Gaussian Process Nowcasting: Application to COVID-19 Mortality Reporting. Supplementary Material.

Iwona Hawryluk[1] **Henrique Hoeltgebaum**[2] **Swapnil Mishra**[1] **Xenia Miscouridou**[2] **Ricardo P Schnekenberg**[3]
Charles Whittaker[1] **Michaela Vollmer**[1] **Seth Flaxman**[2] **Samir Bhatt**[1,**] **Thomas A. Mellan**[1,**]

[1]Department of Infectious Disease Epidemiology, School of Public Health, Imperial College London, UK
[2]Department of Mathematics, Imperial College London, UK
[3] Nuffield Department of Clinical Neuroscience, University of Oxford, UK
[**] Contributed equally

1 MODEL SPECIFICATION

Table S1: Priors for the analysed models. Here T is the maximum number of weeks for which the data is available, that is rows in the reporting triangle, and D is the maximum reporting delay, that is number of columns in the reporting triangle. Γ denotes a Gamma distribution. * the same hyperparameters were used for the models with Matérn(1/2) and Matérn(3/2) kernels.

	1D SE	1D SE+SE	1D SE+Mat*	1D SE+SE data-split	2D additive	NobBS
r	$\Gamma(500,2)$	$\Gamma(500,2)$	$\Gamma(500,2)$	$\Gamma(500,2)$	$\Gamma(200,2)$	$\Gamma(500,2)$
$\alpha_{1,\text{long}}$	$\mathcal{N}(1,1)$	$\mathcal{N}(15,2)$	$\mathcal{N}(15,2)$	$\mathcal{N}(15,2)$	-	-
$\alpha_{2,\text{long}}$	-	-	-	$\mathcal{N}(20,2)$	-	-
$\alpha_{1,\text{short}}$	-	$\mathcal{N}(5,2)$	$\mathcal{N}(5,1)$	$\mathcal{N}(5,1)$	-	-
$\alpha_{1,\text{short}}$	-	-	-	$\mathcal{N}(3,1)$	-	-
$\alpha_{1,t}$	-	-	-	-	$\mathcal{N}(T,1)$	-
$\alpha_{2,t}$	-	-	-	-	$\mathcal{N}(D,1)$	-
$\alpha_{1,d}$	-	-	-	-	$\mathcal{N}(0,1)$	-
$\alpha_{2,d}$	-	-	-	-	$\mathcal{N}(0,1)$	-
$\rho_{1,\text{long}}$	$\mathcal{N}(3,1)$	$\mathcal{N}(T,0.1)$	$\mathcal{N}(T,0.1)$	$\mathcal{N}(T,0.1)$	-	-
$\rho_{2,\text{long}}$	-	-	-	$\mathcal{N}(D,0.1)$	-	-
$\rho_{1,\text{short}}$	-	$\mathcal{N}(1,0.01)$	$\mathcal{N}(1,0.01)$	$\mathcal{N}(1,0.01)$	-	-
$\rho_{2,\text{short}}$	-	-	-	$\mathcal{N}(1,0.01)$	-	-
$\rho_{1,t}$	-	-	-	-	T	-
$\rho_{2,t}$	-	-	-	-	1	-
$\rho_{1,d}$	-	-	-	-	D	-
$\rho_{2,d}$	-	-	-	-	1	-
δ_1	$\mathcal{N}(0,1e-6)$	$\mathcal{N}(0,1e-6)$	$\mathcal{N}(0,1e-6)$	$\mathcal{N}(0,1e-6)$	$\mathcal{N}(0,1e-7)$	-
δ_2	-	-	-	$\mathcal{N}(0,1e-6)$	$\mathcal{N}(0,1e-7)$	-
z	$\mathcal{N}(0,0.1)$	$\mathcal{N}(0,0.1)$	$\mathcal{N}(0,0.1)$	$\mathcal{N}(0,0.1)$	$\mathcal{N}(0,0.1)$	-
τ	-	-	-	-	-	$\Gamma(0.01,0.01)$
$a[1]$	-	-	-	-	-	$\mathcal{N}(0,\sqrt{\frac{1}{0.001}})$
$a[t]$	-	-	-	-	-	$\mathcal{N}(\alpha[t-1],\sqrt{\frac{1}{\tau}})$
β	-	-	-	-	-	Dirichlet(0.1)

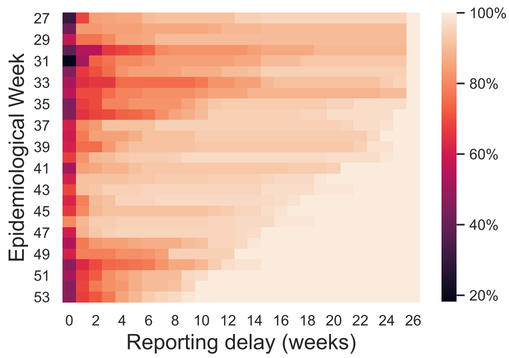

Figure S1: The change in the distribution of delays for Manaus (state Amazonas). Each cell in this heatmap shows the percent of all deaths which occurred at week t, and were reported with given delay d. Here we assume, that up till epidemiological week 53 deaths have been recorded in the database. We use all available SIVEP data up to the release on 5-April-2021.

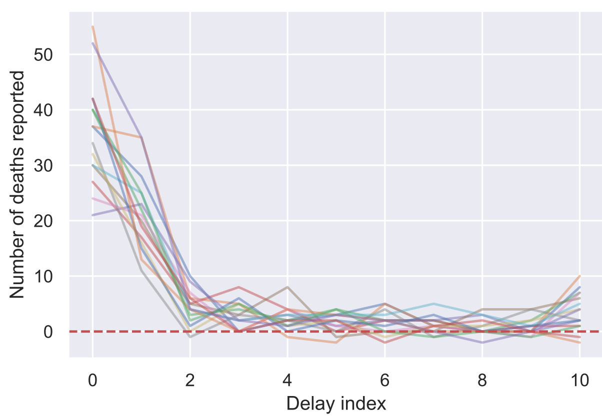

Figure S2: Example of weekly reporting delay with negative signal for Amazonas, epidemiological weeks 27 to 42. Each week's mortality data are plotted as a single line. Some lines fall under the $y = 0$ line (red dashed line), indicating that there were some deaths that were incorrectly assigned to a given week, which was corrected by the following releases of the data.

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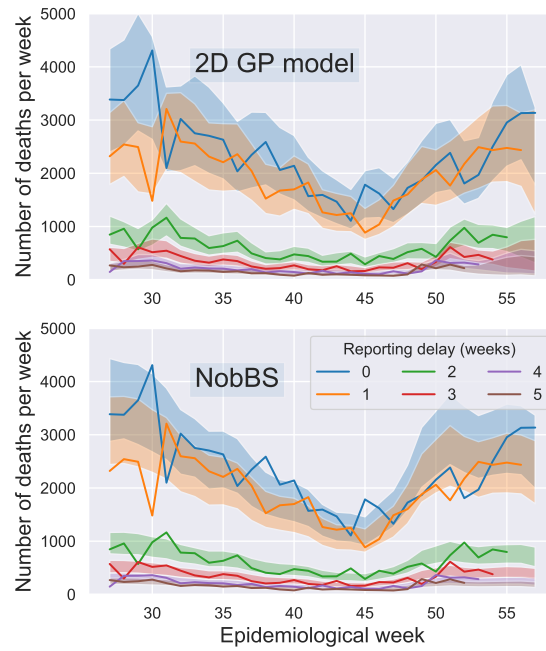

Figure S3: Reported and nowcasted numbers of deaths with reporting delay 0 to 5 weeks generated by the 2D additive GP and NobBS models. The reported data are shown with solid lines, and the 95% CrI for the nowcasts with the ribbons.

