GNNRank: Learning Global Rankings from Pairwise Comparisons via Directed Graph Neural Networks

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Abstract

Recovering global rankings from pairwise comparisons has wide applications from time synchronization to sports team ranking. Pairwise comparisons corresponding to matches in a competition can be construed as edges in a directed graph (digraph), whose nodes represent e.g. competitors with an unknown rank. In this paper, we introduce neural networks into the ranking recovery problem by proposing the so-called GNNRank, a trainable GNN-based framework with digraph embedding. Moreover, new objectives are devised to encode ranking upsets/violations. The framework involves a ranking score estimation approach, and adds an inductive bias by unfolding the Fiedler vector computation of the graph constructed from a learnable similarity matrix. Experimental results on extensive data sets show that our methods attain competitive and often superior performance against baselines, as well as showing promising transfer ability. Codes and preprocessed data are at: `https://github.com/SherylHYX/GNNRank`.

1. Introduction

Recovering global rankings from pairwise comparisons reflecting relative latent strengths or scores is a fundamental problem in information retrieval (Schütze et al., 2008; Liu, 2011) and beyond. When analyzing large-scale data sets, one often seeks various forms of rankings (i.e. orderings) of the data for the purpose of identifying the most important entries, efficient computation of search & sort operations, or for extracting the main features. There is a swarm of applications employing ranking techniques ranging from Amazon's Mechanical Turk system for crowdsourcing (Raykar & Yu, 2011) to the movie recommendation system provided by Netflix (Cremonesi et al., 2010), and modeling outcomes of football matches (Tsokos et al., 2019).

A very rich literature on ranking traces back to (Kendall & Smith, 1940), who studied recovering the ranking of a set of players from pairwise comparisons reflecting a total ordering. The last decades have seen a flurry of methods for ranking from pairwise comparisons, mostly based on spectral methods leveraging the eigenvectors of suitably defined matrix operators built directly from the data, which will be detailed in the related work. In particular, Serial-Rank (Fogel et al., 2014) shows promising ability in ranking for a set of totally ordered items without ties, by introducing a specific inductive bias from the so-called *seriation* problem (Atkins et al., 1998): a Fiedler vector of a certain similarity matrix could recover true rankings given enough pairwise comparisons in the noiseless setting. The *Fiedler value* of a symmetric and nonnegative matrix (or a graph) is defined to be the second smallest eigenvalue of the combinatorial Laplacian of the matrix (Fiedler, 1973), and its corresponding eigenvector is called a *Fiedler vector*. The Fiedler vector is able to encode useful information of a graph, such as bi-partitions of a graph (Fiedler, 1973) and the seriation problem (Atkins et al., 1998), and can be employed for partitioning hypergraphs (Chen et al., 2017).

Moreover, there has been promising progress on combinatorial optimization in machine learning (Wang et al., 2021; Maurya et al., 2021), especially graph neural networks (GNNs) (Zhou et al., 2020; Wu et al., 2020), due to their potential in data exploration. Compared with their great success in many combinatorial tasks, the capability of GNNs in ranking tasks is not well developed. The few existing works are restricted to specific settings e.g. top-n personalized recommendations (Wang et al., 2021), and approximating centrality measures (Maurya et al., 2021). Another technical gap is the inability to learn a model by directly optimizing the ranking objective, which we aim to fill in our work.

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We propose GNNRank, an end-to-end ranking framework compatible with existing GNN models e.g. (He et al., 2021; Tong et al., 2020) that is able to learn directed graph (digraph) node embeddings. We devise differentiable objectives to encode ranking upsets/violations. Following the standard protocol in (d'Aspremont et al., 2021), an upset of an edge is a ranking violation; the inferred relative ranking is in the opposite direction to what the original measurement indicates. GNNRank consists of a family of ranking score estimation techniques, and adds an inductive bias by unfolding the Fiedler vector computation of the graph constructed from a *learnable* similarity matrix.

Our main contributions are as follows:

- To the best of our knowledge, this is *the first neural network framework* (specifically GNN) to recover global rankings from pairwise comparisons whereby a direct optimization of the ranking objectives is enabled, without supervision. Our method differs from the learning-free methods including SerialRank because we design two novel differentiable losses tailored to neural networks, while the metric $\mathcal{L}_{upset,naive}$ used in previous works is piecewise constant and only used for post-evaluation instead of training the algorithms. Along the way, we adapt the widely used $\mathcal{L}_{upset,naive}$ metric to a more fined-grained evaluation method. Thus the motivation is also different from existing works.
- For jointly solving global ranking and GNN training, we design and introduce an inductive bias as achieved by the proximal gradient steps to our neural network, as unfolded from the Fiedler vector calculation, as an effective way of encoding latent orderings. The technique of constructing the Q matrix and the transformation of the optimization problem for differentiable computation of Fiedler eigenvectors as part of a deep learning model is nontrivial and, to the best of our knowledge, novel.
- Our methods empirically attain competitive, and often superior, accuracy compared to state-of-the-art methods, and its cost-effectiveness is especially pronounced when the trained GNN model is transferred to new datasets for ranking recovery. Compared to some existing methods e.g. Minimum Violation Rank (MVR) (Ali et al., 1986) with expensive optimizations, our methods have better asymptotic time complexity.
- From a computational perspective, when the data may have a temporal dimension or when there are similarities across different data sets, one can apply an already trained model to new data sets that are similar to the one the model has been trained on.
- We provide theoretical convergence guarantees for our method, with a technically novel proof, which in our view is a considerable advantage compared to other, ad-hoc, neural solvers.

2. Related Work

Ranking Methods One of the most popular models in the ranking literature is the Bradley-Terry-Luce (BTL) model (Bradley & Terry, 1952), (Luce, 1959). In its basic version, the probability that player i beats player j is given by $P_{ij} = \frac{w_i}{w_i+w_j}$, where the parameter vector $w \in \mathbb{R}^n_+$ is estimated from the data, and w_i is a proxy for the strength of player i. (Firth, 2005) facilitates the specification and fitting of Bradley-Terry logit models to pairwise comparisons.

Employing the stationary distribution of a suitably defined Markov chain for the ranking task traces its roots in early work on the topic of *network centrality*. Such measures have been designed to quantify the extent to which nodes of the graph (or other network structures) are most important (Newman, 2010); see for example (Page et al., 1998), (Bonacich, 1987), and (Negahban et al., 2017).

David's Score (David, 1987) computes rankings from proportions of wins and losses. Minimum Violation Rank (MVR) (Ali et al., 1986) encompasses a suite of methods that aim to directly minimize certain penalty functions at the level of each upset. In our experiments, we compare against the algorithm of (Gupte et al., 2011), that considers a linear relaxation of an integer program that minimizes a so-called agony loss. However, MVR is computationally expensive as also will be shown in our experiments.

SyncRank (Cucuringu, 2016) formulates the ranking problem with incomplete noisy pairwise information as an instance of the group synchronization problem over the group SO(2) of planar rotations (Singer, 2011), which has attracted significant attention in recent years. The SpringRank algorithm of (De Bacco et al., 2018) borrows intuition from statistical physics, and proposes to infer hierarchical rankings in directed networks by solving a linear system of equations. (d'Aspremont et al., 2021) introduces simple algorithms for ranking and synchronization based on singular value decompositions, with theoretical guarantees. SerialRank (Fogel et al., 2014) first computes the Laplacian from a certain similarity matrix $\mathbf{S}'$. The corresponding Fiedler vector of $\mathbf{S}'$ then serves as the final ranking estimate. The intuition is that the more similar two players are (in terms of the pattern of incoming/outgoing edges), the more similar their ranking should be; indeed, if two players defeat, and are defeated, by the same set of other players, then they are likely to have a similar ranking/strength. In this classical ordering problem (called *seriation* (Atkins et al., 1998)), one is given a similarity matrix between a set of items and assumes that the items can be ordered along a chain such that the similarity between items decreases with their distance in the chain.

Apart from the above classical learning-free optimizers for ranking problems, there also exist learning-based (mostly neural) models for similar tasks. The early work (Scarselli et al., 2005) applies a certain form of GNN to ranking web pages, while it requires label supervision for ground-truth ranks for training. While the authors in (Damke & Hüllermeier, 2021) propose the family of so-called RankGNNs, their competitors are graphs. (Rigutini et al., 2011) applies a neural network approach for preference learning, (Köppel et al., 2019) generalizes (Burges et al., 2005), but these methods require queries as input, which solve a different problem from ours. (Maurya et al., 2021) proposes the first GNN-based model to approximate betweenness and closeness centrality, facilitating locating influential nodes in the graphs in terms of information spread and connectivity. The pairwise direction is rarely considered in these works but it is important for the problem studied in this paper. Thus, our task differs fundamentally from the (abundant) learning-to-rank literature.

Directed Graph Neural Networks Directed GNNs are useful in learning digraph node embeddings. (Tong et al., 2020) builds aggregators based on higher-order proximity. (Zhang et al., 2021) constructs a complex Hermitian Laplacian matrix. (He et al., 2021) introduces imbalance objectives for digraph clustering. In GNNRank, existing digraph neural networks can be readily incorporated.

Unfolding Techiques Algorithm unfolding (Gregor & LeCun, 2010) was first introduced to unfold the iterations as a cascade of layers while adding learnable parameters. The unfolding idea has later been applied to problems such as semantic segmentation (Liu et al., 2017) and efficient power allocation (Chowdhury et al., 2021). The work (Yang et al., 2021) GRshould we use different citation types using citep and citet ? discusses a new family of GNN layers designed to mimic and integrate the update rules of classical iterative algorithms. The unfolding idea inspires us to add a useful inductive bias from the calculation of the Fiedler vector via our bi-level optimization pipeline (Talbi, 2013). Most importantly, unfolding allows us to pass gradients through the optimization of a useful function.

3. Approach

3.1. Problem Definition

Without loss of generality, we consider pairwise comparisons in a competition, which can be encoded in a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the node set $\mathcal{V}$ denotes competitors, and the edge set $\mathcal{E}$ represents pairwise comparisons. The outcomes of the matches can be captured by the adjacency matrix $\mathbf{A}$. A single edge $e \in \mathcal{E}$ from node v_i to node v_j, with edge weight $\mathbf{A}_{i,j} \geqslant 0$ that is not reciprocal, denotes that node v_i is stronger than node v_j by $\mathbf{A}_{i,j}$ For a reciprocal edge, $\mathbf{A}_{i,j}$ and $\mathbf{A}_{j,i}$ could be different; they could denote the (sum of) match scores for both competitors in matches between them, or the sum of absolute wins across different matches between them, where $\mathbf{A}_{i,j}$ is the sum of absolute wins for v_i. Recovering global rankings from pairwise comparisons amounts to assigning an integer R_i to each node $v_i \in \mathcal{V}$, denoting its position among competitors, where the lower the rank, the stronger the node is. To this end, many existing methods (including our proposed methods) first learn a real-valued ranking score r_i for v_i, where the higher the score, the stronger the node is. The scores are then ordered to provide the final integer ranking.

3.2. Motivation and Connection to SerialRank

The whole GNNRank framework, described in Algo. 2, uses steps from Algo. 1 for the lower-level optimization within the whole bi-level optimization pipeline; a diagram is provided in Fig. 1. Details are provided later in this section.

Indeed, we highlight an intrinsic connection to SerialRank (Fogel et al., 2014), which operates by first computing the Laplacian from a certain similarity matrix $\mathbf{S}' = \frac{1}{2}(n\mathbf{1}\mathbf{1}^\top + \mathbf{C}\mathbf{C}^\top)$, where $\mathbf{C}$ is the binary comparison matrix with $\mathbf{C}_{i,j} = \text{sign}(\mathbf{A}_{i,j} - \mathbf{A}_{j,i})$, and $\mathbf{A}$ the digraph adjacency matrix. The corresponding Fiedler vector of $\mathbf{S}'$ then serves as the final ranking estimate, after a global sign reconciliation. While often effective in practice, SerialRank is heavily dependent on the quality of the underlying similarity matrix $\mathbf{S}'$.

To address this issue, we introduce a parameterized GNN model that allows us to compute trainable measures of similarity that are useful for subsequent ranking. However, for training purposes, we of course need to somehow backpropagate gradients through the computation of a Fiedler vector to update the GNN parameters. Because it is generally difficult to directly pass gradients through eigenvector computations, we instead express the Fiedler vector as the solution of a constrained optimization problem. We then approximate the solution of this problem using proximal gradient steps, each of which are themselves differentiable with respect to the underlying optimization variables, and ultimately by the chain rule, the GNN parameters. Note that Algo. 1 can be viewed as a *differentiable* function that inputs $\mathbf{r}'$ and $\mathbf{L}$, and outputs $\mathbf{r}$. Therefore, the gradients can backpropagate *through* Algo. 1 into $\mathbf{L}$ and $\mathbf{r}'$, hence the similarity matrix $\mathbf{S}$ and the rest of the model parameters.

In broader contexts, this process is sometimes referred to as *unfolded optimization* (Gregor & LeCun, 2010), and is applicable in situations whereby a high-level loss function (in our case a ranking loss) is defined with respect to the minimization of some lower-level, parameter-dependent optimization problem (e.g., Fiedler vector computation) that has been unfolded across differentiable iterations/updates.

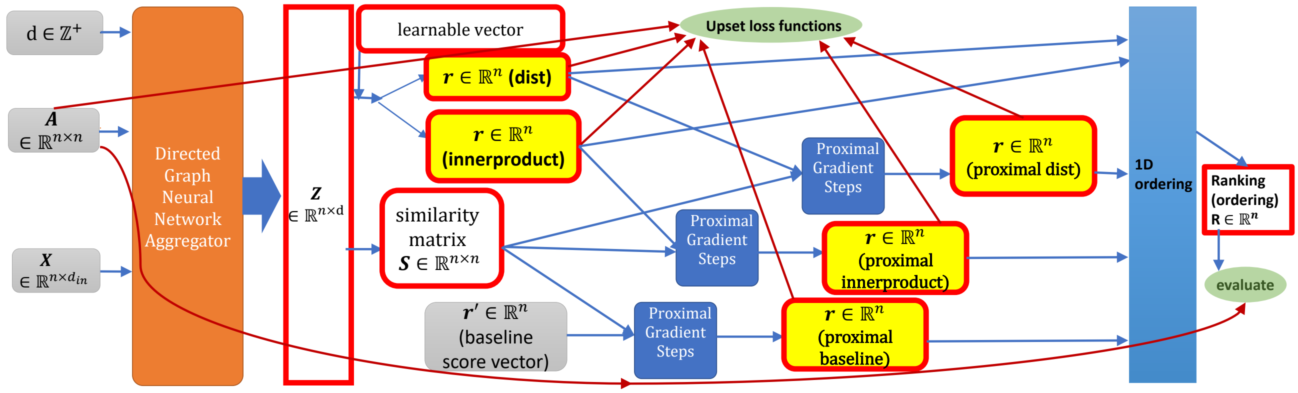

Figure 1. Overview of GNNRank based on directed graph neural networks and the proximal gradient steps corresponding to Algo. 2: starting from an adjacency matrix $\mathbf{A}$ which encodes pairwise comparisons, input feature matrix $\mathbf{X}$ and embedding dimension d, GNNRank first applies a directed graph neural network model to learn node embeddings $\mathbf{Z}$ for each competitor (node). Then it calculates the inner product or the similarity score with respect to a learnable vector to produce non-proximal outcomes for ranking scores ("innerproduct" or "dist"). Proximal variants start from a similarity matrix constructed from the learnable embeddings $\mathbf{Z}$, then utilize proximal gradient steps to output ranking scores. Depending on the initial guess score vector $\mathbf{r}'$, the proximal variants have names "proximal innerproduct", "proximal dist" or "proximal baseline". Ordering the scores in the score vector $\mathbf{r}$ induces the final ranking/ordering vector $\mathbf{R} \in \mathbb{R}^n$. The loss function is applied to a variant's output score vector $\mathbf{r}$, given the input adjacency matrix $\mathbf{A}$, while the final evaluation is based on $\mathbf{R}$ and $\mathbf{A}$. Red frames indicate trainable tensors/vectors/matrices. Grey squares correspond to fixed inputs.

Within our proposed framework, this process allows to combine the inductive bias of Fiedler-vector-based rankings with the flexibility of GNNs for modeling relations between entities. Also, SerialRank can be viewed as a special case of GNNRank; $\mathbf{S}'_{i,j}$ counts the number of competitors that beat both v_i and v_j, plus that of competitors beaten by both v_i and v_j, minus that of competitors beating one of v_i and v_j but beaten by the other, plus half the number of nodes. Information from common neighbors could be aggregated by directed GNNs, and a kernel such as a Gaussian RBF kernel applied to the digraph embeddings could approximate $\mathbf{S}'$.

3.3. Loss Functions and Objectives

Define the skew-symmetric matrix $\mathbf{M}' = \mathbf{A} - \mathbf{A}^\top$, and let t be the number of nonzero elements in $\mathbf{M}'$. For a vector $\mathbf{r} = [r_1, \ldots, r_n]^\top$ with *real-valued* ranking scores (often viewed as skill levels) as entries, a naive upset is defined as the fraction of relationships disagreeing with their expected sign, in the spirit of (d'Aspremont et al., 2021). Formally, let $\mathbf{1}$ be an all-one column vector, and define the matrix $\mathbf{T}' = \mathbf{r}\mathbf{1}^\top - \mathbf{1}\mathbf{r}^\top \in \mathbb{R}^{n\times n}$. Then we have $\mathbf{T}'_{i,j} = r_i - r_j, \forall i,j \in \{1, \cdots, n\}$. To not penalize entries where the initial pairwise rankings are not available, we only compare $\mathbf{T}'$ with $\mathbf{M}'$ at locations where $\mathbf{M}'$ has nonzero entries. A naive upset loss is defined as

$$\mathcal{L}_{\text{upset, naive}} = \sum_{i,j:\mathbf{M}'_{i,j}\neq 0} (\text{sign}(\mathbf{T}'_{i,j}) \neq \text{sign}(\mathbf{M}'_{i,j}))/t. \tag{1}$$

Being piecewise constant, $\mathcal{L}_{\text{upset, naive}}$ is not useful in gradient descent. To account for the difference in the scaling between the output scores and the input adjacency matrix, we define element-wise divisions $\mathbf{M} = \frac{\mathbf{A}-\mathbf{A}^\top}{\mathbf{A}+\mathbf{A}^\top}$, and $\mathbf{T} = \frac{\mathbf{r}\mathbf{1}^\top - \mathbf{1}\mathbf{r}^\top}{\mathbf{r}\mathbf{1}^\top + \mathbf{1}\mathbf{r}^\top} \in \mathbb{R}^{n\times n}$. Then $\mathbf{T}_{i,j} = \frac{r_i - r_j}{r_i + r_j}, \forall i,j \in \{1, \cdots, n\}$. Similarly, we only compare $\mathbf{T}$ with $\mathbf{M}$ at locations where $\mathbf{M}$ has nonzero entries.

With $\tilde{\mathbf{T}}$ where $\tilde{\mathbf{T}}_{i,j} = \mathbf{T}_{i,j}$ if $\mathbf{M}_{i,j} \neq 0$ and $\tilde{\mathbf{T}}_{i,j} = 0$ if $\mathbf{M}_{i,j} = 0$, the differentiable upset loss is then defined as

$$\mathcal{L}_{\text{upset, ratio}} = \left\| \tilde{\mathbf{T}} - \mathbf{M} \right\|_F^2 / t(\mathbf{M}), \tag{2}$$

where the subscript F means Frobenius norm, and $t(\mathbf{M})$ is the number of nonzero elements in $\mathbf{M}$.

Note that $\mathcal{L}_{\text{upset, ratio}}$ requires $r_i \geqslant 0$ for each r_i to represent the skill level. Hence we use transformations e.g. $r_i \leftarrow \text{sigmoid}(r_i)$ for general $r_i \in \mathbb{R}$, and $r_i \leftarrow \frac{r_i+1}{2}$ when we know $r_i \geqslant -1$, e.g. when the score $\mathbf{r} = (r_i)$ has unit norm.

Another choice is the margin loss $\mathcal{L}_{\text{upset, margin}}$, with a tunable nonnegative parameter $\epsilon \geqslant 0$ (default 0.01), defined as

$$\mathcal{L}_{\text{upset, margin}} = \sum_{i,j} (\mathbf{M}_{i,j} + |\mathbf{M}_{i,j}|) \cdot \text{ReLU}(r_j - r_i + \epsilon)/t(\mathbf{M}), \tag{3}$$

where ReLU is the Rectified Linear Unit.

The training loss used is either $\mathcal{L}_{\text{upset, ratio}}$, $\mathcal{L}_{\text{upset, margin}}$ or their sum, where the choice is set as a hyperparameter.

Evaluation. For evaluation, in addition to $\mathcal{L}_{\text{upset, naive}}$, we introduce another objective similar to $\mathcal{L}_{\text{upset, naive}}$ but penalizing predicted signs opposite to the actual signs more than predicted signs being zero, i.e. we distinguish predictions as ties or being opposite signs, which can also take integer rankings as input

$$\mathcal{L}_{\text{upset, simple}} = \left\|\text{sign}(\mathbf{T}') - \text{sign}(\mathbf{M}')\right)\right\|_F^2 /t, \quad (4)$$

where the sign function acts element-wise, and t is the number of nonzero elements in $\mathbf{M}'$. However, although $\mathcal{L}_{\text{upset, simple}}$ distinguishes ties and opposite signs, this loss can easily be made equal to one by assigning the same score to all nodes (i.e., making $\mathbf{r}$ a constant vector which corresponds to all ties), which means that whenever we achieve a value larger than one, the model performs even worse than trivial guess.

When ground-truth rankings are available, we use the Kendall tau (Brandenburg et al., 2013) values for evaluation, and select the model based on the lowest $\mathcal{L}_{\text{upset, simple}}$.

3.4. Obtaining Directed Graph Embeddings

For obtaining a directed graph embedding, any GNN method which can take into account directionality and output node embeddings could be applied, e.g. DIMPA by (He et al., 2021), the inception block model (IB) (Tong et al., 2020), and MagNet (Zhang et al., 2021). In our experiments, we employ DIMPA and IB, to aid in the ranking task. Denoting the final node embedding by $\mathbf{Z} \in \mathbb{R}^{n\times d}$, the embedding vector $\mathbf{z}_i$ for a node v_i is $\mathbf{z}_i = (\mathbf{Z})_{(i,:)} \in \mathbb{R}^d$, the i^{th} row of $\mathbf{Z}$.

3.5. Obtaining Final Scores and Rankings

To obtain the final ranking score, we unfold the calculation of a Fiedler vector for the graph constructed from our symmetric similarity matrix $\mathbf{S}$ with proximal gradient steps.

Obtaining the similarity matrix. From the high-dimensional embedding matrix $\mathbf{Z}$, we calculate the symmetric similarity matrix $\mathbf{S}$ with $\mathbf{S}_{i,j} = \exp(-|\mathbf{z}_j - \mathbf{z}_i|_2^2/(\sigma^2 d))$ where $\sigma \in \mathbb{R}$ is the same trainable parameter as in "dist". Denote by $\mathbf{D}$ the diagonal matrix with $\mathbf{D}_{i,i} = \sum_j \mathbf{S}_{i,j}$. We consider the unnormalized Laplacian $\mathbf{L} = \mathbf{D} - \mathbf{S}$, and apply proximal gradient to approximate a Fiedler vector of $\mathbf{S}$, which then serves as $\mathbf{r}$.

Transformation of the Optimization Problem. Computing a Fiedler vector of the similarity matrix $\mathbf{S}$ is equivalent to solving the optimization problem (Von Luxburg, 2007)

$$\min_{\mathbf{r}} \mathbf{r}^\top \mathbf{L}\mathbf{r} \text{ s.t. } \quad \|\mathbf{r}\|_2^2 = 1, \quad \mathbf{r}^\top \mathbf{1} = 0, \quad (5)$$

where $\mathbf{L}$ is the graph Laplacian matrix. We observe that the constraints describe an intersection of a unit sphere and a hyperplane. By rotating the problem and the constraints so that the hyperplane becomes cardinal, we can effectively fix one dimension to zero and solve

$$\min_{\mathbf{y}} \mathbf{y}^\top \mathbf{Q}\mathbf{L}\mathbf{Q}^\top \mathbf{y} \text{ s.t. } \quad \|\mathbf{y}\|_2^2 = 1, y_1 = 0. \quad (6)$$

Here $\mathbf{Q}$ is an orthogonal matrix. We choose a $\mathbf{Q}$ such that $\mathbf{Q}\mathbf{1} = \sqrt{n}\mathbf{e}_1$, where $\mathbf{e}_1 = \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^\top$. Let $\mathbf{y} = \mathbf{Q}\mathbf{r}$, and thus $\mathbf{r} = \mathbf{Q}^\top\mathbf{y}$. The problem becomes

$$\begin{aligned} &\min_{\mathbf{y}} \mathbf{y}^\top \mathbf{Q}\mathbf{L}\mathbf{Q}^\top \mathbf{y} \\ &\text{s.t.} \quad \|\mathbf{r}\|_2^2 = 1 \qquad \mathbf{r}^\top\mathbf{1} = \sqrt{n}\mathbf{y}^\top\mathbf{e}_1 = \sqrt{n}y_1 = 0. \end{aligned}$$

Since we fix $y_1 = 0$, this is equivalent to

$$\min_{\mathbf{y}' \in \mathbb{R}^{n-1}} \mathbf{y}'^\top \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{2:n,2:n} \mathbf{y}' \quad \text{s.t.} \quad \left\|\mathbf{y}'\right\|_2^2 = 1, \quad (7)$$

where $[\cdot]_{2:n,2:n}$ represents the matrix with its first row and first column removed. To illuminate this equivalence, since the constraint $\mathbf{r}^\top\mathbf{1} = 0$ is equivalent to $y_1 = 0$, we need to ensure that $y_1 = 0$ is maintained throughout. If we start with $\mathbf{y} \in \mathbb{R}^n$ where $y_1 = 0$, and let $\mathbf{y}' = [y_2, \ldots, y_n]^\top \in \mathbb{R}^{n-1}$ then

$$\begin{aligned} &\mathbf{y}^\top \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{1:n,1:n} \mathbf{y} = \sum_{1\leqslant i\leqslant n, 1\leqslant j\leqslant n} \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{i,j} y_i y_j \\ &= \sum_{2\leqslant i\leqslant n, 2\leqslant j\leqslant n} \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{i,j} y_i y_j = \mathbf{y}'^\top \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{2:n,2:n} \mathbf{y}'. \end{aligned}$$

One possible $\mathbf{Q}$, with details of the construction given in Appendix B, is the following upper Hessenberg matrix, which can be efficiently precomputed

$$\mathbf{Q}_{ij} = \begin{cases} \sqrt{\frac{1}{n}} & i = 1 \\ -\sqrt{\frac{n-i+1}{n-i+2}} & i \geqslant 2, j = i-1 \\ \sqrt{\frac{1}{(n-i+1)(n-i+2)}} & i \geqslant 2, j \geqslant i \\ 0 & \text{otherwise.} \end{cases}$$

Proximal Gradient Steps. To enforce a zero in the first entry of the initial guess, we zero-center the input score vector, then left multiply it by $\mathbf{Q}$, so this resulting vector has 0 in its first entry. We then remove the first entry of the resulting vector, and discard the first row and first column for the matrix $\mathbf{Q}\mathbf{L}\mathbf{Q}^\top$. The gradient of the objective $\mathbf{y}^\top\widetilde{\mathbf{L}}\mathbf{y}$ with respect to $\mathbf{y}$ is $(\widetilde{\mathbf{L}} + \widetilde{\mathbf{L}}^\top)\mathbf{y} = 2\widetilde{\mathbf{L}}\mathbf{y}$ since $\widetilde{\mathbf{L}}$ is symmetric. We set the number of proximal steps as $\Gamma = 5$, and initial learning rates inside proximal gradient steps $\alpha_\gamma = 1, \gamma = 1, \ldots, \Gamma$. Define the spherical projection operation $\mathcal{P}_{\mathcal{S}^{n-1}}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ by $\mathcal{P}_{\mathcal{S}^{n-1}}(\mathbf{x}) = \frac{\mathbf{x}}{\|\mathbf{x}\|_2}$ if $\mathbf{x} \neq \mathbf{0}$ and $\mathcal{P}_{\mathcal{S}^{n-1}}(\mathbf{0}) = \mathbf{e}_1$, where $\mathbf{0} = [0, \ldots, 0]^\top$. Algo. 1 details the proximal gradient steps, with proximal operator $\mathcal{P}_{\mathcal{S}^{n-2}}$, which guarantees descent of the optimization objective in (5) given suitable α's (see Thm. 3.1).

3.6. Initialization and Pretraining Considerations

Algo. 1 requires an initial guess $\mathbf{r}' \in \mathbb{R}^n$. To achieve this, we introduce two non-proximal variants, whose output score

Algorithm 1 Proximal Gradient Steps

Input: Initial score $\mathbf{r}' \in \mathbb{R}^n$, Laplacian $\mathbf{L} \in \mathbb{R}^{n\times n}$ and $\mathbf{Q} \in \mathbb{R}^{n\times n}$
Parameter: (Initial) learning rate set $\{\alpha_\gamma > 0\}_{\gamma=1}^{\Gamma}$ that could either be fixed or trainable (default: trainable).
Output: Updated score vector $\mathbf{r} = [r_1, \dots, r_n]^\top$.

Let $\mathbf{y} = \mathbf{r}' - \sum_{i=1}^{n} r_i'/n$;
$\mathbf{y} \leftarrow \mathbf{Q}'\mathbf{y} \in \mathbb{R}^{n-1}$ ($\mathbf{Q}'$ is $\mathbf{Q}$ with the first row removed);
$\mathbf{y} \leftarrow \mathcal{P}_{\mathcal{S}^{n-1}}(\mathbf{y})$ to have unit 2-norm;
$\tilde{\mathbf{L}} \leftarrow \left[\mathbf{Q}\mathbf{L}\mathbf{Q}^\top\right]_{2:n,2:n}$.
for $\gamma < \Gamma$ **do**
  $\mathbf{y} \leftarrow \mathbf{y} - \alpha_\gamma(2\tilde{\mathbf{L}})\mathbf{y}/n$;
  $\mathbf{y} \leftarrow \mathcal{P}_{\mathcal{S}^{n-2}}(\mathbf{y})$ to have unit 2-norm;
  $\gamma \leftarrow \gamma + 1$.
end for
$\mathbf{y} \leftarrow \text{CONCAT}(0, \mathbf{y}) \in \mathbb{R}^n$;
$\mathbf{r} = \mathbf{Q}^\top\mathbf{y}$
return $\mathbf{r}$;

Algorithm 2 GNNRank: Proposed Ranking Framework

Input: Digraph adjacency matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, node feature matrix $\mathbf{X}$, variant name var, (optional) initial guess from baseline $\mathbf{r}'$.
Parameter: Learnable vector $\mathbf{a} \in \mathbb{R}^n, b \in \mathbb{R}, \sigma \in \mathbb{R}$, GNN parameters, parameters from Algo. 1.
Output: Score vector $\mathbf{r}$.

$\mathbf{Z} \in \mathbb{R}^{n\times d} = \text{GNN}(\mathbf{A}, \mathbf{X})$;
if $var \in$ {"dist", "proximal dist"} **then**
  Compute $\mathbf{r} : r_i = \exp(-|\mathbf{a} - \mathbf{z}_i|_2^2/(\sigma^2 d))$;
else if $var \in$ {"innerproduct", "proximal innerproduct"} **then**
  Compute $\mathbf{r} : r_i = \text{sigmoid}(z_i \cdot \mathbf{a} + b)$;
end if
if $var \in$ {"proximal dist", "proximal innerproduct", "proximal baseline"} **then**
  Compute $\mathbf{S} : \mathbf{S}_{i,j} = \exp(-|\mathbf{z}_j - \mathbf{z}_i|_2^2/(\sigma^2 d))$;
  Compute Laplacian $\mathbf{L}$ and $\mathbf{Q}$ by Sec. 3.5;
  if $var ==$ {"proximal baseline"} **then**
    $\mathbf{r} = \mathbf{r}'$;
  end if
  $\mathbf{r} \leftarrow$ Proximal Gradient Steps($\mathbf{r}, \mathbf{L}, \mathbf{Q}$) from Algo. 1;
end if
return $\mathbf{r}$;

vector could serve as $\mathbf{r}'$. These non-proximal variants are also evaluated in our experiments.

•(1) *"innerproduct"* variant: With a trainable vector $\mathbf{a}$ that has its dimension equal to the embedding dimension, we obtain the scores by the inner product of $\mathbf{z}_i$ with $\mathbf{a}$, plus a trainable bias b, followed by a sigmoid layer to force positive score values: $r_i = \text{sigmoid}(\mathbf{z}_i \cdot \mathbf{a} + b)$.

•(2) *"dist"* variant: $r_i = \exp(-|\mathbf{a} - \mathbf{z}_i|_2^2/(\sigma^2 d))$ where $\mathbf{a} \in \mathbb{R}^d, \sigma \in \mathbb{R}$ are trainable parameters. This variant applies a trainable Gaussian RBF kernel to describe the scores.

For each non-proximal variant, the corresponding proximal variant is called •(3) *"proximal dist"* or •(4) *"proximal innerproduct"*. We can also adopt the initial guess from a certain existing baseline's output, with variant name •(5) *"proximal baseline"*.

For a reasonable similarity matrix to start with, we apply some pretraining. One option is to train the ranking model with a non-proximal variant, "dist" or "innerproduct", for the early training epochs. As the proximal variants are inspired by unfolding the Fiedler vector calculation introduced in SerialRank (Fogel et al., 2014), another option is to add a term to the loss function in early training epochs that compares the similarity matrix constructed by $\mathbf{Z}$ using a GNN with the normalized version (divided by the max entry so that the maximum is 1) of $\mathbf{S}'$, the similarity matrix from SerialRank. The corresponding additional term added to the loss function $\mathcal{L}_{\text{upset, ratio}}$ from Eq. (2) or $\mathcal{L}_{\text{upset, margin}}$ from Eq. (3), or their sum, is $\mathcal{L}_{\text{similarity}} = n^{-2}\|\mathbf{S} - \mathbf{S}'/\max(\{\mathbf{S}'_{i,j}\})\|_F^2$.

3.7. Convergence Analysis

An analysis of the convergence of our proximal gradient steps is provided in Appendix C, along with additional theoretical and practical considerations. We summarize our main result below; the proof is available in Appendix C.

Theorem 3.1. *Let $\{\alpha_\gamma > 0\}_{\gamma=1}^{\Gamma}$ in Algo. 1 be fixed (equal to α) and let ρ be the Fiedler eigenvalue of $\mathbf{S}$. Denote a Fiedler eigenvector by $\mathbf{r}^*$. Assume that $\mathbf{r}^*$ is a strict local minimizer of problem (5). If $0 < \alpha < \frac{1}{4(n-1)}$, then with our definition of $\mathbf{S}$, Algo. 1 converges locally uniformly to $\mathbf{r}^*$.*

4. Experiments

Implementation details are provided in Appendix A. Experiments were conducted on a compute node with 8 Nvidia Tesla T4, 96 Intel Xeon Platinum 8259CL CPUs @ 2.50GHz and 378GB RAM.

4.1. Data sets and Protocol

Real-World Data. We consider 10 real-world data sets (78 digraphs constructed in total). Digraphs based on pairwise comparisons are constructed as follows in most cases: for each match, an edge is added if the match result is not a tie, with the edge weight the difference between scores (absolute wins, that is, the absolute difference of the scores), and ties are treated the same as no match between the pair of players.

When raw data with individual match scores on both competitors are available, we in addition construct a *"finer"* version which takes ties into account, as follows: for each match, we have a reciprocal edge with potentially different weights on the two directions. If there are multiple matches, weights are added. To distinguish zero (a tie) from no game, for our methods, we add a small value (here 0.1) to every existing edge.

Real-world data sets, detailed in Appendix D, include

- NCAA College Basketball[1] (used to construct the networks *Basketball* and *Basketball finer*) are the outcomes of US College basketball matches from seasons 1985-2014. We construct two separate digraphs for each season, a regular version and a "finer" version introduced in the last paragraph.
- England Football Premier League[2] (*Football* and *Football finer*) are Premier League football match results for six seasons, from 2009 to 2014, between 20 teams. As in NCAA College Basketball, we construct two separate digraphs for each season.
- The Animal Dominance Network (*Animal*) (Hobson & DeDeo, 2015) describes the number of net aggressive wins of 21 captive monk parakeets.
- Microsoft Halo 2 Tournament on Head-to-Head Games (*HeadToHead*) collects game outcomes during the Beta testing period for the Xbox game Halo 2[3].
- Faculty Hiring Networks (*Faculty: Business*, *Faculty: CS* and *Faculty: History*) (Clauset et al., 2015) contains three North American academic hiring networks tracking the flow of academics between universities.
- Lead-Lag Relationships on Stocks (*Finance*) (Bennett et al., 2021) contains lead-lag relationships on 1315 stocks from 2001-2019. An edge (i, j) in the digraph encodes the t-value of the coefficient in the regression of the daily returns of stock i on the lag-1 (previous day) returns of stock j.

Table 1 gives the number of nodes (n), the number of directed edges ($|\mathcal{E}|$), the number of reciprocal edges ($|\mathcal{E}^r|$) (self-loops are counted once and for $u \neq v$, a reciprocal edge $u \to v, v \to u$ is counted twice) and their percentage among all edges, for the real-world networks, illustrating the variability in network size and density (defined as $|\mathcal{E}|/[n(n-1)]$). When input features are unavailable, we stack the real and imaginary parts of the top K eigenvectors of $(\mathbf{A} - \mathbf{A}^\top) \cdot i$ in line with the protocol in (Cucuringu et al., 2020) for GNNs. We report the embedding dimension d.

Synthetic Data We perform experiments on graphs with $n = 350$ nodes for Erdős-Rényi Outlier (ERO) models as in (d'Aspremont et al., 2021), with edge density $p \in \{0.05, 1\}$, noise level $\eta \in \{0, 0.1, \ldots, 0.8\}$ (corresponding to γ in (d'Aspremont et al., 2021)) and style "uniform" or "gamma" depending on the distribution from which the ground-truth scores are generated. [4] We fix $K = 5, d = 16$ for the ERO.

[1] https://www.ncaa.com/sports/basketball-men/d1

[2] https://www.premierleague.com/

[3] Credits for using the Halo 2 Beta data set are given to Microsoft Research Ltd. and Bungie.

[4] This synthetic graph is an ER graph with tunable noise level, a test case which is informative without being too unrealistic.

Table 1. Summary statistics for the real-world networks.

Data	n	$\lvert\mathcal{E}\rvert$	density	$\lvert\mathcal{E}^r\rvert$	$\frac{\lvert\mathcal{E}^r\rvert}{\lvert\mathcal{E}\rvert}$(%)	K	d
HeadToHead	602	5010	1.38e-02	464	9.26	48	32
Finance	1315	1729225	1.00e+00	1729225	100	20	64
Animal	21	193	4.60e-01	64	33.16	3	8
Faculty:Business	113	1787	1.41e-01	0	0.00	5	16
Faculty:CS	206	1407	3.33e-02	0	0.00	9	16
Faculty:History	145	1204	5.77e-02	0	0.00	12	16
Football(avg)	20	201	5.29e-01	71	32.17	9	8
Basketball(avg)	316	3506	3.51e-02	986	28.57	20	16
Football finer (avg)	20	367	9.65e-01	367	100	9	8
Basketball finer (avg)	316	6139	6.12e-02	6139	100	20	16

4.2. Main Experimental Results

In our numerical experiments, we compare against **11 baselines**, where results are averaged over 10 runs: •Eigenvector Centrality (Eig.Cent.) (Bonacich, 1987), •PageRank (Page et al., 1998), •Rank Centrality (Rank-Cent.) (Negahban et al., 2017), •Minimum Violation Rank (MVR) (Ali et al., 1986), •SerialRank (Fogel et al., 2014), •SyncRank (Cucuringu, 2016), •SVD_NRS and SVD_RS by (d'Aspremont et al., 2021), •Bradley-Terry-Luce (BTL) model (Firth, 2005), •David's Score (DavidScore) (David, 1987), and •SpringRank (De Bacco et al., 2018). Models are selected based on the lowest $\mathcal{L}_{\text{upset, simple}}$ obtained without label supervision, where non-proximal results (for "dist" and "innerproduct" variants) are listed in the "GNNRank-N" column and proximal methods (for "proximal dist", "proximal innerproduct" and "proximal baseline" variants) listed with "GNNRank-P", in all tables. We report the best-performing variant for each data set within GNNRank-N and GNNRank-P, respectively. Performance with respect to each of the GNNRank variants, for each individual input digraph, and on different objectives, are given in Appendix E. Table 2 compares our two groups of methods against baselines on 10 real-world data sets, where basketball data sets are averaged over 30 seasons, and football ones are averaged over 6 seasons. Our best-performing variant of the proximal method also outperforms its inspiration SerialRank (Fogel et al., 2014) on all real-world data sets by including more trainable parameters. Our proximal method achieves state-of-the-art performance when using a typically good baseline, such as SyncRank, as an initial guess to the score vector before applying proximal gradient steps.

