Supplementary Material for: Hierarchical Probabilistic Model for Blind Source Separation via Legendre Transformation

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A FEATURE EXTRACTION FOR A 2D POINT CLOUD EXPERIMENT

We demonstrate the effectiveness of IGBSS in identification of independent components on a 2-dimensional point cloud to be used for feature extraction or dimensionality reduction. In our experiment, we generate a 2-dimensional point cloud using two standard Student's t-distribution with 1.3 degree of freedom and have scaled the first dimension by $1/5$ and the second dimension by $1/10$ to the point cloud, illustrated in Figure S2a. Then we have randomly generated a mixing matrix for our experiment to generate a mixed signal shown in Figure S2b. We run the experiment on our model IGBSS using min-max normalization as a pre-processing step and compare it to PCA and ICA. We apply the reverse transformation of the min-max normalization on the recovered signal.

We have plotted experimental results in Figure S2. From the results, we can see that PCA is able to recover the same scale of the point cloud. However, the sign of the signal is not recovered as we have recovered reversed sign of the signal. PCA also recovers signals which are orthogonal to the largest variance. Therefore the axes of the point cloud recovered by PCA does not align with the source signal in Figure S2a, that is, the axes do not run parallel to the x- and y-axes but instead is still in the same orientation as the mixed signal. This is not what we want as the signal is still mixed, and we would like to recover the signal in the same orientation as the source signal in blind source separation. ICA aims to recover statistically independent signals that are generally considered as the axes with the largest variances and not necessarily orthogonal to each other. However, the limitations of ICA is that it is unable to recover the sign and the scale of the signal. Therefore the scale of the recovered signal does not match with the source signal. In our experiment, we have plotted the results with unit variance as the recovered signal is generally unnormalized in ICA.

Since our experiment is synthetically generated, we are able to quantitatively measure the the error in each approach by normalizing both the recovered signal and the source signal by its standard deviation then computing the root mean squared error (RMSE) and the signal-to-noise ratio (SNR). The results of this is shown in Table S1. Our proposed approach IGBSS has clear advantages, where it is able to recover the same orientation as the source signal as well as preserve the signal.

Table S1: Signal-to-Noise Ratio (SNR) and Root Mean Square Error (RMSE) between the recovered signal and the latent source signal for the 2-dimensional point cloud experiment.

Model	PCA	ICA	IGBSS
RMSE	2.011	1.445	**1.421**
SNR	25.997	27.431	**27.503**

B SIGN INVERSION IN ICA

We demonstrate the problem of the sign inversion in ICA. We use the same experimental set-up explained in Section 3.1 on blind source separation for affine transformation. We run the experiment on the dataset used for the experiment 1 for the first order experiment and have shown the output of several runs in FastICA to show the problem of the sign inversion in Figure S1. For the 6 runs, we can see that none of the experiments were able to obtain the correct sign of the signal. This means that applying FastICA to applications where the sign of the signal is important is problematic.

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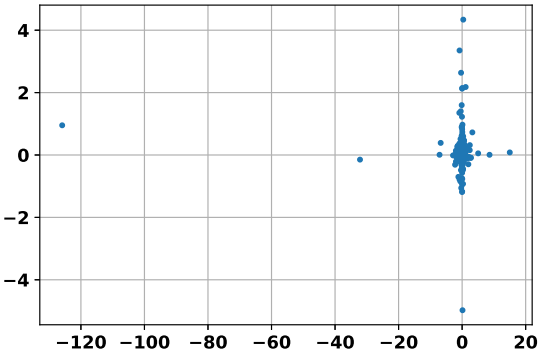
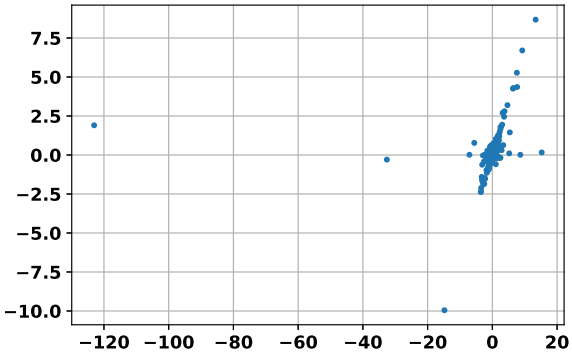
(a) GT (b) Mixed (c) Run 1 (d) Run 2 (e) Run 3 (f) Run 4 (g) Run 5 (h) Run 6

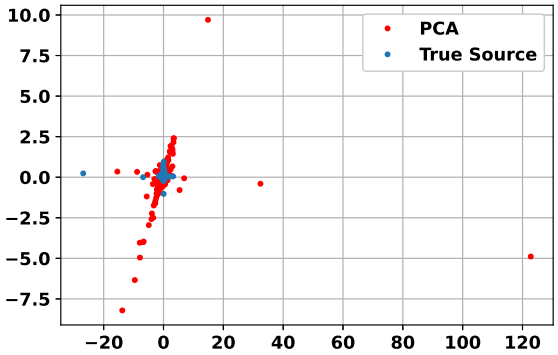
Figure S1: Six different runs of FastICA with the same experimental input experimental dataset as exp1 with first order interactions. The different results can demonstrate that the FastICA model is non-convex leading to potential problemic results such as the sign inversion.



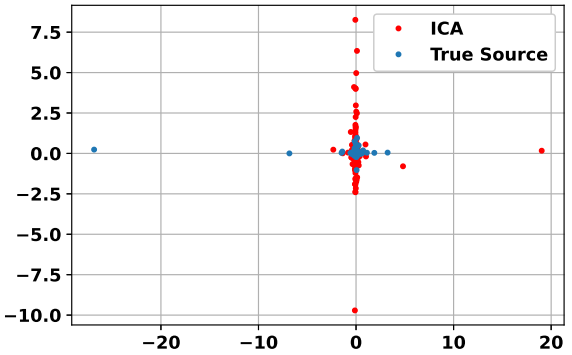
(a) Source Signal



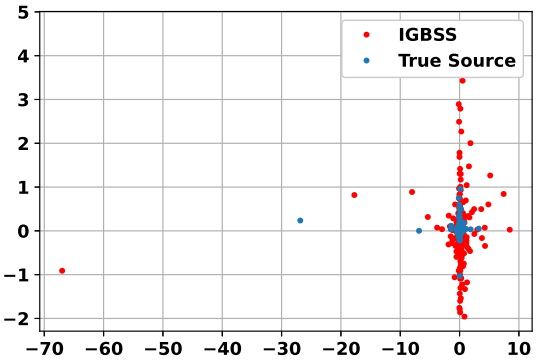
(b) Mixed Signal



(c) PCA Recovery



(d) ICA Recovery



(e) IGBSS Recovery

Figure S2: 2-dimensional point cloud experiment.