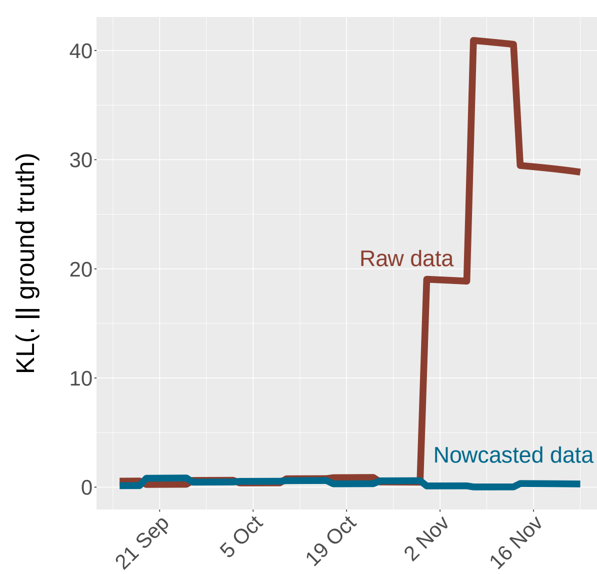

Figure S4: Kullback-Leibler divergence (relative entropy) between the R_t value estimated using raw data and nowcasted data.

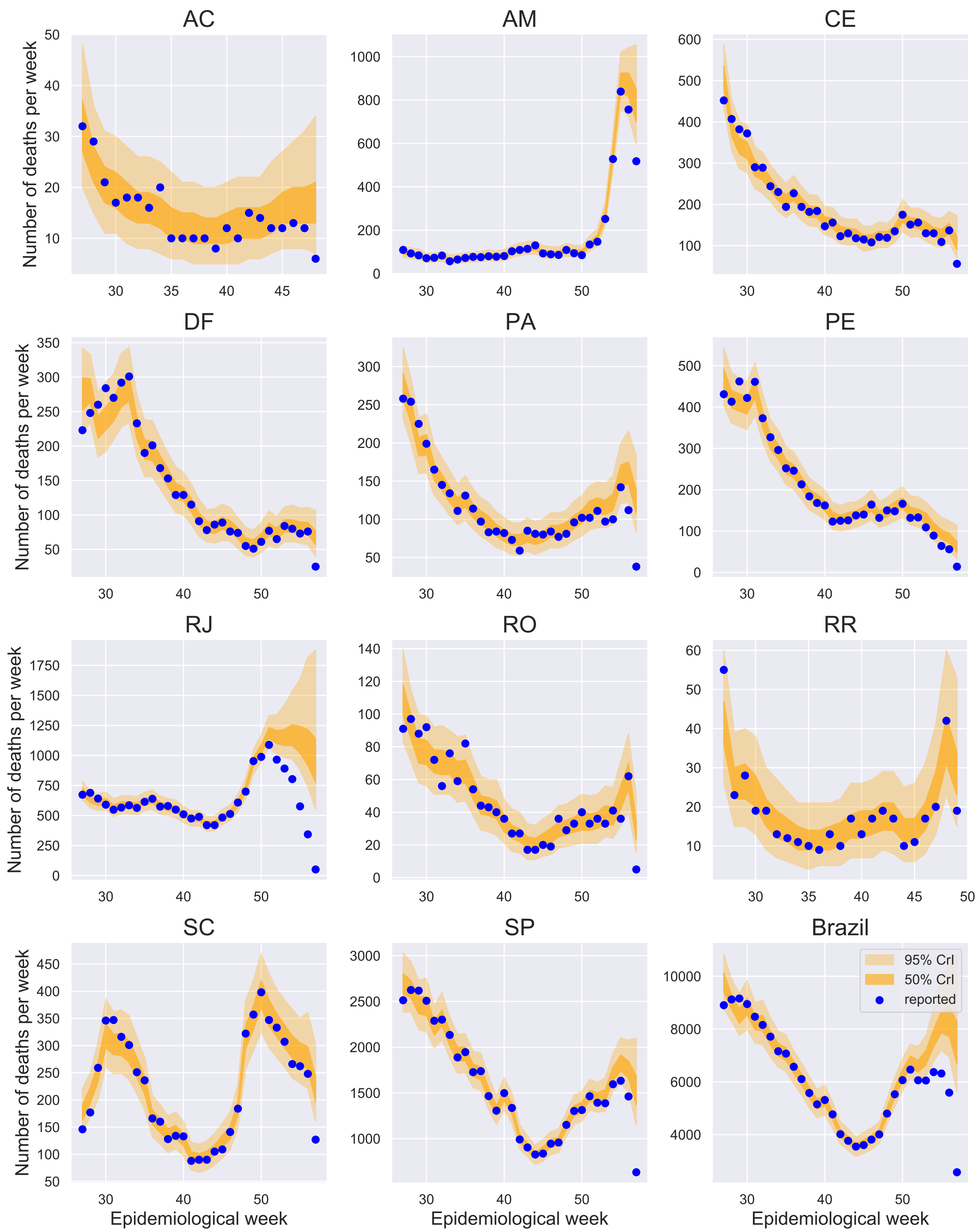

Figure S5: Nowcasts made by the 1D SE+SE data-split GP model, using data up to 8-Feb-2021 for Acre (AC), Amazonas (AM), Ceará (CE), Distrito Federal (DF), Pará (PA), Pernambuco (PE), Rio de Janeiro (RJ), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP) and whole Brazil.

2 RETROSPECTIVE TESTING

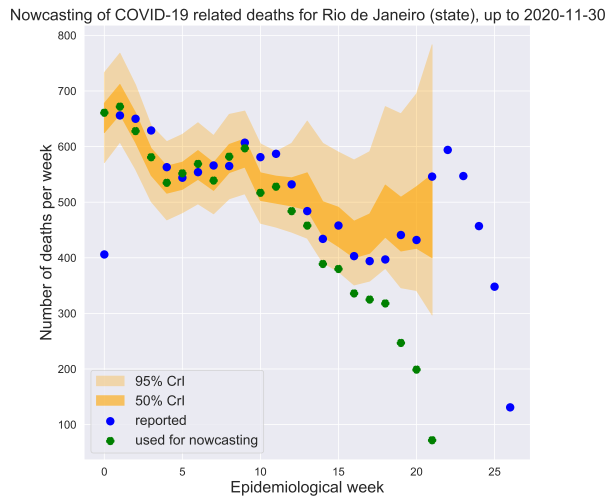

Figure S6: Nowcasted and reported deaths due to COVID-19 death for Rio de Janeiro (state), generated with a 1D SE+SE data-split GP model. Reported deaths are shown in blue, nowcasted CrI in orange. Here the nowcasting was performed with all data available up till the SIVEP data release on the 30-Nov-2020. At that point, looking only at the reported data might indicate that the number of deaths keep decreasing, however using nowcasting would have revealed the uptick in the number of deaths, which was not yet observed in the data at the time of the 30-Nov-2020 release.