Table 3 shows Kendall tau selected from the lowest $\mathcal{L}_{\text{upset, naive}}$ on synthetic models. For some dense digraphs, SerialRank (which motivated our proximal gradient steps) attains leading performance, while for some other cases it fails. GNNRank-P outperforms across all synthetic models shown here. Full results are in Appendix E.2.

We conclude that both non-proximal and proximal methods can achieve leading performance on real-world data sets, while on the synthetic models listed here, the best method in GNNRank-P performs much better than the best method in GNNRank-N. Performance results for each of the variants are provided in Appendix E.3; the individual variants also attain comparable and often superior performance compared to the baselines.

We observe across all data sets that our proximal methods: •(1) can improve on existing baseline methods when using them as initial guesses, and never perform significantly worse than the corresponding baseline, hence they can be used to enhance existing methods; •(2) do not rely on baseline methods for an initial guess but can instead use GNNRank-N outcomes, such as "proximal dist" and "proximal innerproduct"; •(3) can outperform SerialRank by unfolding its Fiedler vector calculations with a trainable similarity matrix and proximal gradient steps.

4.3. Discussion

Ablation Study. Table 4 shows results on varying the current choices: •1) For GNNRank-N methods: forcing the loss to be the sum $\mathcal{L}_{\text{upset, simple}} + \mathcal{L}_{\text{upset, margin}}$, $\mathcal{L}_{\text{upset, simple}}$ only, or $\mathcal{L}_{\text{upset, margin}}$, respectively; •2) for GNNRank-P methods: in addition to the variants for GNNRank-N methods, removing pretraining, fixing all α_γ in Algo. 1, and changing the number of Fiedler proximal steps from default 5 to 3 or 7.

It is shown that for GNNRank-N methods, using $\mathcal{L}_{\text{upset, margin}}$ usually harms performance. For GNNRank-P methods, using $\mathcal{L}_{\text{upset, margin}}$ in addition to $\mathcal{L}_{\text{upset, ratio}}$ usually boosts performance. Pretraining generally leads to better performance, so does making $\{\alpha_\gamma\}_{\gamma=1}^{\Gamma}$ trainable. The number of proximal gradient steps does not need to be large probably due to fast convergence, so we use 5 throughout.Full comparison tables are in Appendix E.4 including results on $\mathcal{L}_{\text{upset, naive}}$, with similar conclusions.

In addition, note that essentially we could use any neural network method to obtain the node embeddings, yet digraph GNNs are natural to be employed given the input data structure. To validate the benefit of using a digraph GNN, we adopt a two-layer Multilayer perceptron to obtain node embeddings, but obtained on average 2% worse $\mathcal{L}_{upset,simple}$ for both the non-proximal and proximal variants across all real-world data sets, respectively.

Inductive Learning. We observe that our proximal methods, if trained only once and then applied to similar data sets, still perform comparably to multiply trained analogs. This can save training time and validates the inductive learning ability for our framework. To this end, Appendix E.5 shows results on the performance of the "IB proximal baseline" variant, trained with "emb baseline" on the *Basketball finer* data set. On average, directly applying gives $\mathcal{L}_{\text{upset, simple}} = 0.75 \pm 0.02$ and $\mathcal{L}_{\text{upset, naive}} = 0.19 \pm 0.01$ while training specifically for the season gives $\mathcal{L}_{\text{upset, simple}} = 0.74 \pm 0.00$ and $\mathcal{L}_{\text{upset, naive}} = 0.19 \pm 0.00$.

Variants and Hyperparameters. The results in Tables 2 and 3 are selected within either non-proximal or proximal categories, depending on whether they have proximal gradient steps within the architecture. They show the lowest reported evaluation metric (except for Kendall tau, when we select variants based on the lowest $\mathcal{L}_{\text{upset, naive}}$) for all variants within the group. Appendix G gives the variant selected based on minimizing either $\mathcal{L}_{\text{upset, simple}}$ or $\mathcal{L}_{\text{upset, ratio}}$ for non-proximal and proximal groups, respectively. We find that each variant has its scenarios where it shows competitive or even outstanding performance; that "dist" seems to outperform "innerproduct" within non-proximal methods; and that "proximal baseline" is usually the best among proximal methods, with SyncRank output as initial guess, pretrained with a SerialRank similarity matrix. This shows that our proximal method, initialized with a good baseline, e.g. Sync-Rank, and pretrained with information from SerialRank, can boost the corresponding baseline method by using a learnable similarity matrix.

Boosting Baselines. Appendix F shows improvements on $\mathcal{L}_{upset,simple}$ and $\mathcal{L}_{upset,naive}$ by "proximal baseline" when setting a certain baseline as $\mathbf{r}'$. Across all data sets, "proximal baseline" improves the most (by 1.02 and 0.24, respectively) with SyncRank as initial guess, while the average improvement for SpringRank, SerialRank, BTL, Eig.Cent., PageRank and SVD_NRS are 0.07, 0.82, 0.22, 0.19, 0.21, and 0.12, respectively, for $\mathcal{L}_{upset,simple}$, and for 0.00, 0.18, 0.04, 0.03, 0.03, and 0.01, respectively, for $\mathcal{L}_{upset,naive}$.

5. Conclusion and Outlook

We have proposed a general framework based on directed graph neural networks to recover global rankings from pairwise comparisons. Future directions include learning a more powerful model to work for different input digraphs, minimizing upsets under some constraints, training with some supervision of ground-truth rankings, and exploring the interplay with low-rank matrix completion. Incorporating side information, in the form of node level covariates, and comparing to the, currently rather limited, existing literature on ranking with covariates, is another interesting direction.

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Table 2. Performance on $\mathcal{L}_{\text{upset, simple}}$ (top half) and $\mathcal{L}_{\text{upset, naive}}$ (bottom half), averaged over 10 runs with one standard deviation. "avg" for time series first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best is marked in **bold red** while the second best is in underline blue. When MVR does not generate results after one week, we fill in "NAN".

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	MVR	GNNRank-N	GNNRank-P
HeadToHead	1.00±0.00	1.94±0.00	2.01±0.00	1.12±0.01	1.16±0.00	1.47±0.00	1.36±0.00	2.00±0.02	1.79±0.00	1.42±0.00	nan±nan	0.99±0.00	**0.96±0.00**
Finance	1.63±0.00	1.98±0.00	1.61±0.00	1.78±0.01	1.63±0.00	1.74±0.00	1.75±0.00	1.88±0.00	1.64±0.00	1.64±0.00	nan±nan	**1.00±0.00**	**1.00±0.00**
Animal	0.50±0.00	1.62±0.24	1.98±0.48	0.45±0.00	0.33±0.00	0.55±0.00	0.63±0.00	1.96±0.00	1.03±0.00	0.53±0.00	2.02±0.32	0.41±0.09	**0.25±0.00**
Faculty: Business	0.41±0.00	0.83±0.00	1.19±0.00	0.41±0.01	0.49±0.00	0.49±0.00	0.49±0.00	2.01±0.03	0.68±0.00	0.46±0.00	0.78±0.05	0.38±0.01	**0.36±0.00**
Faculty: CS	0.33±0.00	0.98±0.10	1.40±0.00	0.34±0.01	0.61±0.00	0.51±0.00	0.44±0.00	1.99±0.27	0.93±0.00	0.58±0.00	0.87±0.09	0.33±0.03	**0.32±0.00**
Faculty: History	0.32±0.00	0.57±0.00	2.16±0.80	0.30±0.01	0.57±0.00	0.40±0.00	0.37±0.00	2.13±0.30	0.95±0.00	0.38±0.00	0.84±0.17	**0.28±0.01**	0.30±0.01
Basketball (avg)	0.78±0.00	1.72±0.00	1.98±0.00	0.91±0.03	0.79±0.00	0.88±0.00	0.88±0.00	1.95±0.00	0.99±0.00	0.89±0.00	nan±nan	0.80±0.00	**0.73±0.00**
Basketball finer (avg)	0.81±0.00	1.73±0.01	1.96±0.00	1.39±0.02	0.85±0.00	1.19±0.00	1.15±0.00	1.97±0.00	1.00±0.00	0.90±0.00	nan±nan	0.84±0.00	**0.74±0.00**
Football (avg)	0.91±0.00	1.63±0.12	1.20±0.00	0.94±0.02	0.94±0.00	1.07±0.00	1.08±0.00	1.78±0.03	1.00±0.00	0.90±0.00	1.72±0.09	0.81±0.03	**0.78±0.02**
Football finer (avg)	0.98±0.00	1.68±0.03	1.16±0.00	1.01±0.02	0.93±0.00	1.13±0.00	1.21±0.00	1.91±0.01	1.00±0.00	0.90±0.00	2.06±0.04	0.89±0.06	**0.82±0.01**
HeadToHead	0.25±0.00	0.48±0.00	0.50±0.00	0.28±0.00	0.29±0.00	0.37±0.00	0.34±0.00	0.50±0.01	0.45±0.00	0.36±0.00	nan±nan	0.27±0.00	**0.24±0.00**
Finance	0.41±0.00	0.50±0.00	**0.40±0.00**	0.45±0.00	0.41±0.00	0.44±0.00	0.44±0.00	0.47±0.00	0.41±0.00	0.41±0.00	nan±nan	0.41±0.00	**0.40±0.00**
Animal	0.13±0.00	0.40±0.06	0.58±0.11	0.11±0.00	0.08±0.00	0.14±0.00	0.16±0.00	0.49±0.00	0.26±0.00	0.13±0.00	0.50±0.08	0.10±0.02	**0.06±0.00**
Faculty: Business	0.10±0.00	0.21±0.00	0.30±0.00	0.10±0.00	0.12±0.00	0.12±0.00	0.12±0.00	0.50±0.01	0.17±0.00	0.12±0.00	0.19±0.01	0.10±0.00	**0.09±0.00**
Faculty: CS	**0.08±0.00**	0.24±0.02	0.35±0.00	**0.08±0.00**	0.15±0.00	0.13±0.00	0.11±0.00	0.50±0.07	0.23±0.00	0.15±0.00	0.22±0.02	**0.08±0.01**	**0.08±0.00**
Faculty: History	0.08±0.00	0.14±0.00	0.54±0.20	0.08±0.00	0.15±0.00	0.10±0.00	0.09±0.00	0.53±0.08	0.24±0.00	0.10±0.00	0.21±0.04	**0.07±0.00**	**0.07±0.00**
Basketball (avg)	0.20±0.00	0.43±0.00	0.49±0.00	0.23±0.01	0.20±0.00	0.22±0.00	0.22±0.00	0.49±0.00	0.25±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball finer (avg)	0.20±0.00	0.43±0.00	0.49±0.00	0.35±0.00	0.21±0.00	0.30±0.00	0.29±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Football (avg)	0.23±0.00	0.41±0.03	0.30±0.00	0.23±0.01	0.24±0.00	0.27±0.00	0.27±0.00	0.44±0.01	0.25±0.00	0.22±0.00	0.43±0.02	0.22±0.01	**0.21±0.00**
Football finer (avg)	0.25±0.00	0.42±0.01	0.29±0.00	0.25±0.01	0.23±0.00	0.28±0.00	0.30±0.00	0.48±0.00	0.25±0.00	0.22±0.00	0.51±0.01	0.24±0.01	**0.21±0.00**

Table 3. Performance on Kendall Tau based on the lowest $\mathcal{L}_{\text{upset, naive}}$ on ERO models, averaged over 10 runs with one standard deviation. "avg" for time series first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best is marked in **bold red** while the second best is in underline blue. As MVR does not generate results after one week, we leave it out here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	GNNRank-N	GNNRank-P
ERO(p=0.05, style=uniform,η=0.1)	0.75±0.00	0.04±0.00	0.03±0.00	0.70±0.01	0.77±0.00	0.56±0.00	0.58±0.00	0.01±0.05	0.74±0.00	0.77±0.00	0.76±0.01	**0.79±0.01**
ERO(p=0.05, style=gamma,η=0.2)	0.61±0.00	0.01±0.00	-0.01±0.00	0.61±0.00	0.74±0.00	0.52±0.00	0.51±0.00	-0.01±0.01	0.45±0.00	0.64±0.00	0.52±0.01	**0.77±0.00**
ERO(p=0.05, style=uniform,η=0.3)	0.61±0.00	0.05±0.00	0.01±0.00	0.59±0.01	0.68±0.00	0.44±0.00	0.41±0.00	0.05±0.00	0.60±0.00	0.62±0.00	0.62±0.00	**0.70±0.02**
ERO(p=0.05, style=gamma,η=0.4)	0.51±0.00	0.08±0.00	-0.00±0.00	0.52±0.00	0.65±0.00	0.43±0.00	0.43±0.00	0.09±0.01	0.23±0.00	0.44±0.00	0.38±0.08	**0.66±0.01**
ERO(p=1, style=uniform,η=0.5)	0.85±0.00	0.07±0.00	**0.92±0.00**	0.81±0.03	0.91±0.00	0.80±0.00	0.73±0.00	0.24±0.00	0.89±0.00	0.87±0.00	0.90±0.01	**0.92±0.00**
ERO(p=1, style=gamma,η=0.6)	0.72±0.00	0.09±0.00	**0.89±0.00**	0.67±0.01	0.88±0.00	0.65±0.00	0.64±0.00	0.05±0.02	0.74±0.00	0.73±0.00	0.77±0.00	**0.89±0.00**

Table 4. $\mathcal{L}_{\text{upset, simple}}$ comparison for different variants on real-world data, averaged over 10 runs, and plus/minus one standard deviation. The best for each group (GNNRank-N or GNNRank-P) is marked in **bold red** while the second best is in underline blue.

Methods	GNNRank-N			GNNRank-P						
Data/Variant	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	no pretrain	$\{\alpha_\gamma\}_{\gamma=1}^{\Gamma}$ not trainable	$\Gamma=3$	$\Gamma=7$
Animal	0.43±0.06	0.59±0.08	**0.41±0.09**	**0.25±0.00**	**0.25±0.00**	**0.25±0.01**	**0.25±0.00**	**0.25±0.00**	**0.25±0.00**	**0.25±0.00**
Faculty: Business	0.40±0.02	0.49±0.16	**0.38±0.01**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**
Faculty: CS	0.35±0.01	0.36±0.01	**0.33±0.03**	**0.32±0.00**	**0.32±0.00**	**0.32±0.00**	0.33±0.00	**0.32±0.00**	**0.32±0.00**	**0.32±0.00**
Faculty: History	**0.28±0.01**	0.31±0.01	**0.28±0.01**	**0.30±0.01**	**0.30±0.01**	**0.30±0.02**	**0.30±0.01**	**0.30±0.01**	**0.30±0.01**	**0.30±0.01**
Football (avg)	**0.82±0.01**	0.84±0.03	**0.82±0.05**	0.78±0.02	0.78±0.01	0.79±0.01	0.79±0.02	0.79±0.02	**0.77±0.01**	0.78±0.02
Football finer (avg)	**0.90±0.01**	0.97±0.06	0.91±0.07	**0.82±0.01**	0.84±0.01	**0.82±0.01**	0.84±0.02	**0.82±0.00**	**0.82±0.01**	**0.82±0.01**

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A. Implementation Details

A.1. Setup

We use all data for training for at most 1000 epochs, and stop early if the loss does not decrease for 200 epochs. For proximal variants, we use 50 epochs for pretraining. We use Adam (Kingma & Ba, 2014) and Stochastic Gradient Descend (SGD) as the optimizers and ℓ_2 regularization with weight decay $5 \cdot 10^{-4}$ to avoid overfitting. We use as learning rate 0.01 throughout for GNNRank-N methods as well as pretraining with Adam, and 10 times that of pretraining learning rate for GNNRank-P methods with SGD. We run a grid search on hyperparameters.

For real-world data sets, we conduct 10 repeated runs, while for synthetic data, we generate 5 synthetic networks under the same setting, each with 2 repeated runs.

Note that we do not evaluate our method by the loss which is used to devise the method, as that would not be fair; instead we employ $\mathcal{L}_{upset,naive}$ or $\mathcal{L}_{upset,simple}$ which are never used in training. Thus, the comparison is fair. When ground truths are given, we follow (d'Aspremont et al., 2021) to use Kendall Tau values for comparison.

A.2. Codes, Data and Hardware

To fully reproduce our results, codes and preprocessed data are available at `https://github.com/SherylHYX/GNNRank`. Experiments were conducted on a compute node with 8 Nvidia Tesla T4, 96 Intel Xeon Platinum 8259CL CPUs @ 2.50GHz and 378GB RAM. Most experiments can be completed within a week, including all variants, hyperparameter searches and ablation studies, except for those on MVR.

The data sets considered here are relatively small and the same applies to GNNRank's competitive papers. However, even a network with 100 nodes has more than 10^{157} possible rankings; this large scale task needs efficient methods. Although each individual task does not require much resource (often $<$ 5min/run), for the paper we have 78 real-world and 18 synthetic data sets, each requires 10 runs for each of the 36 (proximal innerproduct, proximal dist) + 126 (proximal baseline) + 12 (non-proximal) = 174 variants, thus 167,040 runs, plus an extra ablation study.

B. Details on Finding Q for Proximal Gradient Steps

There are infinitely many choices of valid $\mathbf{Q}$; here we construct one of the special $\mathbf{Q}$'s for which we can compute $\mathbf{QL}$ efficiently. Since $\mathbf{Q}^\top \mathbf{e}_1 = \mathbf{1}/\sqrt{n}$, we can construct $\mathbf{R} = \mathbf{Q}^\top$ as a series of matrix multiplication of $(n-1)$

rotation matrices on two adjacent axes

$$\mathbf{R} = \mathbf{R}_{n-1}\mathbf{R}_{n-2}\cdots\mathbf{R}_2\mathbf{R}_1$$

where $\mathbf{R}_k$ is defined by:

$$[\mathbf{R}_k]_{ij} = \begin{cases} \sqrt{\frac{1}{n-k}} & i=k, j=k \text{ or } i=k+1, j=k+1 \\ -\sqrt{\frac{n-k-1}{n-k}} & i=k, j=k+1 \\ \sqrt{\frac{n-k-1}{n-k}} & i=k+1, j=k \\ 1 & i=j, \text{and } i,j \neq k \text{ or } k+1 \\ 0 & \text{otherwise.} \end{cases}$$

To explain this construction, starting from $\mathbf{e}_1 = \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^\top$, we carry out a rotation on the first and second axis to make the first element $\sqrt{1/n}$. The rotation matrix is

$$\mathbf{R}_1 = \begin{bmatrix} \sqrt{\frac{1}{n}} & -\sqrt{\frac{n-1}{n}} & & & \cdots & \\ \sqrt{\frac{n-1}{n}} & \sqrt{\frac{1}{n}} & & & \cdots & \\ & & 1 & & \cdots & \\ & & & 1 & \cdots & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & & & & & 1 \end{bmatrix}$$

We observe that

$$\mathbf{R}_1\mathbf{e}_1 = \begin{bmatrix}\sqrt{\frac{1}{n}} & \sqrt{\frac{n-1}{n}} & 0 & 0 & \cdots & 0\end{bmatrix}^\top.$$

The second rotation occurs on the second and third axis to render $\mathbf{R}_2\mathbf{R}_1\mathbf{e}_1 = \begin{bmatrix}\sqrt{\frac{1}{n}} & \sqrt{\frac{1}{n}} & \sqrt{\frac{n-2}{n}} & 0 & \cdots & 0\end{bmatrix}^\top$; this $\mathbf{R}_2$ is

$$\mathbf{R}_2 = \begin{bmatrix} 1 & & & & \cdots & \\ & \sqrt{\frac{1}{n-1}} & -\sqrt{\frac{n-2}{n-1}} & & \cdots & \\ & \sqrt{\frac{n-2}{n-1}} & \sqrt{\frac{1}{n-1}} & & \cdots & \\ & & & 1 & \cdots & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & & & & & 1 \end{bmatrix}$$

The general matrices R_k are obtained by continuing this construction.

The matrix $\mathbf{R}$ is

$$\begin{bmatrix} \sqrt{\frac{1}{n}} & -\sqrt{\frac{n-1}{n}} & & & \cdots & \\ \sqrt{\frac{1}{n}} & \sqrt{\frac{1}{n(n-1)}} & -\sqrt{\frac{n-2}{n-1}} & & \cdots & \\ \sqrt{\frac{1}{n}} & \sqrt{\frac{1}{n(n-1)}} & \sqrt{\frac{1}{(n-1)(n-2)}} & -\sqrt{\frac{n-3}{n-2}} & \cdots & \\ \sqrt{\frac{1}{n}} & \sqrt{\frac{1}{n(n-1)}} & \sqrt{\frac{1}{(n-1)(n-2)}} & \sqrt{\frac{1}{(n-2)(n-3)}} & \cdots & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sqrt{\frac{1}{n}} & \sqrt{\frac{1}{n(n-1)}} & \sqrt{\frac{1}{(n-1)(n-2)}} & \sqrt{\frac{1}{(n-2)(n-3)}} & \cdots & \sqrt{\frac{1}{2}} \end{bmatrix}$$

We then put $\mathbf{Q} = \mathbf{R}^\top$.

The resulting $\mathbf{Q}$ is the following upper Hessenberg matrix, which is independent of model parameters and can be efficiently precomputed

$$\mathbf{Q}_{ij} = \begin{cases} \sqrt{\frac{1}{n}} & i=1 \\ -\sqrt{\frac{n-i+1}{n-i+2}} & i \geqslant 2, j=i-1 \\ \sqrt{\frac{1}{(n-i+1)(n-i+2)}} & i \geqslant 2, j \geqslant i \\ 0 & \text{otherwise.} \end{cases}$$

The computation of $\mathbf{QL}$ takes $O(n^2)$ time since $\mathbf{Q}$ is a summation of (1) an upper-triangular matrix $\mathbf{U}$ with the same non-zero value on the same row, and (2) a shift matrix $\mathbf{V}$ (with negative subdiagonal). Then $\mathbf{UL}$ is essentially a cumulative sum from the bottom row to the top row, followed by a row-wise multiplication of the diagonal of $\mathbf{U}$, and $\mathbf{VL}$ is essentially a combination of row-wise multiplication and indexing.

C. Theoretical Analysis and Practical Considerations on Convergence of the Proximal Gradient Steps

First we prove a theorem useful for the main result.

Theorem C.1. *Let $\{\alpha_\gamma > 0\}_{\gamma=1}^{\Gamma}$ in Algo. 1 be fixed and equal to α. Let ρ be the Fiedler eigenvalue of $\mathbf{S}$, i.e., the second smallest eigenvalue of $\mathbf{L}$. Let $\mathbf{r}^*$ be a Fiedler eigenvector corresponding to ρ, let $\mathbf{y}^* = [\mathbf{Q}\mathbf{r}^*]_{2:n}$ and $\widetilde{L} = \left[\mathbf{QLQ}^\top\right]_{2:n,2:n}$. Let $\lambda_1 \geqslant \lambda_2 \cdots \geqslant \lambda_{n-1} = 0$ be the eigenvalues of $\mathbf{P}\widetilde{\mathbf{L}}\mathbf{P}$, where $\mathbf{P} = \mathbf{I} - \mathbf{y}^*\mathbf{y}^{*\top}$. Let $\rho_\alpha = \max_{1 \leqslant i \leqslant n-2} \frac{|1-2\alpha\lambda_i|}{1-2\alpha\rho}$. If $\mathbf{r}^*$ is a strict local minimum of problem (5), and if $\rho_\alpha < 1$ and $\alpha(\lambda_1 + \rho) < 1$, then Algo. 1 converges locally uniformly to $\mathbf{r}^*$.*

Proof. Our problem (7) is a special case of the problem (2) in (Vu et al., 2019) (henceforth (8) further below, of minimizing a quadratic over a sphere), where our fixed point $\mathbf{y}^*$ is determined by $\mathbf{f}_\alpha(\overline{\mathbf{x}}) = \mathcal{P}_{\mathcal{S}^{n-2}}(-\alpha(2\widetilde{\mathbf{L}}\overline{\mathbf{x}})) = \overline{x}$. Recall that the spherical projection operator $\mathcal{P}_{\mathcal{S}^{n-2}}(\cdot) : \mathbb{R}^{n-1} \to \mathbb{R}^{n-1}$ is defined to be $\mathcal{P}_{\mathcal{S}^{n-2}}(\mathbf{x}) = \frac{\mathbf{x}}{\|\mathbf{x}\|_2}$ if $\mathbf{x} \neq \mathbf{0}$, and $\mathcal{P}_{\mathcal{S}^{n-2}}(\mathbf{0}) = [1, 0, \ldots, 0]^\top$, where $\mathbf{0} = [0, \ldots, 0]^\top$. It is shown in Lemma 1 of (Vu et al., 2019) (henceforth Lemma C.3 further below) that $\mathbf{y}^*$ with $||\mathbf{y}^*|| = 1$ being a solution of (7) is equivalent to the existence of a constant $\rho(\overline{\mathbf{x}})$ such that

$$\widetilde{\mathbf{L}}\mathbf{y}^* = \rho(\mathbf{y}^*) \cdot \mathbf{y}^*,$$

rendering $\rho(\mathbf{y}^*)$ to be an eigenvalue of $\widetilde{\mathbf{L}}$. Theorem 1 in (Vu et al., 2019) (henceforth Theorem C.5 below) states that with an initial step size $\alpha > 0$, such that $2\alpha(\rho(\mathbf{y}^*) + \lambda_1) < 2$, as long as $\rho_\alpha = \max_{1\leqslant i\leqslant n-2} \frac{|1-2\alpha\lambda_i|}{1-2\alpha\gamma(\mathbf{y}^*)}) < 1$, the proximal gradients steps will converge to a local minimum.

Note that problem (5) and problem (7) are equivalent, and there is a one-to-one correspondence between $\mathbf{r}^*$ and $\mathbf{y}^*$, namely, $\mathbf{y}^* = [\mathbf{Q}\mathbf{r}^*]_{2:n} \in \mathbb{R}^{n-1}$, and $\mathbf{r}^* = \text{CONCAT}(0, [\mathbf{Q}^\top\mathbf{r}^*]) \in \mathbb{R}^n$. Hence, since $\mathbf{r}^*$ is a strict local minimum of problem (5), we have that $\mathbf{y}^*$ is a strict local minimum of problem (7).

By Lemma C.3 from (Vu et al., 2019), we have $\rho < \lambda_{n-2}$, since $\mathbf{y}^*$ is a strict local minimum. In particular, by Theorem C.5 in (Vu et al., 2019), when $\mathbf{y}^*$ is a strict local minimum to problem (8), if $2\alpha(\rho+\lambda_1) < 2$, i.e., $\alpha(\rho+\lambda_1) < 1$, there is a constant M such that

$$||\mathbf{y}^\gamma - \mathbf{y}^*|| \leqslant M||\mathbf{y}^0 - \mathbf{y}^*||(\rho_\alpha + o(1))^\gamma,$$

where $\mathbf{y}^0$ is the initial guess at proximal gradient step 0, and γ denotes the step number. As $\rho_\alpha < 1$ is assumed, convergence follows. □

For convergence, it suffices to ensure that $\rho_\alpha < 1$, i.e. to have $2\alpha\gamma(\overline{\mathbf{x}}) < 1$, $|1 - 2\alpha\lambda_1| < 1 - \alpha\gamma(\overline{\mathbf{x}})$ and $|1 - 2\alpha\lambda_{n-2}| < 1 - 2\alpha\gamma(\overline{\mathbf{x}})$. In particular, in our problem we take $\rho(\overline{\mathbf{x}}) = \rho$, which is also the smallest eigenvalue of $\widetilde{\mathbf{L}}$, so $\rho < \lambda_{n-2} \leqslant \lambda_1$. To apply Theorem C.1 in order to obtain a convergence guarantee in Algo. 1, as $\lambda_{n-2} > \rho$, (which is equivalent to $\mathbf{r}^*$ being a strict local minimum) it remains to use an α value such that $2\alpha\lambda_1 < 1$. Lemma C.4 of (Vu et al., 2019) also suggests that if $2\alpha\lambda_1 < 1$, then we indeed have $\mathbf{r}^*$ as a global minimizer to problem (5).

Finally we have the ingredients to prove the main result.

Theorem C.2. *Let $\{\alpha_\gamma > 0\}_{\gamma=1}^\Gamma$ in Algo. 1 be fixed (equal to α) and let ρ be the Fiedler eigenvalue of $\mathbf{S}$. Denote a Fiedler eigenvector by $\mathbf{r}^*$. Assume that $\mathbf{r}^*$ is a strict local minimizer of problem (5). If $0 < \alpha < \frac{1}{4(n-1)}$, then with our definition of the similarity matrix, Algo. 1 converges locally uniformly to $\mathbf{r}^*$.*

Proof. Via our similarity matrix construction process, entries in $\mathbf{S}$ are upper-bounded by 1, so the degree matrix $\mathbf{D}$ has entries bounded above by n, the number of nodes, and lower bounded by 0, and that $\mathbf{L}$ is positive semi-definite. We thus have that the eigenvalues of $\mathbf{L}$ are in $[0, (n-1) + (n-1) \times 1]$ by the Gershgorin disc theorem. Since $\mathbf{Q}$ is orthogonal, we have that eigenvalues of $\widetilde{\mathbf{L}}$ are also within the range $[0, 2(n-1)]$. To see this, suppose $\mathbf{x}$ is an eigenvector of $\mathbf{L}$ with eigenvalue λ, i.e., $\mathbf{L}\mathbf{x} = \lambda\mathbf{x}$, so $\mathbf{QLQ}\top(\mathbf{Qx}) = \mathbf{QL}(\mathbf{Q}\top\mathbf{Q})\mathbf{x} = \mathbf{QLx} = \mathbf{Q}(\mathbf{Lx}) = \mathbf{Q}\lambda\mathbf{x} = \lambda(\mathbf{Qx})$. Therefore, the values $\{\lambda_i\}_{i=1}^{n-2}$ in Theorem C.1 are bounded above by $2(n-1)$. Thus, we have that $\rho + \lambda_1 \leqslant 2(n-1) + 2(n-1) = 4(n-1)$.

For convergence, it suffices to ensure that $\rho_\alpha = \max_{1\leqslant i\leqslant n-2} \frac{|1-2\alpha\lambda_i|}{1-2\alpha\rho} < 1$, i.e. to have $2\alpha\rho < 1$, $|1 - 2\alpha\lambda_1| < 1 - 2\alpha\rho$, and $|1 - \alpha\lambda_{n-2}| < 1 - 2\alpha\rho$.

Thus, since we have $\lambda_{n-2} > \rho$ by Lemma C.4 from (Vu et al., 2019) and the proof of Theorem C.1, it remains to use an α value such that $2\alpha\lambda_1 < 1$, then $2\alpha\rho < 2\alpha\lambda_1 < 1$, and $0 < 1 - 2\alpha\lambda_1 = |1 - 2\alpha\lambda_1| \leqslant 1 - 2\alpha\lambda_{n-2} = |1 - 2\alpha\lambda_{n-2}| < 1 - 2\alpha\rho$. With $\alpha < \frac{1}{4(n-1)}$, we indeed have $2\alpha\lambda_1 < 2 \times \frac{1}{4(n-1)} \times 2(n-1) = 1$. □

Our current setting is to initialize all α values to be $\frac{1}{n-1}$. Note that within Algo. 1, the α values are fixed. Although not guaranteed to converge with the initial α, they could be adapted by our outer training loop, i.e. by optimization over the ranking loss function. (Vu et al., 2019) also suggests the value of the optimal step size to be $\frac{2}{\lambda_1+\lambda_{n-2}}$, which during training our method could in principle reproduce if the optimal step size is actually important for making accurate final rankings.

We remark that real-world weighted networks have the corresponding Fiedler eigenvalue of multiplicity 1; this multiplicity is larger than 1, only for special classes of graphs that exhibit certain symmetries. Our pragmatic assumption that the Fiedler eigenvalue has multiplicity 1, which thus means $\mathbf{r}^*$ being a strict local minimum of problem 5, is not a very restrictive one.

For completeness, the remainder of this section recalls the setup and main results from (Vu et al., 2019). The problem considered therein is that of minimizing a quadratic form over the sphere

$$\min_{\mathbf{x}} \mathbf{x}^\top\mathbf{B}\mathbf{x} - \mathbf{b}^\top\mathbf{x} \quad \text{s.t.} \quad \|\mathbf{x}\|_2^2 = 1 \tag{8}$$

with the matrix $\mathbf{B} \in \mathbb{R}^{m\times m}$ assumed symmetric but not positive semi-definite, hence a non-convex objective function. Upon considering the corresponding Lagrangian function, with ν denoting the Lagrange multiplier, the authors prove the following results, which also constitute building blocks in our analysis. The eigenvalues of $\mathbf{A}$ are given by $\lambda_{min}(\mathbf{B}) \leqslant \lambda_{m-1} \leqslant \ldots \leqslant \lambda_1 \leqslant \lambda_{max}(\mathbf{B})$.

Lemma C.3 (Lemma 1 from (Vu et al., 2019))**.** *(Stationary conditions). The vector $\mathbf{x}_*$ is a stationary point of problem (8) if and only if $\mathbf{x}_* \in \mathcal{S}^{m-1} = \{\mathbf{x} \in \mathbb{R}^m : \|\mathbf{x}\| = 1\}$ and there exists a constant $\nu(\mathbf{x}_*)$ such that $\mathbf{r}_* = A\mathbf{x}_* - b = \nu(\mathbf{x}_*) \cdot \mathbf{x}_*$.*

Lemma C.4 (Lemma 2 from (Vu et al., 2019))**.** *A stationary point $\mathbf{x}_*$ of problem (8) is a strict local minimum if and only if $\nu(\mathbf{x}_*) < \lambda_{m-1}(\mathbf{x}_*)$. Furthermore, $\mathbf{x}_*$ is a **global minimizer** of problem (8) if and only if $\nu(\mathbf{x}_*) \leqslant \lambda_{min}(\mathbf{B})$.*

The following result establishes the result that Projected Gradient Descent (PGD) converges to a local minimum at an asymptotic linear rate ρ_β.

Theorem C.5 (Theorem 1 from (Vu et al., 2019))**.** *The vector $\mathbf{x}_*$ is a strict local minimum of problem (8), i.e. $\nu < \lambda_{m-1}$, if and only if there exists $\beta > 0$ such that Algorithm 1 [Projected Gradient Descent (PGD)] with step size α converges locally uniformly to $\mathbf{x}_*$. Furthermore, for any step size $\beta > 0$ such that $\beta(\lambda_1 + \nu) < 2$, the sequence $\{\mathbf{x}^{(t)}\}$ satisfies*

$$\|\mathbf{x}^{(t)} - \mathbf{x}_*\| \leqslant M\|\mathbf{x}^{(0)} - \mathbf{x}_*\| \left(\rho_\alpha + o(1)\right)^t ,$$

for some constant $M > 0$ and $\rho_\beta = \max_{1 \leqslant i \leqslant m-1} \frac{|1-\beta\lambda_i|}{1-\beta\nu}$.

D. Detailed Summary Statistics

Table 5 gives the number of nodes (n), the number of directed edges ($|\mathcal{E}|$), the number of reciprocal edges ($|\mathcal{E}^r|$) (self-loops are counted once and for $u \neq v$, a reciprocal edge $u \to v, v \to u$ is counted twice) as well as their percentage among all edges, for the real-world networks, illustrating the variability in network size and density (defined as $|\mathcal{E}|/[n(n-1)]$). As we do not have input features available, we use the eigengap of the Hermitian matrix $(\mathbf{A} - \mathbf{A}^\top) \cdot i$, where $\mathbf{A}$ is the adjacency matrix and i the imaginary unit, introduced in (Cucuringu et al., 2020), to determine the value K, which is assumed to be the number of clusters if we are solving a clustering problem. We then stack the real and imaginary parts of the top K eigenvectors of $(\mathbf{A} - \mathbf{A}^\top) \cdot i$ into $2K$-dimensional input features for GNNs. We also report the embedding dimension d used in the experiments, for each of the data sets.

E. Full Result Tables

E.1. Results on Individual Digraphs and $\mathcal{L}_{\text{upset, ratio}}$

Tables 6, 7 and 8 provide detailed comparison on both $\mathcal{L}_{\text{upset, simple}}$, $\mathcal{L}_{\text{upset, naive}}$ and $\mathcal{L}_{\text{upset, ratio}}$ for all time-series match data, where Table 8 additionally provides $\mathcal{L}_{\text{upset, ratio}}$ results on other real-world data sets. Again the best-performing variants of the non-proximal and of the proximal method are reported. The proximal method does not perform as well as the non-proximal method in terms of $\mathcal{L}_{\text{upset, ratio}}$, perhaps partially due to the fact that outputs from SerialRank (Fogel et al., 2014) do not align well with skill level but rather relate to relative ordering. It is also natural for GNNRank-N methods that are not motivated by SerialRank to perform well on this metric as $\mathcal{L}_{\text{upset, ratio}}$ is in the training loss function. Therefore, here we only present the results on $\mathcal{L}_{\text{upset, ratio}}$ but do not draw conclusions on which method outperforms others.

Table 5. Summary statistics for the real-world networks.

