wedge \neg X_5 \wedge \neg X_6 \wedge X_7 \vee \\
& \neg X_1 \wedge X_2 \wedge X_3 \wedge \neg X_4 \wedge \neg X_5 \wedge X_6 \wedge X_7 \vee \\
& X_1 \wedge \neg X_2 \wedge X_3 \wedge X_4 \wedge \neg X_5 \wedge X_6 \wedge X_7 \vee \\
& X_1 \wedge \neg X_2 \wedge X_3 \wedge X_4 \wedge X_5 \wedge X_6 \wedge X_7 \vee \\
& X_1 \wedge X_2 \wedge X_3 \wedge \neg X_4 \wedge \neg X_5 \wedge \neg X_6 \wedge \neg X_7 \vee \\
& X_1 \wedge X_2 \wedge X_3 \wedge X_4 \wedge X_5 \wedge X_6 \wedge X_7 \vee \\
& X_1 \wedge X_2 \wedge X_3 \wedge X_4 \wedge \neg X_5 \wedge X_6 \wedge X_7
\end{aligned}$$

D.2 LED PIXELS

The LED with pixels dataset follows the same idea as LED, but with added pixels as latent variables instead of observable segments. We manually observed critical key pixels for each segment (i.e. pixels which are often set to true/1 if the segment is on. We count pixels row-wise from top left to bottom right. The following are the critical key pixels for each segment:

$$\begin{aligned}
S_1 &= \{24, 25, 26, 27, 15, 16, 28, 35, 36\} \\
S_2 &= \{27, 28, 37, 38, 47, 48, 57, 58, 49, 59, 69\} \\
S_3 &= \{77, 78, 87, 88, 109, 98, 99, 108, 118\} \\
S_4 &= \{124, 125, 126, 127, 128, 135, 136, 114, 115, 116\} \\
S_5 &= \{93, 94, 103, 104, 113, 114, 124, 82, 92, 83\} \\
S_6 &= \{33, 34, 43, 53, 52, 63, 73\} \\
S_7 &= \{64, 65, 66, 67, 75, 76, 85, 86, 95, 96, 94\}
\end{aligned}$$

Each S_i corresponds to the critical key pixels of segment i. The formula for the key pixels is then set to

$$\alpha = \bigvee_{i=1}^{7} \left(\bigwedge_{p \in S_i} p \right) \wedge X_i.$$

We also add a constraint for pixels which are never on for a given digit. Let $f(i)$ a function that maps a digit i to the set of all pixels which are always off when d_i is true. We set

$$\beta = \bigvee_{i=0}^{9} d_i \wedge \left(\bigwedge_{p \in f(i)} \neg p \right).$$

The final constraint is then $\phi = \alpha \wedge \beta$.

D.3 SUSHI

For the sushi ranking dataset, we used the same constraints as [Choi et al., 2015]. For the sushi 10-choose-5, we used the same constraints as [Shen et al., 2017].

D.4 DOTA 2

To model the constraints, we used a cardinality constraint of Exactly$(5, 113)$ for the first and equivalently for the second team. To do this, each character i had a pair of variables (X_i, Y_i), where X_i attributed the character for the first team, and Y_i to the second. A cardinality constraint $\sum_{X_i} x_i = 5$ was set to the first team, and $\sum_{Y_i} y_i = 5$ to the second.

References

Arthur Choi, Guy Van den Broeck, and Adnan Darwiche. Tractable learning for structured probability spaces: A case study in learning preference distributions. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence*, pages 2861–2868, 2015.

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