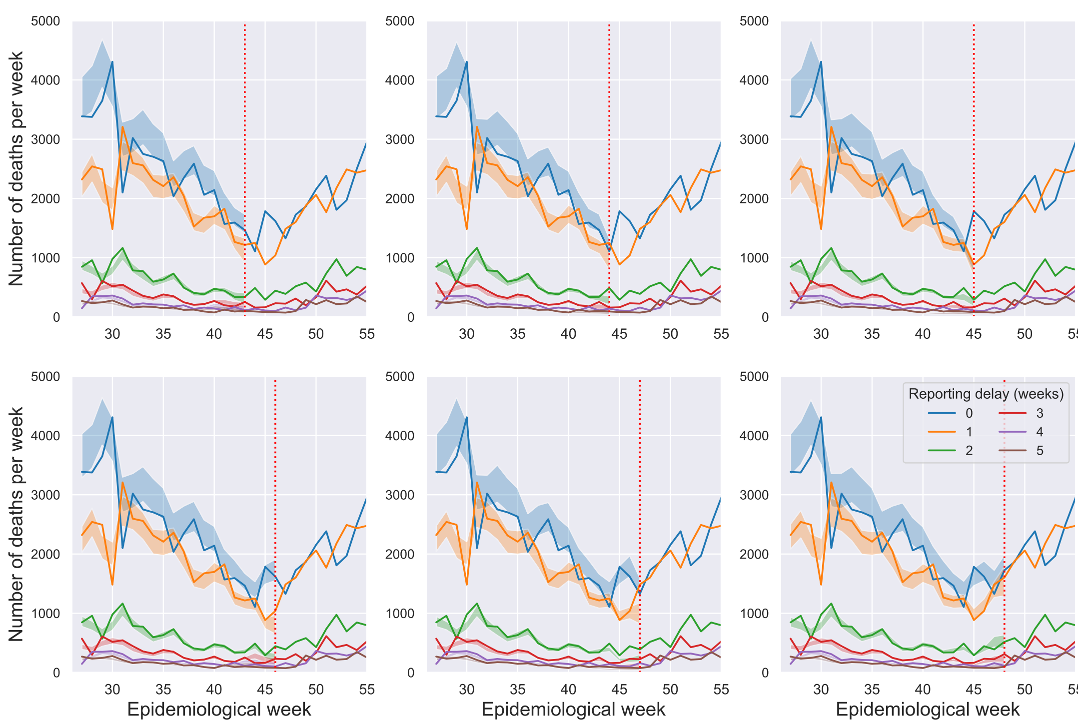

Figure S7: Distribution of reporting delays for each of the retrospective tests. The moment of the "nowcast" is shown with the red dotted line in each plot. Solid line present the data extracted from the release of SIVEP data from 31-May-2021, and the ribbons show 50% CrI of the model fit obtained obtain using the 2D additive kernel GP model.

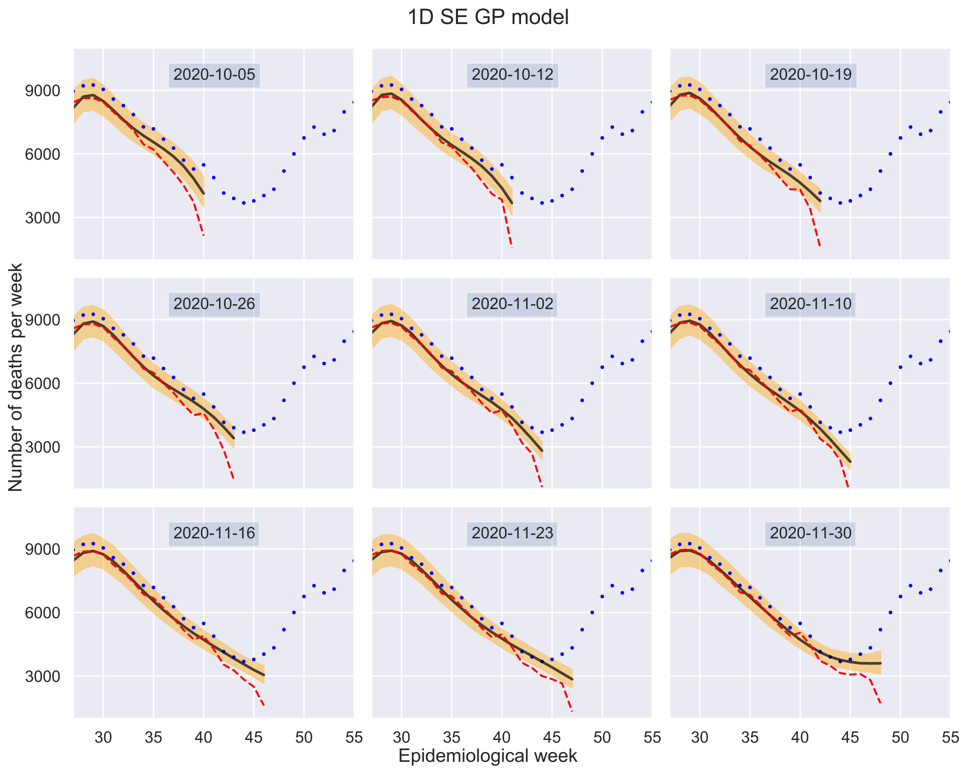

Figure S8: Retrospective tests for the 1D SE GP nowcasting mode. Deaths reported in the 31-May-2021 release are shown with blue dots, data available at the time of nowcasting with red dashed line, nowcasted mean values with black solid line and 95% CrI with orange ribbon.

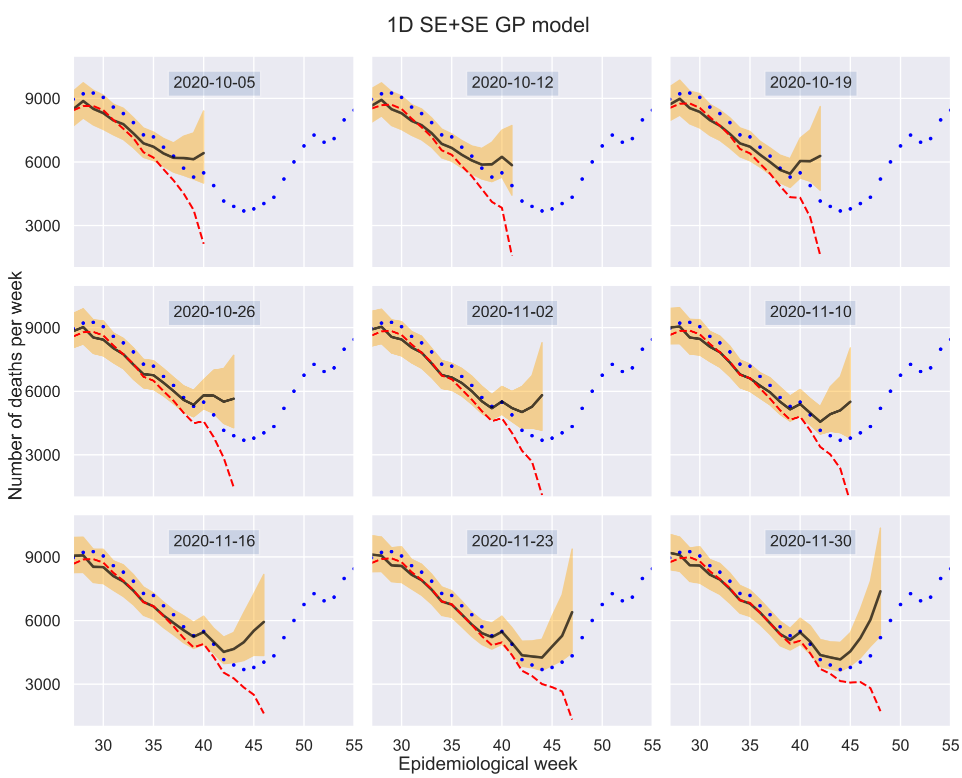

Figure S9: Retrospective tests for the 1D SE+SE GP model.