Data	n	$\lvert\mathcal{E}\rvert$	density	$\lvert\mathcal{E}^r\rvert$	$\frac{\lvert\mathcal{E}^r\rvert}{\lvert\mathcal{E}\rvert}$(%)	K	d
HeadToHead	602	5010	1.38e-02	464	9.26	48	32
Basketball (1985)	282	2904	3.66e-02	998	34.37	20	16
Basketball finer (1985)	282	4814	6.08e-02	4814	100.00	20	16
Basketball (1986)	283	2937	3.68e-02	1014	34.53	20	16
Basketball finer (1986)	283	4862	6.09e-02	4862	100.00	20	16
Basketball (1987)	290	3045	3.63e-02	1012	33.23	20	16
Basketball finer (1987)	290	5088	6.07e-02	5088	100.00	20	16
Basketball (1988)	290	3099	3.70e-02	1034	33.37	20	16
Basketball finer (1988)	290	5170	6.17e-02	5170	100.00	20	16
Basketball (1989)	293	3162	3.70e-02	1014	32.07	20	16
Basketball finer (1989)	293	5318	6.22e-02	5318	100.00	20	16
Basketball (1990)	292	3192	3.76e-02	1042	32.64	20	16
Basketball finer (1990)	292	5350	6.30e-02	5350	100.00	20	16
Basketball (1991)	295	3218	3.71e-02	1018	31.63	20	16
Basketball finer (1991)	295	5420	6.25e-02	5420	100.00	20	16
Basketball (1992)	298	3238	3.66e-02	1036	32.00	20	16
Basketball finer (1992)	298	5444	6.15e-02	5444	100.00	20	16
Basketball (1993)	298	3088	3.49e-02	1024	33.16	20	16
Basketball finer (1993)	298	5160	5.83e-02	5160	100.00	20	16
Basketball (1994)	301	3144	3.48e-02	1044	33.21	20	16
Basketball finer (1994)	301	5252	5.82e-02	5252	100.00	20	16
Basketball (1995)	302	3182	3.50e-02	1034	32.50	20	16
Basketball finer (1995)	302	5336	5.87e-02	5336	100.00	20	16
Basketball (1996)	305	3256	3.51e-02	1026	31.51	20	16
Basketball finer (1996)	305	5498	5.93e-02	5498	100.00	20	16
Basketball (1997)	305	3333	3.59e-02	1044	31.32	20	16
Basketball finer (1997)	305	5628	6.07e-02	5628	100.00	20	16
Basketball (1998)	306	3321	3.56e-02	966	29.09	20	16
Basketball finer (1998)	306	5684	6.09e-02	5684	100.00	20	16
Basketball (1999)	310	3385	3.53e-02	998	29.48	20	16
Basketball finer (1999)	310	5788	6.04e-02	5788	100.00	20	16
Basketball (2000)	318	3475	3.45e-02	852	24.52	20	16
Basketball finer (2000)	318	6274	6.22e-02	6274	100.00	20	16
Basketball (2001)	318	3405	3.38e-02	904	26.55	20	16
Basketball finer (2001)	318	6116	6.07e-02	6116	100.00	20	16
Basketball (2002)	321	3505	3.41e-02	976	27.85	20	16
Basketball finer (2002)	321	6192	6.03e-02	6192	100.00	20	16
Basketball (2003)	327	3560	3.34e-02	954	26.80	20	16
Basketball finer (2003)	327	6356	5.96e-02	6356	100.00	20	16
Basketball (2004)	326	3527	3.33e-02	952	26.99	20	16
Basketball finer (2004)	326	6316	5.96e-02	6316	100.00	20	16
Basketball (2005)	330	3622	3.34e-02	946	26.12	20	16
Basketball finer (2005)	330	6476	5.96e-02	6476	100.00	20	16
Basketball (2006)	334	3695	3.32e-02	924	25.01	20	16
Basketball finer (2006)	334	6680	6.01e-02	6680	100.00	20	16
Basketball (2007)	336	3974	3.53e-02	976	24.56	20	16
Basketball finer (2007)	336	7186	6.38e-02	7186	100.00	20	16
Basketball (2008)	342	4051	3.47e-02	972	23.99	20	16
Basketball finer (2008)	342	7386	6.33e-02	7386	100.00	20	16
Basketball (2009)	347	4155	3.46e-02	1046	25.17	20	16
Basketball finer (2009)	347	7478	6.23e-02	7478	100.00	20	16
Basketball (2010)	347	4133	3.44e-02	916	22.16	20	16
Basketball finer (2010)	347	7538	6.28e-02	7538	100.00	20	16
Basketball (2011)	345	4086	3.44e-02	950	23.25	20	16
Basketball finer (2011)	345	7504	6.32e-02	7504	100.00	20	16
Basketball (2012)	345	4126	3.48e-02	950	23.02	20	16
Basketball finer (2012)	345	7580	6.39e-02	7580	100.00	20	16
Basketball (2013)	347	4153	3.46e-02	960	23.12	20	16
Basketball finer (2013)	347	7616	6.34e-02	7616	100.00	20	16
Basketball (2014)	351	4196	3.42e-02	1008	24.02	20	16
Basketball finer (2014)	351	7650	6.23e-02	7650	100.00	20	16
Football (2009)	20	215	5.66e-01	78	36.28	9	8
Football finer (2009)	20	380	1.00e+00	380	100.00	9	8
Football (2010)	20	219	5.76e-01	86	39.27	9	8
Football finer (2010)	20	380	1.00e+00	380	100.00	9	8
Football (2011)	20	226	5.95e-01	92	40.71	9	8
Football finer (2011)	20	380	1.00e+00	380	100.00	9	8
Football (2012)	20	216	5.68e-01	86	39.81	9	8
Football finer (2012)	20	380	1.00e+00	380	100.00	9	8
Football (2013)	20	222	5.84e-01	82	36.94	9	8
Football finer (2013)	20	380	1.00e+00	380	100.00	9	8
Football (2014)	20	107	2.82e-01	0	0.00	9	8
Football finer (2014)	20	300	7.89e-01	300	100.00	9	8
Football(avg)	20	201	5.29e-01	71	32.17	9	8
Basketball(avg)	316	3506	3.51e-02	986	28.57	20	16
Football finer(avg)	20	367	9.65e-01	367	100	9	8
Basketball finer(avg)	316	6139	6.12e-02	6139	100	20	16
Animal	21	193	4.60e-01	64	33.16	3	8
Finance	1315	1729225	1.00e+00	1729225	100	20	64
Faculty:Business	113	1787	1.41e-01	0	0.00	5	16
Faculty:CS	206	1407	3.33e-02	0	0.00	9	16
Faculty:History	145	1204	5.77e-02	0	0.00	12	16

Table 6. Result table on $\mathcal{L}_{\text{upset, simple}}$ for individual directed graphs, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue. When MVR could not generate results after a week, we omit the results and fill in "NAN" here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	MVR	GNNRank-N	GNNRank-P
HeadToHead	1.00±0.00	1.94±0.00	2.01±0.00	1.12±0.01	1.16±0.00	1.47±0.00	1.36±0.00	2.00±0.02	1.79±0.00	1.42±0.00	nan±nan	0.99±0.00	**0.96±0.00**
Finance	1.63±0.00	1.98±0.00	1.61±0.00	1.78±0.01	1.63±0.00	1.74±0.00	1.75±0.00	1.88±0.00	1.64±0.00	1.64±0.00	nan±nan	**1.00±0.00**	**1.00±0.00**
Animal	0.50±0.00	1.62±0.24	1.98±0.48	0.45±0.00	0.33±0.00	0.55±0.00	0.63±0.00	1.96±0.00	1.03±0.00	0.53±0.00	2.02±0.32	0.41±0.09	**0.25±0.00**
Faculty: Business	0.41±0.00	0.83±0.00	1.19±0.00	0.41±0.01	0.49±0.00	0.49±0.00	0.49±0.00	2.01±0.03	0.68±0.00	0.46±0.00	0.78±0.05	0.38±0.01	**0.36±0.00**
Faculty: CS	0.33±0.00	0.98±0.10	1.40±0.00	0.34±0.01	0.61±0.00	0.51±0.00	0.44±0.00	1.99±0.27	0.93±0.00	0.58±0.00	0.87±0.09	0.33±0.03	**0.32±0.00**
Faculty: History	0.32±0.00	0.57±0.00	2.16±0.80	0.30±0.01	0.57±0.00	0.40±0.00	0.37±0.00	2.13±0.30	0.95±0.00	0.38±0.00	0.84±0.17	**0.28±0.01**	0.30±0.01
Basketball(1985)	0.76±0.00	1.62±0.00	2.11±0.00	0.93±0.08	0.75±0.00	0.89±0.00	0.91±0.00	1.92±0.00	1.00±0.00	0.86±0.00	nan±nan	0.77±0.01	**0.71±0.00**
Basketball(1986)	0.78±0.00	1.79±0.00	1.99±0.00	0.96±0.02	0.78±0.00	0.89±0.00	0.88±0.00	1.96±0.00	1.09±0.00	0.86±0.00	nan±nan	0.78±0.01	**0.69±0.00**
Basketball(1987)	0.82±0.00	1.79±0.00	1.88±0.00	0.99±0.02	0.85±0.00	0.99±0.00	0.90±0.00	1.94±0.00	0.99±0.00	0.91±0.00	nan±nan	0.84±0.01	**0.77±0.01**
Basketball(1988)	0.74±0.00	1.66±0.00	1.91±0.00	0.92±0.03	0.77±0.00	0.87±0.00	0.85±0.00	1.93±0.00	0.97±0.00	0.83±0.00	nan±nan	0.76±0.01	**0.70±0.00**
Basketball(1989)	0.75±0.00	1.74±0.00	1.86±0.00	0.94±0.02	0.77±0.00	0.82±0.00	0.85±0.00	1.87±0.00	0.98±0.00	0.90±0.00	nan±nan	0.78±0.02	**0.70±0.00**
Basketball(1990)	0.75±0.00	1.68±0.00	1.94±0.00	0.89±0.08	0.78±0.00	0.90±0.00	0.88±0.00	1.95±0.00	0.91±0.00	0.85±0.00	nan±nan	0.77±0.02	**0.70±0.00**
Basketball(1991)	0.80±0.00	1.80±0.00	2.03±0.00	0.92±0.07	0.77±0.00	0.86±0.00	0.84±0.00	1.95±0.00	1.00±0.00	0.90±0.00	nan±nan	0.78±0.01	**0.70±0.00**
Basketball(1992)	0.75±0.00	1.74±0.00	1.89±0.00	0.86±0.10	0.72±0.00	0.84±0.00	0.80±0.00	2.00±0.00	0.96±0.00	0.84±0.00	nan±nan	0.74±0.01	**0.67±0.00**
Basketball(1993)	0.75±0.00	1.68±0.00	2.04±0.00	0.88±0.06	0.74±0.00	0.83±0.00	0.85±0.00	1.93±0.00	0.99±0.00	0.86±0.00	nan±nan	0.76±0.02	**0.68±0.01**
Basketball(1994)	0.74±0.00	1.67±0.00	2.00±0.00	0.77±0.09	0.74±0.00	0.82±0.00	0.83±0.00	2.04±0.00	0.90±0.00	0.84±0.00	nan±nan	0.75±0.01	**0.69±0.00**
Basketball(1995)	0.77±0.00	1.78±0.00	1.88±0.00	0.84±0.10	0.78±0.00	0.83±0.00	0.87±0.00	2.02±0.00	0.94±0.00	0.86±0.00	nan±nan	0.78±0.01	**0.72±0.01**
Basketball(1996)	0.82±0.00	1.67±0.00	2.05±0.00	0.96±0.07	0.81±0.00	0.90±0.00	0.92±0.00	2.05±0.00	1.08±0.00	0.95±0.00	nan±nan	0.84±0.01	**0.77±0.01**
Basketball(1997)	0.81±0.00	1.72±0.00	1.95±0.00	0.97±0.02	0.83±0.00	0.91±0.00	0.90±0.00	1.89±0.00	0.96±0.00	0.92±0.00	nan±nan	0.83±0.01	**0.76±0.01**
Basketball(1998)	0.77±0.00	1.69±0.00	1.91±0.00	0.96±0.03	0.79±0.00	0.90±0.00	0.85±0.00	1.92±0.00	0.96±0.00	0.89±0.00	nan±nan	0.80±0.01	**0.74±0.01**
Basketball(1999)	0.83±0.00	1.56±0.00	2.02±0.00	0.97±0.06	0.81±0.00	0.93±0.00	0.94±0.00	1.90±0.00	1.16±0.00	0.95±0.00	nan±nan	0.84±0.01	**0.74±0.00**
Basketball(2000)	0.84±0.00	1.77±0.00	1.97±0.00	0.98±0.02	0.86±0.00	0.91±0.00	0.91±0.00	1.91±0.00	1.11±0.00	0.94±0.00	nan±nan	0.82±0.01	**0.77±0.00**
Basketball(2001)	0.75±0.00	1.73±0.00	2.03±0.00	0.91±0.07	0.78±0.00	0.88±0.00	0.84±0.00	2.00±0.00	1.05±0.00	0.94±0.00	nan±nan	0.77±0.01	**0.71±0.00**
Basketball(2002)	0.80±0.00	1.69±0.00	1.91±0.00	0.92±0.05	0.80±0.00	0.85±0.00	0.91±0.00	1.94±0.00	1.06±0.00	0.88±0.00	nan±nan	0.80±0.01	**0.73±0.00**
Basketball(2003)	0.81±0.00	1.85±0.00	2.05±0.00	0.95±0.07	0.80±0.00	0.94±0.00	0.94±0.00	1.93±0.00	0.99±0.00	0.94±0.00	nan±nan	0.83±0.01	**0.78±0.00**
Basketball(2004)	0.75±0.00	1.67±0.00	2.00±0.11	0.89±0.07	0.75±0.00	0.85±0.00	0.87±0.00	1.95±0.00	0.92±0.00	0.86±0.00	nan±nan	0.77±0.01	**0.69±0.00**
Basketball(2005)	0.78±0.00	1.78±0.01	1.98±0.05	0.98±0.02	0.81±0.00	0.85±0.00	0.91±0.00	1.87±0.00	1.05±0.00	0.92±0.00	nan±nan	0.81±0.02	**0.73±0.00**
Basketball(2006)	0.81±0.00	1.78±0.00	2.00±0.00	0.84±0.08	0.77±0.00	0.86±0.00	0.89±0.00	1.99±0.00	1.03±0.00	0.91±0.00	nan±nan	0.82±0.01	**0.74±0.00**
Basketball(2007)	0.79±0.00	1.87±0.00	1.94±0.08	0.94±0.01	0.84±0.00	0.89±0.00	0.89±0.00	1.94±0.00	0.92±0.00	0.89±0.00	nan±nan	0.83±0.01	**0.77±0.00**
Basketball(2008)	0.82±0.00	1.75±0.00	2.01±0.00	0.90±0.07	0.80±0.00	0.90±0.00	0.93±0.00	1.92±0.00	0.98±0.00	0.90±0.00	nan±nan	0.83±0.01	**0.78±0.00**
Basketball(2009)	0.78±0.00	1.74±0.00	2.02±0.00	0.94±0.02	0.76±0.00	0.85±0.00	0.86±0.00	1.89±0.00	0.95±0.00	0.88±0.00	nan±nan	0.78±0.01	**0.72±0.01**
Basketball(2010)	0.79±0.00	1.55±0.00	1.97±0.00	0.85±0.10	0.77±0.00	0.92±0.00	0.91±0.00	2.03±0.00	0.93±0.00	0.91±0.00	nan±nan	0.82±0.01	**0.71±0.00**
Basketball(2011)	0.81±0.00	1.63±0.00	1.96±0.00	0.85±0.08	0.79±0.00	0.88±0.00	0.89±0.00	2.03±0.00	0.95±0.00	0.87±0.00	nan±nan	0.80±0.01	**0.74±0.00**
Basketball(2012)	0.76±0.00	1.81±0.00	1.97±0.00	0.88±0.08	0.77±0.00	0.85±0.00	0.84±0.00	1.96±0.00	0.90±0.00	0.85±0.00	nan±nan	0.78±0.01	**0.69±0.00**
Basketball(2013)	0.80±0.00	1.71±0.00	1.99±0.05	0.95±0.06	0.80±0.00	0.90±0.00	0.87±0.00	1.95±0.00	1.00±0.00	0.90±0.00	nan±nan	0.81±0.01	**0.76±0.00**
Basketball(2014)	0.80±0.00	1.72±0.00	2.06±0.00	0.87±0.09	0.79±0.00	0.88±0.00	0.91±0.00	1.94±0.00	1.01±0.00	0.87±0.00	nan±nan	0.83±0.01	**0.75±0.00**
Basketball finer(1985)	0.76±0.00	1.63±0.00	1.96±0.10	1.46±0.05	0.83±0.00	1.18±0.00	1.16±0.00	1.97±0.00	1.00±0.00	0.87±0.00	nan±nan	0.79±0.01	**0.71±0.00**
Basketball finer(1986)	0.77±0.00	1.81±0.00	1.99±0.00	1.42±0.06	0.84±0.00	1.16±0.00	1.15±0.00	1.99±0.00	1.09±0.00	0.86±0.00	nan±nan	0.81±0.01	**0.69±0.00**
Basketball finer(1987)	0.82±0.00	1.79±0.00	1.87±0.00	1.41±0.06	0.89±0.00	1.17±0.00	1.21±0.00	1.95±0.00	0.99±0.00	0.91±0.00	nan±nan	0.86±0.02	**0.77±0.01**
Basketball finer(1988)	0.78±0.00	1.79±0.00	1.90±0.00	1.43±0.10	0.84±0.00	1.23±0.00	1.19±0.00	1.97±0.00	0.97±0.00	0.83±0.00	nan±nan	0.80±0.01	**0.70±0.01**
Basketball finer(1989)	0.77±0.00	1.67±0.00	1.86±0.00	1.43±0.05	0.83±0.00	1.13±0.00	1.14±0.00	1.94±0.00	0.99±0.00	0.90±0.00	nan±nan	0.82±0.02	**0.70±0.00**
Basketball finer(1990)	0.79±0.00	1.67±0.00	1.93±0.00	1.45±0.05	0.82±0.00	1.28±0.00	1.17±0.00	1.98±0.00	0.91±0.00	0.84±0.00	nan±nan	0.80±0.02	**0.71±0.00**
Basketball finer(1991)	0.81±0.00	1.83±0.00	2.03±0.00	1.36±0.06	0.83±0.00	1.38±0.00	1.31±0.00	1.97±0.00	0.99±0.00	0.89±0.00	nan±nan	0.83±0.01	**0.71±0.00**
Basketball finer(1992)	0.73±0.00	1.72±0.00	1.88±0.00	1.33±0.06	0.77±0.00	1.26±0.00	1.21±0.00	1.87±0.00	0.95±0.00	0.84±0.00	nan±nan	0.75±0.02	**0.67±0.00**
Basketball finer(1993)	0.75±0.00	1.66±0.00	2.03±0.00	1.35±0.05	0.78±0.00	1.18±0.00	1.10±0.00	1.97±0.00	0.98±0.00	0.86±0.00	nan±nan	0.80±0.01	**0.68±0.01**
Basketball finer(1994)	0.74±0.00	1.69±0.00	2.01±0.00	1.35±0.08	0.78±0.00	1.23±0.00	1.10±0.00	1.94±0.00	0.90±0.00	0.83±0.00	nan±nan	0.77±0.01	**0.67±0.00**
Basketball finer(1995)	0.79±0.00	1.78±0.00	1.89±0.00	1.35±0.06	0.83±0.00	1.19±0.00	1.13±0.00	1.92±0.01	0.95±0.00	0.87±0.00	nan±nan	0.78±0.01	**0.72±0.01**
Basketball finer(1996)	0.81±0.00	1.67±0.00	1.95±0.00	1.44±0.06	0.88±0.00	1.22±0.00	1.20±0.00	1.94±0.00	1.08±0.00	0.95±0.00	nan±nan	0.86±0.01	**0.77±0.00**
Basketball finer(1997)	0.83±0.00	1.77±0.00	1.94±0.00	1.40±0.04	0.86±0.00	1.19±0.00	1.16±0.00	2.05±0.00	0.96±0.00	0.92±0.00	nan±nan	0.85±0.01	**0.75±0.01**
Basketball finer(1998)	0.78±0.00	1.70±0.00	1.92±0.00	1.36±0.07	0.83±0.00	1.14±0.00	1.13±0.00	1.91±0.00	0.97±0.00	0.90±0.00	nan±nan	0.82±0.02	**0.73±0.01**
Basketball finer(1999)	0.81±0.00	1.64±0.00	2.02±0.00	1.38±0.07	0.86±0.00	1.17±0.00	1.11±0.00	1.99±0.00	1.17±0.00	0.94±0.00	nan±nan	0.86±0.01	**0.73±0.00**
Basketball finer(2000)	0.84±0.00	1.75±0.00	1.97±0.00	1.39±0.05	0.90±0.00	1.26±0.00	1.18±0.00	1.92±0.00	1.12±0.00	0.95±0.00	nan±nan	0.88±0.01	**0.78±0.00**
Basketball finer(2001)	0.81±0.00	1.69±0.00	2.06±0.00	1.41±0.06	0.86±0.00	1.25±0.00	1.18±0.00	2.03±0.00	1.08±0.00	0.97±0.00	nan±nan	0.86±0.01	**0.73±0.00**
Basketball finer(2002)	0.87±0.00	1.75±0.00	1.86±0.00	1.43±0.08	0.89±0.00	1.20±0.00	1.13±0.00	2.03±0.00	1.07±0.00	0.92±0.00	nan±nan	0.83±0.01	**0.77±0.01**
Basketball finer(2003)	0.87±0.00	1.78±0.00	1.98±0.07	1.46±0.09	0.91±0.00	1.18±0.00	1.14±0.00	2.00±0.00	1.02±0.00	0.95±0.00	nan±nan	0.88±0.02	**0.78±0.00**
Basketball finer(2004)	0.77±0.00	1.71±0.06	1.87±0.00	1.41±0.06	0.80±0.00	1.17±0.00	1.13±0.00	1.98±0.02	0.95±0.00	0.88±0.00	nan±nan	0.84±0.01	**0.71±0.01**
Basketball finer(2005)	0.84±0.00	1.82±0.00	1.93±0.00	1.38±0.06	0.88±0.00	1.14±0.00	1.09±0.00	2.00±0.00	1.08±0.00	0.95±0.00	nan±nan	0.86±0.01	**0.75±0.01**
Basketball finer(2006)	0.86±0.00	1.76±0.00	1.97±0.00	1.40±0.07	0.85±0.00	1.21±0.00	1.11±0.00	1.96±0.00	1.06±0.00	0.94±0.00	nan±nan	0.88±0.02	**0.76±0.00**
Basketball finer(2007)	0.86±0.00	1.85±0.00	1.97±0.14	1.39±0.05	0.93±0.00	1.15±0.00	1.09±0.00	1.92±0.00	0.95±0.00	0.93±0.00	nan±nan	0.87±0.01	**0.80±0.01**
Basketball finer(2008)	0.85±0.00	1.72±0.00	1.98±0.00	1.36±0.08	0.88±0.00	1.20±0.00	1.13±0.00	1.96±0.00	0.99±0.00	0.91±0.00	nan±nan	0.86±0.02	**0.78±0.00**
Basketball finer(2009)	0.84±0.00	1.71±0.00	2.00±0.04	1.37±0.04	0.90±0.00	1.17±0.00	1.12±0.00	2.06±0.00	0.99±0.00	0.91±0.00	nan±nan	0.85±0.01	**0.75±0.00**
Basketball finer(2010)	0.82±0.00	1.68±0.00	1.98±0.00	1.34±0.06	0.84±0.00	1.10±0.00	1.13±0.00	1.97±0.00	0.94±0.00	0.92±0.00	nan±nan	0.85±0.01	**0.75±0.00**
Basketball finer(2011)	0.85±0.00	1.65±0.00	1.96±0.00	1.36±0.06	0.87±0.00	1.12±0.00	1.14±0.00	2.01±0.00	0.96±0.00	0.89±0.00	nan±nan	0.87±0.01	**0.77±0.00**
Basketball finer(2012)	0.80±0.00	1.68±0.00	1.97±0.00	1.37±0.07	0.83±0.00	1.17±0.00	1.10±0.00	1.91±0.00	0.93±0.00	0.89±0.00	nan±nan	0.83±0.01	**0.74±0.01**
Basketball finer(2013)	0.83±0.00	1.73±0.00	2.06±0.00	1.37±0.07	0.88±0.00	1.15±0.00	1.13±0.00	1.97±0.00	1.01±0.00	0.92±0.00	nan±nan	0.85±0.00	**0.78±0.01**
Basketball finer(2014)	0.84±0.00	1.77±0.19	2.07±0.00	1.42±0.09	0.86±0.00	1.18±0.00	1.13±0.00	1.97±0.00	1.02±0.00	0.88±0.00	nan±nan	0.87±0.01	**0.78±0.00**
Football(2009)	**0.61±0.00**	1.44±0.44	0.93±0.00	0.67±0.07	0.80±0.00	0.93±0.00	1.05±0.00	1.80±0.09	0.80±0.00	0.66±0.00	1.55±0.16	0.75±0.06	**0.61±0.00**
Football(2010)	1.19±0.00	1.81±0.15	1.54±0.00	1.18±0.04	1.29±0.00	1.34±0.00	1.37±0.00	1.84±0.13	1.24±0.00	1.17±0.00	1.94±0.20	**0.90±0.02**	0.95±0.05
Football(2011)	**0.80±0.00**	1.57±0.37	0.95±0.00	0.86±0.04	0.87±0.00	0.99±0.00	1.02±0.00	1.60±0.00	0.92±0.00	0.85±0.00	1.80±0.19	0.83±0.03	**0.80±0.01**
Football(2012)	0.97±0.00	1.50±0.03	1.41±0.00	0.99±0.05	0.89±0.00	0.94±0.00	0.99±0.00	1.65±0.00	1.02±0.00	0.86±0.00	1.73±0.18	**0.75±0.03**	0.80±0.08
Football(2013)	0.75±0.00	1.64±0.10	0.78±0.00	0.77±0.05	0.68±0.00	0.92±0.00	0.87±0.00	1.93±0.09	0.90±0.00	0.75±0.00	1.59±0.11	0.64±0.04	**0.56±0.00**
Football(2014)	1.12±0.00	1.81±0.18	1.57±0.00	1.16±0.01	1.09±0.00	1.27±0.00	1.20±0.00	1.83±0.11	1.12±0.00	1.08±0.00	1.72±0.20	**0.96±0.12**	**0.96±0.07**
Football finer(2009)	0.71±0.00	1.78±0.00	0.93±0.00	0.79±0.10	0.71±0.00	1.05±0.00	1.10±0.00	1.89±0.05	0.80±0.00	0.66±0.00	1.91±0.17	0.76±0.18	**0.65±0.02**
Football finer(2010)	1.29±0.00	1.63±0.02	1.44±0.00	1.31±0.09	1.29±0.00	1.37±0.00	1.49±0.00	1.96±0.00	1.24±0.00	1.17±0.00	2.11±0.01	1.00±0.01	**0.99±0.02**
Football finer(2011)	0.87±0.00	1.58±0.02	1.07±0.00	0.93±0.04	0.92±0.00	1.02±0.00	1.14±0.00	1.87±0.00	0.92±0.00	0.85±0.00	2.14±0.02	0.92±0.16	**0.84±0.02**
Football finer(2012)	0.97±0.00	1.76±0.18	1.10±0.00	0.99±0.05	0.92±0.00	1.10±0.00	1.20±0.00	2.03±0.06	1.02±0.00	**0.86±0.00**	2.15±0.01	0.93±0.07	**0.86±0.03**
Football finer(2013)	0.90±0.00	1.57±0.01	0.87±0.00	0.85±0.06	0.73±0.00	0.99±0.00	1.07±0.00	2.01±0.00	0.90±0.00	0.75±0.00	1.94±0.14	0.73±0.19	**0.57±0.01**
Football finer(2014)	1.16±0.00	1.74±0.04	1.57±0.00	1.19±0.04	1.05±0.00	1.23±0.00	1.23±0.00	1.68±0.00	1.12±0.00	1.08±0.00	2.09±0.17	**1.00±0.00**	**1.00±0.03**

E.2. Full Results on Synthetic Data

Table 9 shows Kandall Tau values and corresponding $\mathcal{L}_{\text{upset, naive}}$ on ERO models, extending the results of Table 3 in the main text.

Table 7. Result table on $\mathcal{L}_{\text{upset, naive}}$ for individual directed graphs, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue. When MVR could not generate results after a week, we omit the results and fill in "NAN" here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	MVR	GNNRank-N	GNNRank-P
HeadToHead	0.25±0.00	0.48±0.00	0.50±0.00	0.28±0.00	0.29±0.00	0.37±0.00	0.34±0.00	0.50±0.01	0.45±0.00	0.36±0.00	nan±nan	0.27±0.00	**0.24±0.00**
Finance	0.41±0.00	0.50±0.00	**0.40±0.00**	0.45±0.00	0.41±0.00	0.44±0.00	0.44±0.00	0.47±0.00	0.41±0.00	0.41±0.00	nan±nan	0.41±0.00	**0.40±0.00**
Animal	0.13±0.00	0.40±0.06	0.58±0.11	0.11±0.00	0.08±0.00	0.14±0.00	0.16±0.00	0.49±0.00	0.26±0.00	0.13±0.00	0.50±0.08	0.10±0.02	**0.06±0.00**
Faculty: Business	0.10±0.00	0.21±0.00	0.30±0.00	0.10±0.00	0.12±0.00	0.12±0.00	0.12±0.00	0.50±0.01	0.17±0.00	0.12±0.00	0.19±0.01	0.10±0.00	**0.09±0.00**
Faculty: CS	**0.08±0.00**	0.24±0.02	0.35±0.00	**0.08±0.00**	0.15±0.00	0.13±0.00	0.11±0.00	0.50±0.07	0.23±0.00	0.15±0.00	0.22±0.02	**0.08±0.01**	**0.08±0.00**
Faculty: History	0.08±0.00	0.14±0.00	0.54±0.20	0.08±0.00	0.15±0.00	0.10±0.00	0.09±0.00	0.53±0.08	0.24±0.00	0.10±0.00	0.21±0.04	**0.07±0.00**	**0.07±0.00**
Basketball(1985)	0.19±0.00	0.40±0.00	0.53±0.00	0.23±0.02	0.19±0.00	0.22±0.00	0.23±0.00	0.48±0.00	0.25±0.00	0.22±0.00	nan±nan	0.19±0.00	**0.18±0.00**
Basketball(1986)	0.19±0.00	0.45±0.00	0.50±0.00	0.24±0.01	0.19±0.00	0.22±0.00	0.22±0.00	0.49±0.00	0.27±0.00	0.21±0.00	nan±nan	0.20±0.00	**0.17±0.00**
Basketball(1987)	0.21±0.00	0.45±0.00	0.47±0.00	0.25±0.00	0.21±0.00	0.25±0.00	0.22±0.00	0.48±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(1988)	0.19±0.00	0.42±0.00	0.48±0.00	0.23±0.01	0.19±0.00	0.22±0.00	0.21±0.00	0.48±0.00	0.24±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(1989)	0.19±0.00	0.44±0.00	0.47±0.00	0.24±0.01	0.19±0.00	0.20±0.00	0.21±0.00	0.47±0.00	0.25±0.00	0.23±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(1990)	0.19±0.00	0.42±0.00	0.48±0.00	0.22±0.02	0.20±0.00	0.23±0.00	0.22±0.00	0.49±0.00	0.23±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(1991)	0.20±0.00	0.45±0.00	0.51±0.00	0.23±0.02	0.19±0.00	0.22±0.00	0.21±0.00	0.49±0.00	0.25±0.00	0.22±0.00	nan±nan	0.19±0.00	**0.18±0.00**
Basketball(1992)	0.19±0.00	0.43±0.00	0.47±0.00	0.21±0.02	0.18±0.00	0.21±0.00	0.20±0.00	0.50±0.00	0.24±0.00	0.21±0.00	nan±nan	0.18±0.00	**0.17±0.00**
Basketball(1993)	0.19±0.00	0.42±0.00	0.51±0.00	0.22±0.02	0.19±0.00	0.21±0.00	0.21±0.00	0.48±0.00	0.25±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(1994)	0.19±0.00	0.42±0.00	0.50±0.00	0.19±0.02	0.18±0.00	0.20±0.00	0.21±0.00	0.51±0.00	0.23±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(1995)	0.19±0.00	0.44±0.00	0.47±0.00	0.21±0.02	0.19±0.00	0.21±0.00	0.22±0.00	0.50±0.00	0.24±0.00	0.22±0.00	nan±nan	0.19±0.00	**0.18±0.00**
Basketball(1996)	0.21±0.00	0.42±0.00	0.51±0.00	0.24±0.02	0.20±0.00	0.23±0.00	0.23±0.00	0.51±0.00	0.27±0.00	0.24±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(1997)	0.20±0.00	0.43±0.00	0.49±0.00	0.24±0.01	0.21±0.00	0.23±0.00	0.23±0.00	0.47±0.00	0.24±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(1998)	0.19±0.00	0.42±0.00	0.48±0.00	0.24±0.01	0.20±0.00	0.22±0.00	0.21±0.00	0.48±0.00	0.24±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball(1999)	0.21±0.00	0.39±0.00	0.50±0.00	0.24±0.02	0.20±0.00	0.23±0.00	0.24±0.00	0.47±0.00	0.29±0.00	0.24±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball(2000)	0.21±0.00	0.44±0.00	0.49±0.00	0.25±0.01	0.21±0.00	0.23±0.00	0.23±0.00	0.48±0.00	0.28±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(2001)	0.19±0.00	0.43±0.00	0.51±0.00	0.23±0.02	0.19±0.00	0.22±0.00	0.21±0.00	0.50±0.00	0.26±0.00	0.23±0.00	nan±nan	0.19±0.00	**0.18±0.00**
Basketball(2002)	0.20±0.00	0.42±0.00	0.48±0.00	0.23±0.01	0.20±0.00	0.21±0.00	0.23±0.00	0.49±0.00	0.26±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball(2003)	0.20±0.00	0.46±0.00	0.51±0.00	0.24±0.02	0.20±0.00	0.24±0.00	0.23±0.00	0.48±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(2004)	0.19±0.00	0.42±0.00	0.50±0.03	0.22±0.02	0.19±0.00	0.21±0.00	0.22±0.00	0.49±0.00	0.23±0.00	0.22±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(2005)	0.19±0.00	0.45±0.00	0.49±0.01	0.24±0.01	0.20±0.00	0.21±0.00	0.23±0.00	0.47±0.00	0.26±0.00	0.23±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball(2006)	0.20±0.00	0.44±0.00	0.50±0.00	0.21±0.02	**0.19±0.00**	0.22±0.00	0.22±0.00	0.50±0.00	0.26±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(2007)	0.20±0.00	0.47±0.00	0.48±0.02	0.24±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.49±0.00	0.23±0.00	0.22±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball(2008)	**0.20±0.00**	0.44±0.00	0.50±0.00	0.22±0.02	**0.20±0.00**	0.22±0.00	0.23±0.00	0.48±0.00	0.24±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.20±0.00**
Basketball(2009)	0.19±0.00	0.43±0.00	0.51±0.00	0.24±0.00	0.19±0.00	0.21±0.00	0.21±0.00	0.47±0.00	0.24±0.00	0.22±0.00	nan±nan	0.19±0.00	**0.18±0.00**
Basketball(2010)	0.20±0.00	0.39±0.00	0.49±0.00	0.21±0.02	0.19±0.00	0.23±0.00	0.23±0.00	0.51±0.00	0.23±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball(2011)	0.20±0.00	0.41±0.00	0.49±0.00	0.21±0.02	0.20±0.00	0.22±0.00	0.22±0.00	0.51±0.00	0.24±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball(2012)	0.19±0.00	0.45±0.00	0.49±0.00	0.22±0.02	0.19±0.00	0.21±0.00	0.21±0.00	0.49±0.00	0.22±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball(2013)	0.20±0.00	0.43±0.00	0.50±0.01	0.24±0.02	0.20±0.00	0.22±0.00	0.22±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.20±0.00	**0.19±0.00**
Basketball(2014)	0.20±0.00	0.43±0.00	0.52±0.00	0.22±0.02	0.20±0.00	0.22±0.00	0.23±0.00	0.48±0.00	0.25±0.00	0.22±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(1985)	0.19±0.00	0.41±0.00	0.49±0.02	0.36±0.01	0.21±0.00	0.29±0.00	0.29±0.00	0.49±0.00	0.25±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball finer(1986)	0.19±0.00	0.45±0.00	0.50±0.00	0.36±0.02	0.21±0.00	0.29±0.00	0.29±0.00	0.50±0.00	0.27±0.00	0.21±0.00	nan±nan	0.20±0.00	**0.17±0.00**
Basketball finer(1987)	0.20±0.00	0.45±0.00	0.47±0.00	0.35±0.01	0.22±0.00	0.29±0.00	0.30±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.01	**0.19±0.00**
Basketball finer(1988)	0.19±0.00	0.45±0.00	0.48±0.00	0.36±0.03	0.21±0.00	0.31±0.00	0.30±0.00	0.49±0.00	0.24±0.00	0.21±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball finer(1989)	0.19±0.00	0.42±0.00	0.46±0.00	0.36±0.01	0.21±0.00	0.28±0.00	0.29±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.20±0.01	**0.18±0.00**
Basketball finer(1990)	0.20±0.00	0.42±0.00	0.48±0.00	0.36±0.01	0.21±0.00	0.32±0.00	0.29±0.00	0.50±0.00	0.23±0.00	0.21±0.00	nan±nan	0.20±0.01	**0.18±0.00**
Basketball finer(1991)	0.20±0.00	0.46±0.00	0.51±0.00	0.34±0.02	0.21±0.00	0.35±0.00	0.33±0.00	0.49±0.00	0.25±0.00	0.22±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball finer(1992)	0.18±0.00	0.43±0.00	0.47±0.00	0.33±0.01	0.19±0.00	0.31±0.00	0.30±0.00	0.47±0.00	0.24±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball finer(1993)	0.19±0.00	0.42±0.00	0.51±0.00	0.34±0.01	0.20±0.00	0.29±0.00	0.27±0.00	0.49±0.00	0.25±0.00	0.21±0.00	nan±nan	0.20±0.00	**0.17±0.00**
Basketball finer(1994)	0.18±0.00	0.42±0.00	0.50±0.00	0.34±0.02	0.19±0.00	0.31±0.00	0.27±0.00	0.49±0.00	0.22±0.00	0.21±0.00	nan±nan	0.19±0.00	**0.17±0.00**
Basketball finer(1995)	0.20±0.00	0.44±0.00	0.47±0.00	0.34±0.02	0.21±0.00	0.30±0.00	0.28±0.00	0.48±0.00	0.24±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball finer(1996)	0.20±0.00	0.42±0.00	0.49±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.30±0.00	0.49±0.00	0.27±0.00	0.24±0.00	nan±nan	0.22±0.00	**0.19±0.00**
Basketball finer(1997)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.01	0.21±0.00	0.30±0.00	0.29±0.00	0.51±0.00	0.24±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(1998)	0.20±0.00	0.42±0.00	0.48±0.00	0.34±0.02	0.21±0.00	0.29±0.00	0.28±0.00	0.48±0.00	0.24±0.00	0.22±0.00	nan±nan	0.20±0.00	**0.18±0.00**
Basketball finer(1999)	0.20±0.00	0.41±0.00	0.50±0.00	0.34±0.02	0.22±0.00	0.29±0.00	0.28±0.00	0.50±0.00	0.29±0.00	0.24±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball finer(2000)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.01	0.23±0.00	0.32±0.00	0.30±0.00	0.48±0.00	0.28±0.00	0.24±0.00	nan±nan	0.22±0.00	**0.19±0.00**
Basketball finer(2001)	0.20±0.00	0.42±0.00	0.51±0.00	0.35±0.01	0.21±0.00	0.31±0.00	0.30±0.00	0.51±0.00	0.27±0.00	0.24±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball finer(2002)	0.22±0.00	0.44±0.00	0.47±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.51±0.00	0.27±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(2003)	0.22±0.00	0.45±0.00	0.50±0.02	0.36±0.02	0.23±0.00	0.29±0.00	0.29±0.00	0.50±0.00	0.26±0.00	0.24±0.00	nan±nan	0.22±0.00	**0.19±0.00**
Basketball finer(2004)	0.19±0.00	0.43±0.01	0.47±0.00	0.35±0.01	0.20±0.00	0.29±0.00	0.28±0.00	0.49±0.00	0.24±0.00	0.22±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball finer(2005)	0.21±0.00	0.46±0.00	0.48±0.00	0.34±0.01	0.22±0.00	0.29±0.00	0.27±0.00	0.50±0.00	0.27±0.00	0.24±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(2006)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.02	0.21±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.27±0.00	0.23±0.00	nan±nan	0.22±0.01	**0.19±0.00**
Basketball finer(2007)	0.21±0.00	0.46±0.00	0.49±0.03	0.35±0.01	0.23±0.00	0.29±0.00	0.27±0.00	0.48±0.00	0.24±0.00	0.23±0.00	nan±nan	0.22±0.00	**0.20±0.00**
Basketball finer(2008)	0.21±0.00	0.43±0.00	0.49±0.00	0.34±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.20±0.00**
Basketball finer(2009)	0.21±0.00	0.43±0.00	0.50±0.01	0.34±0.01	0.22±0.00	0.29±0.00	0.28±0.00	0.52±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(2010)	0.20±0.00	0.42±0.00	0.49±0.00	0.33±0.02	0.21±0.00	0.28±0.00	0.28±0.00	0.49±0.00	0.24±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.19±0.00**
Basketball finer(2011)	0.21±0.00	0.41±0.00	0.49±0.00	0.34±0.01	0.22±0.00	0.28±0.00	0.28±0.00	0.50±0.00	0.24±0.00	0.22±0.00	nan±nan	0.22±0.00	**0.19±0.00**
Basketball finer(2012)	0.20±0.00	0.42±0.00	0.49±0.00	0.34±0.02	0.21±0.00	0.29±0.00	0.28±0.00	0.48±0.00	0.23±0.00	0.22±0.00	nan±nan	0.21±0.00	**0.18±0.00**
Basketball finer(2013)	0.21±0.00	0.43±0.00	0.52±0.00	0.34±0.02	0.22±0.00	0.29±0.00	0.28±0.00	0.49±0.00	0.25±0.00	0.23±0.00	nan±nan	0.21±0.00	**0.20±0.00**
Basketball finer(2014)	0.21±0.00	0.44±0.05	0.52±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.26±0.00	0.22±0.00	nan±nan	0.22±0.00	**0.20±0.00**
Football(2009)	**0.15±0.00**	0.36±0.11	0.23±0.00	0.17±0.02	0.20±0.00	0.23±0.00	0.26±0.00	0.45±0.02	0.20±0.00	0.16±0.00	0.39±0.04	0.19±0.01	**0.15±0.00**
Football(2010)	0.30±0.00	0.45±0.04	0.39±0.00	0.30±0.01	0.32±0.00	0.34±0.00	0.34±0.00	0.46±0.03	0.31±0.00	**0.29±0.00**	0.49±0.05	0.30±0.01	**0.29±0.00**
Football(2011)	**0.20±0.00**	0.39±0.09	0.24±0.00	0.22±0.01	0.22±0.00	0.25±0.00	0.25±0.00	0.40±0.00	0.23±0.00	0.21±0.00	0.45±0.05	0.22±0.01	**0.20±0.00**
Football(2012)	0.24±0.00	0.38±0.01	0.35±0.00	0.25±0.01	0.22±0.00	0.24±0.00	0.25±0.00	0.41±0.00	0.25±0.00	0.22±0.00	0.43±0.04	**0.20±0.02**	0.21±0.00
Football(2013)	0.19±0.00	0.41±0.03	0.19±0.00	0.19±0.01	0.17±0.00	0.23±0.00	0.22±0.00	0.48±0.02	0.22±0.00	0.19±0.00	0.40±0.03	0.16±0.01	**0.14±0.00**
Football(2014)	0.28±0.00	0.45±0.05	0.39±0.00	0.29±0.00	0.28±0.00	0.32±0.00	0.30±0.00	0.46±0.03	0.28±0.00	0.27±0.00	0.43±0.05	**0.24±0.01**	0.26±0.01
Football finer(2009)	0.18±0.00	0.45±0.00	0.23±0.00	0.20±0.03	0.18±0.00	0.26±0.00	0.27±0.00	0.47±0.01	0.20±0.00	**0.16±0.00**	0.48±0.04	0.19±0.05	**0.16±0.01**
Football finer(2010)	0.32±0.00	0.41±0.01	0.36±0.00	0.33±0.02	0.32±0.00	0.34±0.00	0.37±0.00	0.49±0.00	0.31±0.00	**0.29±0.00**	0.53±0.00	0.31±0.02	**0.29±0.00**
Football finer(2011)	0.22±0.00	0.40±0.01	0.27±0.00	0.23±0.01	0.23±0.00	0.25±0.00	0.28±0.00	0.47±0.00	0.23±0.00	**0.21±0.00**	0.53±0.00	0.23±0.04	**0.21±0.01**
Football finer(2012)	0.24±0.00	0.44±0.04	0.27±0.00	0.25±0.01	0.23±0.00	0.27±0.00	0.30±0.00	0.51±0.01	0.25±0.00	0.22±0.00	0.54±0.00	0.24±0.02	**0.21±0.01**
Football finer(2013)	0.22±0.00	0.39±0.00	0.22±0.00	0.21±0.02	0.18±0.00	0.25±0.00	0.27±0.00	0.50±0.00	0.22±0.00	0.19±0.00	0.49±0.04	0.18±0.05	**0.14±0.00**
Football finer(2014)	0.29±0.00	0.44±0.01	0.39±0.00	0.30±0.01	**0.26±0.00**	0.31±0.00	0.31±0.00	0.42±0.00	0.28±0.00	0.27±0.00	0.52±0.04	0.27±0.01	0.27±0.02