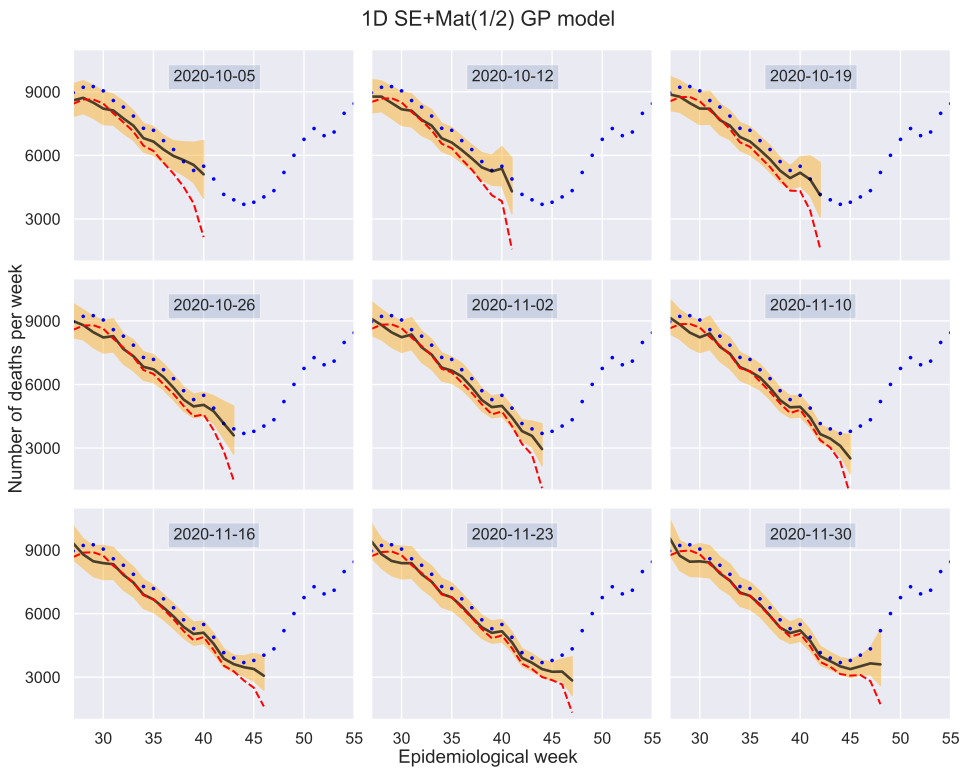

Figure S10: Retrospective tests for the 1D SE+Mat(1/2) GP model.

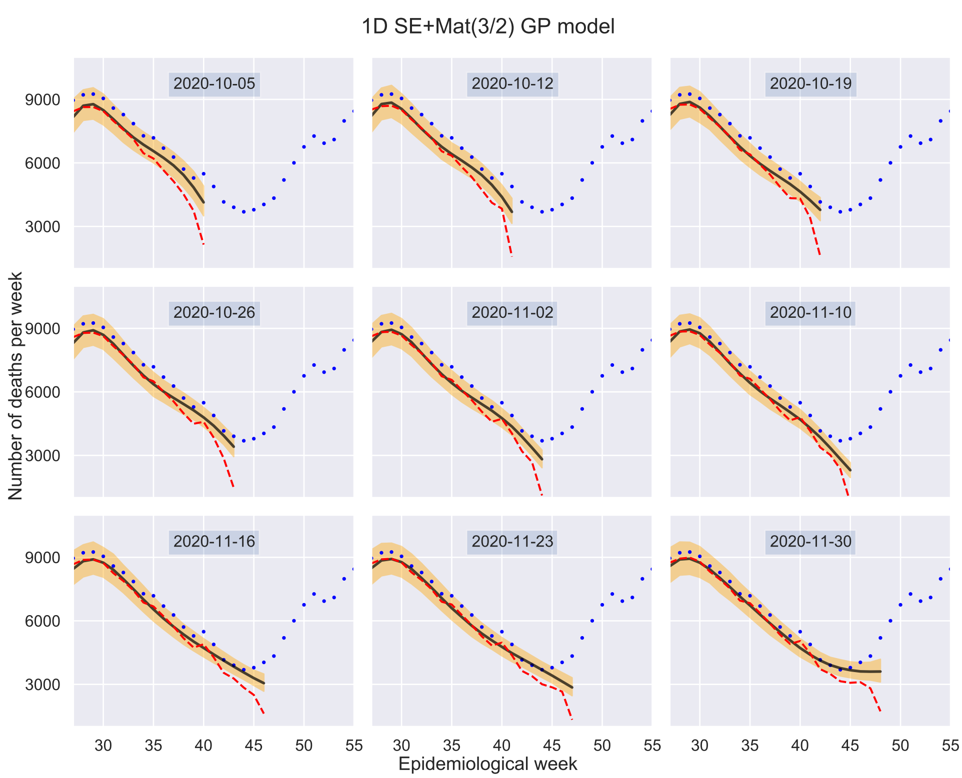

Figure S11: Retrospective tests for the 1D SE+Mat(3/2) GP model.

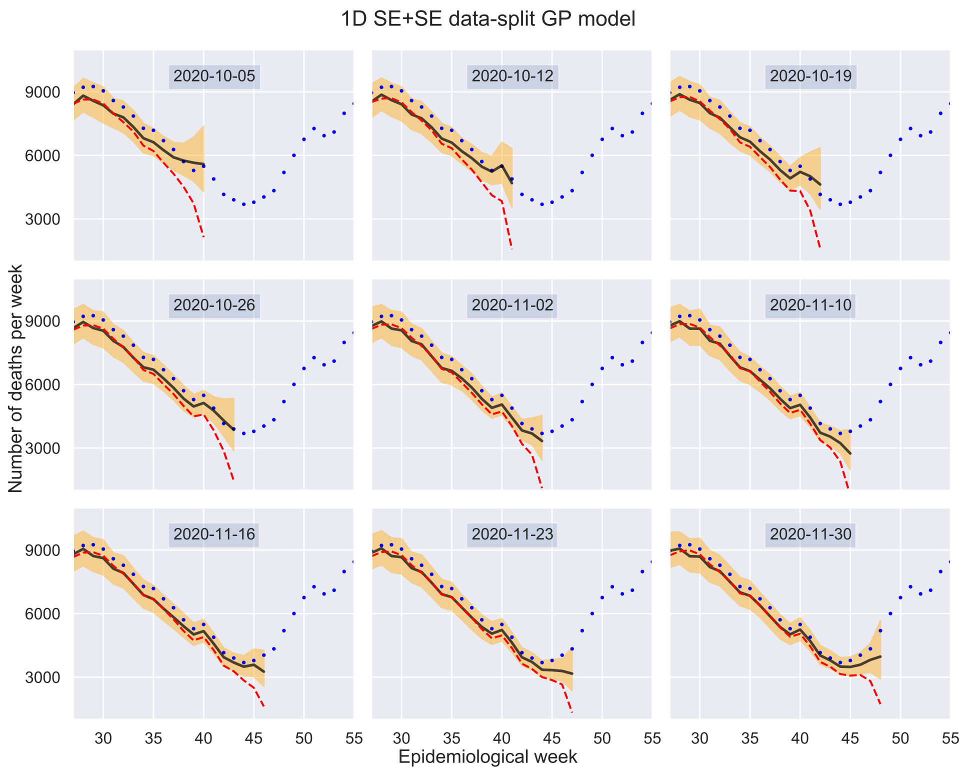

Figure S12: Retrospective tests for the 1D SE+SE data-split GP model.

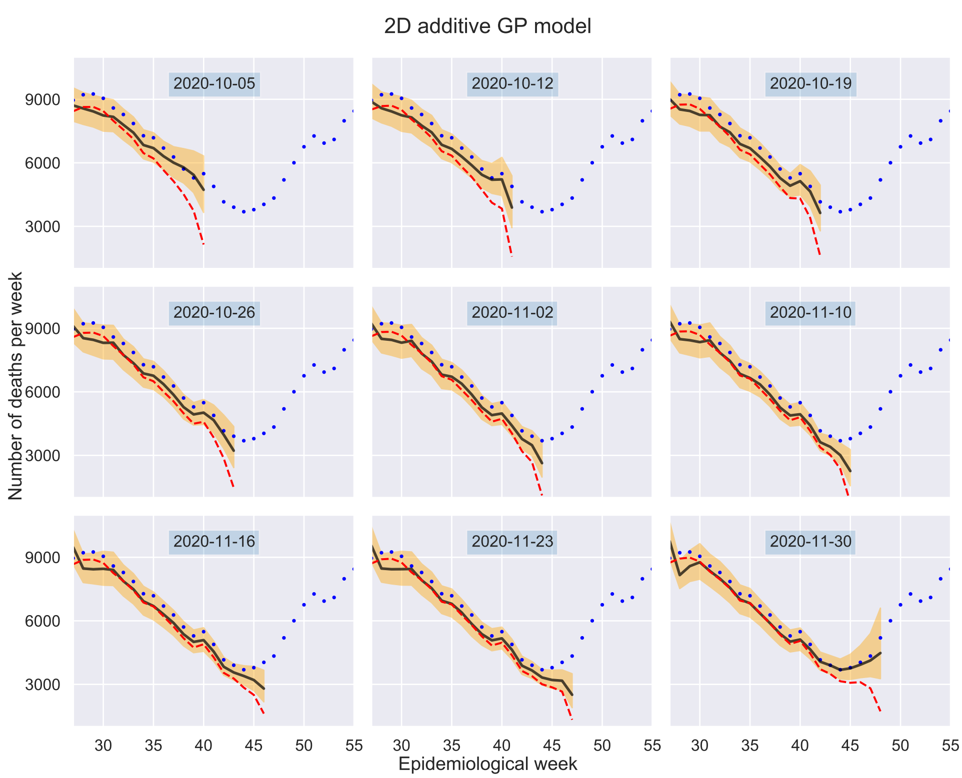

Figure S13: Retrospective tests for the 2D additive kernel GP model.

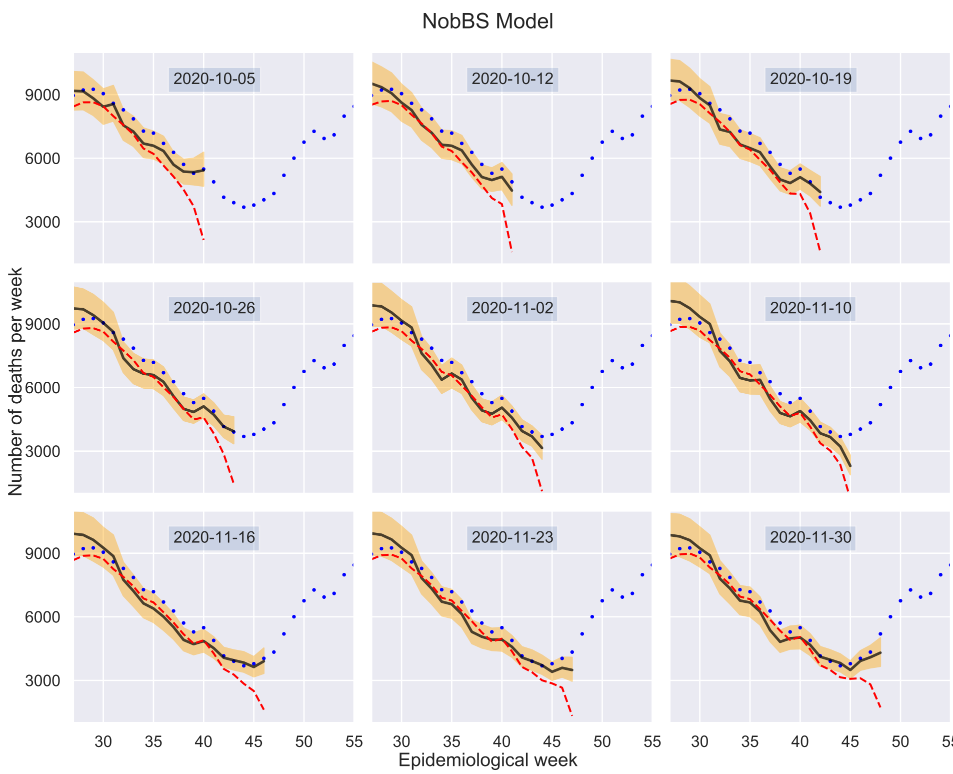

Figure S14: Retrospective tests for the NobBS model.

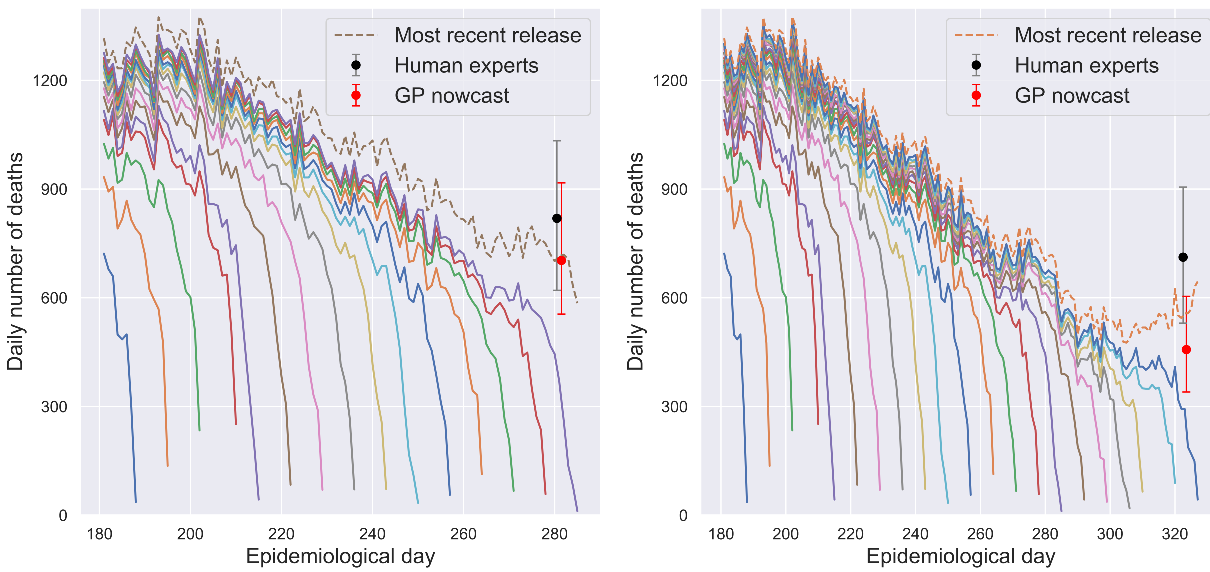

Figure S15: Human experts and 1D SE+SE data-split GP model estimates of a true number of deaths on 8-Oct-2020 (left) and 19-Nov-2020 (right) plotted together with the reported data.