E.3. Results on Different Variants

Tables 10, 11 and 12 compare different variants in the proposed GNNRank framework with themselves, together with

Table 8. Result table on $\mathcal{L}_{\text{upset, ratio}}$ for individual directed graphs as input, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue. MVR could not generate scores, so we omit the results.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	GNNRank-N	GNNRank-P
HeadToHead	0.86±0.00	0.97±0.00	0.97±0.00	0.71±0.01	0.89±0.00	0.98±0.04	0.83±0.00	1.19±0.09	1.14±0.00	0.95±0.00	**0.68±0.00**	0.94±0.00
Finance	**0.26±0.00**	0.27±0.00	0.27±0.00	0.28±0.00	0.64±0.00	0.27±0.00	**0.26±0.00**	0.59±0.00	**0.26±0.00**	**0.26±0.00**	**0.26±0.00**	0.27±0.00
Animal	0.42±0.00	0.80±0.02	0.88±0.00	**0.22±0.00**	0.41±0.00	0.43±0.14	0.50±0.00	1.34±0.00	0.62±0.00	0.42±0.00	0.24±0.03	0.66±0.00
Faculty: Business	0.67±0.00	0.98±0.00	0.99±0.00	0.34±0.00	0.52±0.00	0.74±0.25	0.47±0.00	1.40±0.13	0.64±0.00	0.62±0.00	**0.31±0.00**	0.89±0.00
Faculty: CS	0.65±0.00	0.99±0.00	1.00±0.00	0.28±0.00	0.51±0.00	0.63±0.24	0.42±0.00	1.33±0.16	0.73±0.00	0.72±0.00	**0.26±0.02**	0.86±0.00
Faculty: History	0.61±0.00	0.99±0.00	1.00±0.00	0.23±0.00	0.49±0.00	0.60±0.28	0.37±0.00	1.35±0.11	0.68±0.00	0.72±0.00	**0.21±0.00**	0.84±0.02
Basketball(1985)	0.67±0.00	0.86±0.00	0.86±0.00	0.54±0.04	0.62±0.00	0.67±0.17	0.55±0.00	1.32±0.00	0.67±0.00	0.63±0.00	**0.46±0.01**	0.82±0.00
Basketball(1986)	0.69±0.00	0.85±0.00	0.86±0.00	0.54±0.01	0.61±0.00	0.73±0.16	0.54±0.00	1.30±0.00	0.75±0.00	0.63±0.00	**0.42±0.00**	0.81±0.00
Basketball(1987)	0.71±0.00	0.86±0.00	0.86±0.00	0.57±0.01	0.65±0.00	0.67±0.15	0.56±0.00	1.37±0.00	0.70±0.00	0.68±0.00	**0.48±0.01**	0.82±0.00
Basketball(1988)	0.65±0.00	0.86±0.00	0.86±0.00	0.53±0.01	0.59±0.00	0.70±0.17	0.54±0.00	1.33±0.00	0.70±0.00	0.63±0.00	**0.45±0.01**	0.82±0.00
Basketball(1989)	0.71±0.00	0.87±0.00	0.87±0.00	0.55±0.01	0.63±0.00	0.77±0.15	0.55±0.00	1.34±0.00	0.70±0.00	0.66±0.00	**0.46±0.01**	0.83±0.00
Basketball(1990)	0.67±0.00	0.86±0.00	0.87±0.00	0.51±0.05	0.62±0.00	0.74±0.16	0.55±0.00	1.35±0.00	0.65±0.00	0.62±0.00	**0.44±0.01**	0.82±0.00
Basketball(1991)	0.67±0.00	0.86±0.00	0.87±0.00	0.53±0.03	0.64±0.00	0.74±0.16	0.55±0.00	1.34±0.00	0.68±0.00	0.63±0.00	**0.45±0.00**	0.82±0.00
Basketball(1992)	0.69±0.00	0.87±0.00	0.87±0.00	0.50±0.06	0.59±0.00	0.73±0.17	0.54±0.00	1.35±0.00	0.70±0.00	0.61±0.00	**0.43±0.00**	0.83±0.00
Basketball(1993)	0.67±0.00	0.86±0.00	0.86±0.00	0.51±0.03	0.60±0.00	0.79±0.11	0.52±0.00	1.28±0.00	0.68±0.00	0.62±0.00	**0.44±0.01**	0.82±0.00
Basketball(1994)	0.56±0.00	0.86±0.00	0.86±0.00	0.43±0.05	0.59±0.00	0.64±0.19	0.53±0.00	1.35±0.00	0.61±0.00	0.60±0.00	**0.42±0.00**	0.82±0.00
Basketball(1995)	0.67±0.00	0.85±0.00	0.86±0.00	0.47±0.06	0.59±0.00	0.58±0.16	0.53±0.00	1.36±0.00	0.64±0.00	0.61±0.00	**0.44±0.00**	0.81±0.00
Basketball(1996)	0.68±0.00	0.86±0.00	0.87±0.00	0.55±0.03	0.65±0.00	0.57±0.13	0.56±0.00	1.35±0.00	0.74±0.00	0.68±0.00	**0.48±0.00**	0.82±0.00
Basketball(1997)	0.71±0.00	0.86±0.00	0.86±0.00	0.56±0.00	0.63±0.00	0.71±0.16	0.56±0.00	1.30±0.00	0.69±0.00	0.68±0.00	**0.48±0.00**	0.82±0.00
Basketball(1998)	0.70±0.00	0.88±0.00	0.88±0.00	0.56±0.01	0.64±0.00	0.71±0.17	0.56±0.00	1.36±0.00	0.70±0.00	0.67±0.00	**0.46±0.00**	0.84±0.00
Basketball(1999)	0.60±0.00	0.87±0.00	0.88±0.00	0.55±0.03	0.65±0.00	0.68±0.17	0.57±0.00	1.32±0.00	0.80±0.00	0.66±0.00	**0.48±0.00**	0.83±0.00
Basketball(2000)	0.70±0.00	0.90±0.00	0.90±0.00	0.58±0.01	0.65±0.00	0.77±0.16	0.59±0.00	1.40±0.01	0.77±0.00	0.67±0.00	**0.50±0.00**	0.86±0.00
Basketball(2001)	0.70±0.00	0.89±0.00	0.89±0.00	0.55±0.04	0.64±0.00	0.72±0.18	0.56±0.00	1.40±0.00	0.74±0.00	0.67±0.00	**0.47±0.00**	0.85±0.00
Basketball(2002)	0.70±0.00	0.88±0.00	0.88±0.00	0.56±0.03	0.66±0.00	0.65±0.17	0.58±0.00	1.33±0.00	0.77±0.00	0.68±0.00	**0.48±0.01**	0.84±0.00
Basketball(2003)	0.72±0.00	0.88±0.00	0.89±0.00	0.56±0.04	0.68±0.00	0.67±0.16	0.59±0.00	1.40±0.00	0.73±0.00	0.71±0.00	**0.51±0.00**	0.85±0.00
Basketball(2004)	0.68±0.00	0.88±0.00	0.89±0.00	0.54±0.04	0.64±0.00	0.64±0.18	0.56±0.00	1.35±0.00	0.67±0.00	0.64±0.00	**0.46±0.00**	0.85±0.00
Basketball(2005)	0.73±0.00	0.89±0.00	0.89±0.00	0.57±0.01	0.65±0.00	0.69±0.18	0.58±0.00	1.29±0.01	0.73±0.00	0.68±0.00	**0.49±0.01**	0.85±0.00
Basketball(2006)	0.73±0.00	0.89±0.00	0.90±0.00	0.50±0.05	0.65±0.00	0.69±0.18	0.58±0.00	1.36±0.00	0.76±0.00	0.69±0.00	**0.49±0.01**	0.85±0.00
Basketball(2007)	0.72±0.00	0.90±0.00	0.90±0.00	0.57±0.00	0.65±0.00	0.62±0.16	0.57±0.00	1.41±0.00	0.67±0.00	0.67±0.00	**0.49±0.00**	0.86±0.00
Basketball(2008)	0.74±0.00	0.90±0.00	0.90±0.00	0.55±0.04	0.65±0.00	0.73±0.17	0.58±0.00	1.33±0.01	0.70±0.00	0.67±0.00	**0.51±0.01**	0.86±0.00
Basketball(2009)	0.70±0.00	0.90±0.00	0.90±0.00	0.57±0.01	0.65±0.00	0.65±0.18	0.56±0.00	1.24±0.00	0.69±0.00	0.66±0.00	**0.47±0.00**	0.86±0.00
Basketball(2010)	0.74±0.00	0.91±0.00	0.91±0.00	0.52±0.06	0.66±0.00	0.71±0.18	0.58±0.00	1.35±0.00	0.69±0.00	0.67±0.00	**0.49±0.00**	0.87±0.00
Basketball(2011)	0.73±0.00	0.90±0.00	0.90±0.00	0.52±0.05	0.66±0.00	0.63±0.16	0.58±0.00	1.34±0.00	0.70±0.00	0.68±0.00	**0.49±0.00**	0.86±0.00
Basketball(2012)	0.73±0.00	0.90±0.00	0.91±0.00	0.54±0.05	0.64±0.00	0.73±0.19	0.56±0.00	1.42±0.00	0.66±0.00	0.64±0.00	**0.48±0.01**	0.86±0.00
Basketball(2013)	0.74±0.00	0.90±0.00	0.91±0.00	0.58±0.03	0.67±0.00	0.70±0.17	0.59±0.00	1.34±0.00	0.73±0.00	0.70±0.00	**0.51±0.01**	0.86±0.00
Basketball(2014)	0.72±0.00	0.90±0.00	0.90±0.00	0.52±0.06	0.66±0.00	0.65±0.18	0.58±0.00	1.41±0.00	0.72±0.00	0.67±0.00	**0.49±0.01**	0.86±0.00
Basketball finer(1985)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.47±0.00	0.01±0.00	0.01±0.00	0.57±0.00	0.46±0.00	0.46±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1986)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.60±0.00	0.48±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1987)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.51±0.00	0.01±0.00	0.01±0.00	0.58±0.00	0.46±0.00	0.48±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1988)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.52±0.00	0.01±0.00	0.01±0.00	0.57±0.00	0.56±0.00	0.52±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1989)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.56±0.00	0.49±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1990)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.55±0.00	0.48±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1991)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.54±0.00	0.49±0.00	0.47±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1992)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.55±0.00	0.44±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1993)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.51±0.00	0.01±0.00	0.01±0.00	0.58±0.00	0.47±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1994)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.49±0.00	0.01±0.00	0.01±0.00	0.58±0.00	0.46±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1995)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.50±0.00	0.01±0.00	0.01±0.00	0.55±0.00	0.47±0.00	0.49±0.00	**0.00±0.00**	0.01±0.00
Basketball finer(1996)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.48±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.45±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1997)	0.04±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.48±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1998)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.47±0.00	0.45±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1999)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.48±0.00	0.46±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2000)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.55±0.00	0.50±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2001)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.51±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2002)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.51±0.00	0.50±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2003)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.47±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2004)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.45±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2005)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.60±0.00	0.43±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2006)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.46±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2007)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.53±0.00	**0.01±0.00**	**0.01±0.00**	0.53±0.00	0.49±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2008)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.47±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2009)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.50±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2010)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.48±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2011)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.46±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2012)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.48±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2013)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.46±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2014)	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.53±0.00	0.01±0.00	0.01±0.00	0.56±0.00	0.48±0.00	0.47±0.00	**0.00±0.00**	0.01±0.00
Football(2009)	0.64±0.00	0.79±0.03	0.73±0.00	0.48±0.02	0.55±0.00	0.63±0.09	0.60±0.00	1.16±0.05	0.49±0.00	0.49±0.00	**0.46±0.01**	0.69±0.00
Football(2010)	0.73±0.00	0.80±0.04	0.81±0.01	0.66±0.02	0.76±0.00	0.75±0.04	0.74±0.00	1.25±0.01	0.74±0.00	0.73±0.00	**0.65±0.00**	0.73±0.01
Football(2011)	0.66±0.00	0.79±0.01	0.70±0.00	**0.53±0.02**	0.70±0.00	0.65±0.08	0.60±0.00	1.17±0.00	0.59±0.00	0.58±0.00	**0.53±0.01**	0.69±0.00
Football(2012)	0.58±0.00	0.81±0.00	0.83±0.00	0.53±0.03	0.60±0.00	0.68±0.09	0.63±0.00	1.18±0.00	0.58±0.00	0.58±0.00	**0.51±0.01**	0.71±0.00
Football(2013)	0.66±0.00	0.81±0.00	0.73±0.00	0.48±0.04	0.59±0.00	0.64±0.10	0.62±0.00	0.94±0.00	0.56±0.00	0.53±0.00	**0.46±0.01**	0.71±0.00
Football(2014)	0.85±0.00	0.95±0.01	0.97±0.01	0.74±0.01	0.98±0.00	0.83±0.07	0.81±0.00	1.49±0.00	0.84±0.00	0.82±0.00	**0.69±0.01**	0.85±0.00
Football finer(2009)	0.22±0.00	0.27±0.00	0.23±0.00	**0.17±0.01**	0.38±0.00	0.23±0.03	0.22±0.00	0.73±0.04	0.21±0.00	0.21±0.00	**0.17±0.00**	0.21±0.00
Football finer(2010)	0.20±0.00	0.23±0.00	0.21±0.00	0.18±0.01	0.56±0.00	0.22±0.01	0.22±0.00	0.67±0.00	0.24±0.00	0.22±0.00	**0.17±0.00**	0.18±0.00
Football finer(2011)	0.19±0.00	0.25±0.00	0.23±0.00	**0.17±0.00**	0.49±0.00	0.22±0.02	0.21±0.00	0.58±0.00	0.22±0.00	0.22±0.00	**0.17±0.00**	0.19±0.00
Football finer(2012)	0.19±0.00	0.23±0.04	0.21±0.01	0.16±0.00	0.48±0.00	0.19±0.02	0.19±0.00	0.66±0.04	0.18±0.00	0.17±0.00	**0.15±0.00**	0.17±0.00
Football finer(2013)	0.24±0.00	0.30±0.00	0.29±0.01	0.21±0.01	0.43±0.00	0.27±0.03	0.26±0.00	0.52±0.00	0.24±0.00	0.22±0.00	**0.19±0.04**	0.24±0.01
Football finer(2014)	0.36±0.00	0.45±0.00	0.45±0.00	0.35±0.00	0.62±0.00	0.40±0.03	0.39±0.00	0.98±0.00	0.37±0.00	0.37±0.00	**0.34±0.00**	0.38±0.00

Table 9. Performance on Kendall Tau (top half) based on the lowest $\mathcal{L}_{\text{upset, naive}}$ (bottom half for corresponding values) on ERO models, averaged over 10 runs with one standard deviation. "avg" for time series first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best is marked in **bold red** while the second best is in <u>underline blue</u>. As MVR does not generate results after one week, we leave it out here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	GNNRank-N	GNNRank-P
ERO(p=0.05, style=uniform,η=0)	0.86±0.00	0.06±0.00	0.03±0.00	0.79±0.05	0.80±0.00	**0.91±0.00**	0.87±0.00	0.02±0.09	0.78±0.00	0.83±0.00	<u>0.88±0.00</u>	0.86±0.00
ERO(p=0.05, style=gamma,η=0)	0.81±0.00	0.06±0.00	0.00±0.00	0.87±0.01	0.81±0.00	**0.92±0.00**	<u>0.90±0.00</u>	-0.00±0.01	0.58±0.00	0.79±0.00	0.87±0.00	0.84±0.01
ERO(p=0.05, style=uniform,η=0.1)	0.75±0.00	0.04±0.00	0.03±0.00	0.70±0.01	<u>0.77±0.00</u>	0.56±0.00	0.58±0.00	0.01±0.05	0.74±0.00	<u>0.77±0.00</u>	0.76±0.01	**0.79±0.01**
ERO(p=0.05, style=gamma,η=0.1)	0.69±0.00	0.04±0.00	0.01±0.00	0.68±0.00	<u>0.78±0.00</u>	0.58±0.00	0.60±0.00	0.06±0.00	0.52±0.00	0.72±0.00	0.61±0.01	**0.81±0.01**
ERO(p=0.05, style=uniform,η=0.2)	0.68±0.00	0.07±0.00	0.03±0.00	0.64±0.01	<u>0.73±0.00</u>	0.50±0.00	0.48±0.00	0.02±0.04	0.67±0.00	0.70±0.00	0.68±0.01	**0.76±0.01**
ERO(p=0.05, style=gamma,η=0.2)	0.61±0.00	0.01±0.00	-0.01±0.00	0.61±0.00	<u>0.74±0.00</u>	0.52±0.00	0.51±0.00	-0.01±0.01	0.45±0.00	0.64±0.00	0.52±0.01	**0.77±0.00**
ERO(p=0.05, style=uniform,η=0.3)	0.61±0.00	0.05±0.00	0.01±0.00	0.59±0.01	<u>0.68±0.00</u>	0.44±0.00	0.41±0.00	0.05±0.00	0.60±0.00	0.62±0.00	0.62±0.00	**0.70±0.02**
ERO(p=0.05, style=gamma,η=0.3)	0.56±0.00	0.11±0.00	-0.00±0.00	0.56±0.01	<u>0.71±0.00</u>	0.46±0.00	0.44±0.00	0.11±0.00	0.34±0.00	0.56±0.00	0.43±0.03	**0.72±0.00**
ERO(p=0.05, style=uniform,η=0.4)	0.55±0.00	0.08±0.00	0.01±0.00	0.54±0.04	**0.63±0.00**	0.40±0.00	0.35±0.00	0.02±0.00	0.51±0.00	0.54±0.00	0.52±0.00	<u>0.62±0.02</u>
ERO(p=0.05, style=gamma,η=0.4)	0.51±0.00	0.08±0.00	-0.00±0.00	0.52±0.00	<u>0.65±0.00</u>	0.43±0.00	0.43±0.00	0.09±0.01	0.23±0.00	0.44±0.00	0.38±0.08	**0.66±0.01**
ERO(p=0.05, style=uniform,η=0.5)	0.47±0.00	0.08±0.00	0.01±0.00	0.47±0.05	**0.57±0.00**	0.37±0.00	0.32±0.00	0.04±0.00	0.25±0.00	0.36±0.00	0.42±0.02	<u>0.56±0.00</u>
ERO(p=0.05, style=gamma,η=0.5)	0.44±0.00	0.07±0.00	0.01±0.00	0.45±0.01	<u>0.58±0.00</u>	0.37±0.00	0.37±0.00	0.04±0.00	0.22±0.00	0.23±0.00	0.22±0.01	**0.59±0.02**
ERO(p=0.05, style=uniform,η=0.6)	0.39±0.00	0.03±0.00	-0.00±0.00	0.39±0.06	**0.48±0.00**	0.31±0.00	0.27±0.00	-0.03±0.00	0.15±0.00	0.15±0.00	0.33±0.00	<u>0.46±0.00</u>
ERO(p=0.05, style=gamma,η=0.6)	0.36±0.00	0.03±0.00	0.01±0.00	0.37±0.01	<u>0.49±0.00</u>	0.31±0.00	0.33±0.00	0.01±0.00	0.14±0.00	0.09±0.00	0.24±0.03	**0.50±0.01**
ERO(p=0.05, style=uniform,η=0.7)	0.31±0.00	0.08±0.00	-0.01±0.00	0.31±0.06	<u>0.38±0.00</u>	0.26±0.00	0.23±0.00	0.04±0.00	0.07±0.00	0.06±0.00	0.23±0.01	**0.39±0.03**
ERO(p=0.05, style=gamma,η=0.7)	0.29±0.00	-0.00±0.00	0.02±0.00	0.29±0.01	**0.39±0.00**	0.25±0.00	0.28±0.00	-0.00±0.00	0.02±0.00	0.03±0.00	0.05±0.02	**0.39±0.02**
ERO(p=0.05, style=uniform,η=0.8)	0.21±0.00	0.02±0.00	-0.02±0.00	0.22±0.07	<u>0.25±0.00</u>	0.20±0.00	0.15±0.00	-0.03±0.00	0.00±0.00	0.01±0.00	0.11±0.00	**0.27±0.04**
ERO(p=0.05, style=gamma,η=0.8)	0.18±0.00	0.07±0.00	0.00±0.00	0.18±0.01	<u>0.24±0.00</u>	0.16±0.00	0.19±0.00	-0.02±0.02	0.03±0.00	-0.03±0.00	0.05±0.02	**0.27±0.03**
ERO(p=1, style=uniform,η=0)	**1.00±0.00**	0.08±0.00	**1.00±0.00**	0.85±0.05	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**	-0.00±0.06	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**
ERO(p=1, style=gamma,η=0)	**1.00±0.00**	0.08±0.00	**1.00±0.00**	0.95±0.00	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**	0.01±0.09	**1.00±0.00**	0.99±0.00	**1.00±0.00**	**1.00±0.00**
ERO(p=1, style=uniform,η=0.1)	0.94±0.00	0.05±0.00	**0.98±0.00**	0.85±0.04	**0.98±0.00**	0.86±0.00	0.63±0.00	0.52±0.00	0.97±0.00	0.94±0.00	0.93±0.01	**0.98±0.00**
ERO(p=1, style=gamma,η=0.1)	0.88±0.00	0.10±0.00	**0.98±0.00**	0.85±0.00	**0.98±0.00**	0.72±0.00	0.57±0.00	0.38±0.00	0.92±0.00	0.90±0.00	0.89±0.01	**0.98±0.00**
ERO(p=1, style=uniform,η=0.2)	0.91±0.00	0.07±0.00	**0.97±0.00**	0.84±0.03	0.96±0.00	0.85±0.00	0.69±0.00	0.33±0.09	0.95±0.00	0.93±0.00	0.91±0.00	**0.97±0.00**
ERO(p=1, style=gamma,η=0.2)	0.85±0.00	0.05±0.00	**0.97±0.00**	0.83±0.00	0.96±0.00	0.71±0.00	0.60±0.00	0.28±0.00	0.89±0.00	0.87±0.00	0.86±0.00	**0.97±0.00**
ERO(p=1, style=uniform,η=0.3)	0.89±0.00	0.09±0.00	**0.96±0.00**	0.84±0.03	0.94±0.00	0.83±0.00	0.71±0.00	0.29±0.00	0.94±0.00	0.91±0.00	0.90±0.01	**0.96±0.00**
ERO(p=1, style=gamma,η=0.3)	0.82±0.00	0.06±0.00	**0.95±0.00**	0.80±0.00	**0.95±0.00**	0.70±0.00	0.64±0.00	0.32±0.00	0.85±0.00	0.84±0.00	0.85±0.00	**0.95±0.00**
ERO(p=1, style=uniform,η=0.4)	0.88±0.00	0.06±0.00	**0.94±0.00**	0.83±0.03	0.93±0.00	0.83±0.00	0.73±0.00	0.11±0.00	0.92±0.00	0.90±0.00	0.91±0.01	**0.94±0.00**
ERO(p=1, style=gamma,η=0.4)	0.79±0.00	0.04±0.00	**0.94±0.00**	0.78±0.00	0.93±0.00	0.69±0.00	0.65±0.00	0.14±0.00	0.83±0.00	0.82±0.00	0.83±0.01	**0.94±0.00**
ERO(p=1, style=uniform,η=0.5)	0.85±0.00	0.07±0.00	**0.92±0.00**	0.81±0.03	0.91±0.00	0.80±0.00	0.73±0.00	0.24±0.00	0.89±0.00	0.87±0.00	0.90±0.01	**0.92±0.00**
ERO(p=1, style=gamma,η=0.5)	0.76±0.00	0.03±0.00	**0.92±0.00**	0.74±0.01	0.90±0.00	0.68±0.00	0.66±0.00	0.16±0.01	0.79±0.00	0.78±0.00	0.81±0.01	**0.92±0.00**
ERO(p=1, style=uniform,η=0.6)	0.83±0.00	0.08±0.00	**0.89±0.00**	0.78±0.04	0.88±0.00	0.78±0.00	0.73±0.00	-0.00±0.00	0.86±0.00	0.85±0.00	0.87±0.01	**0.89±0.00**
ERO(p=1, style=gamma,η=0.6)	0.72±0.00	0.09±0.00	**0.89±0.00**	0.67±0.01	0.88±0.00	0.65±0.00	0.64±0.00	0.05±0.02	0.74±0.00	0.73±0.00	0.77±0.00	**0.89±0.00**
ERO(p=1, style=uniform,η=0.7)	0.77±0.00	0.08±0.00	0.80±0.00	0.72±0.05	**0.83±0.00**	0.74±0.00	0.70±0.00	0.18±0.00	0.80±0.00	0.79±0.00	**0.83±0.01**	**0.83±0.00**
ERO(p=1, style=gamma,η=0.7)	0.67±0.00	0.02±0.00	0.75±0.00	0.61±0.01	**0.83±0.00**	0.62±0.00	0.61±0.00	0.09±0.00	0.67±0.00	0.67±0.00	0.71±0.00	**0.83±0.00**
ERO(p=1, style=uniform,η=0.8)	0.68±0.00	0.06±0.00	0.11±0.00	0.62±0.05	**0.75±0.00**	0.65±0.00	0.62±0.00	0.15±0.00	0.67±0.00	0.66±0.00	0.70±0.01	<u>0.74±0.01</u>
ERO(p=1, style=gamma,η=0.8)	0.57±0.00	0.06±0.00	0.04±0.00	0.51±0.01	**0.75±0.00**	0.55±0.00	0.55±0.00	0.04±0.00	0.54±0.00	0.54±0.00	0.56±0.00	<u>0.74±0.00</u>
ERO(p=0.05, style=uniform,η=0)	0.06±0.00	0.47±0.00	0.47±0.00	0.09±0.03	0.06±0.00	**0.03±0.00**	0.05±0.00	0.49±0.03	0.10±0.00	0.07±0.00	<u>0.04±0.00</u>	0.05±0.00
ERO(p=0.05, style=gamma,η=0)	0.08±0.00	0.48±0.00	0.51±0.00	0.04±0.00	0.05±0.00	**0.02±0.00**	<u>0.03±0.00</u>	0.50±0.00	0.20±0.00	0.09±0.00	0.04±0.00	0.04±0.01
ERO(p=0.05, style=uniform,η=0.1)	0.13±0.00	0.46±0.00	0.47±0.00	0.15±0.01	<u>0.11±0.00</u>	0.22±0.00	0.21±0.00	0.50±0.01	0.16±0.00	0.14±0.00	0.13±0.01	**0.09±0.00**
ERO(p=0.05, style=gamma,η=0.1)	0.16±0.00	0.49±0.00	0.51±0.00	0.16±0.00	<u>0.11±0.00</u>	0.22±0.00	0.21±0.00	0.47±0.00	0.24±0.00	0.15±0.00	0.19±0.00	**0.09±0.00**
ERO(p=0.05, style=uniform,η=0.2)	0.18±0.00	0.48±0.00	0.50±0.00	0.20±0.01	<u>0.15±0.00</u>	0.25±0.00	0.27±0.00	0.49±0.01	0.21±0.00	0.19±0.00	0.19±0.00	**0.13±0.00**
ERO(p=0.05, style=gamma,η=0.2)	0.21±0.00	0.49±0.00	0.52±0.00	0.21±0.00	<u>0.15±0.00</u>	0.28±0.00	0.27±0.00	0.51±0.01	0.30±0.00	0.21±0.00	0.24±0.00	**0.14±0.00**
ERO(p=0.05, style=uniform,η=0.3)	0.22±0.00	0.46±0.00	0.51±0.00	0.23±0.01	<u>0.20±0.00</u>	0.28±0.00	0.31±0.00	0.47±0.00	0.26±0.00	0.24±0.00	0.23±0.00	**0.18±0.01**
ERO(p=0.05, style=gamma,η=0.3)	0.25±0.00	0.48±0.00	0.51±0.00	0.26±0.00	<u>0.20±0.00</u>	0.32±0.00	0.33±0.00	0.46±0.00	0.37±0.00	0.26±0.00	0.31±0.01	**0.19±0.00**
ERO(p=0.05, style=uniform,η=0.4)	0.26±0.00	0.48±0.00	0.51±0.00	0.27±0.01	**0.23±0.00**	0.31±0.00	0.34±0.00	0.50±0.00	0.31±0.00	0.28±0.00	0.28±0.00	**0.23±0.01**
ERO(p=0.05, style=gamma,η=0.4)	0.29±0.00	0.47±0.00	0.51±0.00	0.28±0.00	<u>0.24±0.00</u>	0.34±0.00	0.34±0.00	0.48±0.01	0.43±0.00	0.32±0.00	0.34±0.04	**0.23±0.01**
ERO(p=0.05, style=uniform,η=0.5)	0.29±0.00	0.48±0.00	0.51±0.00	0.30±0.01	**0.27±0.00**	0.34±0.00	0.36±0.00	0.49±0.00	0.40±0.00	0.35±0.00	0.32±0.00	**0.27±0.00**
ERO(p=0.05, style=gamma,η=0.5)	0.32±0.00	0.48±0.00	0.49±0.00	0.33±0.00	<u>0.28±0.00</u>	0.37±0.00	0.37±0.00	0.48±0.00	0.43±0.00	0.43±0.00	0.38±0.01	**0.27±0.01**
ERO(p=0.05, style=uniform,η=0.6)	0.32±0.00	0.47±0.00	0.51±0.00	0.33±0.01	<u>0.31±0.00</u>	0.37±0.00	0.39±0.00	0.51±0.00	0.44±0.00	0.42±0.00	0.35±0.00	**0.30±0.00**
ERO(p=0.05, style=gamma,η=0.6)	0.35±0.00	0.48±0.00	0.50±0.00	0.35±0.00	<u>0.32±0.00</u>	0.40±0.00	0.40±0.00	0.48±0.00	0.47±0.00	0.46±0.00	0.40±0.01	**0.30±0.01**
ERO(p=0.05, style=uniform,η=0.7)	<u>0.34±0.00</u>	0.48±0.00	0.49±0.00	0.35±0.01	<u>0.34±0.00</u>	0.39±0.00	0.40±0.00	0.49±0.00	0.48±0.00	0.47±0.00	0.38±0.01	**0.32±0.00**
ERO(p=0.05, style=gamma,η=0.7)	0.36±0.00	0.49±0.00	0.50±0.00	0.37±0.01	<u>0.35±0.00</u>	0.42±0.00	0.40±0.00	0.49±0.00	0.49±0.00	0.49±0.00	0.43±0.01	**0.33±0.01**
ERO(p=0.05, style=uniform,η=0.8)	<u>0.36±0.00</u>	0.48±0.00	0.49±0.00	0.37±0.00	<u>0.36±0.00</u>	0.40±0.00	0.41±0.00	0.51±0.00	0.50±0.00	0.49±0.00	0.42±0.00	**0.34±0.00**
ERO(p=0.05, style=gamma,η=0.8)	0.38±0.00	0.48±0.00	0.50±0.00	0.39±0.01	<u>0.36±0.00</u>	0.42±0.00	0.41±0.00	0.50±0.01	0.49±0.00	0.50±0.00	0.43±0.02	**0.34±0.01**
ERO(p=1, style=uniform,η=0)	**0.00±0.00**	0.46±0.00	**0.00±0.00**	0.07±0.02	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**	0.50±0.03	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**
ERO(p=1, style=gamma,η=0)	**0.00±0.00**	0.46±0.00	**0.00±0.00**	0.02±0.00	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**	0.50±0.05	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**
ERO(p=1, style=uniform,η=0.1)	0.08±0.00	0.48±0.00	**0.06±0.00**	0.12±0.02	**0.06±0.00**	0.11±0.00	0.21±0.00	0.27±0.00	**0.06±0.00**	0.08±0.00	0.08±0.00	**0.06±0.00**
ERO(p=1, style=gamma,η=0.1)	0.10±0.00	0.46±0.00	**0.06±0.00**	0.11±0.00	**0.06±0.00**	0.18±0.00	0.24±0.00	0.33±0.00	0.09±0.00	0.09±0.00	0.10±0.00	**0.06±0.00**
ERO(p=1, style=uniform,η=0.2)	0.13±0.00	0.47±0.00	**0.11±0.00**	0.16±0.01	0.12±0.00	0.16±0.00	0.22±0.00	0.37±0.04	0.12±0.00	0.13±0.00	0.14±0.00	**0.11±0.00**
ERO(p=1, style=gamma,η=0.2)	0.16±0.00	0.48±0.00	**0.11±0.00**	0.17±0.00	0.12±0.00	0.21±0.00	0.26±0.00	0.39±0.00	0.14±0.00	0.15±0.00	0.15±0.00	**0.11±0.00**
ERO(p=1, style=uniform,η=0.3)	0.19±0.00	0.47±0.00	**0.16±0.00**	0.21±0.01	0.17±0.00	0.21±0.00	0.25±0.00	0.40±0.00	0.17±0.00	0.18±0.00	0.18±0.00	**0.16±0.00**
ERO(p=1, style=gamma,η=0.3)	0.21±0.00	0.48±0.00	**0.16±0.00**	0.21±0.00	0.17±0.00	0.25±0.00	0.27±0.00	0.39±0.00	0.20±0.00	0.20±0.00	0.20±0.00	**0.16±0.00**
ERO(p=1, style=uniform,η=0.4)	0.23±0.00	0.48±0.00	**0.21±0.00**	0.25±0.01	0.22±0.00	0.25±0.00	0.28±0.00	0.47±0.00	0.22±0.00	0.23±0.00	0.23±0.00	**0.21±0.00**
ERO(p=1, style=gamma,η=0.4)	0.26±0.00	0.49±0.00	**0.22±0.00**	0.26±0.00	**0.22±0.00**	0.29±0.00	0.30±0.00	0.46±0.00	0.25±0.00	0.25±0.00	0.25±0.00	**0.22±0.00**
ERO(p=1, style=uniform,η=0.5)	0.28±0.00	0.49±0.00	**0.27±0.00**	0.30±0.01	**0.27±0.00**	0.30±0.00	0.31±0.00	0.44±0.00	**0.27±0.00**	0.28±0.00	**0.27±0.00**	**0.27±0.00**
ERO(p=1, style=gamma,η=0.5)	0.30±0.00	0.49±0.00	**0.27±0.00**	0.31±0.00	**0.27±0.00**	0.32±0.00	0.33±0.00	0.46±0.00	0.30±0.00	0.30±0.00	0.29±0.00	**0.27±0.00**
ERO(p=1, style=uniform,η=0.6)	0.33±0.00	0.48±0.00	**0.32±0.00**	0.34±0.01	**0.32±0.00**	0.34±0.00	0.35±0.00	0.50±0.00	**0.32±0.00**	0.33±0.00	**0.32±0.00**	**0.32±0.00**
ERO(p=1, style=gamma,η=0.6)	0.35±0.00	0.48±0.00	**0.32±0.00**	0.36±0.00	**0.32±0.00**	0.36±0.00	0.36±0.00	0.49±0.00	0.34±0.00	0.34±0.00	0.34±0.00	**0.32±0.00**
ERO(p=1, style=uniform,η=0.7)	0.38±0.00	0.49±0.00	**0.37±0.00**	0.39±0.01	**0.37±0.00**	0.38±0.00	0.39±0.00	0.47±0.00	**0.37±0.00**	0.38±0.00	**0.37±0.00**	**0.37±0.00**
ERO(p=1, style=gamma,η=0.7)	0.39±0.00	0.50±0.00	0.38±0.00	0.40±0.00	**0.37±0.00**	0.40±0.00	0.40±0.00	0.49±0.00	0.39±0.00	0.39±0.00	0.38±0.00	**0.37±0.00**
ERO(p=1, style=uniform,η=0.8)	<u>0.42±0.00</u>	0.49±0.00	0.49±0.00	0.43±0.00	<u>0.42±0.00</u>	0.43±0.00	0.43±0.00	0.48±0.00	<u>0.42±0.00</u>	<u>0.42±0.00</u>	<u>0.42±0.00</u>	**0.41±0.00**
ERO(p=1, style=gamma,η=0.8)	0.43±0.00	0.49±0.00	0.49±0.00	0.44±0.00	<u>0.42±0.00</u>	0.43±0.00	0.43±0.00	0.50±0.00	0.43±0.00	0.43±0.00	0.43±0.00	**0.41±0.00**

the best and the worst baseline method, across all real-world data sets. Again, $\mathcal{L}_{\text{upset, ratio}}$ results are only shown to illustrate the performance in terms of minimizing the loss function $\mathcal{L}_{\text{upset, ratio}}$, but are not used to compare methods. We conclude that "proximal baseline" with IB as aggregation GNN usually performs the best among the variants. Compared with baselines, our different variants can attain comparable and often superior performance, and are never strongly outperformed.