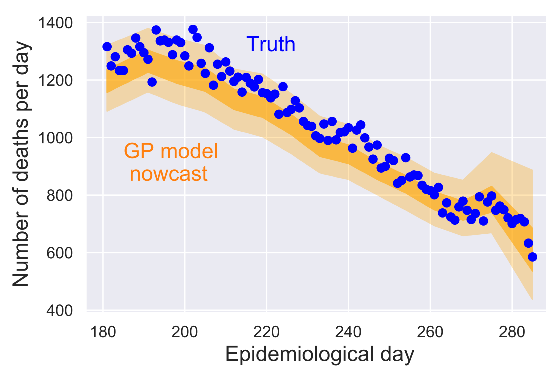

Figure S16: Example of the interpolation of numbers of deaths per day based on the weekly nowcasted values. The 50% and 95% CrI for the nowcast are shown with the ribbon.

3 MODEL DIAGNOSTICS

For each of the model runs shown in this section, 4 chains were run for 1000 iterations, with 500 iterations used for burn-in. All fits presented are done for nowcasting using all data available up to 11-Jan-2021.

Table S2: Diagnostics for the 1D SE+SE data-split GP model.

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
$\rho_{1,\text{long}}$	28.004	0.101	27.820	28.194	0.002	0.001	2794.0	1241.0	1.00
$\rho_{2,\text{long}}$	29.009	0.102	28.826	29.209	0.002	0.002	2205.0	1377.0	1.01
$\rho_{1,\text{short}}$	1.003	0.010	0.984	1.021	<0.001	<0.001	2985.0	1388.0	1.00
$\rho_{2,\text{short}}$	1.013	0.009	0.996	1.032	<0.001	<0.001	2096.0	1691.0	1.00
$\alpha_{1,\text{long}}$	19.165	1.570	16.248	22.085	0.074	0.053	468.0	482.0	1.01
$\alpha_{2,\text{long}}$	23.366	1.234	21.145	25.692	0.084	0.060	219.0	288.0	1.02
$\alpha_{1,\text{short}}$	4.958	0.481	4.048	5.830	0.024	0.017	412.0	460.0	1.01
$\alpha_{2,\text{short}}$	5.941	0.279	5.447	6.463	0.014	0.010	378.0	886.0	1.01
δ_1	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	1448.0	795.0	1.00
δ_2	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	583.0	1188.0	1.01
r	246.499	11.073	225.669	267.226	0.217	0.154	2632.0	1399.0	1.00

Table S3: Diagnostics for the 2D additive GP model.

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
$\alpha_{1,t}$	29.169	1.015	27.467	31.164	0.018	0.013	3288.0	1544.0	1.01
$\alpha_{2,t}$	5.067	0.942	3.360	6.824	0.043	0.031	469.0	764.0	1.01
$\alpha_{1,d}$	0.548	0.119	0.346	0.778	0.005	0.004	471.0	680.0	1.01
$\alpha_{2,d}$	0.616	0.096	0.460	0.805	0.004	0.003	654.0	914.0	1.01
δ_1	<0.001	<0.001	<0.001	<0.001	0.000	<0.001	1776.0	854.0	1.00
δ_2	<0.001	<0.001	<0.001	<0.001	<0.0010	<0.001	1672.0	1110.0	1.01
r	89.733	6.478	77.809	102.541	0.176	0.126	1390.0	1365.0	1.00

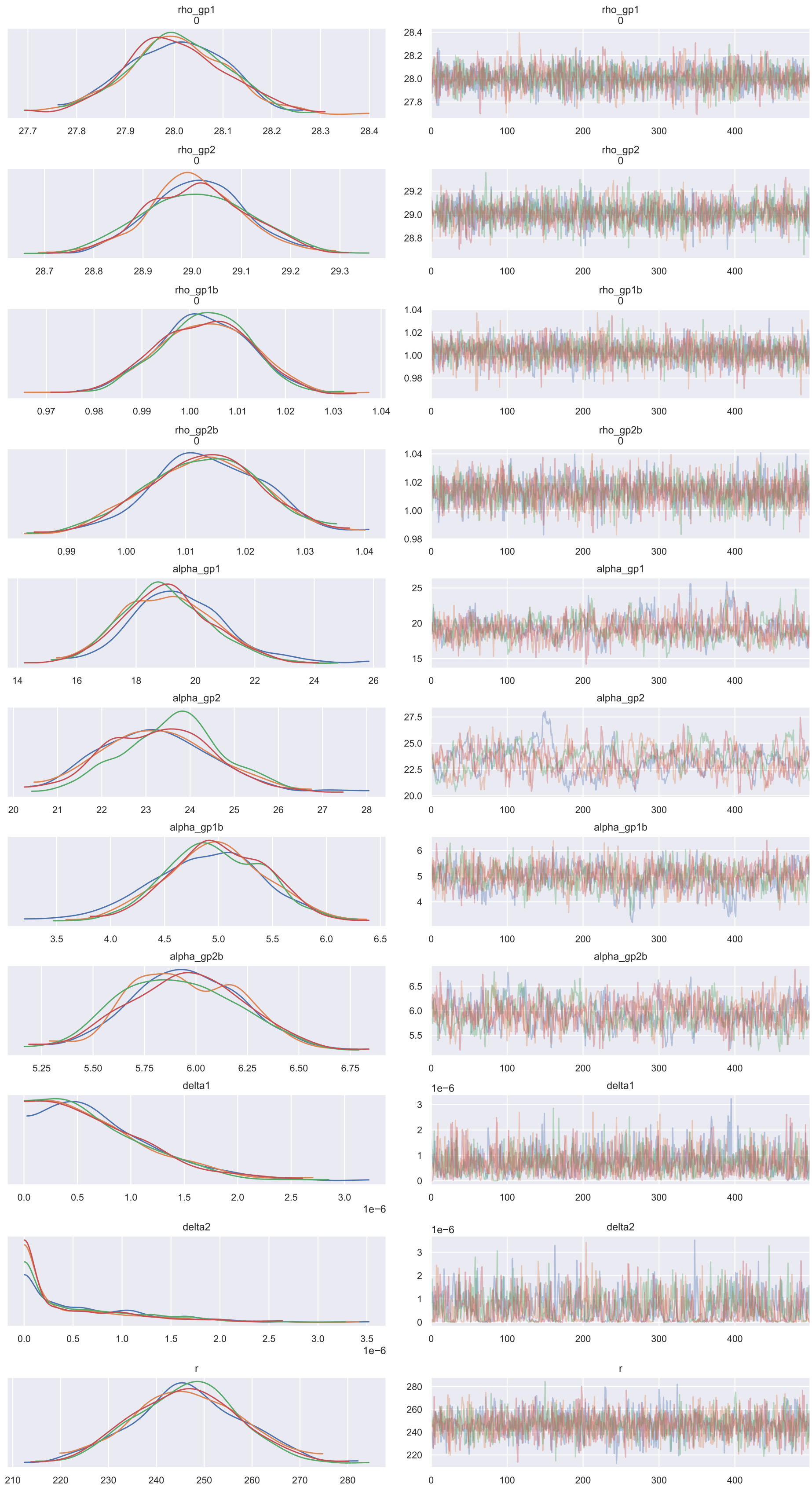

Figure S17: Traceplots for the 1D SE+SE data-split GP model.