Table 10. Performance on $\mathcal{L}_{\text{upset, simple}}$ for each variant in the proposed GNNRank framework, compared with the worst and the best baseline method, averaged over 10 runs, and plus/minus one standard deviation. "avg" for time series data sets first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best is marked in **bold red** while the second best is highlighted in underline blue.

Type	Baselines		DIMPA					IB				
Data	worst baseline	best baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline
HeadToHead	2.01±0.00	1.00±0.00	1.07±0.01	1.07±0.01	1.12±0.01	1.09±0.01	0.97±0.00	0.99±0.00	0.99±0.00	0.99±0.00	0.99±0.00	**0.96±0.00**
Finance	1.98±0.00	1.61±0.00	1.63±0.00	1.63±0.00	1.63±0.00	1.63±0.00	1.61±0.00	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**	**1.00±0.00**	1.61±0.00
Animal	2.02±0.32	0.33±0.00	0.41±0.09	0.48±0.19	0.41±0.05	0.42±0.05	0.42±0.05	0.61±0.07	0.57±0.08	0.90±0.33	0.83±0.13	**0.25±0.00**
Faculty: Business	2.01±0.03	0.41±0.00	0.41±0.02	0.38±0.01	0.42±0.02	0.47±0.03	0.39±0.01	0.92±0.15	0.94±0.07	0.89±0.02	0.90±0.04	**0.36±0.00**
Faculty: CS	1.99±0.27	0.33±0.00	0.33±0.03	0.34±0.01	0.38±0.01	0.36±0.01	0.33±0.01	0.92±0.01	0.95±0.06	0.92±0.03	0.92±0.03	**0.32±0.00**
Faculty: History	2.16±0.80	0.30±0.01	0.30±0.01	**0.28±0.01**	0.30±0.01	0.31±0.01	0.30±0.01	0.92±0.01	0.94±0.08	0.93±0.06	0.90±0.06	0.30±0.01
Basketball(1985)	2.11±0.00	0.75±0.00	0.77±0.01	0.80±0.09	0.85±0.02	0.81±0.02	**0.71±0.01**	0.94±0.01	1.04±0.02	0.95±0.01	1.12±0.13	**0.71±0.00**
Basketball(1986)	1.99±0.00	0.78±0.00	0.79±0.01	0.78±0.01	0.83±0.02	0.82±0.01	0.70±0.00	0.90±0.02	1.01±0.02	0.92±0.02	1.02±0.03	**0.69±0.00**
Basketball(1987)	1.94±0.00	0.82±0.00	0.86±0.01	0.84±0.01	0.90±0.02	0.89±0.02	**0.77±0.01**	0.93±0.01	1.05±0.04	0.96±0.02	1.08±0.04	**0.77±0.00**
Basketball(1988)	1.93±0.00	0.74±0.00	0.76±0.01	0.78±0.08	0.81±0.02	0.80±0.01	**0.70±0.01**	0.91±0.01	1.01±0.01	0.95±0.03	1.02±0.02	**0.70±0.00**
Basketball(1989)	1.87±0.00	0.75±0.00	0.78±0.02	0.81±0.08	0.85±0.03	0.82±0.01	0.71±0.01	0.94±0.02	1.00±0.01	0.93±0.02	1.02±0.02	**0.70±0.00**
Basketball(1990)	1.95±0.00	0.75±0.00	0.77±0.02	0.81±0.01	0.84±0.02	0.81±0.02	**0.70±0.01**	0.93±0.02	1.01±0.03	0.95±0.02	1.02±0.03	**0.70±0.00**
Basketball(1991)	2.03±0.00	0.77±0.00	0.80±0.02	0.78±0.01	0.85±0.03	0.83±0.02	0.73±0.01	0.93±0.01	1.01±0.02	0.97±0.01	1.03±0.03	**0.70±0.00**
Basketball(1992)	2.00±0.00	0.72±0.00	0.76±0.01	0.74±0.01	0.80±0.01	0.78±0.02	0.69±0.01	0.93±0.01	1.02±0.02	0.97±0.02	1.05±0.04	**0.67±0.00**
Basketball(1993)	2.04±0.00	0.74±0.00	0.76±0.02	0.76±0.01	0.83±0.02	0.81±0.02	**0.68±0.01**	0.91±0.03	1.01±0.02	0.95±0.03	1.05±0.06	**0.68±0.00**
Basketball(1994)	2.04±0.00	0.74±0.00	0.76±0.02	0.75±0.01	0.82±0.02	0.80±0.02	0.71±0.01	0.90±0.02	1.02±0.03	0.94±0.01	1.08±0.08	**0.69±0.00**
Basketball(1995)	2.02±0.00	0.77±0.00	0.79±0.01	0.78±0.01	0.83±0.02	0.81±0.02	**0.72±0.01**	0.93±0.02	1.03±0.02	0.96±0.01	1.05±0.03	**0.72±0.00**
Basketball(1996)	2.05±0.00	0.81±0.00	0.84±0.01	0.84±0.01	0.92±0.01	0.89±0.01	**0.77±0.01**	0.92±0.02	1.03±0.05	0.94±0.01	1.09±0.08	**0.77±0.00**
Basketball(1997)	1.95±0.00	0.81±0.00	0.84±0.01	0.83±0.01	0.90±0.02	0.87±0.02	**0.76±0.01**	0.94±0.01	1.05±0.04	0.96±0.01	1.11±0.09	0.77±0.00
Basketball(1998)	1.92±0.00	0.77±0.00	0.81±0.02	0.80±0.01	0.88±0.02	0.84±0.02	**0.74±0.01**	0.90±0.02	1.01±0.01	0.94±0.02	1.04±0.06	0.75±0.00
Basketball(1999)	2.02±0.00	0.81±0.00	0.84±0.01	0.86±0.06	0.91±0.02	0.88±0.03	0.76±0.01	0.96±0.01	1.01±0.01	0.98±0.01	1.03±0.02	**0.74±0.00**
Basketball(2000)	1.97±0.00	0.84±0.00	0.84±0.01	0.82±0.01	0.92±0.01	0.90±0.02	**0.77±0.01**	0.94±0.01	1.01±0.01	0.95±0.02	1.04±0.04	**0.77±0.00**
Basketball(2001)	2.03±0.00	0.75±0.00	0.78±0.02	0.77±0.01	0.87±0.02	0.83±0.02	**0.71±0.01**	0.96±0.02	1.01±0.01	0.95±0.01	1.03±0.03	**0.71±0.00**
Basketball(2002)	1.94±0.00	0.80±0.00	0.81±0.01	0.80±0.01	0.86±0.02	0.84±0.03	**0.73±0.01**	0.91±0.01	1.02±0.02	0.93±0.03	1.06±0.11	**0.73±0.00**
Basketball(2003)	2.05±0.00	0.80±0.00	0.85±0.02	0.83±0.01	0.91±0.02	0.88±0.02	**0.78±0.01**	0.92±0.01	1.00±0.01	0.95±0.03	1.01±0.02	**0.78±0.00**
Basketball(2004)	2.00±0.11	0.75±0.00	0.77±0.01	0.78±0.01	0.84±0.02	0.81±0.02	0.70±0.01	0.93±0.01	0.99±0.01	0.96±0.01	1.05±0.15	**0.69±0.00**
Basketball(2005)	1.98±0.05	0.78±0.00	0.82±0.01	0.81±0.02	0.88±0.02	0.86±0.01	**0.73±0.00**	0.91±0.01	1.02±0.03	0.95±0.02	1.08±0.08	0.74±0.00
Basketball(2006)	2.00±0.00	0.77±0.00	0.82±0.01	0.83±0.01	0.90±0.02	0.88±0.02	0.75±0.01	0.94±0.02	1.01±0.02	0.95±0.01	1.04±0.05	**0.74±0.00**
Basketball(2007)	1.94±0.00	0.79±0.00	0.84±0.01	0.83±0.01	0.88±0.01	0.87±0.02	**0.77±0.01**	0.93±0.01	1.00±0.01	0.95±0.01	1.02±0.02	**0.77±0.00**
Basketball(2008)	2.01±0.00	0.80±0.00	0.83±0.01	0.84±0.04	0.88±0.02	0.86±0.01	0.79±0.01	0.92±0.02	1.00±0.01	0.95±0.02	1.01±0.04	**0.78±0.00**
Basketball(2009)	2.02±0.00	0.76±0.00	0.78±0.00	0.78±0.01	0.83±0.01	0.83±0.01	**0.72±0.01**	0.93±0.01	1.00±0.01	0.93±0.02	1.01±0.02	**0.72±0.00**
Basketball(2010)	2.03±0.00	0.77±0.00	0.83±0.01	0.82±0.01	0.89±0.01	0.86±0.01	0.73±0.01	0.89±0.01	1.01±0.02	0.92±0.02	1.01±0.02	**0.71±0.00**
Basketball(2011)	2.03±0.00	0.79±0.00	0.82±0.01	0.80±0.01	0.86±0.01	0.85±0.01	0.75±0.00	0.91±0.02	1.00±0.01	0.93±0.02	1.00±0.01	**0.74±0.00**
Basketball(2012)	1.97±0.00	0.76±0.00	0.78±0.02	0.78±0.01	0.84±0.02	0.83±0.02	0.70±0.01	0.90±0.03	1.00±0.01	0.94±0.03	1.01±0.01	**0.69±0.00**
Basketball(2013)	1.99±0.05	0.80±0.00	0.81±0.01	0.83±0.06	0.87±0.03	0.85±0.01	**0.76±0.00**	0.94±0.01	1.00±0.01	0.94±0.01	1.01±0.01	0.77±0.00
Basketball(2014)	2.06±0.00	0.79±0.00	0.84±0.01	0.83±0.01	0.89±0.02	0.88±0.01	**0.75±0.00**	0.94±0.02	1.01±0.01	0.97±0.01	1.02±0.02	0.76±0.00
Basketball finer(1985)	1.97±0.00	0.76±0.00	0.81±0.02	0.79±0.01	0.80±0.02	0.79±0.01	0.72±0.01	1.00±0.00	1.09±0.06	1.00±0.00	1.21±0.11	**0.71±0.00**
Basketball finer(1986)	1.99±0.00	0.77±0.00	0.84±0.02	0.81±0.01	0.91±0.03	0.81±0.01	**0.69±0.01**	1.00±0.00	1.11±0.06	1.00±0.00	1.19±0.10	**0.69±0.00**
Basketball finer(1987)	1.95±0.00	0.82±0.00	0.86±0.02	0.90±0.01	0.87±0.03	0.90±0.01	**0.77±0.01**	1.00±0.00	1.09±0.06	1.00±0.00	1.23±0.09	**0.77±0.00**
Basketball finer(1988)	1.97±0.00	0.78±0.00	0.81±0.01	0.80±0.01	0.81±0.01	0.80±0.01	**0.70±0.01**	1.00±0.00	1.10±0.11	1.00±0.00	1.22±0.14	**0.70±0.00**
Basketball finer(1989)	1.94±0.00	0.77±0.00	0.82±0.02	0.82±0.01	0.82±0.02	0.82±0.01	0.73±0.05	1.00±0.00	1.07±0.06	1.00±0.00	1.19±0.08	**0.70±0.00**
Basketball finer(1990)	1.98±0.00	0.79±0.00	0.80±0.02	0.80±0.01	0.81±0.02	0.81±0.01	0.72±0.01	1.00±0.00	1.10±0.06	1.00±0.00	1.20±0.12	**0.71±0.00**
Basketball finer(1991)	2.03±0.00	0.81±0.00	0.83±0.01	0.84±0.01	0.83±0.07	0.84±0.01	0.72±0.01	1.00±0.00	1.11±0.06	1.00±0.00	1.20±0.11	**0.71±0.00**
Basketball finer(1992)	1.88±0.00	0.73±0.00	0.75±0.02	0.76±0.01	0.75±0.02	0.76±0.01	0.68±0.01	1.00±0.00	1.08±0.06	1.00±0.00	1.21±0.07	**0.67±0.00**
Basketball finer(1993)	2.03±0.00	0.75±0.00	0.80±0.01	0.80±0.01	0.81±0.06	0.80±0.01	**0.68±0.01**	1.00±0.00	1.08±0.06	1.00±0.00	1.22±0.08	0.69±0.00
Basketball finer(1994)	2.01±0.00	0.74±0.00	0.77±0.01	0.78±0.01	0.77±0.01	0.78±0.01	0.69±0.01	1.00±0.00	1.10±0.06	1.00±0.00	1.21±0.06	**0.67±0.00**
Basketball finer(1995)	1.92±0.01	0.79±0.00	0.82±0.09	0.78±0.01	0.82±0.07	0.78±0.01	**0.72±0.01**	1.00±0.00	1.08±0.03	1.00±0.00	1.19±0.07	0.73±0.00
Basketball finer(1996)	1.95±0.00	0.81±0.00	0.86±0.01	0.90±0.01	0.87±0.01	0.90±0.01	**0.77±0.00**	1.00±0.00	1.07±0.04	1.00±0.00	1.15±0.08	**0.77±0.00**
Basketball finer(1997)	2.05±0.00	0.83±0.00	0.85±0.01	0.86±0.01	0.86±0.01	0.86±0.01	**0.75±0.01**	1.00±0.00	1.06±0.05	1.00±0.00	1.21±0.11	0.77±0.00
Basketball finer(1998)	1.92±0.00	0.78±0.00	0.82±0.02	0.86±0.01	0.86±0.02	0.86±0.01	**0.73±0.01**	1.00±0.00	1.08±0.05	1.00±0.00	1.23±0.07	0.74±0.00
Basketball finer(1999)	2.02±0.00	0.81±0.00	0.86±0.01	0.86±0.01	0.87±0.02	0.86±0.01	0.74±0.01	1.00±0.00	1.06±0.04	1.00±0.00	1.18±0.07	**0.73±0.00**
Basketball finer(2000)	1.97±0.00	0.84±0.00	0.88±0.01	0.90±0.01	0.88±0.01	0.90±0.01	**0.78±0.01**	1.00±0.00	1.09±0.06	1.00±0.00	1.24±0.03	**0.78±0.00**
Basketball finer(2001)	2.06±0.00	0.81±0.00	0.86±0.01	0.87±0.00	0.86±0.01	0.87±0.00	**0.73±0.01**	1.00±0.00	1.08±0.05	1.00±0.00	1.18±0.09	**0.73±0.00**
Basketball finer(2002)	2.03±0.00	0.87±0.00	0.83±0.01	0.84±0.01	0.83±0.02	0.84±0.01	**0.77±0.01**	1.00±0.00	1.06±0.04	1.00±0.00	1.18±0.07	0.78±0.00
Basketball finer(2003)	2.00±0.00	0.87±0.00	0.88±0.02	0.89±0.01	0.89±0.02	0.89±0.01	0.79±0.00	1.00±0.00	1.08±0.06	1.00±0.00	1.18±0.10	**0.78±0.00**
Basketball finer(2004)	1.98±0.02	0.77±0.00	0.84±0.01	0.84±0.01	0.93±0.01	0.84±0.01	**0.71±0.01**	1.00±0.00	1.04±0.04	1.00±0.00	1.19±0.09	0.72±0.00
Basketball finer(2005)	2.00±0.00	0.84±0.00	0.86±0.01	0.87±0.01	0.87±0.01	0.87±0.01	**0.75±0.01**	1.00±0.00	1.08±0.08	1.00±0.00	1.17±0.08	**0.75±0.00**
Basketball finer(2006)	1.97±0.00	0.85±0.00	0.88±0.02	0.90±0.01	0.88±0.02	0.90±0.00	0.77±0.01	1.00±0.00	1.08±0.05	1.00±0.00	1.21±0.08	**0.76±0.00**
Basketball finer(2007)	1.97±0.14	0.86±0.00	0.87±0.01	0.89±0.01	0.88±0.01	0.89±0.01	**0.80±0.01**	1.00±0.00	1.07±0.08	1.00±0.00	1.18±0.09	**0.80±0.00**
Basketball finer(2008)	1.98±0.00	0.85±0.00	0.86±0.02	0.87±0.02	0.86±0.02	0.88±0.02	0.79±0.01	1.00±0.00	1.07±0.07	1.00±0.00	1.20±0.11	**0.78±0.00**
Basketball finer(2009)	2.06±0.00	0.84±0.00	0.85±0.01	0.85±0.01	0.86±0.02	0.85±0.01	0.76±0.01	1.00±0.00	1.09±0.06	1.00±0.00	1.20±0.13	**0.75±0.00**
Basketball finer(2010)	1.98±0.00	0.82±0.00	0.85±0.01	0.85±0.01	0.86±0.02	0.85±0.01	**0.75±0.01**	1.00±0.00	1.02±0.03	1.00±0.00	1.12±0.05	**0.75±0.00**
Basketball finer(2011)	2.01±0.00	0.85±0.00	0.87±0.01	0.87±0.01	0.87±0.01	0.87±0.01	0.78±0.01	1.00±0.00	1.08±0.05	1.00±0.00	1.19±0.07	**0.77±0.00**
Basketball finer(2012)	1.97±0.00	0.80±0.00	0.83±0.01	0.85±0.01	0.84±0.01	0.85±0.01	**0.74±0.01**	1.00±0.00	1.10±0.07	1.00±0.00	1.19±0.07	0.75±0.00
Basketball finer(2013)	2.06±0.00	0.83±0.00	0.86±0.01	0.85±0.00	0.86±0.01	0.85±0.00	**0.78±0.01**	1.00±0.00	1.11±0.04	1.00±0.00	1.22±0.07	0.79±0.00
Basketball finer(2014)	2.07±0.00	0.84±0.00	0.88±0.03	0.87±0.01	0.89±0.03	0.87±0.00	**0.78±0.00**	1.00±0.00	1.11±0.10	1.00±0.00	1.22±0.09	0.79±0.00
Football(2009)	1.80±0.09	**0.61±0.00**	0.76±0.02	0.75±0.06	0.98±0.23	0.87±0.14	0.65±0.03	0.79±0.04	0.83±0.11	0.80±0.05	0.88±0.13	**0.61±0.00**
Football(2010)	1.94±0.20	1.17±0.00	1.20±0.05	1.21±0.04	1.15±0.14	1.26±0.07	1.17±0.04	**0.90±0.02**	0.96±0.05	0.96±0.04	0.95±0.05	1.17±0.01
Football(2011)	1.80±0.19	**0.80±0.00**	0.87±0.02	0.91±0.15	1.16±0.21	1.01±0.12	0.82±0.03	0.83±0.03	0.96±0.07	0.85±0.09	0.91±0.08	**0.80±0.01**
Football(2012)	1.73±0.18	0.86±0.00	0.81±0.09	0.83±0.17	0.98±0.11	0.97±0.10	0.85±0.04	**0.75±0.03**	0.86±0.12	0.80±0.08	0.82±0.11	0.83±0.01
Football(2013)	1.93±0.09	0.68±0.00	0.64±0.04	0.66±0.08	0.73±0.15	0.83±0.24	0.59±0.02	0.71±0.05	0.84±0.18	0.76±0.07	0.80±0.08	**0.56±0.00**
Football(2014)	1.83±0.11	1.08±0.00	0.97±0.06	0.98±0.17	1.22±0.12	1.18±0.12	1.06±0.06	**0.96±0.12**	1.08±0.27	**0.96±0.07**	1.03±0.03	1.08±0.00
Football finer(2009)	1.91±0.17	0.66±0.00	0.76±0.03	0.76±0.18	0.80±0.08	0.83±0.09	0.68±0.02	0.99±0.04	0.98±0.05	0.95±0.12	0.98±0.04	**0.65±0.02**
Football finer(2010)	2.11±0.01	1.17±0.00	1.23±0.09	1.25±0.16	1.23±0.04	1.27±0.06	1.20±0.04	1.00±0.01	1.00±0.01	1.00±0.00	**0.99±0.02**	1.17±0.01
Football finer(2011)	2.14±0.02	0.85±0.00	0.92±0.16	1.00±0.09	1.00±0.09	0.95±0.10	0.85±0.03	0.99±0.04	0.99±0.02	0.99±0.04	0.97±0.09	**0.84±0.02**
Football finer(2012)	2.15±0.01	**0.86±0.00**	0.93±0.07	0.99±0.05	0.95±0.05	0.97±0.06	0.89±0.07	1.00±0.00	0.99±0.03	0.99±0.03	0.98±0.05	**0.86±0.03**
Football finer(2013)	2.01±0.00	0.73±0.00	0.74±0.04	0.73±0.19	0.82±0.10	0.88±0.23	0.62±0.04	0.99±0.04	0.98±0.05	0.94±0.07	0.94±0.08	**0.57±0.01**
Football finer(2014)	2.09±0.17	1.05±0.00	1.07±0.05	1.10±0.15	1.22±0.11	1.20±0.26	1.07±0.06	**1.00±0.00**	1.03±0.10	**1.00±0.03**	1.03±0.08	1.07±0.02

E.4. Ablation Study Full Tables

Tables 13, 14 and 15 extend results of the ablation study to all seasons, and additionally report $\mathcal{L}_{\text{upset, ratio}}$ and $\mathcal{L}_{\text{upset, naive}}$ results. Again, $\mathcal{L}_{\text{upset, ratio}}$ results are only shown to illustrate the performance in terms of minimizing the loss function $\mathcal{L}_{\text{upset, ratio}}$, but are not used to compare methods.

E.5. Inductive Learning Full Tables

Tables 16 and 18 contain results on the performance of the "IB proximal baseline" variant, trained with the "emb baseline" on the *Basketball finer* data set. The second to last column contains results of directly applying the model trained for season 1985 without further training, while the last column is the result for the model specifically trained

Table 11. Performance on $\mathcal{L}_{\text{upset, naive}}$ for each variant in the proposed GNNRank framework, compared with the worst and the best baseline method, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue.

Type	Baselines		DIMPA					IB				
Data	worst baseline	best baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline
HeadToHead	0.50±0.00	0.25±0.00	0.27±0.00	0.27±0.00	0.28±0.00	0.27±0.00	**0.24±0.00**	0.55±0.07	0.49±0.05	0.64±0.08	0.51±0.03	**0.24±0.00**
Finance	0.50±0.00	**0.40±0.00**	0.41±0.00	0.41±0.00	0.41±0.00	0.41±0.00	**0.40±0.00**	0.44±0.00	0.46±0.02	0.45±0.01	0.45±0.01	**0.40±0.00**
Animal	0.58±0.11	0.08±0.00	0.10±0.02	0.12±0.05	0.10±0.01	0.10±0.01	0.11±0.01	0.52±0.12	0.36±0.09	0.48±0.09	0.39±0.10	**0.06±0.00**
Faculty: Business	0.50±0.01	0.10±0.00	0.10±0.00	0.10±0.00	0.11±0.00	0.12±0.01	0.10±0.00	0.24±0.07	0.24±0.02	0.25±0.07	0.23±0.01	**0.09±0.00**
Faculty: CS	0.50±0.07	**0.08±0.00**	**0.08±0.01**	0.09±0.00	0.09±0.00	0.09±0.00	**0.08±0.00**	0.48±0.02	0.44±0.07	0.44±0.02	0.46±0.03	**0.08±0.00**
Faculty: History	0.54±0.20	0.08±0.00	**0.07±0.00**	**0.07±0.00**	**0.07±0.00**	0.08±0.00	0.08±0.00	0.50±0.08	0.43±0.16	0.51±0.24	0.46±0.21	0.08±0.00
Basketball(1985)	0.53±0.00	0.19±0.00	0.19±0.00	0.20±0.02	0.21±0.01	0.20±0.00	**0.18±0.00**	0.56±0.10	0.49±0.13	0.79±0.23	0.54±0.13	**0.18±0.00**
Basketball(1986)	0.50±0.00	0.19±0.00	0.20±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.18±0.00	0.61±0.15	0.46±0.11	0.77±0.18	0.54±0.07	**0.17±0.00**
Basketball(1987)	0.48±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.23±0.01	0.22±0.00	**0.19±0.00**	0.67±0.21	0.46±0.10	0.75±0.24	0.57±0.06	**0.19±0.00**
Basketball(1988)	0.48±0.00	0.19±0.00	0.19±0.00	0.19±0.02	0.20±0.01	0.20±0.00	0.18±0.00	0.72±0.06	0.46±0.14	0.82±0.09	0.56±0.09	**0.17±0.00**
Basketball(1989)	0.47±0.00	0.19±0.00	0.19±0.00	0.20±0.02	0.21±0.01	0.21±0.00	0.18±0.00	0.50±0.16	0.49±0.09	0.67±0.27	0.52±0.11	**0.17±0.00**
Basketball(1990)	0.49±0.00	0.19±0.00	0.19±0.00	0.20±0.00	0.21±0.00	0.20±0.01	0.18±0.00	0.61±0.12	0.46±0.09	0.80±0.21	0.53±0.09	**0.17±0.00**
Basketball(1991)	0.51±0.00	0.19±0.00	0.20±0.00	0.19±0.00	0.21±0.01	0.21±0.00	**0.18±0.00**	0.72±0.06	0.48±0.07	0.88±0.15	0.55±0.08	**0.18±0.00**
Basketball(1992)	0.50±0.00	0.18±0.00	0.19±0.00	0.18±0.00	0.20±0.00	0.20±0.00	**0.17±0.00**	0.56±0.20	0.43±0.08	0.70±0.27	0.54±0.08	**0.17±0.00**
Basketball(1993)	0.51±0.00	0.19±0.00	0.19±0.00	0.19±0.00	0.21±0.00	0.20±0.01	**0.17±0.00**	0.67±0.11	0.51±0.07	0.85±0.21	0.57±0.10	**0.17±0.00**
Basketball(1994)	0.51±0.00	0.18±0.00	0.19±0.00	0.19±0.00	0.20±0.01	0.20±0.00	0.18±0.00	0.52±0.22	0.43±0.08	0.63±0.29	0.52±0.08	**0.17±0.00**
Basketball(1995)	0.50±0.00	0.19±0.00	0.20±0.00	0.19±0.00	0.21±0.00	0.20±0.01	**0.18±0.00**	0.57±0.17	0.46±0.08	0.77±0.24	0.53±0.07	**0.18±0.00**
Basketball(1996)	0.51±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.23±0.00	0.22±0.00	**0.19±0.00**	0.65±0.15	0.49±0.08	0.84±0.20	0.57±0.04	**0.19±0.00**
Basketball(1997)	0.49±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.66±0.17	0.48±0.07	0.82±0.19	0.53±0.06	**0.19±0.00**
Basketball(1998)	0.48±0.00	0.19±0.00	0.20±0.00	0.20±0.00	0.22±0.01	0.21±0.00	**0.18±0.00**	0.65±0.20	0.45±0.08	0.78±0.26	0.56±0.08	0.19±0.00
Basketball(1999)	0.50±0.00	0.20±0.00	0.21±0.00	0.22±0.01	0.23±0.00	0.22±0.01	0.19±0.00	0.79±0.07	0.45±0.07	0.90±0.10	0.60±0.12	**0.18±0.00**
Basketball(2000)	0.49±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.23±0.00	0.22±0.01	**0.19±0.00**	0.62±0.14	0.45±0.06	0.80±0.17	0.54±0.09	**0.19±0.00**
Basketball(2001)	0.51±0.00	0.19±0.00	0.20±0.01	0.19±0.00	0.22±0.01	0.21±0.00	**0.18±0.00**	0.49±0.16	0.46±0.05	0.67±0.30	0.51±0.10	**0.18±0.00**
Basketball(2002)	0.49±0.00	0.20±0.00	0.20±0.00	0.20±0.00	0.22±0.00	0.21±0.01	**0.18±0.00**	0.48±0.21	0.49±0.06	0.61±0.28	0.50±0.08	**0.18±0.00**
Basketball(2003)	0.51±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.23±0.00	0.22±0.00	0.20±0.00	0.55±0.13	0.52±0.10	0.79±0.18	0.54±0.06	**0.19±0.00**
Basketball(2004)	0.50±0.03	0.19±0.00	0.19±0.00	0.19±0.00	0.21±0.00	0.20±0.00	**0.17±0.00**	0.74±0.12	0.45±0.06	0.86±0.07	0.60±0.09	**0.17±0.00**
Basketball(2005)	0.49±0.01	0.19±0.00	0.21±0.00	0.20±0.00	0.22±0.01	0.21±0.00	**0.18±0.00**	0.73±0.08	0.48±0.08	0.84±0.20	0.57±0.11	**0.18±0.00**
Basketball(2006)	0.50±0.00	**0.19±0.00**	0.21±0.00	0.21±0.00	0.23±0.00	0.22±0.01	**0.19±0.00**	0.60±0.18	0.46±0.06	0.76±0.26	0.56±0.08	**0.19±0.00**
Basketball(2007)	0.49±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.55±0.16	0.49±0.09	0.76±0.25	0.55±0.07	**0.19±0.00**
Basketball(2008)	0.50±0.00	**0.20±0.00**	0.21±0.00	0.21±0.01	0.22±0.00	0.21±0.00	**0.20±0.00**	0.72±0.17	0.47±0.09	0.86±0.20	0.57±0.10	**0.20±0.00**
Basketball(2009)	0.51±0.00	0.19±0.00	0.20±0.00	0.19±0.00	0.21±0.00	0.21±0.00	**0.18±0.00**	0.48±0.15	0.47±0.07	0.70±0.23	0.51±0.12	**0.18±0.00**
Basketball(2010)	0.51±0.00	0.19±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	**0.18±0.00**	0.54±0.20	0.41±0.04	0.68±0.23	0.52±0.15	**0.18±0.00**
Basketball(2011)	0.51±0.00	0.20±0.00	0.20±0.00	0.20±0.00	0.22±0.00	0.21±0.00	0.19±0.00	0.50±0.13	0.46±0.14	0.73±0.21	0.55±0.14	**0.18±0.00**
Basketball(2012)	0.49±0.00	0.19±0.00	0.19±0.00	0.19±0.00	0.21±0.00	0.21±0.00	0.18±0.00	0.64±0.10	0.48±0.13	0.81±0.21	0.56±0.11	**0.17±0.00**
Basketball(2013)	0.50±0.01	0.20±0.00	0.20±0.00	0.21±0.02	0.22±0.01	0.21±0.00	**0.19±0.00**	0.49±0.16	0.46±0.10	0.66±0.26	0.50±0.08	**0.19±0.00**
Basketball(2014)	0.52±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.76±0.05	0.48±0.12	0.89±0.10	0.59±0.12	**0.19±0.00**
Basketball finer(1985)	0.49±0.00	0.19±0.00	0.20±0.01	0.20±0.00	0.20±0.01	0.20±0.00	**0.18±0.00**	0.99±0.02	0.81±0.12	0.99±0.01	0.75±0.06	**0.18±0.00**
Basketball finer(1986)	0.50±0.00	0.19±0.00	0.21±0.00	0.20±0.00	0.23±0.01	0.20±0.00	**0.17±0.00**	0.99±0.01	0.82±0.10	0.98±0.06	0.76±0.06	**0.17±0.00**
Basketball finer(1987)	0.49±0.00	0.20±0.00	0.21±0.01	0.23±0.00	0.22±0.01	0.23±0.00	**0.19±0.00**	0.99±0.02	0.79±0.12	0.98±0.06	0.74±0.03	**0.19±0.00**
Basketball finer(1988)	0.49±0.00	0.19±0.00	0.20±0.00	0.20±0.00	0.20±0.00	0.20±0.00	**0.18±0.00**	0.99±0.01	0.76±0.15	0.97±0.09	0.73±0.09	**0.18±0.00**
Basketball finer(1989)	0.49±0.00	0.19±0.00	0.20±0.01	0.20±0.00	0.20±0.01	0.20±0.00	**0.18±0.01**	0.98±0.05	0.75±0.15	1.00±0.01	0.71±0.12	**0.18±0.00**
Basketball finer(1990)	0.50±0.00	0.20±0.00	0.20±0.01	0.20±0.00	0.20±0.01	0.20±0.00	**0.18±0.00**	0.98±0.05	0.79±0.15	1.00±0.01	0.74±0.04	**0.18±0.00**
Basketball finer(1991)	0.51±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.23±0.09	0.21±0.00	**0.18±0.00**	0.96±0.10	0.78±0.14	0.97±0.08	0.71±0.11	**0.18±0.00**
Basketball finer(1992)	0.47±0.00	0.18±0.00	0.19±0.00	0.19±0.00	0.19±0.01	0.19±0.00	**0.17±0.00**	0.94±0.10	0.77±0.18	0.99±0.02	0.72±0.14	**0.17±0.00**
Basketball finer(1993)	0.51±0.00	0.19±0.00	0.20±0.00	0.20±0.00	0.22±0.01	0.20±0.00	**0.17±0.00**	0.98±0.04	0.79±0.15	1.00±0.01	0.75±0.05	**0.17±0.00**
Basketball finer(1994)	0.50±0.00	0.18±0.00	0.19±0.00	0.20±0.00	0.19±0.00	0.20±0.00	**0.17±0.00**	0.98±0.04	0.80±0.14	0.98±0.06	0.75±0.10	**0.17±0.00**
Basketball finer(1995)	0.48±0.00	0.20±0.00	0.21±0.00	0.20±0.00	0.22±0.00	0.20±0.00	**0.18±0.00**	0.94±0.08	0.77±0.18	0.99±0.02	0.73±0.15	**0.18±0.00**
Basketball finer(1996)	0.49±0.00	0.20±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.97±0.06	0.79±0.15	0.97±0.07	0.74±0.13	**0.19±0.00**
Basketball finer(1997)	0.51±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.96±0.07	0.79±0.16	0.99±0.03	0.74±0.13	**0.19±0.00**
Basketball finer(1998)	0.48±0.00	0.20±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.18±0.00**	0.99±0.02	0.83±0.12	1.00±0.00	0.75±0.03	**0.18±0.00**
Basketball finer(1999)	0.50±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.21±0.00	**0.18±0.00**	0.99±0.03	0.84±0.10	0.99±0.01	0.77±0.04	**0.18±0.00**
Basketball finer(2000)	0.49±0.00	0.21±0.00	0.22±0.00	0.23±0.00	0.22±0.00	0.23±0.00	0.20±0.00	1.00±0.01	0.84±0.11	1.00±0.00	0.75±0.08	**0.19±0.00**
Basketball finer(2001)	0.51±0.00	0.20±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.18±0.00**	0.97±0.09	0.78±0.12	0.99±0.02	0.75±0.05	**0.18±0.00**
Basketball finer(2002)	0.51±0.00	0.22±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.19±0.00**	0.98±0.07	0.83±0.07	0.97±0.10	0.77±0.06	**0.19±0.00**
Basketball finer(2003)	0.50±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.20±0.00	0.98±0.05	0.84±0.02	1.00±0.01	0.77±0.04	**0.19±0.00**
Basketball finer(2004)	0.49±0.00	0.19±0.00	0.21±0.00	0.21±0.00	0.23±0.00	0.21±0.00	**0.18±0.00**	0.98±0.05	0.79±0.15	1.00±0.01	0.72±0.13	**0.18±0.00**
Basketball finer(2005)	0.50±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	0.99±0.02	0.83±0.11	0.98±0.06	0.74±0.05	**0.19±0.00**
Basketball finer(2006)	0.49±0.00	0.21±0.00	0.22±0.01	0.23±0.00	0.22±0.01	0.23±0.00	**0.19±0.00**	0.99±0.03	0.80±0.13	0.99±0.02	0.73±0.05	**0.19±0.00**
Basketball finer(2007)	0.49±0.03	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.20±0.00**	1.00±0.00	0.82±0.02	1.00±0.00	0.76±0.05	**0.20±0.00**
Basketball finer(2008)	0.49±0.00	0.21±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.20±0.00**	1.00±0.00	0.84±0.02	1.00±0.00	0.77±0.05	**0.20±0.00**
Basketball finer(2009)	0.52±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.19±0.00**	0.98±0.05	0.82±0.11	1.00±0.00	0.76±0.07	**0.19±0.00**
Basketball finer(2010)	0.49±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.19±0.00**	0.99±0.02	0.83±0.01	1.00±0.01	0.75±0.05	**0.19±0.00**
Basketball finer(2011)	0.50±0.00	0.21±0.00	0.22±0.00	0.22±0.00	0.22±0.00	0.22±0.00	**0.19±0.00**	1.00±0.00	0.84±0.04	1.00±0.00	0.75±0.04	**0.19±0.00**
Basketball finer(2012)	0.49±0.00	0.20±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.18±0.00**	1.00±0.01	0.83±0.10	0.99±0.04	0.77±0.05	0.19±0.00
Basketball finer(2013)	0.52±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.20±0.00**	0.99±0.03	0.82±0.11	0.98±0.06	0.76±0.04	**0.20±0.00**
Basketball finer(2014)	0.52±0.00	0.21±0.00	0.22±0.01	0.22±0.00	0.22±0.01	0.22±0.00	**0.20±0.00**	1.00±0.01	0.82±0.02	1.00±0.01	0.76±0.04	**0.20±0.00**
Football(2009)	0.45±0.02	**0.15±0.00**	0.19±0.01	0.19±0.01	0.24±0.06	0.22±0.04	0.16±0.01	0.29±0.07	0.24±0.05	0.36±0.09	0.27±0.04	**0.15±0.00**
Football(2010)	0.49±0.05	**0.29±0.00**	0.30±0.01	0.30±0.01	0.31±0.01	0.31±0.02	**0.29±0.01**	0.37±0.05	0.38±0.04	0.48±0.10	0.38±0.04	**0.29±0.00**
Football(2011)	0.45±0.05	**0.20±0.00**	0.22±0.01	0.23±0.04	0.29±0.05	0.25±0.03	0.21±0.01	0.43±0.23	0.34±0.07	0.46±0.22	0.34±0.08	**0.20±0.00**
Football(2012)	0.43±0.04	0.22±0.00	**0.20±0.02**	0.21±0.04	0.24±0.03	0.24±0.02	0.21±0.01	0.25±0.02	0.30±0.08	0.32±0.07	0.30±0.09	0.21±0.00
Football(2013)	0.48±0.02	0.17±0.00	0.16±0.01	0.16±0.02	0.18±0.04	0.21±0.06	0.15±0.01	0.34±0.06	0.24±0.02	0.46±0.12	0.34±0.06	**0.14±0.00**
Football(2014)	0.46±0.03	0.27±0.00	**0.24±0.01**	0.25±0.04	0.31±0.03	0.30±0.03	0.26±0.01	0.28±0.02	0.32±0.04	0.27±0.03	0.29±0.04	0.27±0.00
Football finer(2009)	0.48±0.04	**0.16±0.00**	0.19±0.01	0.19±0.05	0.21±0.03	0.21±0.02	0.17±0.00	0.58±0.33	0.29±0.07	0.59±0.28	0.32±0.10	**0.16±0.01**
Football finer(2010)	0.53±0.00	**0.29±0.00**	0.31±0.02	0.31±0.04	0.31±0.01	0.32±0.02	0.30±0.01	0.64±0.24	0.43±0.06	0.62±0.23	0.41±0.04	**0.29±0.00**
Football finer(2011)	0.53±0.00	**0.21±0.00**	0.23±0.04	0.25±0.02	0.25±0.02	0.24±0.03	**0.21±0.01**	0.52±0.25	0.32±0.07	0.48±0.22	0.33±0.07	**0.21±0.01**
Football finer(2012)	0.54±0.00	0.22±0.00	0.24±0.02	0.25±0.01	0.24±0.01	0.24±0.02	0.22±0.02	0.77±0.31	0.39±0.12	0.59±0.29	0.36±0.05	**0.21±0.01**
Football finer(2013)	0.50±0.00	0.18±0.00	0.19±0.01	0.18±0.05	0.20±0.03	0.22±0.06	0.15±0.01	0.67±0.29	0.28±0.04	0.52±0.25	0.32±0.06	**0.14±0.00**
Football finer(2014)	0.52±0.04	**0.26±0.00**	0.27±0.01	0.28±0.04	0.30±0.03	0.30±0.06	0.27±0.02	0.40±0.03	0.45±0.06	0.43±0.03	0.44±0.06	0.27±0.01
ERO(p=0.05, style=uniform,η=0)	0.49±0.03	**0.03±0.00**	0.04±0.00	0.06±0.03	0.05±0.01	0.06±0.04	0.05±0.01	0.06±0.00	0.05±0.00	0.05±0.00	0.05±0.00	0.05±0.00
ERO(p=0.05, style=gamma,η=0)	0.51±0.00	**0.02±0.00**	0.04±0.00	0.07±0.06	0.04±0.00	0.04±0.00	0.04±0.01	0.10±0.00	0.09±0.00	0.09±0.00	0.09±0.00	0.07±0.01
ERO(p=0.05, style=uniform,η=0.1)	0.50±0.01	0.11±0.00	0.14±0.01	0.14±0.02	0.17±0.06	0.14±0.01	**0.09±0.00**	0.13±0.00	0.13±0.01	0.13±0.01	0.13±0.00	0.13±0.00
ERO(p=0.05, style=gamma,η=0.1)	0.51±0.00	0.11±0.00	0.19±0.00	0.21±0.00	0.24±0.01	0.23±0.01	**0.09±0.00**	0.20±0.00	0.22±0.03	0.20±0.00	0.20±0.01	0.11±0.01
ERO(p=0.05, style=uniform,η=0.2)	0.50±0.00	0.15±0.00	0.19±0.00	0.20±0.02	0.21±0.05	0.20±0.00	**0.13±0.00**	0.19±0.00	0.19±0.00	0.19±0.01	0.20±0.00	0.19±0.01
ERO(p=0.05, style=gamma,η=0.2)	0.52±0.00	0.15±0.00	0.24±0.00	0.27±0.00	0.32±0.01	0.28±0.01	**0.14±0.00**	0.27±0.00	0.29±0.00	0.28±0.01	0.29±0.01	**0.14±0.01**
ERO(p=0.05, style=uniform,η=0.3)	0.51±0.00	0.20±0.00	0.24±0.00	0.24±0.02	0.26±0.00	0.25±0.00	**0.18±0.01**	0.23±0.00	0.23±0.00	0.26±0.03	0.25±0.01	0.24±0.01
ERO(p=0.05, style=gamma,η=0.3)	0.51±0.00	0.20±0.00	0.31±0.01	0.32±0.01	0.35±0.01	0.32±0.01	**0.19±0.00**	0.32±0.00	0.34±0.01	0.32±0.01	0.33±0.01	**0.19±0.00**
ERO(p=0.05, style=uniform,η=0.4)	0.51±0.00	**0.23±0.00**	0.29±0.00	0.29±0.02	0.36±0.00	0.30±0.03	**0.23±0.01**	0.28±0.00	0.28±0.01	0.32±0.00	0.31±0.02	0.27±0.02
ERO(p=0.05, style=gamma,η=0.4)	0.51±0.00	0.24±0.00	0.34±0.04	0.36±0.01	0.40±0.04	0.36±0.01	**0.23±0.01**	0.34±0.01	0.37±0.01	0.35±0.01	0.37±0.04	**0.23±0.01**
ERO(p=0.05, style=uniform,η=0.5)	0.51±0.00	**0.27±0.00**	0.32±0.00	0.35±0.00	0.40±0.00	0.35±0.00	**0.27±0.00**	0.33±0.01	0.32±0.00	0.34±0.01	0.34±0.01	0.29±0.01
ERO(p=0.05, style=gamma,η=0.5)	0.49±0.00	0.28±0.00	0.39±0.02	0.39±0.00	0.42±0.02	0.39±0.00	**0.27±0.01**	0.38±0.01	0.40±0.01	0.38±0.01	0.40±0.01	**0.27±0.01**
ERO(p=0.05, style=uniform,η=0.6)	0.51±0.00	0.31±0.00	0.36±0.01	0.38±0.00	0.42±0.01	0.38±0.00	**0.30±0.00**	0.35±0.00	0.35±0.00	0.37±0.00	0.37±0.00	0.31±0.00
ERO(p=0.05, style=gamma,η=0.6)	0.50±0.00	0.32±0.00	0.40±0.01	0.42±0.00	0.44±0.01	0.42±0.00	**0.30±0.01**	0.41±0.01	0.42±0.01	0.44±0.01	0.44±0.02	**0.30±0.01**
ERO(p=0.05, style=uniform,η=0.7)	0.49±0.00	0.34±0.00	0.39±0.01	0.40±0.00	0.43±0.00	0.40±0.00	**0.32±0.00**	0.39±0.00	0.38±0.01	0.40±0.00	0.39±0.01	0.33±0.01
ERO(p=0.05, style=gamma,η=0.7)	0.50±0.00	0.35±0.00	0.46±0.01	0.45±0.01	0.46±0.01	0.45±0.01	**0.33±0.01**	0.43±0.02	0.43±0.01	0.45±0.01	0.46±0.02	**0.33±0.01**
ERO(p=0.05, style=uniform,η=0.8)	0.51±0.00	0.36±0.00	0.43±0.01	0.42±0.00	0.43±0.01	0.42±0.00	**0.34±0.00**	0.42±0.00	0.42±0.00	0.42±0.00	0.42±0.00	**0.34±0.00**
ERO(p=0.05, style=gamma,η=0.8)	0.50±0.00	0.36±0.00	0.45±0.01	0.44±0.00	0.45±0.01	0.44±0.00	**0.34±0.01**	0.43±0.02	0.44±0.01	0.45±0.04	0.46±0.05	**0.34±0.01**
ERO(p=1, style=uniform,η=0)	0.50±0.03	**0.00±0.00**	0.01±0.02	0.04±0.07	0.01±0.01	0.04±0.09	**0.00±0.01**	**0.00±0.00**	0.04±0.11	0.01±0.01	0.05±0.06	**0.00±0.00**
ERO(p=1, style=gamma,η=0)	0.50±0.05	**0.00±0.00**	0.03±0.03	0.01±0.03	0.03±0.04	0.04±0.05	0.02±0.00	**0.00±0.00**	0.01±0.01	**0.00±0.00**	**0.00±0.00**	**0.00±0.00**
ERO(p=1, style=uniform,η=0.1)	0.48±0.00	**0.06±0.00**	0.09±0.01	0.08±0.00	0.08±0.00	0.08±0.00	0.08±0.01	0.11±0.01	0.13±0.09	0.10±0.01	0.11±0.03	**0.06±0.00**
ERO(p=1, style=gamma,η=0.1)	0.46±0.00	**0.06±0.00**	0.10±0.00	0.12±0.01	0.10±0.00	0.13±0.01	0.11±0.01	0.20±0.15	0.13±0.02	0.21±0.18	0.12±0.02	**0.06±0.00**
ERO(p=1, style=uniform,η=0.2)	0.47±0.00	**0.11±0.00**	0.14±0.00	0.15±0.02	0.14±0.00	0.14±0.00	0.13±0.00	0.21±0.03	0.23±0.03	0.22±0.04	0.21±0.08	**0.11±0.00**
ERO(p=1, style=gamma,η=0.2)	0.48±0.00	**0.11±0.00**	0.15±0.00	0.16±0.02	0.18±0.01	0.17±0.01	0.17±0.00	0.29±0.13	0.20±0.05	0.27±0.11	0.18±0.00	**0.11±0.00**
ERO(p=1, style=uniform,η=0.3)	0.47±0.00	**0.16±0.00**	0.18±0.00	0.19±0.00	0.20±0.01	0.19±0.00	0.19±0.00	0.31±0.19	0.25±0.02	0.27±0.08	0.23±0.02	**0.16±0.00**
ERO(p=1, style=gamma,η=0.3)	0.48±0.00	**0.16±0.00**	0.20±0.00	0.22±0.02	0.23±0.01	0.22±0.01	0.21±0.00	0.39±0.13	0.25±0.01	0.36±0.08	0.24±0.01	**0.16±0.00**
ERO(p=1, style=uniform,η=0.4)	0.48±0.00	**0.21±0.00**	0.23±0.00	0.23±0.00	0.26±0.03	0.26±0.06	0.23±0.00	0.29±0.03	0.30±0.02	0.28±0.03	0.27±0.02	**0.21±0.00**
ERO(p=1, style=gamma,η=0.4)	0.49±0.00	**0.22±0.00**	0.25±0.00	0.26±0.02	0.27±0.00	0.27±0.01	0.25±0.00	0.52±0.28	0.30±0.02	0.36±0.07	0.29±0.00	**0.22±0.00**
ERO(p=1, style=uniform,η=0.5)	0.49±0.00	**0.27±0.00**	**0.27±0.00**	0.28±0.02	0.32±0.03	0.28±0.00	0.28±0.00	0.33±0.01	0.35±0.04	0.32±0.02	0.33±0.02	**0.27±0.00**
ERO(p=1, style=gamma,η=0.5)	0.49±0.00	**0.27±0.00**	0.29±0.00	0.31±0.02	0.32±0.01	0.32±0.02	0.30±0.00	0.66±0.25	0.37±0.05	0.37±0.04	0.33±0.00	**0.27±0.00**
ERO(p=1, style=uniform,η=0.6)	0.50±0.00	**0.32±0.00**	**0.32±0.00**	0.33±0.00	0.37±0.02	0.33±0.00	0.33±0.00	0.37±0.01	0.40±0.04	0.37±0.01	0.37±0.01	**0.32±0.00**
ERO(p=1, style=gamma,η=0.6)	0.49±0.00	**0.32±0.00**	0.34±0.00	0.35±0.02	0.36±0.02	0.35±0.03	0.34±0.02	0.61±0.25	0.42±0.07	0.38±0.01	0.38±0.01	**0.32±0.00**
ERO(p=1, style=uniform,η=0.7)	0.49±0.00	**0.37±0.00**	**0.37±0.00**	**0.37±0.00**	0.40±0.00	**0.37±0.00**	0.38±0.00	0.41±0.01	0.43±0.03	0.41±0.01	0.42±0.02	**0.37±0.00**
ERO(p=1, style=gamma,η=0.7)	0.50±0.00	**0.37±0.00**	0.39±0.00	0.38±0.00	0.40±0.01	0.38±0.00	**0.37±0.00**	0.62±0.16	0.44±0.02	0.48±0.17	0.42±0.01	**0.37±0.00**
ERO(p=1, style=uniform,η=0.8)	0.49±0.00	0.42±0.00	0.42±0.00	0.42±0.00	0.43±0.00	0.42±0.00	0.42±0.00	0.44±0.01	0.47±0.01	0.45±0.01	0.46±0.03	**0.41±0.00**
ERO(p=1, style=gamma,η=0.8)	0.50±0.00	0.42±0.00	0.43±0.00	0.43±0.00	0.44±0.01	0.43±0.00	**0.41±0.00**	0.49±0.05	0.46±0.02	0.45±0.00	0.45±0.01	**0.41±0.00**

Table 12. Performance on $\mathcal{L}_{\text{upset, ratio}}$ for each variant in the proposed GNNRank framework, compared with the worst and the best baseline method, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue.

Type	Baselines		DIMPA					IB				
Data	worst baseline	best baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline	dist	innerproduct	proximal dist	proximal innerproduct	proximal baseline
HeadToHead	1.19±0.09	0.71±0.01	**0.68±0.00**	0.69±0.01	0.95±0.00	0.95±0.00	0.94±0.00	0.96±0.00	0.96±0.00	0.96±0.00	0.96±0.00	0.94±0.00
Finance	0.64±0.00	**0.26±0.00**	**0.26±0.00**	**0.26±0.00**	0.27±0.00	0.27±0.00	0.27±0.00	0.27±0.00	0.27±0.00	0.27±0.00	0.27±0.00	0.27±0.00
Animal	1.34±0.00	**0.22±0.00**	0.24±0.03	0.35±0.24	0.68±0.02	0.67±0.01	0.66±0.00	0.60±0.14	0.45±0.02	0.79±0.01	0.72±0.02	0.66±0.00
Faculty: Business	1.40±0.13	0.34±0.00	0.33±0.01	**0.31±0.00**	0.89±0.00	0.90±0.00	0.89±0.01	0.80±0.20	0.80±0.07	0.93±0.02	0.93±0.01	0.89±0.00
Faculty: CS	1.33±0.16	0.28±0.00	**0.26±0.02**	**0.26±0.01**	0.92±0.01	0.91±0.00	0.86±0.00	0.90±0.00	0.90±0.06	0.95±0.00	0.95±0.00	0.90±0.00
Faculty: History	1.35±0.11	0.23±0.00	0.23±0.01	**0.21±0.00**	0.85±0.00	0.88±0.00	0.84±0.02	0.89±0.04	0.83±0.11	0.96±0.00	0.93±0.01	0.87±0.00
Basketball(1985)	1.32±0.00	0.54±0.04	**0.46±0.01**	0.48±0.11	0.82±0.00	0.82±0.00	0.82±0.00	0.81±0.02	0.90±0.02	0.85±0.01	0.85±0.00	0.82±0.00
Basketball(1986)	1.30±0.00	0.54±0.01	0.43±0.00	**0.42±0.00**	0.82±0.00	0.82±0.00	0.81±0.00	0.78±0.02	0.87±0.02	0.85±0.01	0.85±0.00	0.81±0.00
Basketball(1987)	1.37±0.00	0.56±0.00	0.49±0.01	**0.48±0.01**	0.83±0.00	0.83±0.00	0.83±0.00	0.82±0.01	0.91±0.04	0.85±0.01	0.86±0.00	0.82±0.00
Basketball(1988)	1.33±0.00	0.53±0.01	**0.45±0.01**	0.47±0.01	0.83±0.00	0.83±0.00	0.82±0.00	0.82±0.01	0.87±0.01	0.86±0.01	0.86±0.00	0.82±0.00
Basketball(1989)	1.34±0.00	0.55±0.01	**0.46±0.01**	0.50±0.01	0.83±0.00	0.83±0.00	0.83±0.00	0.80±0.04	0.87±0.01	0.86±0.01	0.86±0.00	0.83±0.00
Basketball(1990)	1.35±0.00	0.51±0.05	**0.44±0.01**	0.47±0.01	0.83±0.00	0.83±0.00	0.82±0.00	0.82±0.01	0.88±0.03	0.86±0.01	0.86±0.00	0.82±0.00
Basketball(1991)	1.34±0.00	0.53±0.03	0.46±0.00	**0.45±0.00**	0.83±0.00	0.83±0.00	0.82±0.00	0.84±0.01	0.88±0.02	0.86±0.00	0.86±0.01	0.82±0.00
Basketball(1992)	1.35±0.00	0.50±0.06	0.44±0.01	**0.43±0.00**	0.83±0.00	0.83±0.00	0.83±0.00	0.81±0.04	0.90±0.02	0.86±0.01	0.86±0.00	0.83±0.00
Basketball(1993)	1.28±0.00	0.51±0.03	0.45±0.01	**0.44±0.01**	0.82±0.00	0.82±0.00	0.82±0.00	0.80±0.06	0.88±0.02	0.85±0.01	0.85±0.00	0.82±0.00
Basketball(1994)	1.35±0.00	0.43±0.05	0.43±0.01	**0.42±0.00**	0.82±0.00	0.82±0.00	0.82±0.00	0.77±0.07	0.89±0.03	0.85±0.01	0.85±0.00	0.82±0.00
Basketball(1995)	1.36±0.00	0.47±0.06	0.45±0.01	**0.44±0.00**	0.82±0.00	0.82±0.00	0.81±0.00	0.81±0.02	0.89±0.02	0.85±0.01	0.85±0.00	0.81±0.00
Basketball(1996)	1.35±0.00	0.55±0.03	**0.48±0.00**	**0.48±0.00**	0.83±0.00	0.83±0.00	0.82±0.00	0.80±0.03	0.90±0.05	0.85±0.01	0.86±0.00	0.82±0.00
Basketball(1997)	1.30±0.00	0.56±0.00	0.49±0.00	**0.48±0.00**	0.83±0.00	0.83±0.00	0.82±0.00	0.82±0.02	0.91±0.04	0.85±0.01	0.86±0.00	0.82±0.00
Basketball(1998)	1.36±0.00	0.56±0.00	0.47±0.01	**0.46±0.00**	0.84±0.00	0.84±0.00	0.84±0.00	0.82±0.04	0.90±0.01	0.87±0.01	0.87±0.00	0.84±0.00
Basketball(1999)	1.32±0.00	0.55±0.03	**0.48±0.00**	0.51±0.01	0.84±0.00	0.84±0.00	0.83±0.00	0.85±0.01	0.89±0.01	0.87±0.00	0.87±0.00	0.83±0.00
Basketball(2000)	1.40±0.01	0.58±0.01	0.51±0.01	**0.50±0.00**	0.86±0.00	0.86±0.00	0.86±0.00	0.85±0.02	0.91±0.01	0.89±0.01	0.89±0.00	0.86±0.00
Basketball(2001)	1.40±0.00	0.55±0.04	0.48±0.01	**0.47±0.00**	0.85±0.00	0.85±0.00	0.85±0.00	0.83±0.03	0.90±0.01	0.88±0.01	0.88±0.00	0.85±0.00
Basketball(2002)	1.33±0.00	0.56±0.03	0.49±0.01	**0.48±0.01**	0.85±0.00	0.85±0.00	0.84±0.00	0.80±0.04	0.91±0.02	0.87±0.01	0.88±0.00	0.84±0.00
Basketball(2003)	1.40±0.00	0.56±0.04	0.52±0.01	**0.51±0.00**	0.85±0.00	0.85±0.00	0.85±0.00	0.82±0.02	0.89±0.01	0.88±0.01	0.88±0.00	0.85±0.00
Basketball(2004)	1.35±0.00	0.54±0.04	0.47±0.01	**0.46±0.00**	0.85±0.00	0.85±0.00	0.85±0.00	0.85±0.02	0.88±0.01	0.88±0.00	0.88±0.00	0.85±0.00
Basketball(2005)	1.29±0.01	0.57±0.01	0.50±0.01	**0.49±0.01**	0.86±0.00	0.86±0.00	0.85±0.00	0.84±0.01	0.91±0.03	0.88±0.01	0.88±0.00	0.85±0.00
Basketball(2006)	1.36±0.00	0.50±0.05	0.50±0.01	**0.49±0.01**	0.86±0.00	0.86±0.00	0.86±0.00	0.83±0.04	0.91±0.01	0.88±0.01	0.89±0.00	0.85±0.00
Basketball(2007)	1.41±0.00	0.57±0.00	**0.49±0.00**	**0.49±0.00**	0.86±0.00	0.86±0.00	0.86±0.00	0.83±0.03	0.91±0.01	0.89±0.01	0.89±0.00	0.86±0.00
Basketball(2008)	1.33±0.01	0.55±0.04	**0.51±0.01**	0.53±0.10	0.86±0.00	0.86±0.00	0.86±0.00	0.85±0.03	0.90±0.01	0.89±0.01	0.89±0.00	0.86±0.00
Basketball(2009)	1.24±0.00	0.56±0.00	0.48±0.00	**0.47±0.00**	0.86±0.00	0.86±0.00	0.86±0.00	0.83±0.03	0.90±0.01	0.89±0.01	0.89±0.00	0.86±0.00
Basketball(2010)	1.35±0.00	0.52±0.06	**0.49±0.00**	**0.49±0.00**	0.88±0.00	0.87±0.00	0.87±0.00	0.83±0.05	0.92±0.02	0.90±0.01	0.91±0.00	0.87±0.00
Basketball(2011)	1.34±0.00	0.52±0.05	0.50±0.01	**0.49±0.00**	0.87±0.00	0.87±0.00	0.86±0.00	0.83±0.03	0.91±0.01	0.89±0.01	0.89±0.00	0.86±0.00
Basketball(2012)	1.42±0.00	0.54±0.05	**0.48±0.01**	**0.48±0.01**	0.87±0.00	0.87±0.00	0.86±0.00	0.85±0.04	0.91±0.01	0.90±0.01	0.90±0.00	0.86±0.00
Basketball(2013)	1.34±0.00	0.58±0.03	**0.51±0.01**	0.53±0.10	0.87±0.00	0.87±0.00	0.86±0.00	0.84±0.03	0.91±0.01	0.89±0.01	0.90±0.00	0.86±0.00
Basketball(2014)	1.41±0.00	0.52±0.06	**0.49±0.01**	**0.49±0.01**	0.86±0.00	0.86±0.00	0.86±0.00	0.86±0.01	0.91±0.01	0.89±0.00	0.89±0.00	0.86±0.00
Basketball finer(1985)	0.57±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.19±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1986)	0.60±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.19±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1987)	0.58±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.21±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1988)	0.57±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.22±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1989)	0.56±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.22±0.03	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1990)	0.55±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.22±0.01	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1991)	0.54±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.23±0.03	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1992)	0.55±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.22±0.03	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1993)	0.58±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.20±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1994)	0.58±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.21±0.03	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1995)	0.55±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.21±0.02	0.01±0.00	0.01±0.00	0.01±0.00
Basketball finer(1996)	0.56±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(1997)	0.56±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(1998)	0.56±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.21±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(1999)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.18±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2000)	0.55±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.21±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2001)	0.59±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2002)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2003)	0.58±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.21±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2004)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2005)	0.60±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2006)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2007)	0.53±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.22±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2008)	0.59±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2009)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2010)	0.58±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.20±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2011)	0.59±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.21±0.02	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2012)	0.57±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.22±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2013)	0.56±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.22±0.03	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**
Basketball finer(2014)	0.56±0.00	0.01±0.00	0.01±0.00	**0.00±0.00**	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00	0.20±0.04	0.01±0.00	0.01±0.00	0.01±0.00
Football(2009)	1.16±0.05	0.48±0.02	**0.46±0.01**	0.49±0.11	0.73±0.04	0.71±0.03	0.69±0.00	0.73±0.05	0.70±0.16	0.81±0.02	0.79±0.03	0.69±0.00
Football(2010)	1.25±0.01	0.66±0.02	**0.65±0.00**	0.68±0.07	0.73±0.00	0.75±0.00	0.73±0.01	0.75±0.02	0.76±0.03	0.79±0.01	0.78±0.01	0.74±0.00
Football(2011)	1.17±0.00	**0.53±0.02**	**0.53±0.01**	0.54±0.07	0.73±0.01	0.71±0.01	0.69±0.00	0.70±0.04	0.79±0.07	0.77±0.03	0.77±0.04	0.69±0.00
Football(2012)	1.18±0.00	0.53±0.03	**0.51±0.01**	0.53±0.08	0.74±0.05	0.72±0.01	0.71±0.00	0.64±0.03	0.77±0.09	0.74±0.02	0.76±0.05	0.71±0.00
Football(2013)	0.94±0.00	0.48±0.04	**0.46±0.01**	0.47±0.12	0.72±0.02	0.72±0.03	0.71±0.00	0.65±0.06	0.69±0.11	0.80±0.03	0.79±0.04	0.71±0.00
Football(2014)	1.49±0.00	0.74±0.01	**0.69±0.01**	**0.69±0.07**	0.87±0.01	0.87±0.02	0.85±0.00	0.87±0.02	0.96±0.05	0.88±0.01	0.90±0.03	0.85±0.00
Football finer(2009)	0.73±0.04	**0.17±0.01**	**0.17±0.00**	**0.17±0.03**	0.21±0.00	0.22±0.01	0.21±0.00	0.30±0.01	0.30±0.01	0.29±0.01	0.28±0.02	0.21±0.00
Football finer(2010)	0.67±0.00	0.18±0.01	0.19±0.01	**0.17±0.00**	0.18±0.00	0.18±0.01	0.18±0.00	0.25±0.01	0.25±0.00	0.24±0.01	0.23±0.01	0.19±0.00
Football finer(2011)	0.58±0.00	**0.17±0.00**	**0.17±0.00**	0.19±0.05	0.19±0.00	0.20±0.00	0.19±0.00	0.28±0.01	0.27±0.02	0.26±0.01	0.26±0.02	0.20±0.00
Football finer(2012)	0.66±0.04	0.16±0.00	**0.15±0.00**	0.16±0.03	0.17±0.00	0.17±0.00	0.17±0.00	0.25±0.01	0.25±0.00	0.24±0.01	0.23±0.02	0.17±0.00
Football finer(2013)	0.52±0.00	0.21±0.01	0.21±0.01	**0.19±0.04**	0.25±0.00	0.25±0.02	0.24±0.01	0.33±0.01	0.33±0.01	0.33±0.01	0.31±0.02	0.25±0.00
Football finer(2014)	0.98±0.00	0.35±0.00	**0.34±0.00**	0.35±0.05	0.38±0.00	0.39±0.03	0.38±0.00	0.47±0.01	0.47±0.01	0.46±0.02	0.46±0.01	0.38±0.00

for that season. For each year we have 10 runs, and we average over the total of 30 years for each run. On average, directly applying the original trained model gives $\mathcal{L}_{\text{upset, simple}} = 0.75 \pm 0.02$, $\mathcal{L}_{\text{upset, naive}} = 0.19 \pm 0.01$ and $\mathcal{L}_{\text{upset, ratio}} = 0.01 \pm 0.00$, while training specifically for the that season gives $\mathcal{L}_{\text{upset, simple}} = 0.74 \pm 0.00$, $\mathcal{L}_{\text{upset, naive}} = 0.19 \pm 0.00$ and $\mathcal{L}_{\text{upset, ratio}} = 0.01 \pm 0.00$. Thus, in this example, applying the general model produces almost the same superior performance.

F. Improvement on Baselines when Employed as Initial Guess for "Proximal Baseline" Variant

Table 19 shows improvements on $\mathcal{L}_{upset,simple}$ by "proximal baseline" when setting a certain baseline as $\mathbf{r}'$. Across all data sets, "proximal baseline" improves the most, by 1.02, with SyncRank as initial guess, while the average improvement for SpringRank, SerialRank, BTL, Eig.Cent.,

Table 13. $\mathcal{L}_{\text{upset, simple}}$ comparison for different variants on selected real-world data, averaged over 10 runs, and plus/minus one standard deviation. "avg" for time series data first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best for each group of variants (GNNRank-N or GNNRank-P) is marked in **bold red** while the second best is highlighted in underline blue.

Methods	GNNRank-N			GNNRank-P						
Data/Variant	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	no pretrain	$\{\alpha_\gamma\}_{\gamma=1}^{\Gamma}$not trainable	$\Gamma = 3$	$\Gamma = 7$
Animal	0.43±0.06	0.59±0.08	**0.41±0.09**	**0.25±0.00**	**0.25±0.00**	**0.25±0.01**	**0.25±0.00**	**0.25±0.00**	**0.25±0.00**	**0.25±0.00**
Faculty: Business	0.40±0.02	0.49±0.16	**0.38±0.01**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**	**0.36±0.00**
Faculty: CS	0.35±0.01	0.36±0.01	**0.33±0.03**	**0.32±0.00**	**0.32±0.00**	**0.32±0.00**	0.33±0.00	**0.32±0.00**	**0.32±0.00**	**0.32±0.00**
Faculty: History	**0.28±0.01**	0.31±0.01	**0.28±0.01**	**0.30±0.01**	**0.30±0.01**	**0.30±0.02**	**0.30±0.01**	**0.30±0.01**	**0.30±0.01**	**0.30±0.01**
Football(2009)	0.76±0.03	0.79±0.04	**0.75±0.06**	**0.61±0.00**	**0.61±0.00**	**0.61±0.00**	**0.61±0.00**	**0.61±0.00**	**0.61±0.00**	**0.61±0.00**
Football(2010)	**0.90±0.02**	0.93±0.05	0.92±0.03	**0.95±0.05**	0.96±0.06	0.96±0.05	**0.95±0.06**	0.96±0.06	**0.95±0.05**	**0.95±0.04**
Football(2011)	0.86±0.10	**0.83±0.03**	0.84±0.08	**0.80±0.00**	**0.80±0.00**	**0.80±0.01**	**0.80±0.00**	**0.80±0.01**	**0.80±0.00**	**0.80±0.00**
Football(2012)	0.81±0.09	**0.75±0.03**	0.78±0.03	0.80±0.08	0.80±0.08	0.84±0.00	0.84±0.00	0.80±0.08	**0.78±0.03**	0.80±0.06
Football(2013)	0.65±0.03	0.75±0.05	**0.64±0.04**	**0.56±0.00**	**0.56±0.00**	**0.56±0.00**	**0.56±0.00**	**0.56±0.00**	**0.56±0.00**	**0.56±0.00**
Football(2014)	**0.96±0.12**	1.00±0.11	0.98±0.17	0.98±0.06	0.96±0.07	0.98±0.09	0.99±0.10	0.98±0.06	**0.95±0.05**	0.97±0.08
Football finer(2009)	**0.76±0.03**	0.93±0.38	**0.76±0.18**	0.65±0.02	0.65±0.02	0.65±0.03	0.66±0.00	0.65±0.03	**0.63±0.03**	0.64±0.02
Football finer(2010)	**1.00±0.01**	**1.00±0.01**	**1.00±0.01**	**0.99±0.02**	1.00±0.01	1.00±0.00	**0.99±0.02**	1.00±0.01	1.00±0.00	1.00±0.00
Football finer(2011)	0.99±0.03	**0.92±0.16**	0.99±0.04	0.85±0.01	0.84±0.02	0.85±0.00	0.85±0.00	0.85±0.01	**0.83±0.03**	0.84±0.03
Football finer(2012)	**0.93±0.07**	1.00±0.02	0.96±0.03	**0.86±0.02**	**0.86±0.03**	**0.86±0.02**	**0.86±0.00**	**0.86±0.03**	**0.86±0.01**	**0.86±0.00**
Football finer(2013)	0.74±0.04	0.98±0.04	**0.73±0.19**	0.59±0.02	0.69±0.03	**0.57±0.01**	0.58±0.00	0.58±0.01	**0.57±0.02**	**0.57±0.02**
Football finer(2014)	**1.00±0.00**	1.01±0.04	**1.00±0.00**	**1.00±0.00**	1.01±0.03	**1.00±0.03**	1.08±0.12	**1.00±0.03**	**1.00±0.00**	**1.00±0.00**

Table 14. $\mathcal{L}_{\text{upset, naive}}$ comparison for different variants on selected real-world data, averaged over 10 runs, and plus/minus one standard deviation. "avg" for time series data first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best is marked in **bold red** while the second best is highlighted in underline blue.

Methods	GNNRank-N			GNNRank-P						
Data/Variant	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	no pretrain	$\{\alpha_\gamma\}_{\gamma=1}^{\Gamma}$not trainable	$\Gamma = 3$	$\Gamma = 7$
Animal	0.11±0.02	0.15±0.02	**0.10±0.02**	**0.06±0.00**	**0.06±0.00**	**0.06±0.00**	**0.06±0.00**	**0.06±0.00**	**0.06±0.00**	**0.06±0.00**
Faculty: Business	**0.10±0.00**	0.12±0.04	**0.10±0.00**	**0.09±0.00**	**0.09±0.00**	**0.09±0.00**	**0.09±0.00**	**0.09±0.00**	**0.09±0.00**	**0.09±0.00**
Faculty: CS	0.09±0.00	0.09±0.00	**0.08±0.01**	**0.08±0.00**	**0.08±0.00**	**0.08±0.00**	**0.08±0.00**	**0.08±0.00**	**0.08±0.00**	**0.08±0.00**
Faculty: History	**0.07±0.00**	0.08±0.00	**0.07±0.00**	**0.07±0.00**	0.08±0.00	0.08±0.00	**0.07±0.00**	**0.07±0.00**	0.08±0.00	0.08±0.00
Football(2009)	**0.19±0.01**	0.26±0.06	**0.19±0.01**	**0.15±0.00**	**0.15±0.00**	**0.15±0.00**	**0.15±0.00**	**0.15±0.00**	**0.15±0.00**	**0.15±0.00**
Football(2010)	**0.30±0.01**	0.36±0.06	**0.30±0.01**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.01**	**0.29±0.00**
Football(2011)	**0.22±0.01**	0.26±0.07	**0.22±0.01**	**0.20±0.00**	**0.20±0.00**	**0.20±0.00**	**0.20±0.00**	**0.20±0.00**	**0.20±0.00**	**0.20±0.00**
Football(2012)	**0.20±0.02**	0.29±0.06	0.21±0.04	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	0.21±0.00	**0.20±0.00**	0.21±0.00
Football(2013)	**0.16±0.01**	0.20±0.07	**0.16±0.01**	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**
Football(2014)	**0.24±0.01**	0.30±0.02	0.25±0.04	**0.26±0.01**	0.27±0.00	0.27±0.02	0.27±0.00	**0.26±0.01**	0.27±0.02	0.27±0.00
Football finer(2009)	**0.19±0.01**	0.23±0.09	**0.19±0.05**	**0.16±0.01**	**0.16±0.01**	**0.16±0.01**	**0.16±0.01**	**0.16±0.01**	**0.16±0.01**	**0.16±0.00**
Football finer(2010)	**0.31±0.01**	0.35±0.05	**0.31±0.02**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**	**0.29±0.00**
Football finer(2011)	0.25±0.01	**0.23±0.04**	0.25±0.01	**0.21±0.00**	**0.21±0.01**	**0.21±0.00**	**0.21±0.00**	**0.21±0.01**	**0.21±0.01**	**0.21±0.01**
Football finer(2012)	**0.24±0.02**	0.25±0.04	**0.24±0.01**	**0.21±0.00**	**0.21±0.01**	0.22±0.01	0.22±0.00	**0.21±0.01**	0.22±0.00	0.22±0.00
Football finer(2013)	0.19±0.01	0.26±0.12	**0.18±0.05**	0.15±0.00	0.17±0.01	**0.14±0.00**	0.15±0.00	**0.14±0.00**	**0.14±0.00**	**0.14±0.00**
Football finer(2014)	**0.27±0.01**	0.35±0.09	0.28±0.04	**0.27±0.02**	**0.27±0.00**	**0.27±0.02**	**0.27±0.00**	**0.27±0.02**	**0.27±0.00**	**0.27±0.01**

Table 15. $\mathcal{L}_{\text{upset, ratio}}$ comparison for different variants on selected real-world data, averaged over 10 runs, and plus/minus one standard deviation. "avg" for time series data first average over all seasons, then consider mean and standard deviation over the 10 averaged values. The best for each group of variants (GNNRank-N or GNNRank-P) is marked in **bold red** while the second best is highlighted in underline blue.