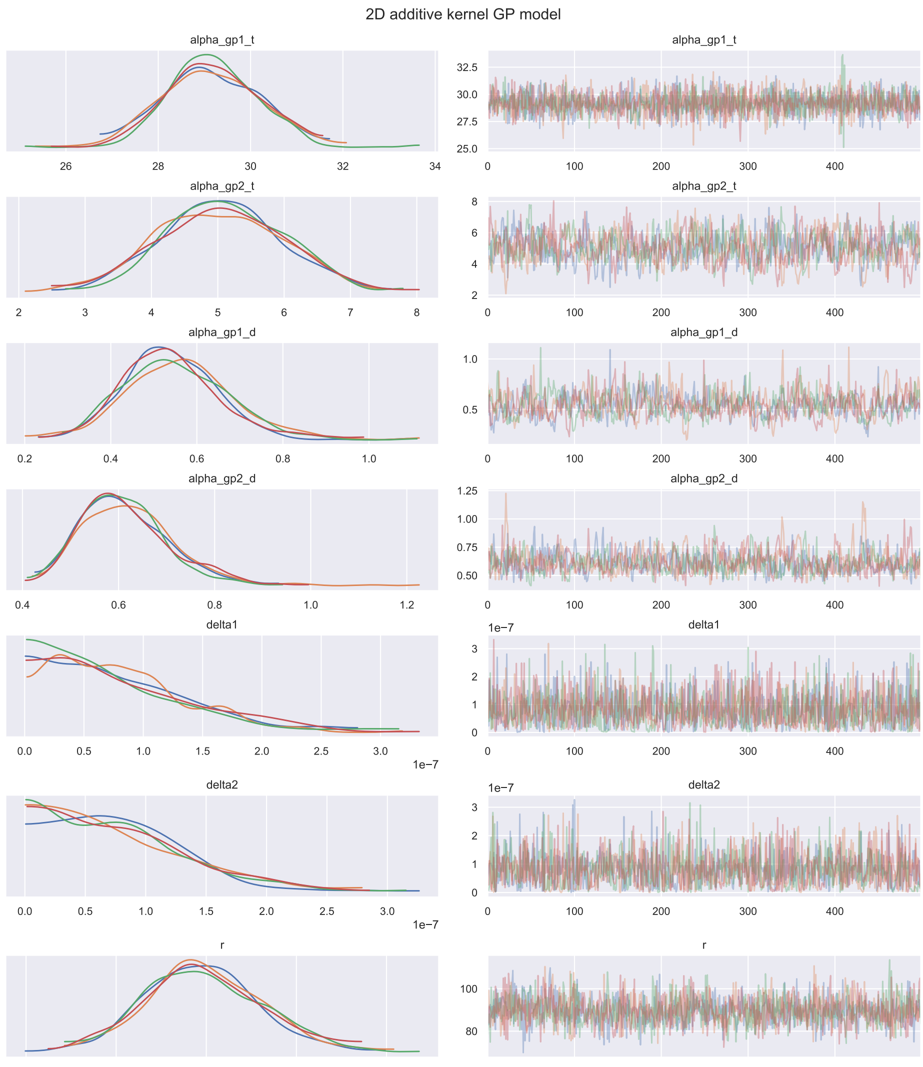

Figure S18: Traceplots for the 2D additive GP model.

4 SENSITIVITY ANALYSIS

For all sensitivity analyses, 1D SE+SE data-split GP model has been used. Each time we run the models with 4 chains for 1000 iterations, with 400 iterations used for burn-in. All fits presented are done for nowcasting using all data available up to 11-Jan-2021.

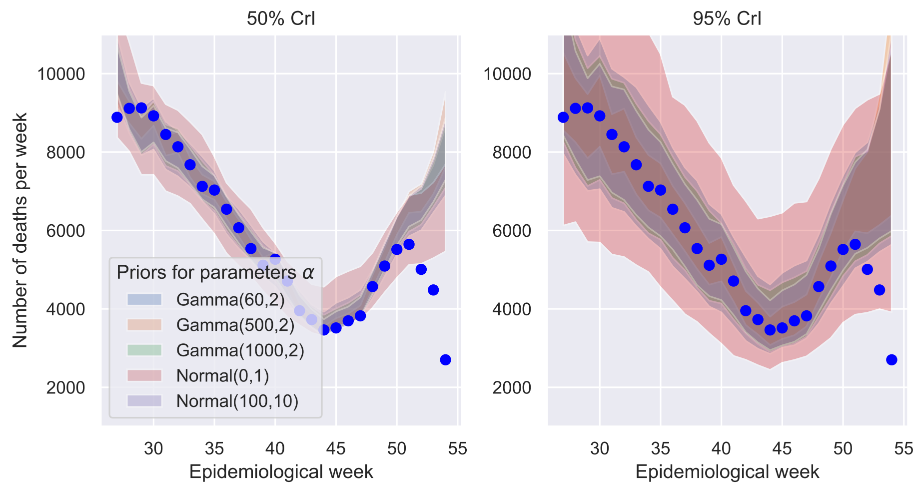

Figure S19: Model fits with different r prior density.

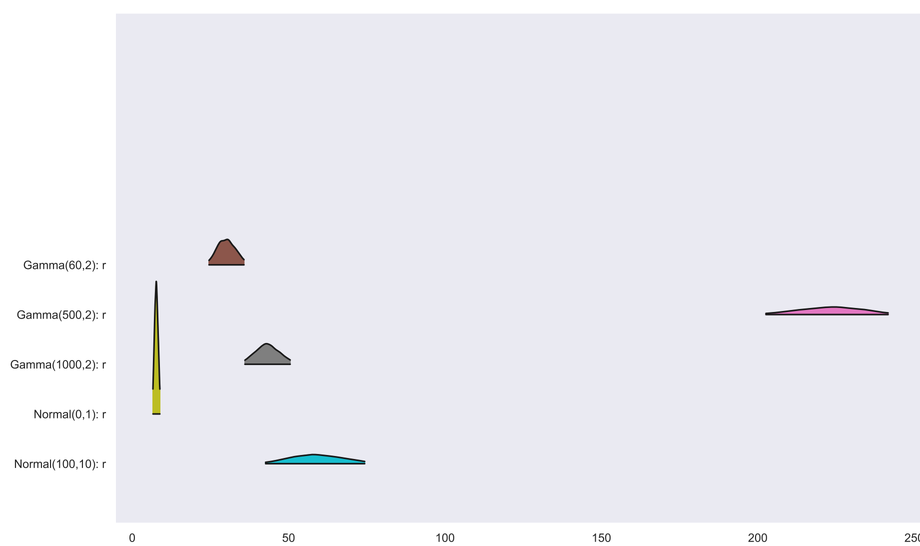

Figure S20: Model fits with different r prior density.

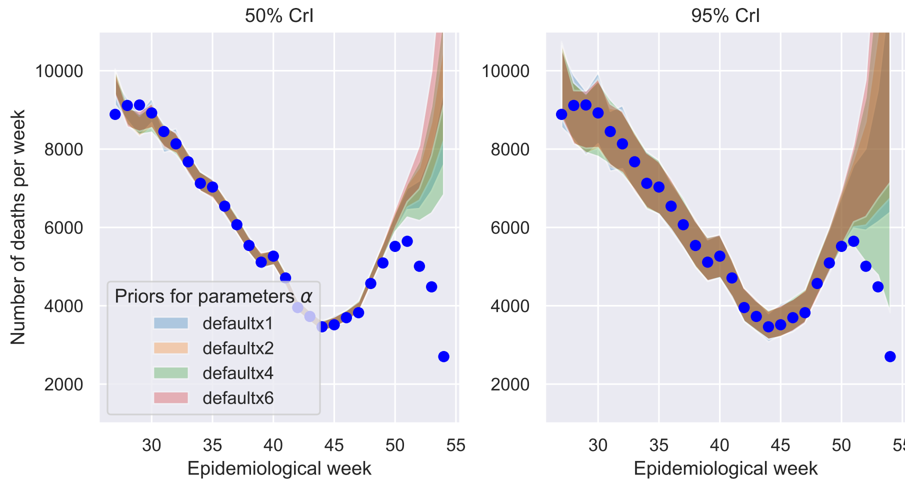

Figure S21: Model fits with different $\alpha_{\text{long},1}$, $\alpha_{\text{long},2}$, $\alpha_{\text{short},1}$ and $\alpha_{\text{short},2}$ prior density variance.