Methods	GNNRank-N			GNNRank-P						
Data/Variant	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	loss sum	$\mathcal{L}_{\text{upset,margin}}$	$\mathcal{L}_{\text{upset, ratio}}$	no pretrain	$\{\alpha_\gamma\}_{\gamma=1}^{\Gamma}$not trainable	$\Gamma = 3$	$\Gamma = 7$
Animal	**0.24±0.01**	0.41±0.09	**0.24±0.03**	**0.66±0.00**	**0.66±0.00**	**0.66±0.00**	**0.66±0.00**	**0.66±0.00**	**0.66±0.00**	**0.66±0.00**
Faculty: Business	0.32±0.02	0.71±0.14	**0.31±0.00**	**0.89±0.00**	**0.89±0.01**	**0.89±0.00**	**0.89±0.01**	**0.89±0.00**	**0.89±0.01**	**0.89±0.00**
Faculty: CS	0.27±0.01	0.69±0.05	**0.26±0.02**	0.90±0.00	**0.86±0.00**	0.90±0.00	0.90±0.00	**0.86±0.00**	**0.86±0.00**	**0.86±0.00**
Faculty: History	**0.21±0.00**	0.60±0.10	**0.21±0.00**	0.86±0.00	**0.84±0.02**	0.85±0.00	0.87±0.00	**0.84±0.02**	**0.84±0.02**	**0.84±0.02**
Football(2009)	**0.46±0.01**	0.72±0.14	0.48±0.01	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**
Football(2010)	**0.65±0.00**	0.75±0.02	0.68±0.07	**0.73±0.01**	0.74±0.00	0.74±0.00	**0.73±0.00**	**0.73±0.01**	**0.73±0.00**	**0.73±0.01**
Football(2011)	**0.53±0.01**	0.70±0.04	0.54±0.02	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**	**0.69±0.00**
Football(2012)	**0.51±0.01**	0.65±0.04	0.53±0.08	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**
Football(2013)	**0.46±0.01**	0.56±0.16	**0.46±0.01**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**	**0.71±0.00**
Football(2014)	**0.69±0.01**	0.92±0.07	**0.69±0.07**	**0.85±0.00**	**0.85±0.00**	**0.85±0.00**	**0.85±0.00**	**0.85±0.00**	**0.85±0.00**	**0.85±0.00**
Football finer(2009)	**0.17±0.00**	0.28±0.07	**0.17±0.03**	0.21±0.00	0.21±0.00	0.21±0.01	0.21±0.00	0.21±0.00	**0.20±0.00**	0.21±0.01
Football finer(2010)	0.19±0.03	0.25±0.00	**0.17±0.00**	**0.18±0.00**	0.19±0.00	**0.18±0.00**	**0.18±0.00**	**0.18±0.00**	**0.18±0.00**	**0.18±0.00**
Football finer(2011)	**0.17±0.00**	0.22±0.09	0.18±0.01	**0.19±0.00**	0.20±0.02	**0.19±0.00**	**0.19±0.00**	**0.19±0.00**	**0.19±0.00**	**0.19±0.00**
Football finer(2012)	**0.15±0.00**	0.23±0.02	0.16±0.03	**0.17±0.00**	**0.17±0.00**	**0.17±0.00**	**0.17±0.00**	**0.17±0.00**	**0.17±0.00**	**0.17±0.00**
Football finer(2013)	0.21±0.01	0.29±0.03	**0.19±0.04**	0.25±0.01	0.25±0.00	**0.24±0.01**	0.25±0.00	**0.24±0.01**	**0.24±0.00**	0.25±0.00
Football finer(2014)	**0.34±0.00**	0.46±0.04	0.35±0.05	**0.38±0.00**	**0.38±0.00**	**0.38±0.00**	**0.38±0.00**	**0.38±0.00**	**0.38±0.00**	**0.38±0.00**

Table 16. Result table on $\mathcal{L}_{\text{upset, simple}}$ for each year in the time series matches, applying the trained model for 1985 on all seasons without further training, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in <u>underline blue</u>. As MVR could not generate results after a week, we omit the results here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	Directly Apply	Train Specifically
Basketball finer(1985)	0.76±0.00	1.63±0.00	1.96±0.10	1.46±0.05	0.83±0.00	1.18±0.00	1.16±0.00	1.97±0.00	1.00±0.00	0.87±0.00	**0.71±0.00**	**0.71±0.00**
Basketball finer(1986)	0.77±0.00	1.81±0.00	1.99±0.00	1.42±0.06	0.84±0.00	1.16±0.00	1.15±0.00	1.99±0.00	1.09±0.00	0.86±0.00	**0.69±0.00**	**0.69±0.00**
Basketball finer(1987)	0.82±0.00	1.79±0.00	1.87±0.00	1.41±0.06	0.89±0.00	1.17±0.00	1.21±0.00	1.95±0.00	0.99±0.00	0.91±0.00	**0.77±0.00**	**0.77±0.00**
Basketball finer(1988)	0.78±0.00	1.79±0.00	1.90±0.00	1.43±0.10	0.84±0.00	1.23±0.00	1.19±0.00	1.97±0.00	0.97±0.00	0.83±0.00	**0.70±0.00**	**0.70±0.00**
Basketball finer(1989)	<u>0.77±0.00</u>	1.67±0.00	1.86±0.00	1.43±0.05	0.83±0.00	1.13±0.00	1.14±0.00	1.94±0.00	0.99±0.00	0.90±0.00	0.88±0.52	**0.70±0.00**
Basketball finer(1990)	0.79±0.00	1.67±0.00	1.93±0.00	1.45±0.05	0.82±0.00	1.28±0.00	1.17±0.00	1.98±0.00	0.91±0.00	0.84±0.00	**0.71±0.00**	**0.71±0.00**
Basketball finer(1991)	0.81±0.00	1.83±0.00	2.03±0.00	1.36±0.06	0.83±0.00	1.38±0.00	1.31±0.00	1.97±0.00	0.99±0.00	0.89±0.00	**0.71±0.00**	**0.71±0.00**
Basketball finer(1992)	0.73±0.00	1.72±0.00	1.88±0.00	1.33±0.06	0.77±0.00	1.26±0.00	1.21±0.00	1.87±0.00	0.95±0.00	0.84±0.00	**0.67±0.00**	**0.67±0.00**
Basketball finer(1993)	0.75±0.00	1.66±0.00	2.03±0.00	1.35±0.05	0.78±0.00	1.18±0.00	1.10±0.00	1.97±0.00	0.98±0.00	0.86±0.00	**0.69±0.00**	**0.69±0.00**
Basketball finer(1994)	0.74±0.00	1.69±0.00	2.01±0.00	1.35±0.08	0.78±0.00	1.23±0.00	1.10±0.00	1.94±0.00	0.90±0.00	0.83±0.00	**0.67±0.00**	**0.67±0.00**
Basketball finer(1995)	0.79±0.00	1.78±0.00	1.89±0.00	1.35±0.06	0.83±0.00	1.19±0.00	1.13±0.00	1.92±0.01	0.95±0.00	0.87±0.00	**0.73±0.00**	**0.73±0.00**
Basketball finer(1996)	0.81±0.00	1.67±0.00	1.95±0.00	1.44±0.06	0.88±0.00	1.22±0.00	1.20±0.00	1.94±0.00	1.08±0.00	0.95±0.00	**0.77±0.00**	**0.77±0.00**
Basketball finer(1997)	0.83±0.00	1.77±0.00	1.94±0.00	1.40±0.04	0.86±0.00	1.19±0.00	1.16±0.00	2.05±0.00	0.96±0.00	0.92±0.00	**0.77±0.00**	**0.77±0.00**
Basketball finer(1998)	0.78±0.00	1.70±0.00	1.92±0.00	1.36±0.07	0.83±0.00	1.14±0.00	1.13±0.00	1.91±0.00	0.97±0.00	0.90±0.00	**0.74±0.00**	**0.74±0.00**
Basketball finer(1999)	0.81±0.00	1.64±0.00	2.02±0.00	1.38±0.07	0.86±0.00	1.17±0.00	1.11±0.00	1.99±0.00	1.17±0.00	0.94±0.00	**0.73±0.00**	**0.73±0.00**
Basketball finer(2000)	0.84±0.00	1.75±0.00	1.97±0.00	1.39±0.05	0.90±0.00	1.26±0.00	1.18±0.00	1.92±0.00	1.12±0.00	0.95±0.00	**0.78±0.00**	**0.78±0.00**
Basketball finer(2001)	0.81±0.00	1.69±0.00	2.06±0.00	1.41±0.06	0.86±0.00	1.25±0.00	1.18±0.00	2.03±0.00	1.08±0.00	0.97±0.00	**0.73±0.00**	**0.73±0.00**
Basketball finer(2002)	0.87±0.00	1.75±0.00	1.86±0.00	1.43±0.08	0.89±0.00	1.20±0.00	1.13±0.00	2.03±0.00	1.07±0.00	0.92±0.00	**0.78±0.00**	**0.78±0.00**
Basketball finer(2003)	0.87±0.00	1.78±0.00	1.98±0.07	1.46±0.09	0.91±0.00	1.18±0.00	1.14±0.00	2.00±0.00	1.02±0.00	0.95±0.00	**0.78±0.00**	**0.78±0.00**
Basketball finer(2004)	0.77±0.00	1.71±0.06	1.87±0.00	1.41±0.06	0.80±0.00	1.17±0.00	1.13±0.00	1.98±0.02	0.95±0.00	0.88±0.00	**0.72±0.00**	**0.72±0.00**
Basketball finer(2005)	<u>0.84±0.00</u>	1.82±0.00	1.93±0.00	1.38±0.06	0.88±0.00	1.14±0.00	1.09±0.00	2.00±0.00	1.08±0.00	0.95±0.00	0.93±0.52	**0.75±0.00**
Basketball finer(2006)	0.86±0.00	1.76±0.00	1.97±0.00	1.40±0.07	0.85±0.00	1.21±0.00	1.11±0.00	1.96±0.00	1.06±0.00	0.94±0.00	**0.76±0.00**	**0.76±0.00**
Basketball finer(2007)	0.86±0.00	1.85±0.00	1.97±0.14	1.39±0.05	0.93±0.00	1.15±0.00	1.09±0.00	1.92±0.00	0.95±0.00	0.93±0.00	**0.80±0.00**	**0.80±0.00**
Basketball finer(2008)	0.85±0.00	1.72±0.00	1.98±0.00	1.36±0.08	0.88±0.00	1.20±0.00	1.13±0.00	1.96±0.00	0.99±0.00	0.91±0.00	**0.78±0.00**	**0.78±0.00**
Basketball finer(2009)	0.84±0.00	1.71±0.00	2.00±0.04	1.37±0.04	0.90±0.00	1.17±0.00	1.12±0.00	2.06±0.00	0.99±0.00	0.91±0.00	**0.75±0.00**	**0.75±0.00**
Basketball finer(2010)	0.82±0.00	1.68±0.00	1.98±0.00	1.34±0.06	0.84±0.00	1.10±0.00	1.13±0.00	1.97±0.00	0.94±0.00	0.92±0.00	**0.75±0.00**	**0.75±0.00**
Basketball finer(2011)	0.85±0.00	1.65±0.00	1.96±0.00	1.36±0.06	0.87±0.00	1.12±0.00	1.14±0.00	2.01±0.00	0.96±0.00	0.89±0.00	**0.77±0.00**	**0.77±0.00**
Basketball finer(2012)	0.80±0.00	1.68±0.00	1.97±0.00	1.37±0.07	0.83±0.00	1.17±0.00	1.10±0.00	1.91±0.00	0.93±0.00	0.89±0.00	**0.75±0.00**	**0.75±0.00**
Basketball finer(2013)	0.83±0.00	1.73±0.00	2.06±0.00	1.37±0.07	0.88±0.00	1.15±0.00	1.13±0.00	1.97±0.00	1.01±0.00	0.92±0.00	**0.79±0.00**	**0.79±0.00**
Basketball finer(2014)	0.84±0.00	1.77±0.19	2.07±0.00	1.42±0.09	0.86±0.00	1.18±0.00	1.13±0.00	1.97±0.00	1.02±0.00	0.88±0.00	**0.79±0.00**	**0.79±0.00**

Table 17. Result table on $\mathcal{L}_{\text{upset, naive}}$ for each year in the time series matches, applying the trained model for 1985 directly without further training, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in <u>underline blue</u>. As MVR could not generate scores, we omit the results here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	Directly Apply	Train Specifically
Basketball finer(1985)	0.19±0.00	0.41±0.00	0.49±0.02	0.36±0.01	0.21±0.00	0.29±0.00	0.29±0.00	0.49±0.00	0.25±0.00	0.22±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1986)	0.19±0.00	0.45±0.00	0.50±0.00	0.36±0.02	0.21±0.00	0.29±0.00	0.29±0.00	0.50±0.00	0.27±0.00	0.21±0.00	**0.17±0.00**	**0.17±0.00**
Basketball finer(1987)	0.20±0.00	0.45±0.00	0.47±0.00	0.35±0.01	0.22±0.00	0.29±0.00	0.30±0.00	0.49±0.00	0.25±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(1988)	0.19±0.00	0.45±0.00	0.48±0.00	0.36±0.03	0.21±0.00	0.31±0.00	0.30±0.00	0.49±0.00	0.24±0.00	0.21±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1989)	<u>0.19±0.00</u>	0.42±0.00	0.46±0.00	0.36±0.01	0.21±0.00	0.28±0.00	0.29±0.00	0.49±0.00	0.25±0.00	0.23±0.00	0.22±0.13	**0.18±0.00**
Basketball finer(1990)	0.20±0.00	0.42±0.00	0.48±0.00	0.36±0.01	0.21±0.00	0.32±0.00	0.29±0.00	0.50±0.00	0.23±0.00	0.21±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1991)	0.20±0.00	0.46±0.00	0.51±0.00	0.34±0.02	0.21±0.00	0.35±0.00	0.33±0.00	0.49±0.00	0.25±0.00	0.22±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1992)	0.18±0.00	0.43±0.00	0.47±0.00	0.33±0.01	0.19±0.00	0.31±0.00	0.30±0.00	0.47±0.00	0.24±0.00	0.21±0.00	**0.17±0.00**	**0.17±0.00**
Basketball finer(1993)	0.19±0.00	0.42±0.00	0.51±0.00	0.34±0.01	0.20±0.00	0.29±0.00	0.27±0.00	0.49±0.00	0.25±0.00	0.21±0.00	**0.17±0.00**	**0.17±0.00**
Basketball finer(1994)	0.18±0.00	0.42±0.00	0.50±0.00	0.34±0.02	0.19±0.00	0.31±0.00	0.27±0.00	0.49±0.00	0.22±0.00	0.21±0.00	**0.17±0.00**	**0.17±0.00**
Basketball finer(1995)	0.20±0.00	0.44±0.00	0.47±0.00	0.34±0.02	0.21±0.00	0.30±0.00	0.28±0.00	0.48±0.00	0.24±0.00	0.22±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1996)	0.20±0.00	0.42±0.00	0.49±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.30±0.00	0.49±0.00	0.27±0.00	0.24±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(1997)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.01	0.21±0.00	0.30±0.00	0.29±0.00	0.51±0.00	0.24±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(1998)	0.20±0.00	0.42±0.00	0.48±0.00	0.34±0.02	0.21±0.00	0.29±0.00	0.28±0.00	0.48±0.00	0.24±0.00	0.22±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(1999)	0.20±0.00	0.41±0.00	0.50±0.00	0.34±0.02	0.22±0.00	0.29±0.00	0.28±0.00	0.50±0.00	0.29±0.00	0.24±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(2000)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.01	0.23±0.00	0.32±0.00	0.30±0.00	0.48±0.00	0.28±0.00	0.24±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2001)	0.20±0.00	0.42±0.00	0.51±0.00	0.35±0.01	0.21±0.00	0.31±0.00	0.30±0.00	0.51±0.00	0.27±0.00	0.24±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(2002)	0.22±0.00	0.44±0.00	0.47±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.51±0.00	0.27±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2003)	0.22±0.00	0.45±0.00	0.50±0.02	0.36±0.02	0.23±0.00	0.29±0.00	0.29±0.00	0.50±0.00	0.26±0.00	0.24±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2004)	0.19±0.00	0.43±0.01	0.47±0.00	0.35±0.01	0.20±0.00	0.29±0.00	0.28±0.00	0.49±0.00	0.24±0.00	0.22±0.00	**0.18±0.00**	**0.18±0.00**
Basketball finer(2005)	<u>0.21±0.00</u>	0.46±0.00	0.48±0.00	0.34±0.01	0.22±0.00	0.29±0.00	0.27±0.00	0.50±0.00	0.27±0.00	0.24±0.00	0.23±0.13	**0.19±0.00**
Basketball finer(2006)	0.21±0.00	0.44±0.00	0.49±0.00	0.35±0.02	0.21±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.27±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2007)	0.21±0.00	0.46±0.00	0.49±0.03	0.35±0.01	0.23±0.00	0.29±0.00	0.27±0.00	0.48±0.00	0.24±0.00	0.23±0.00	**0.20±0.00**	**0.20±0.00**
Basketball finer(2008)	0.21±0.00	0.43±0.00	0.49±0.00	0.34±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.25±0.00	0.23±0.00	**0.20±0.00**	**0.20±0.00**
Basketball finer(2009)	0.21±0.00	0.43±0.00	0.50±0.01	0.34±0.01	0.22±0.00	0.29±0.00	0.28±0.00	0.52±0.00	0.25±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2010)	0.20±0.00	0.42±0.00	0.49±0.00	0.33±0.02	0.21±0.00	0.28±0.00	0.28±0.00	0.49±0.00	0.24±0.00	0.23±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2011)	0.21±0.00	0.41±0.00	0.49±0.00	0.34±0.01	0.22±0.00	0.28±0.00	0.28±0.00	0.50±0.00	0.24±0.00	0.22±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2012)	0.20±0.00	0.42±0.00	0.49±0.00	0.34±0.02	0.21±0.00	0.29±0.00	0.28±0.00	0.48±0.00	0.23±0.00	0.22±0.00	**0.19±0.00**	**0.19±0.00**
Basketball finer(2013)	0.21±0.00	0.43±0.00	0.52±0.00	0.34±0.02	0.22±0.00	0.29±0.00	0.28±0.00	0.49±0.00	0.25±0.00	0.23±0.00	**0.20±0.00**	**0.20±0.00**
Basketball finer(2014)	0.21±0.00	0.44±0.05	0.52±0.00	0.36±0.02	0.22±0.00	0.30±0.00	0.28±0.00	0.49±0.00	0.26±0.00	0.22±0.00	**0.20±0.00**	**0.20±0.00**

PageRank and SVD_NRS are 0.07, 0.82, 0.22, 0.19, 0.21, and 0.12, respectively.

Table 20 shows improvements on $\mathcal{L}_{upset,naive}$ by "proximal baseline" when setting a certain baseline as $\mathbf{r}'$. Across all real data sets, "proximal baseline" improves the most, by 0.24, again with SyncRank as initial guess, while the average improvement for SpringRank, SerialRank, BTL, Eig.Cent., PageRank and SVD_NRS are 0.00, 0.18, 0.04, 0.03, 0.03, and 0.01, respectively.

Table 18. Result table on $\mathcal{L}_{\text{upset, ratio}}$ for each year in the time series matches, applying the trained model for 1985 directly without further training, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue. As MVR could not generate scores, we omit the results here.

Data	SpringRank	SyncRank	SerialRank	BTL	DavidScore	Eig.Cent.	PageRank	RankCent.	SVD_RS	SVD_NRS	Directly Apply	Train Specifically
Basketball finer(1985)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.47±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.46±0.00	0.46±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1986)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.60±0.00	0.48±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1987)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.46±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1988)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.56±0.00	0.52±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1989)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.49±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1990)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.55±0.00	0.48±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1991)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.54±0.00	0.49±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1992)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.55±0.00	0.44±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1993)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.47±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1994)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.49±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.46±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1995)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.55±0.00	0.47±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1996)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.48±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.45±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1997)	0.04±0.00	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.48±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1998)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.47±0.00	0.45±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(1999)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.48±0.00	0.46±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2000)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.55±0.00	0.50±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2001)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.51±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2002)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.51±0.00	0.50±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2003)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.50±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.47±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2004)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.45±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2005)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.60±0.00	0.43±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2006)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.46±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2007)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.53±0.00	**0.01±0.00**	**0.01±0.00**	0.53±0.00	0.49±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2008)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.51±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.47±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2009)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.50±0.00	0.49±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2010)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.58±0.00	0.48±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2011)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.59±0.00	0.46±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2012)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.57±0.00	0.48±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2013)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.52±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.46±0.00	0.48±0.00	**0.01±0.00**	**0.01±0.00**
Basketball finer(2014)	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	**0.01±0.00**	0.53±0.00	**0.01±0.00**	**0.01±0.00**	0.56±0.00	0.48±0.00	0.47±0.00	**0.01±0.00**	**0.01±0.00**

G. Variant and Hyperparameter Selection

The results reported in the main text are selected within either non-proximal or proximal categories depending on whether they have proximal gradient steps within the architecture. All selections are carried out via the lowest $\mathcal{L}_{\text{upset, simple}}$ or the lowest $\mathcal{L}_{\text{upset, naive}}$ or the lowest $\mathcal{L}_{\text{upset, ratio}}$ depending on our objective. "–" in the tables in this section means "not applicable". There are a total of two variants for non-proximal variants and three for proximal variants for score generation, and each can be coupled with one of DIMPA (He et al., 2021) and the inception block model (ib) (Tong et al., 2020) (or others like MagNet (Zhang et al., 2021), not tested here) for digraph embedding learning. We fix the learning rate to be 0.01, and vary the the method of pretraining, the coefficient for $\mathcal{L}_{\text{upset, margin}}$ in training, the coefficient for $\mathcal{L}_{\text{upset, ratio}}$ in training, and the baseline selected for "proximal baseline" variant. Choices are SyncRank, SpringRank, SerialRank, BTL, Eig.Cent., PageRank and SVD_NRS for real-world data sets, and SpringRank, BTL, SerialRank for synthetic data. We consider these choices to demonstrate that our "proximal baseline" has the ability to improve over initial guess vectors coming from different types of baselines. See Sec. F for more details on improvements over baselines.

Table 19. Result table on $\mathcal{L}_{\text{upset, simple}}$ improvement with “proximal baseline” training starting from a baseline as initial guess, for individual directed graphs, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue.

Data	SpringRank	SyncRank	SerialRank	BTL	Eig.Cent.	PageRank	SVD_NRS
HeadToHead	-0.01±0.00	-0.98±0.00	**-1.02±0.00**	-0.12±0.03	-0.48±0.00	-0.37±0.00	-0.43±0.00
Finance	-0.63±0.00	**-0.98±0.00**	-0.61±0.00	-0.78±0.01	-0.74±0.00	-0.75±0.00	-0.64±0.00
Animal	-0.09±0.05	-1.36±0.24	**-1.42±0.50**	-0.01±0.01	-0.02±0.02	-0.08±0.07	-0.02±0.07
Faculty: Business	-0.03±0.00	-0.47±0.00	**-0.70±0.04**	-0.00±0.02	-0.02±0.03	-0.04±0.03	-0.03±0.02
Faculty: CS	-0.00±0.00	-0.66±0.10	**-0.72±0.07**	0.00±0.01	0.01±0.03	-0.00±0.00	-0.10±0.04
Faculty: History	-0.02±0.01	-0.27±0.00	**-1.77±0.82**	0.00±0.01	-0.01±0.03	-0.00±0.00	-0.00±0.00
Football(2009)	-0.00±0.00	**-0.70±0.46**	-0.16±0.06	-0.03±0.08	-0.12±0.05	-0.24±0.06	-0.02±0.01
Football(2010)	-0.24±0.06	**-0.85±0.17**	-0.58±0.06	-0.22±0.09	-0.40±0.04	-0.41±0.04	-0.21±0.05
Football(2011)	-0.00±0.01	**-0.74±0.37**	-0.10±0.06	-0.03±0.04	-0.13±0.09	-0.19±0.09	-0.02±0.09
Football(2012)	-0.17±0.06	**-0.72±0.04**	-0.57±0.07	-0.14±0.10	-0.10±0.09	-0.22±0.03	-0.05±0.08
Football(2013)	-0.03±0.03	**-1.08±0.10**	-0.08±0.06	-0.06±0.05	-0.16±0.06	-0.15±0.03	-0.04±0.03
Football(2014)	-0.17±0.05	**-0.83±0.20**	-0.59±0.08	-0.17±0.06	-0.27±0.09	-0.24±0.07	-0.07±0.09
Football finer(2009)	-0.03±0.03	**-1.04±0.05**	-0.18±0.06	-0.06±0.14	-0.17±0.11	-0.24±0.12	-0.02±0.03
Football finer(2010)	-0.30±0.02	**-0.63±0.02**	-0.45±0.02	-0.31±0.09	-0.37±0.00	-0.49±0.00	-0.17±0.00
Football finer(2011)	-0.00±0.02	**-0.75±0.04**	-0.14±0.06	-0.03±0.06	-0.08±0.05	-0.19±0.04	-0.01±0.02
Football finer(2012)	-0.03±0.03	**-0.90±0.18**	-0.12±0.08	-0.03±0.06	-0.14±0.06	-0.29±0.05	-0.01±0.03
Football finer(2013)	-0.13±0.07	**-1.01±0.02**	-0.12±0.07	-0.08±0.05	-0.19±0.11	-0.26±0.09	-0.07±0.03
Football finer(2014)	-0.16±0.00	**-0.74±0.04**	-0.57±0.00	-0.19±0.04	-0.23±0.00	-0.23±0.00	-0.09±0.03
Basketball(1985)	-0.01±0.01	-0.91±0.00	**-1.26±0.04**	-0.10±0.07	-0.05±0.01	-0.07±0.02	-0.07±0.01
Basketball(1986)	-0.01±0.01	-1.10±0.00	**-1.11±0.03**	-0.14±0.02	-0.07±0.01	-0.06±0.02	-0.03±0.02
Basketball(1987)	-0.00±0.01	**-1.02±0.01**	-0.94±0.03	-0.11±0.02	-0.11±0.02	-0.01±0.01	-0.05±0.02
Basketball(1988)	-0.00±0.00	-0.96±0.00	**-1.07±0.04**	-0.11±0.04	-0.06±0.02	-0.04±0.02	-0.03±0.01
Basketball(1989)	0.00±0.00	**-1.04±0.00**	-0.99±0.02	-0.12±0.02	-0.00±0.00	-0.02±0.01	-0.07±0.01
Basketball(1990)	-0.00±0.01	-0.98±0.00	**-1.09±0.03**	-0.10±0.11	-0.09±0.01	-0.07±0.01	-0.05±0.01
Basketball(1991)	-0.02±0.01	-1.10±0.00	**-1.15±0.04**	-0.09±0.06	-0.03±0.01	-0.01±0.02	-0.08±0.01
Basketball(1992)	-0.01±0.01	**-1.07±0.00**	-1.05±0.02	-0.06±0.10	-0.04±0.01	0.00±0.00	-0.07±0.01
Basketball(1993)	-0.01±0.01	-1.00±0.01	**-1.18±0.06**	-0.09±0.07	-0.05±0.01	-0.06±0.01	-0.07±0.01
Basketball(1994)	0.00±0.00	-0.98±0.00	**-1.15±0.02**	-0.02±0.09	-0.02±0.02	-0.03±0.01	-0.05±0.01
Basketball(1995)	0.00±0.00	**-1.06±0.01**	-1.01±0.03	-0.03±0.10	-0.02±0.01	-0.06±0.01	-0.04±0.01
Basketball(1996)	-0.00±0.01	-0.90±0.01	**-1.11±0.02**	-0.08±0.08	-0.02±0.01	-0.02±0.01	-0.07±0.01
Basketball(1997)	-0.01±0.01	-0.96±0.01	**-1.02±0.04**	-0.12±0.04	-0.05±0.01	-0.03±0.01	-0.07±0.01
Basketball(1998)	-0.00±0.01	-0.95±0.01	**-1.00±0.06**	-0.14±0.02	-0.06±0.01	-0.02±0.01	-0.07±0.01
Basketball(1999)	-0.00±0.01	-0.83±0.00	**-1.10±0.02**	-0.09±0.07	-0.07±0.01	-0.07±0.01	-0.07±0.01
Basketball(2000)	-0.01±0.01	-1.00±0.00	**-1.03±0.03**	-0.09±0.02	-0.02±0.01	-0.03±0.01	-0.05±0.01
Basketball(2001)	-0.00±0.00	-1.02±0.00	**-1.10±0.11**	-0.09±0.08	-0.05±0.02	-0.02±0.02	-0.11±0.02
Basketball(2002)	-0.02±0.00	-0.95±0.00	**-1.05±0.02**	-0.09±0.06	-0.01±0.01	-0.06±0.01	-0.05±0.01
Basketball(2003)	0.00±0.00	-1.07±0.00	**-1.13±0.01**	-0.08±0.07	-0.08±0.01	-0.07±0.01	-0.07±0.01
Basketball(2004)	-0.00±0.00	-0.98±0.00	**-1.14±0.11**	-0.09±0.06	-0.05±0.01	-0.06±0.01	-0.07±0.01
Basketball(2005)	0.00±0.00	-1.05±0.01	**-1.06±0.06**	-0.14±0.03	-0.00±0.01	-0.06±0.01	-0.07±0.01
Basketball(2006)	0.00±0.00	-1.04±0.00	**-1.09±0.03**	0.02±0.09	0.00±0.00	-0.03±0.01	-0.04±0.01
Basketball(2007)	-0.00±0.00	**-1.10±0.00**	-1.03±0.08	-0.09±0.02	-0.03±0.01	-0.02±0.01	-0.04±0.01
Basketball(2008)	0.00±0.00	-0.97±0.00	**-1.10±0.03**	-0.03±0.07	-0.04±0.01	-0.05±0.01	-0.06±0.01
Basketball(2009)	-0.00±0.00	-1.02±0.01	**-1.17±0.02**	-0.12±0.02	-0.03±0.01	-0.03±0.01	-0.06±0.01
Basketball(2010)	-0.00±0.00	-0.83±0.00	**-1.06±0.01**	-0.03±0.13	-0.04±0.01	-0.04±0.01	-0.05±0.01
Basketball(2011)	-0.00±0.01	-0.90±0.00	**-1.09±0.03**	0.00±0.08	-0.04±0.01	-0.05±0.01	-0.02±0.01
Basketball(2012)	-0.00±0.00	**-1.11±0.00**	**-1.11±0.05**	-0.07±0.08	-0.04±0.01	-0.03±0.02	-0.04±0.01
Basketball(2013)	-0.00±0.00	-0.95±0.00	**-1.09±0.06**	-0.11±0.06	-0.06±0.01	-0.03±0.01	-0.07±0.01
Basketball(2014)	-0.00±0.01	-0.96±0.00	**-1.14±0.05**	-0.00±0.10	-0.01±0.01	-0.04±0.01	-0.02±0.01
Basketball finer(1985)	-0.01±0.01	-0.92±0.00	**-0.96±0.10**	-0.56±0.07	-0.35±0.01	-0.32±0.02	-0.07±0.01
Basketball finer(1986)	-0.00±0.00	**-1.12±0.00**	-1.01±0.03	-0.52±0.08	-0.30±0.02	-0.27±0.01	-0.05±0.01
Basketball finer(1987)	-0.01±0.00	**-1.02±0.01**	-0.90±0.02	-0.45±0.07	-0.25±0.01	-0.28±0.01	-0.05±0.01
Basketball finer(1988)	-0.01±0.00	**-1.09±0.01**	-1.02±0.02	-0.55±0.11	-0.38±0.02	-0.34±0.01	-0.03±0.01
Basketball finer(1989)	-0.00±0.00	**-0.96±0.00**	-0.87±0.03	-0.51±0.06	-0.27±0.01	-0.29±0.01	-0.07±0.01
Basketball finer(1990)	-0.01±0.01	-0.96±0.00	**-1.02±0.01**	-0.55±0.05	-0.42±0.01	-0.31±0.01	-0.03±0.01
Basketball finer(1991)	-0.01±0.00	**-1.13±0.00**	-1.03±0.00	-0.45±0.06	-0.46±0.02	-0.41±0.01	-0.05±0.01
Basketball finer(1992)	-0.00±0.00	**-1.04±0.00**	-0.99±0.02	-0.46±0.06	-0.41±0.02	-0.37±0.02	-0.07±0.01
Basketball finer(1993)	-0.00±0.01	-0.98±0.01	**-1.08±0.10**	-0.49±0.06	-0.35±0.01	-0.27±0.01	-0.05±0.01
Basketball finer(1994)	0.00±0.00	-1.02±0.00	**-1.11±0.04**	-0.50±0.07	-0.39±0.02	-0.28±0.01	-0.03±0.01
Basketball finer(1995)	-0.01±0.01	**-1.06±0.01**	-0.98±0.01	-0.48±0.08	-0.35±0.01	-0.30±0.01	-0.06±0.01
Basketball finer(1996)	-0.00±0.01	-0.90±0.00	**-0.95±0.00**	-0.44±0.06	-0.27±0.02	-0.25±0.02	-0.06±0.01
Basketball finer(1997)	-0.01±0.01	**-1.02±0.01**	-0.98±0.01	-0.47±0.04	-0.28±0.01	-0.24±0.01	-0.08±0.01
Basketball finer(1998)	-0.01±0.01	**-0.96±0.01**	-0.95±0.01	-0.44±0.06	-0.25±0.01	-0.24±0.01	-0.08±0.01
Basketball finer(1999)	-0.00±0.01	-0.90±0.00	**-1.05±0.02**	-0.44±0.08	-0.27±0.01	-0.19±0.01	-0.08±0.01
Basketball finer(2000)	0.00±0.00	-0.97±0.00	**-0.99±0.02**	-0.42±0.05	-0.31±0.02	-0.25±0.01	-0.06±0.01
Basketball finer(2001)	-0.00±0.00	-0.96±0.00	**-1.06±0.00**	-0.46±0.06	-0.31±0.01	-0.25±0.02	-0.10±0.01
Basketball finer(2002)	-0.04±0.02	**-0.99±0.01**	-0.92±0.02	-0.52±0.09	-0.30±0.02	-0.25±0.01	-0.05±0.01
Basketball finer(2003)	0.00±0.00	**-1.01±0.00**	-0.98±0.07	-0.49±0.10	-0.25±0.01	-0.21±0.01	-0.05±0.01
Basketball finer(2004)	-0.02±0.01	**-1.00±0.06**	-0.93±0.01	-0.48±0.07	-0.29±0.01	-0.24±0.01	-0.07±0.01
Basketball finer(2005)	-0.01±0.01	**-1.08±0.01**	-0.93±0.00	-0.42±0.05	-0.22±0.01	-0.18±0.01	-0.06±0.01
Basketball finer(2006)	-0.00±0.01	**-1.01±0.00**	-0.97±0.00	-0.42±0.07	-0.24±0.02	-0.17±0.01	-0.04±0.01
Basketball finer(2007)	-0.00±0.01	**-1.05±0.01**	-0.97±0.14	-0.46±0.06	-0.25±0.02	-0.18±0.02	-0.06±0.01
Basketball finer(2008)	-0.00±0.00	-0.94±0.00	**-0.98±0.00**	-0.42±0.09	-0.29±0.01	-0.21±0.01	-0.04±0.01
Basketball finer(2009)	-0.00±0.00	-0.96±0.00	**-1.00±0.04**	-0.45±0.04	-0.26±0.01	-0.21±0.02	-0.06±0.01
Basketball finer(2010)	-0.00±0.00	-0.93±0.00	**-0.98±0.02**	-0.41±0.07	-0.18±0.01	-0.21±0.01	-0.05±0.01
Basketball finer(2011)	-0.01±0.00	-0.88±0.00	**-0.99±0.01**	-0.44±0.06	-0.21±0.01	-0.22±0.01	-0.03±0.01
Basketball finer(2012)	0.00±0.00	-0.94±0.01	**-0.98±0.03**	-0.45±0.07	-0.26±0.01	-0.21±0.01	-0.03±0.00
Basketball finer(2013)	-0.00±0.01	-0.95±0.01	**-1.06±0.00**	-0.45±0.08	-0.27±0.01	-0.25±0.01	-0.07±0.01
Basketball finer(2014)	-0.01±0.00	-0.99±0.19	**-1.07±0.00**	-0.47±0.09	-0.24±0.02	-0.20±0.01	-0.02±0.01
ERO(p=0.05, style=uniform,η=0)	-0.05±0.02	**-1.71±0.01**	-1.64±0.01	-0.16±0.11	0.08±0.01	-0.00±0.01	-0.06±0.02
ERO(p=0.05, style=gamma,η=0)	-0.13±0.01	**-1.76±0.03**	-1.61±0.40	0.22±0.03	0.31±0.03	0.22±0.02	0.07±0.01
ERO(p=0.05, style=uniform,η=0.1)	-0.00±0.00	**-1.49±0.02**	-1.30±0.01	-0.05±0.04	-0.27±0.01	-0.24±0.01	-0.00±0.01
ERO(p=0.05, style=gamma,η=0.1)	-0.00±0.00	**-1.60±0.01**	-1.19±0.04	0.05±0.01	-0.05±0.03	0.02±0.03	0.02±0.01
ERO(p=0.05, style=uniform,η=0.2)	-0.01±0.02	**-1.37±0.01**	-1.21±0.04	-0.01±0.05	-0.22±0.04	-0.31±0.04	-0.02±0.03
ERO(p=0.05, style=gamma,η=0.2)	-0.00±0.01	**-1.43±0.02**	-1.14±0.02	0.00±0.01	-0.17±0.03	-0.14±0.02	-0.01±0.01
ERO(p=0.05, style=uniform,η=0.3)	-0.01±0.01	**-1.10±0.02**	-0.98±0.11	0.05±0.05	-0.14±0.05	-0.21±0.06	-0.01±0.03
ERO(p=0.05, style=gamma,η=0.3)	-0.03±0.01	**-1.16±0.02**	-1.05±0.02	-0.04±0.02	-0.29±0.01	-0.30±0.02	-0.05±0.01
ERO(p=0.05, style=uniform,η=0.4)	-0.00±0.00	**-1.00±0.05**	-0.75±0.03	0.04±0.05	-0.01±0.02	-0.09±0.04	0.02±0.01
ERO(p=0.05, style=gamma,η=0.4)	-0.11±0.03	**-0.98±0.05**	**-0.98±0.09**	-0.10±0.03	-0.31±0.06	-0.34±0.02	-0.17±0.15
ERO(p=0.05, style=uniform,η=0.5)	-0.00±0.00	**-0.84±0.01**	-0.64±0.02	0.02±0.15	-0.01±0.03	-0.04±0.05	-0.02±0.04
ERO(p=0.05, style=gamma,η=0.5)	-0.27±0.01	-0.84±0.04	**-0.87±0.13**	-0.25±0.04	-0.38±0.06	-0.44±0.02	-0.66±0.02
ERO(p=0.05, style=uniform,η=0.6)	-0.00±0.00	**-0.66±0.01**	-0.57±0.01	0.00±0.03	-0.00±0.02	-0.05±0.05	-0.18±0.05
ERO(p=0.05, style=gamma,η=0.6)	-0.39±0.02	-0.83±0.09	**-0.92±0.02**	-0.32±0.09	-0.53±0.01	-0.52±0.03	-0.76±0.09
ERO(p=0.05, style=uniform,η=0.7)	-0.01±0.03	**-0.64±0.02**	-0.56±0.26	-0.01±0.04	-0.14±0.27	-0.23±0.30	-0.48±0.27
ERO(p=0.05, style=gamma,η=0.7)	-0.42±0.00	-0.86±0.05	**-0.89±0.05**	-0.38±0.06	-0.59±0.02	-0.54±0.02	-0.87±0.04
ERO(p=0.05, style=uniform,η=0.8)	-0.24±0.20	-0.61±0.23	-0.69±0.23	-0.19±0.21	-0.26±0.28	-0.26±0.24	**-0.72±0.24**
ERO(p=0.05, style=gamma,η=0.8)	-0.51±0.00	-0.84±0.03	**-0.90±0.03**	-0.44±0.05	-0.61±0.03	-0.55±0.05	-0.88±0.09
ERO(p=1, style=uniform,η=0)	0.00±0.00	**-1.85±0.00**	0.00±0.00	-0.23±0.11	0.00±0.00	0.00±0.00	0.00±0.00
ERO(p=1, style=gamma,η=0)	0.00±0.00	**-1.83±0.00**	0.00±0.00	-0.09±0.01	0.00±0.00	0.00±0.00	-0.01±0.00
ERO(p=1, style=uniform,η=0.1)	-0.02±0.01	**-1.67±0.00**	0.00±0.00	-0.12±0.07	-0.09±0.02	-0.49±0.03	-0.00±0.00
ERO(p=1, style=gamma,η=0.1)	-0.01±0.01	**-1.58±0.00**	0.00±0.00	-0.03±0.01	-0.25±0.02	-0.51±0.02	-0.00±0.00
ERO(p=1, style=uniform,η=0.2)	-0.01±0.01	**-1.41±0.00**	-0.00±0.00	-0.11±0.05	-0.10±0.01	-0.34±0.02	-0.00±0.00
ERO(p=1, style=gamma,η=0.2)	-0.01±0.01	**-1.45±0.00**	-0.00±0.00	-0.01±0.02	-0.15±0.02	-0.32±0.03	-0.00±0.00
ERO(p=1, style=uniform,η=0.3)	-0.00±0.00	**-1.21±0.00**	0.00±0.00	-0.06±0.04	-0.07±0.01	-0.23±0.02	-0.00±0.00
ERO(p=1, style=gamma,η=0.3)	-0.00±0.00	**-1.25±0.00**	-0.00±0.00	-0.00±0.01	-0.12±0.03	-0.21±0.03	0.00±0.00
ERO(p=1, style=uniform,η=0.4)	-0.00±0.00	**-1.05±0.00**	-0.00±0.00	-0.05±0.04	-0.04±0.01	-0.14±0.02	-0.00±0.01
ERO(p=1, style=gamma,η=0.4)	-0.02±0.00	**-1.07±0.00**	0.00±0.00	-0.04±0.00	-0.15±0.00	-0.21±0.00	-0.00±0.00
ERO(p=1, style=uniform,η=0.5)	-0.13±0.00	**-0.94±0.00**	-0.06±0.00	-0.18±0.03	-0.19±0.00	-0.26±0.00	-0.11±0.00
ERO(p=1, style=gamma,η=0.5)	-0.21±0.00	**-0.98±0.00**	-0.07±0.00	-0.23±0.01	-0.30±0.00	-0.33±0.00	-0.19±0.00
ERO(p=1, style=uniform,η=0.6)	-0.32±0.00	**-0.94±0.00**	-0.27±0.00	-0.36±0.03	-0.37±0.00	-0.40±0.00	-0.31±0.00
ERO(p=1, style=gamma,η=0.6)	-0.38±0.00	**-0.93±0.00**	-0.27±0.00	-0.42±0.01	-0.44±0.00	-0.46±0.00	-0.37±0.00
ERO(p=1, style=uniform,η=0.7)	-0.51±0.00	**-0.95±0.00**	-0.49±0.00	-0.54±0.03	-0.53±0.00	-0.55±0.00	-0.51±0.00
ERO(p=1, style=gamma,η=0.7)	-0.55±0.00	**-0.98±0.00**	-0.52±0.00	-0.59±0.01	-0.59±0.00	-0.60±0.00	-0.56±0.00
ERO(p=1, style=uniform,η=0.8)	-0.68±0.00	**-0.97±0.00**	-0.95±0.00	-0.71±0.02	-0.70±0.00	-0.71±0.00	-0.70±0.00
ERO(p=1, style=gamma,η=0.8)	-0.71±0.00	-0.96±0.00	**-0.98±0.00**	-0.75±0.01	-0.73±0.00	-0.73±0.00	-0.73±0.00

Table 20. Result table on $\mathcal{L}_{\text{upset, naive}}$ improvement with “proximal baseline” training starting from a baseline as initial guess, for individual directed graphs, averaged over 10 runs, and plus/minus one standard deviation. The best is marked in **bold red** while the second best is highlighted in underline blue.