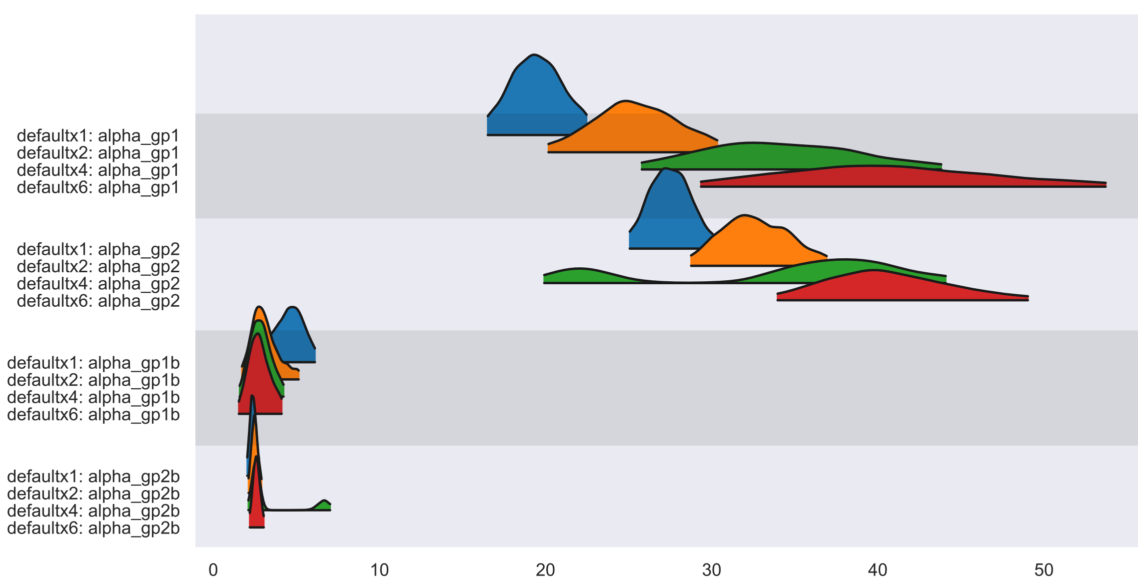

Figure S22: Model fits with different $\alpha_{\text{long},1}$, $\alpha_{\text{long},2}$, $\alpha_{\text{short},1}$ and $\alpha_{\text{short},2}$ prior density variance.

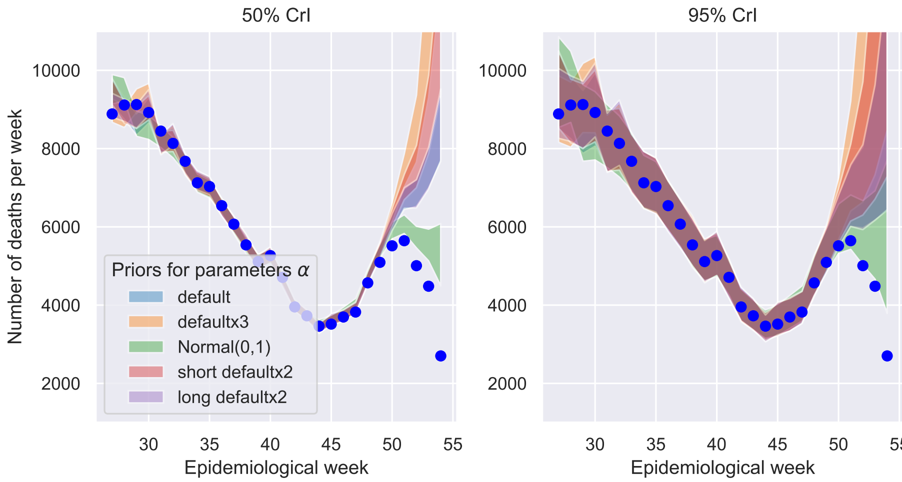

Figure S23: Model fits with different $\alpha_{\text{long},1}$, $\alpha_{\text{long},2}$, $\alpha_{\text{short},1}$ and $\alpha_{\text{short},2}$ prior density. Default means using the default priors described in Table S1, for default x 3 prior we increased the mean in the default priors 3-fold, Normal(0,1) means a standard prior was set to all α-s, and long- and short- default x 2 means increased mean in the default prior long- and short-part respectively.

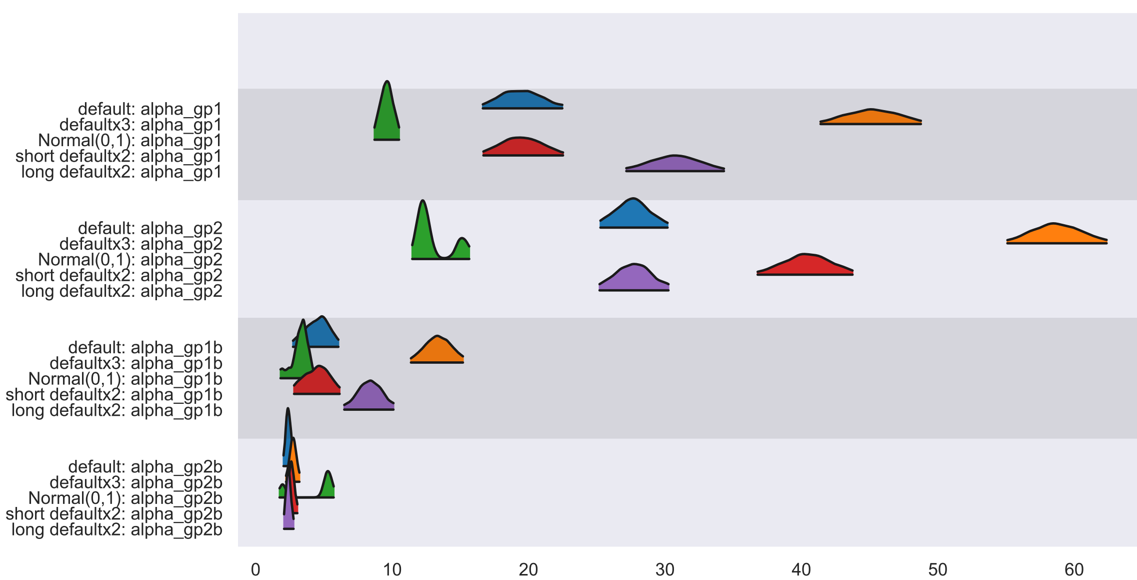

Figure S24: Model fits with different $\alpha_{\text{long},1}$, $\alpha_{\text{long},2}$, $\alpha_{\text{short},1}$ and $\alpha_{\text{short},2}$ prior density. Default means using the default priors described in Table S1, for default x 3 prior we increased the mean in the default priors 3-fold, Normal(0,1) means a standard prior was set to all α-s, and long- and short- default x 2 means increased mean in the default prior long- and short-part respectively.