Data	SpringRank	SyncRank	SerialRank	BTL	Eig.Cent.	PageRank	SVD_NRS
HeadToHead	-0.00±0.00	**-0.24±0.00**	-0.18±0.00	0.00±0.00	-0.05±0.00	-0.02±0.00	-0.04±0.00
Finance	-0.00±0.00	**-0.09±0.00**	0.00±0.00	-0.03±0.00	-0.02±0.00	-0.02±0.00	-0.00±0.00
Animal	-0.02±0.01	-0.34±0.06	**-0.44±0.12**	0.00±0.00	-0.01±0.00	-0.02±0.02	-0.00±0.02
Faculty: Business	-0.01±0.00	-0.12±0.00	**-0.17±0.01**	-0.00±0.00	-0.01±0.01	-0.01±0.01	-0.01±0.00
Faculty: CS	-0.00±0.00	-0.17±0.02	**-0.18±0.02**	0.00±0.00	0.00±0.01	-0.00±0.00	-0.02±0.01
Faculty: History	-0.00±0.00	-0.07±0.00	**-0.44±0.20**	0.00±0.00	-0.00±0.01	-0.00±0.00	-0.00±0.00
Football(2009)	0.00±0.00	**-0.18±0.11**	-0.04±0.02	-0.01±0.02	-0.03±0.01	-0.04±0.02	-0.00±0.00
Football(2010)	-0.01±0.01	**-0.15±0.04**	-0.08±0.01	-0.00±0.01	-0.03±0.02	-0.03±0.02	-0.00±0.00
Football(2011)	-0.00±0.00	**-0.19±0.09**	-0.01±0.01	-0.01±0.01	-0.01±0.01	-0.03±0.02	-0.00±0.01
Football(2012)	-0.01±0.01	**-0.17±0.01**	-0.12±0.03	-0.01±0.02	-0.00±0.01	-0.01±0.00	0.00±0.00
Football(2013)	-0.01±0.01	**-0.27±0.03**	-0.02±0.01	-0.02±0.01	-0.04±0.01	-0.04±0.01	-0.01±0.01
Football(2014)	-0.00±0.01	**-0.18±0.05**	-0.12±0.03	-0.01±0.01	-0.02±0.03	-0.00±0.00	-0.01±0.01
Football finer(2009)	-0.01±0.01	**-0.26±0.01**	-0.04±0.01	-0.01±0.03	-0.04±0.03	-0.06±0.03	-0.01±0.01
Football finer(2010)	-0.02±0.01	**-0.11±0.01**	-0.05±0.02	-0.03±0.03	-0.03±0.02	-0.06±0.01	-0.00±0.00
Football finer(2011)	-0.00±0.01	**-0.19±0.01**	-0.04±0.01	-0.01±0.01	-0.02±0.01	-0.05±0.01	-0.00±0.01
Football finer(2012)	-0.01±0.01	**-0.23±0.04**	-0.03±0.02	-0.01±0.01	-0.04±0.02	-0.07±0.01	-0.00±0.01
Football finer(2013)	-0.03±0.03	**-0.25±0.00**	-0.03±0.02	-0.02±0.02	-0.05±0.03	-0.06±0.02	-0.02±0.01
Football finer(2014)	-0.00±0.01	**-0.17±0.02**	-0.09±0.03	-0.02±0.02	-0.01±0.01	-0.02±0.01	-0.00±0.01
Basketball(1985)	-0.00±0.00	-0.23±0.00	**-0.32±0.01**	-0.03±0.02	-0.01±0.00	-0.02±0.01	-0.02±0.00
Basketball(1986)	-0.00±0.00	**-0.28±0.00**	**-0.28±0.01**	-0.03±0.01	-0.02±0.00	-0.01±0.00	-0.01±0.00
Basketball(1987)	-0.00±0.00	**-0.25±0.00**	-0.23±0.01	-0.03±0.01	-0.03±0.01	-0.00±0.00	-0.01±0.00
Basketball(1988)	0.00±0.00	-0.24±0.00	**-0.27±0.01**	-0.03±0.01	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(1989)	0.00±0.00	**-0.26±0.00**	-0.25±0.01	-0.03±0.00	0.00±0.00	-0.01±0.00	-0.02±0.00
Basketball(1990)	-0.00±0.00	-0.25±0.00	**-0.27±0.01**	-0.03±0.03	-0.02±0.00	-0.02±0.00	-0.01±0.00
Basketball(1991)	-0.00±0.00	-0.27±0.00	**-0.29±0.01**	-0.02±0.01	-0.01±0.00	-0.00±0.00	-0.02±0.00
Basketball(1992)	-0.00±0.00	**-0.27±0.00**	-0.26±0.00	-0.01±0.03	-0.01±0.00	0.00±0.00	-0.02±0.00
Basketball(1993)	-0.00±0.00	-0.25±0.00	**-0.29±0.02**	-0.02±0.02	-0.01±0.00	-0.02±0.00	-0.02±0.00
Basketball(1994)	0.00±0.00	-0.25±0.00	**-0.29±0.00**	-0.01±0.02	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(1995)	0.00±0.00	**-0.26±0.00**	-0.25±0.01	-0.01±0.02	-0.00±0.00	-0.02±0.00	-0.01±0.00
Basketball(1996)	-0.00±0.00	-0.23±0.00	**-0.28±0.01**	-0.02±0.02	-0.01±0.00	-0.01±0.00	-0.02±0.00
Basketball(1997)	-0.00±0.00	-0.24±0.00	**-0.25±0.01**	-0.03±0.01	-0.01±0.00	-0.01±0.00	-0.02±0.00
Basketball(1998)	-0.00±0.00	-0.24±0.00	**-0.25±0.01**	-0.03±0.01	-0.02±0.00	-0.01±0.00	-0.02±0.00
Basketball(1999)	-0.00±0.00	-0.21±0.00	**-0.27±0.01**	-0.02±0.02	-0.02±0.00	-0.02±0.00	-0.02±0.00
Basketball(2000)	-0.00±0.00	-0.25±0.00	**-0.26±0.01**	-0.02±0.01	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2001)	0.00±0.00	-0.26±0.00	**-0.28±0.03**	-0.02±0.02	-0.01±0.00	-0.01±0.00	-0.03±0.01
Basketball(2002)	-0.00±0.00	-0.24±0.00	**-0.26±0.00**	-0.02±0.01	-0.00±0.00	-0.02±0.00	-0.01±0.00
Basketball(2003)	0.00±0.00	-0.27±0.00	**-0.28±0.00**	-0.02±0.02	-0.02±0.00	-0.02±0.00	-0.02±0.00
Basketball(2004)	-0.00±0.00	-0.25±0.00	**-0.28±0.03**	-0.02±0.01	-0.01±0.00	-0.02±0.00	-0.02±0.00
Basketball(2005)	0.00±0.00	**-0.26±0.00**	**-0.26±0.02**	-0.04±0.01	-0.00±0.00	-0.01±0.00	-0.02±0.00
Basketball(2006)	0.00±0.00	-0.26±0.00	**-0.27±0.01**	0.00±0.02	0.00±0.00	-0.01±0.00	-0.01±0.00
Basketball(2007)	-0.00±0.00	**-0.28±0.00**	-0.26±0.02	-0.02±0.01	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2008)	0.00±0.00	-0.24±0.00	**-0.27±0.01**	-0.01±0.02	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2009)	-0.00±0.00	-0.26±0.00	**-0.29±0.01**	-0.03±0.01	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2010)	-0.00±0.00	-0.21±0.00	**-0.26±0.01**	-0.01±0.03	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2011)	-0.00±0.00	-0.22±0.00	**-0.27±0.01**	0.00±0.02	-0.01±0.00	-0.01±0.00	-0.00±0.00
Basketball(2012)	-0.00±0.00	**-0.28±0.00**	**-0.28±0.01**	-0.02±0.02	-0.01±0.00	-0.01±0.00	-0.01±0.00
Basketball(2013)	-0.00±0.00	-0.24±0.00	**-0.27±0.02**	-0.03±0.01	-0.01±0.00	-0.01±0.00	-0.02±0.00
Basketball(2014)	-0.00±0.00	-0.24±0.00	**-0.28±0.01**	-0.00±0.03	-0.00±0.00	-0.01±0.00	-0.00±0.00
Basketball finer(1985)	-0.00±0.00	-0.23±0.00	**-0.24±0.02**	-0.14±0.02	-0.09±0.00	-0.08±0.00	-0.02±0.00
Basketball finer(1986)	0.00±0.00	**-0.28±0.00**	-0.25±0.01	-0.13±0.02	-0.08±0.00	-0.07±0.00	-0.01±0.00
Basketball finer(1987)	-0.00±0.00	**-0.26±0.00**	-0.23±0.00	-0.11±0.02	-0.06±0.00	-0.07±0.00	-0.01±0.00
Basketball finer(1988)	-0.00±0.00	**-0.27±0.00**	-0.25±0.00	-0.14±0.03	-0.09±0.00	-0.09±0.00	-0.01±0.00
Basketball finer(1989)	0.00±0.00	**-0.24±0.00**	-0.22±0.01	-0.13±0.01	-0.07±0.00	-0.07±0.00	-0.02±0.00
Basketball finer(1990)	-0.00±0.00	-0.24±0.00	**-0.25±0.00**	-0.14±0.01	-0.10±0.00	-0.08±0.00	-0.01±0.00
Basketball finer(1991)	-0.00±0.00	**-0.28±0.00**	-0.26±0.01	-0.11±0.02	-0.12±0.00	-0.10±0.00	-0.01±0.00
Basketball finer(1992)	-0.00±0.00	**-0.26±0.00**	-0.25±0.00	-0.12±0.01	-0.10±0.00	-0.09±0.00	-0.02±0.00
Basketball finer(1993)	-0.00±0.00	-0.24±0.00	**-0.27±0.02**	-0.12±0.01	-0.09±0.00	-0.07±0.00	-0.01±0.00
Basketball finer(1994)	0.00±0.00	-0.25±0.00	**-0.28±0.01**	-0.12±0.02	-0.10±0.00	-0.07±0.00	-0.01±0.00
Basketball finer(1995)	-0.00±0.00	**-0.26±0.00**	-0.25±0.00	-0.12±0.02	-0.09±0.00	-0.07±0.00	-0.02±0.00
Basketball finer(1996)	-0.00±0.00	**-0.22±0.00**	**-0.22±0.01**	-0.11±0.01	-0.07±0.01	-0.06±0.00	-0.01±0.00
Basketball finer(1997)	-0.00±0.00	**-0.26±0.00**	-0.24±0.00	-0.12±0.01	-0.07±0.00	-0.06±0.00	-0.02±0.00
Basketball finer(1998)	-0.00±0.00	**-0.24±0.00**	**-0.24±0.00**	-0.11±0.02	-0.06±0.00	-0.06±0.00	-0.02±0.00
Basketball finer(1999)	-0.00±0.00	-0.23±0.00	**-0.26±0.01**	-0.11±0.02	-0.07±0.00	-0.05±0.00	-0.02±0.00
Basketball finer(2000)	0.00±0.00	-0.24±0.00	**-0.25±0.00**	-0.10±0.01	-0.08±0.00	-0.06±0.00	-0.01±0.00
Basketball finer(2001)	-0.00±0.00	-0.24±0.00	**-0.25±0.01**	-0.11±0.01	-0.08±0.00	-0.06±0.00	-0.02±0.00
Basketball finer(2002)	-0.01±0.00	**-0.25±0.00**	-0.23±0.01	-0.13±0.02	-0.08±0.00	-0.06±0.00	-0.01±0.00
Basketball finer(2003)	0.00±0.00	**-0.25±0.00**	-0.22±0.02	-0.12±0.02	-0.06±0.00	-0.05±0.00	-0.01±0.00
Basketball finer(2004)	-0.00±0.00	**-0.25±0.01**	-0.23±0.00	-0.12±0.02	-0.07±0.00	-0.06±0.00	-0.02±0.00
Basketball finer(2005)	-0.00±0.00	**-0.27±0.00**	-0.22±0.00	-0.10±0.01	-0.05±0.00	-0.04±0.00	-0.02±0.00
Basketball finer(2006)	-0.00±0.00	**-0.25±0.00**	-0.24±0.00	-0.10±0.02	-0.06±0.00	-0.04±0.00	-0.01±0.00
Basketball finer(2007)	-0.00±0.00	**-0.26±0.00**	-0.23±0.03	-0.12±0.01	-0.06±0.00	-0.04±0.00	-0.01±0.00
Basketball finer(2008)	-0.00±0.00	-0.23±0.00	**-0.24±0.01**	-0.11±0.02	-0.07±0.00	-0.05±0.00	-0.01±0.00
Basketball finer(2009)	-0.00±0.00	-0.24±0.00	**-0.25±0.01**	-0.11±0.01	-0.07±0.00	-0.05±0.00	-0.01±0.00
Basketball finer(2010)	-0.00±0.00	-0.23±0.00	**-0.25±0.00**	-0.10±0.02	-0.05±0.00	-0.05±0.00	-0.01±0.00
Basketball finer(2011)	-0.00±0.00	-0.22±0.00	**-0.25±0.00**	-0.11±0.01	-0.05±0.00	-0.06±0.00	-0.01±0.00
Basketball finer(2012)	0.00±0.00	**-0.24±0.00**	**-0.24±0.01**	-0.11±0.02	-0.07±0.00	-0.05±0.00	-0.01±0.00
Basketball finer(2013)	-0.00±0.00	-0.24±0.00	**-0.26±0.02**	-0.11±0.02	-0.07±0.00	-0.06±0.00	-0.02±0.00
Basketball finer(2014)	-0.00±0.00	-0.25±0.05	**-0.26±0.01**	-0.12±0.02	-0.06±0.01	-0.05±0.00	-0.01±0.00
ERO(p=0.05, style=uniform,η=0)	-0.01±0.01	**-0.43±0.00**	-0.40±0.01	-0.04±0.03	0.02±0.00	-0.00±0.00	-0.01±0.00
ERO(p=0.05, style=gamma,η=0)	-0.03±0.00	**-0.44±0.01**	-0.40±0.10	0.06±0.01	0.08±0.01	0.05±0.01	0.02±0.00
ERO(p=0.05, style=uniform,η=0.1)	-0.00±0.00	**-0.37±0.00**	-0.32±0.00	-0.01±0.01	-0.06±0.00	-0.06±0.00	-0.00±0.00
ERO(p=0.05, style=gamma,η=0.1)	-0.00±0.00	**-0.40±0.00**	-0.26±0.02	0.01±0.00	-0.01±0.00	0.01±0.00	0.01±0.00
ERO(p=0.05, style=uniform,η=0.2)	-0.00±0.01	**-0.34±0.00**	-0.30±0.01	-0.00±0.01	-0.05±0.01	-0.08±0.01	-0.00±0.01
ERO(p=0.05, style=gamma,η=0.2)	-0.00±0.00	**-0.36±0.00**	-0.21±0.01	0.00±0.00	-0.02±0.00	-0.02±0.00	-0.00±0.00
ERO(p=0.05, style=uniform,η=0.3)	-0.00±0.00	**-0.28±0.01**	-0.25±0.03	0.01±0.01	-0.03±0.01	-0.05±0.02	-0.00±0.01
ERO(p=0.05, style=gamma,η=0.3)	-0.01±0.00	**-0.29±0.00**	-0.16±0.02	-0.00±0.00	-0.01±0.01	-0.01±0.00	-0.00±0.00
ERO(p=0.05, style=uniform,η=0.4)	-0.00±0.00	**-0.25±0.01**	-0.18±0.01	0.01±0.01	-0.00±0.00	-0.02±0.01	0.00±0.00
ERO(p=0.05, style=gamma,η=0.4)	-0.00±0.01	**-0.24±0.01**	-0.11±0.03	0.00±0.01	-0.01±0.00	-0.01±0.00	-0.00±0.00
ERO(p=0.05, style=uniform,η=0.5)	-0.00±0.00	**-0.21±0.00**	-0.16±0.00	0.01±0.04	-0.00±0.01	-0.01±0.01	-0.00±0.01
ERO(p=0.05, style=gamma,η=0.5)	-0.01±0.01	**-0.21±0.01**	-0.06±0.04	-0.01±0.01	-0.01±0.00	-0.00±0.00	-0.03±0.03
ERO(p=0.05, style=uniform,η=0.6)	-0.00±0.00	**-0.17±0.00**	-0.14±0.00	0.00±0.01	-0.00±0.00	-0.01±0.01	-0.04±0.01
ERO(p=0.05, style=gamma,η=0.6)	-0.01±0.01	**-0.18±0.01**	-0.04±0.00	-0.01±0.01	-0.01±0.01	-0.01±0.01	-0.01±0.01
ERO(p=0.05, style=uniform,η=0.7)	-0.00±0.01	**-0.16±0.00**	-0.09±0.00	-0.00±0.01	0.00±0.00	-0.00±0.00	-0.06±0.00
ERO(p=0.05, style=gamma,η=0.7)	-0.00±0.01	**-0.16±0.01**	-0.05±0.01	-0.01±0.01	-0.01±0.01	0.00±0.00	-0.02±0.01
ERO(p=0.05, style=uniform,η=0.8)	-0.00±0.00	**-0.14±0.00**	-0.07±0.00	0.00±0.01	-0.00±0.00	0.00±0.00	-0.07±0.00
ERO(p=0.05, style=gamma,η=0.8)	-0.01±0.01	**-0.14±0.01**	-0.04±0.01	-0.01±0.01	-0.01±0.01	0.00±0.01	-0.03±0.02
ERO(p=1, style=uniform,η=0)	0.00±0.00	**-0.46±0.00**	0.00±0.00	-0.06±0.03	0.00±0.00	0.00±0.00	0.00±0.00
ERO(p=1, style=gamma,η=0)	0.00±0.00	**-0.46±0.00**	0.00±0.00	-0.02±0.00	0.00±0.00	0.00±0.00	-0.00±0.00
ERO(p=1, style=uniform,η=0.1)	-0.00±0.00	**-0.42±0.00**	0.00±0.00	-0.03±0.02	-0.02±0.01	-0.12±0.01	-0.00±0.00
ERO(p=1, style=gamma,η=0.1)	-0.00±0.00	**-0.39±0.00**	0.00±0.00	-0.01±0.00	-0.05±0.02	-0.11±0.01	-0.00±0.00
ERO(p=1, style=uniform,η=0.2)	-0.00±0.00	**-0.35±0.00**	-0.00±0.00	-0.03±0.01	-0.03±0.00	-0.09±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.2)	-0.00±0.00	**-0.36±0.00**	-0.00±0.00	-0.00±0.00	-0.04±0.01	-0.08±0.01	-0.00±0.00
ERO(p=1, style=uniform,η=0.3)	-0.00±0.00	**-0.30±0.00**	0.00±0.00	-0.02±0.01	-0.02±0.00	-0.06±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.3)	-0.00±0.00	**-0.31±0.00**	-0.00±0.00	-0.00±0.00	-0.03±0.01	-0.05±0.01	0.00±0.00
ERO(p=1, style=uniform,η=0.4)	-0.00±0.00	**-0.26±0.00**	-0.00±0.00	-0.01±0.01	-0.01±0.00	-0.04±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.4)	-0.00±0.00	**-0.27±0.00**	0.00±0.00	-0.00±0.00	-0.02±0.01	-0.03±0.01	0.00±0.00
ERO(p=1, style=uniform,η=0.5)	-0.00±0.00	**-0.22±0.00**	-0.00±0.00	-0.01±0.01	-0.01±0.00	-0.03±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.5)	-0.00±0.00	**-0.22±0.00**	-0.00±0.00	0.00±0.01	-0.01±0.01	-0.01±0.01	0.00±0.00
ERO(p=1, style=uniform,η=0.6)	-0.00±0.00	**-0.17±0.00**	-0.00±0.00	-0.01±0.01	-0.01±0.00	-0.02±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.6)	-0.00±0.00	**-0.16±0.00**	-0.00±0.00	-0.00±0.00	-0.01±0.00	-0.01±0.00	-0.00±0.00
ERO(p=1, style=uniform,η=0.7)	-0.01±0.00	**-0.12±0.00**	-0.00±0.00	-0.00±0.01	-0.00±0.00	-0.00±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.7)	-0.01±0.00	**-0.13±0.00**	0.00±0.00	-0.00±0.00	-0.00±0.00	-0.00±0.00	-0.00±0.00
ERO(p=1, style=uniform,η=0.8)	-0.00±0.00	**-0.08±0.00**	-0.05±0.00	-0.00±0.01	-0.00±0.00	-0.00±0.00	-0.00±0.00
ERO(p=1, style=gamma,η=0.8)	-0.00±0.00	**-0.08±0.00**	-0.05±0.00	-0.00±0.00	-0.00±0.00	0.00±0.00	-0.00±0.00

Table 21. GNN selection among GNNRank-N methods for the lowest $\mathcal{L}_{\text{upset, simple}}$.

Data	Variant	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	ib dist	proximal baseline	innerproduct	1	SVD_NRS	1
Finance	ib dist	innerproduct	–	0	–	1
Animal	DIMPA dist	dist	–	0	–	1
Faculty: Business	DIMPA innerproduct	innerproduct	–	0	–	1
Faculty: CS	DIMPA dist	dist	–	0	–	1
Faculty: History	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1985)	DIMPA dist	dist	–	0	–	1
Basketball(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1987)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1988)	DIMPA dist	dist	–	0	–	1
Basketball(1989)	DIMPA dist	dist	–	0	–	1
Basketball(1990)	DIMPA dist	dist	–	0	–	1
Basketball(1991)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1992)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1993)	DIMPA dist	dist	–	1	–	1
Basketball(1994)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1996)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1997)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1998)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1999)	DIMPA dist	dist	–	0	–	1
Basketball(2000)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2001)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2002)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2003)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2004)	DIMPA dist	dist	–	0	–	1
Basketball(2005)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2006)	DIMPA dist	dist	–	0	–	1
Basketball(2007)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2008)	DIMPA dist	dist	–	0	–	1
Basketball(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2010)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2011)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2012)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2013)	DIMPA dist	dist	–	0	–	1
Basketball(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1985)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1987)	DIMPA dist	dist	–	1	–	1
Basketball finer(1988)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1989)	DIMPA dist	dist	–	1	–	1
Basketball finer(1990)	DIMPA dist	dist	–	1	–	1
Basketball finer(1991)	DIMPA dist	dist	–	1	–	0
Basketball finer(1992)	DIMPA dist	dist	–	1	–	1
Basketball finer(1993)	DIMPA dist	dist	–	1	–	0
Basketball finer(1994)	DIMPA dist	dist	–	1	–	1
Basketball finer(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1996)	DIMPA dist	dist	–	1	–	1
Basketball finer(1997)	DIMPA dist	dist	–	1	–	1
Basketball finer(1998)	DIMPA dist	dist	–	1	–	0
Basketball finer(1999)	DIMPA dist	dist	–	1	–	1
Basketball finer(2000)	DIMPA dist	dist	–	1	–	1
Basketball finer(2001)	DIMPA dist	dist	–	1	–	1
Basketball finer(2002)	DIMPA dist	dist	–	1	–	1
Basketball finer(2003)	DIMPA dist	dist	–	1	–	1
Basketball finer(2004)	DIMPA dist	dist	–	1	–	0
Basketball finer(2005)	DIMPA dist	dist	–	1	–	1
Basketball finer(2006)	DIMPA dist	dist	–	1	–	1
Basketball finer(2007)	DIMPA dist	dist	–	1	–	0
Basketball finer(2008)	DIMPA dist	dist	–	1	–	1
Basketball finer(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2010)	DIMPA dist	dist	–	1	–	1
Basketball finer(2011)	DIMPA dist	dist	–	1	–	1
Basketball finer(2012)	DIMPA dist	dist	–	1	–	0
Basketball finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Football(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Football(2010)	ib dist	proximal innerproduct	dist	1	–	1
Football(2011)	ib dist	proximal baseline	dist	1	BTL	0
Football(2012)	ib dist	dist	–	1	–	0
Football(2013)	DIMPA dist	dist	–	0	–	1
Football(2014)	ib dist	proximal baseline	innerproduct	1	Eign.Cent.	1
Football finer(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2010)	ib innerproduct	proximal innerproduct	innerproduct	1	–	0
Football finer(2011)	DIMPA dist	dist	–	1	–	0
Football finer(2012)	DIMPA dist	proximal baseline	dist	1	BTL	1
Football finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2014)	ib dist	proximal innerproduct	SerialRank similarity	1	–	1

Table 22. GNN selection among GNNRank-P methods for the lowest $\mathcal{L}_{\text{upset, simple}}$.

Data	Variant	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Finance	ib proximal dist	innerproduct	–	0	–	1
Animal	ib proximal baseline	proximal baseline	innerproduct	1	SyncRank	0
Faculty: Business	ib proximal baseline	proximal baseline	innerproduct	0	SyncRank	1
Faculty: CS	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Faculty: History	DIMPA proximal dist	dist	–	1	–	1
Basketball(1985)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1986)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1987)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1
Basketball(1988)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1989)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(1990)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1991)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1992)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1993)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball(1994)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1995)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(1996)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball(1997)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1
Basketball(1998)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	0
Basketball(1999)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2000)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2001)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2002)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2003)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2004)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2005)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(2006)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2007)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2008)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2009)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(2010)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2011)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2012)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2013)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2014)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(1985)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1986)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1987)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(1988)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(1989)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1990)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1991)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1992)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1993)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(1994)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1995)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball finer(1996)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1997)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball finer(1998)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball finer(1999)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2000)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2001)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2002)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(2003)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2004)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	1
Basketball finer(2005)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(2006)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2007)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(2008)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2009)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2010)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2011)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2012)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(2013)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball finer(2014)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	1
Football(2009)	ib proximal baseline	dist	–	0	SpringRank	1
Football(2010)	ib proximal innerproduct	proximal baseline	innerproduct	1	SpringRank	1
Football(2011)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SpringRank	1
Football(2012)	ib proximal dist	proximal baseline	SerialRank similarity	1	PageRank	0
Football(2013)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Football(2014)	ib proximal dist	proximal baseline	innerproduct	1	PageRank	0
Football finer(2009)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	1
Football finer(2010)	ib proximal innerproduct	proximal baseline	innerproduct	1	SerialRank	1
Football finer(2011)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	0
Football finer(2012)	ib proximal baseline	proximal baseline	dist	1	SVD_NRS	0
Football finer(2013)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Football finer(2014)	ib proximal dist	proximal baseline	innerproduct	0	SVD_NRS	1

Table 23. GNN selection among GNNRank-N methods for the lowest $\mathcal{L}_{\text{upset, naive}}$.

Data	Variant	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	DIMPA dist	dist	–	0	–	1
Finance	DIMPA innerproduct	innerproduct	–	1	–	0
Animal	DIMPA dist	dist	–	0	–	1
Faculty: Business	DIMPA innerproduct	innerproduct	–	0	–	1
Faculty: CS	DIMPA dist	dist	–	0	–	1
Faculty: History	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1985)	DIMPA dist	dist	–	0	–	1
Basketball(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1987)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1988)	DIMPA dist	dist	–	0	–	1
Basketball(1989)	DIMPA dist	dist	–	0	–	1
Basketball(1990)	DIMPA dist	dist	–	0	–	1
Basketball(1991)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1992)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1993)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1994)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1996)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1997)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1998)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1999)	DIMPA dist	dist	–	0	–	1
Basketball(2000)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2001)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2002)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2003)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2004)	DIMPA dist	dist	–	0	–	1
Basketball(2005)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2006)	DIMPA dist	dist	–	0	–	1
Basketball(2007)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2008)	DIMPA dist	dist	–	0	–	1
Basketball(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2010)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2011)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2012)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2013)	DIMPA dist	dist	–	0	–	1
Basketball(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1985)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1987)	DIMPA dist	dist	–	1	–	1
Basketball finer(1988)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1989)	DIMPA dist	dist	–	1	–	1
Basketball finer(1990)	DIMPA dist	dist	–	1	–	1
Basketball finer(1991)	DIMPA dist	dist	–	1	–	0
Basketball finer(1992)	DIMPA dist	dist	–	1	–	1
Basketball finer(1993)	DIMPA dist	dist	–	1	–	0
Basketball finer(1994)	DIMPA dist	dist	–	1	–	1
Basketball finer(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1996)	DIMPA dist	dist	–	1	–	1
Basketball finer(1997)	DIMPA dist	dist	–	1	–	1
Basketball finer(1998)	DIMPA dist	dist	–	1	–	0
Basketball finer(1999)	DIMPA dist	dist	–	1	–	1
Basketball finer(2000)	DIMPA dist	dist	–	1	–	1
Basketball finer(2001)	DIMPA dist	dist	–	1	–	1
Basketball finer(2002)	DIMPA dist	dist	–	1	–	1
Basketball finer(2003)	DIMPA dist	dist	–	1	–	1
Basketball finer(2004)	DIMPA dist	dist	–	1	–	0
Basketball finer(2005)	DIMPA dist	dist	–	1	–	1
Basketball finer(2006)	DIMPA dist	dist	–	1	–	1
Basketball finer(2007)	DIMPA dist	dist	–	1	–	0
Basketball finer(2008)	DIMPA dist	dist	–	1	–	1
Basketball finer(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2010)	DIMPA dist	dist	–	1	–	1
Basketball finer(2011)	DIMPA dist	dist	–	1	–	1
Basketball finer(2012)	DIMPA dist	dist	–	1	–	0
Basketball finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Football(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Football(2010)	DIMPA dist	proximal dist	dist	1	–	1
Football(2011)	DIMPA dist	dist	–	0	–	1
Football(2012)	DIMPA dist	dist	–	1	–	1
Football(2013)	DIMPA dist	dist	–	0	–	1
Football(2014)	DIMPA dist	dist	–	1	–	1
Football finer(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2010)	DIMPA dist	proximal dist	dist	0	–	1
Football finer(2011)	DIMPA dist	dist	–	1	–	0
Football finer(2012)	DIMPA dist	innerproduct	–	1	–	1
Football finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2014)	DIMPA dist	dist	–	1	–	1
ERO(p=0.05, style=uniform,η=0)	DIMPA dist	dist	–	0	–	1
ERO(p=0.05, style=gamma,η=0)	DIMPA dist	dist	–	0	–	1
ERO(p=0.05, style=uniform,η=0.1)	ib innerproduct	innerproduct	–	0	–	1
ERO(p=0.05, style=gamma,η=0.1)	DIMPA dist	dist	–	0	–	1
ERO(p=0.05, style=uniform,η=0.2)	DIMPA dist	dist	–	1	–	1
ERO(p=0.05, style=gamma,η=0.2)	ib innerproduct	proximal baseline	innerproduct	0	SpringRank	1
ERO(p=0.05, style=uniform,η=0.3)	ib dist	dist	–	1	–	1
ERO(p=0.05, style=gamma,η=0.3)	ib innerproduct	proximal baseline	innerproduct	1	SyncRank	1
ERO(p=0.05, style=uniform,η=0.4)	ib innerproduct	innerproduct	–	1	–	0
ERO(p=0.05, style=gamma,η=0.4)	ib innerproduct	proximal baseline	innerproduct	1	SyncRank	1
ERO(p=0.05, style=uniform,η=0.5)	ib innerproduct	innerproduct	–	1	–	0
ERO(p=0.05, style=gamma,η=0.5)	ib innerproduct	proximal innerproduct	innerproduct	1	–	1
ERO(p=0.05, style=uniform,η=0.6)	ib innerproduct	innerproduct	–	1	–	0
ERO(p=0.05, style=gamma,η=0.6)	ib innerproduct	innerproduct	–	0	–	1
ERO(p=0.05, style=uniform,η=0.7)	ib innerproduct	innerproduct	–	1	–	0
ERO(p=0.05, style=gamma,η=0.7)	ib innerproduct	innerproduct	–	0	–	1
ERO(p=0.05, style=uniform,η=0.8)	ib innerproduct	innerproduct	–	1	–	0
ERO(p=0.05, style=gamma,η=0.8)	ib innerproduct	innerproduct	–	0	–	1
ERO(p=1, style=uniform,η=0)	ib dist	dist	–	1	–	0
ERO(p=1, style=gamma,η=0)	ib dist	dist	–	1	–	0
ERO(p=1, style=uniform,η=0.1)	DIMPA innerproduct	proximal baseline	innerproduct	0	SVD_NRS	1
ERO(p=1, style=gamma,η=0.1)	DIMPA dist	dist	–	0	–	1
ERO(p=1, style=uniform,η=0.2)	DIMPA dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.2)	DIMPA dist	dist	–	0	–	1
ERO(p=1, style=uniform,η=0.3)	DIMPA dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.3)	DIMPA dist	dist	–	1	–	1
ERO(p=1, style=uniform,η=0.4)	DIMPA innerproduct	innerproduct	–	0	–	1
ERO(p=1, style=gamma,η=0.4)	DIMPA dist	dist	–	1	–	1
ERO(p=1, style=uniform,η=0.5)	ib dist	innerproduct	–	0	–	1
ERO(p=1, style=gamma,η=0.5)	ib dist	dist	–	1	–	1
ERO(p=1, style=uniform,η=0.6)	ib dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.6)	ib dist	dist	–	1	–	0
ERO(p=1, style=uniform,η=0.7)	ib dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.7)	ib dist	dist	–	0	–	1
ERO(p=1, style=uniform,η=0.8)	ib dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.8)	ib dist	dist	–	0	–	1

Table 24. GNN selection among GNNRank-P methods for the lowest $\mathcal{L}_{\text{upset, naive}}$.

Data	Variant	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Finance	ib proximal baseline	proximal baseline	SerialRank similarity	0	SerialRank	1
Animal	ib proximal baseline	proximal baseline	innerproduct	1	SyncRank	0
Faculty: Business	ib proximal baseline	proximal baseline	innerproduct	0	SyncRank	1
Faculty: CS	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Faculty: History	DIMPA proximal dist	dist	–	1	–	1
Basketball(1985)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1986)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1987)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1
Basketball(1988)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1989)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(1990)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1991)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1992)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1993)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball(1994)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(1995)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(1996)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball(1997)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1
Basketball(1998)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	0
Basketball(1999)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2000)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2001)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2002)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2003)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2004)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2005)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(2006)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2007)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2008)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2009)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball(2010)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2011)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2012)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball(2013)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball(2014)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(1985)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1986)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1987)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball finer(1988)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(1989)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1990)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1991)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1992)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1993)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(1994)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1995)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball finer(1996)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(1997)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball finer(1998)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
Basketball finer(1999)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2000)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2001)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2002)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(2003)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2004)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	1
Basketball finer(2005)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(2006)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2007)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
Basketball finer(2008)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2009)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2010)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2011)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Basketball finer(2012)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
Basketball finer(2013)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0
Basketball finer(2014)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	1
Football(2009)	ib proximal baseline	dist	–	0	SpringRank	1
Football(2010)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SVD_NRS	1
Football(2011)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SpringRank	1
Football(2012)	ib proximal baseline	proximal baseline	dist	1	SyncRank	0
Football(2013)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Football(2014)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	1
Football finer(2009)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	1
Football finer(2010)	ib proximal baseline	proximal baseline	dist	1	SVD_NRS	0
Football finer(2011)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	0
Football finer(2012)	ib proximal baseline	proximal baseline	dist	1	SVD_NRS	0
Football finer(2013)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
Football finer(2014)	DIMPA proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1
ERO(p=0.05, style=uniform,η=0)	ib proximal baseline	proximal baseline	innerproduct	1	SyncRank	1
ERO(p=0.05, style=gamma,η=0)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=uniform,η=0.1)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=gamma,η=0.1)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=uniform,η=0.2)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=gamma,η=0.2)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=uniform,η=0.3)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=gamma,η=0.3)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
ERO(p=0.05, style=uniform,η=0.4)	DIMPA proximal baseline	proximal baseline	innerproduct	1	SyncRank	0
ERO(p=0.05, style=gamma,η=0.4)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1
ERO(p=0.05, style=uniform,η=0.5)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
ERO(p=0.05, style=gamma,η=0.5)	ib proximal innerproduct	innerproduct	–	1	–	1
ERO(p=0.05, style=uniform,η=0.6)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1
ERO(p=0.05, style=gamma,η=0.6)	ib proximal dist	innerproduct	–	0	–	1
ERO(p=0.05, style=uniform,η=0.7)	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	0
ERO(p=0.05, style=gamma,η=0.7)	ib proximal innerproduct	innerproduct	–	0	–	1
ERO(p=0.05, style=uniform,η=0.8)	ib proximal dist	proximal baseline	dist	1	SpringRank	1
ERO(p=0.05, style=gamma,η=0.8)	ib proximal innerproduct	innerproduct	–	0	–	1
ERO(p=1, style=uniform,η=0)	ib proximal baseline	proximal baseline	innerproduct	1	SerialRank	0
ERO(p=1, style=gamma,η=0)	ib proximal baseline	proximal baseline	dist	0	SerialRank	1
ERO(p=1, style=uniform,η=0.1)	ib proximal baseline	proximal baseline	innerproduct	0	SerialRank	1
ERO(p=1, style=gamma,η=0.1)	ib proximal baseline	proximal baseline	innerproduct	0	SerialRank	1
ERO(p=1, style=uniform,η=0.2)	ib proximal baseline	proximal baseline	innerproduct	0	SerialRank	1
ERO(p=1, style=gamma,η=0.2)	ib proximal baseline	proximal baseline	innerproduct	1	SerialRank	0
ERO(p=1, style=uniform,η=0.3)	ib proximal baseline	proximal baseline	innerproduct	0	SerialRank	1
ERO(p=1, style=gamma,η=0.3)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SerialRank	0
ERO(p=1, style=uniform,η=0.4)	ib proximal baseline	proximal baseline	innerproduct	1	SerialRank	0
ERO(p=1, style=gamma,η=0.4)	ib proximal baseline	proximal baseline	innerproduct	0	SerialRank	1
ERO(p=1, style=uniform,η=0.5)	ib proximal dist	innerproduct	–	0	–	1
ERO(p=1, style=gamma,η=0.5)	ib proximal dist	dist	–	1	–	1
ERO(p=1, style=uniform,η=0.6)	ib proximal dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.6)	ib proximal dist	dist	–	1	–	0
ERO(p=1, style=uniform,η=0.7)	ib proximal dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.7)	ib proximal dist	dist	–	0	–	1
ERO(p=1, style=uniform,η=0.8)	ib proximal dist	dist	–	0	–	1
ERO(p=1, style=gamma,η=0.8)	ib proximal dist	dist	–	0	–	1

Table 25. GNN selection among GNNRank-N methods for the lowest $\mathcal{L}_{\text{upset, ratio}}$.

Data	Variant	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	DIMPA dist	dist	–	0	–	1
Finance	DIMPA innerproduct	innerproduct	–	0	–	1
Animal	DIMPA dist	dist	–	0	–	1
Faculty: Business	DIMPA innerproduct	innerproduct	–	0	–	1
Faculty: CS	DIMPA dist	dist	–	0	–	1
Faculty: History	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1985)	DIMPA dist	dist	–	0	–	1
Basketball(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1987)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1988)	DIMPA dist	dist	–	0	–	1
Basketball(1989)	DIMPA dist	dist	–	0	–	1
Basketball(1990)	DIMPA dist	dist	–	0	–	1
Basketball(1991)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1992)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1993)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1994)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1996)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1997)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1998)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(1999)	DIMPA dist	dist	–	0	–	1
Basketball(2000)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2001)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2002)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2003)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2004)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2005)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2006)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2007)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2008)	DIMPA dist	dist	–	0	–	1
Basketball(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2010)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2011)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2012)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball(2013)	DIMPA dist	dist	–	0	–	1
Basketball(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1985)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1986)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1987)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1988)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1989)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1990)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1991)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1992)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1993)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1994)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1995)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1996)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1997)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1998)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(1999)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2000)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2001)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2002)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2003)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2004)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2005)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2006)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2007)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2008)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2009)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2010)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2011)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2012)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Basketball finer(2014)	DIMPA innerproduct	innerproduct	–	0	–	1
Football(2009)	DIMPA dist	dist	–	1	–	1
Football(2010)	DIMPA dist	dist	–	1	–	1
Football(2011)	DIMPA dist	dist	–	1	–	1
Football(2012)	DIMPA dist	dist	–	1	–	1
Football(2013)	DIMPA dist	dist	–	1	–	1
Football(2014)	DIMPA dist	dist	–	1	–	1
Football finer(2009)	DIMPA dist	dist	–	1	–	1
Football finer(2010)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2011)	DIMPA dist	dist	–	1	–	1
Football finer(2012)	DIMPA dist	dist	–	1	–	1
Football finer(2013)	DIMPA innerproduct	innerproduct	–	0	–	1
Football finer(2014)	DIMPA dist	dist	–	1	–	1

Table 26. GNN selection among GNNRank-P methods for the lowest $\mathcal{L}_{\text{upset, ratio}}$.

Data	Variant	learning rate	train with	pretrain with	$\mathcal{L}_{\text{upset,margin}}$ coefficient	baseline	$\mathcal{L}_{\text{upset,ratio}}$ coefficient
HeadToHead	DIMPA proximal baseline	proximal baseline	dist	1	SyncRank	1	
Finance	DIMPA proximal baseline	proximal innerproduct	dist	1	SpringRank	1	
Animal	ib proximal baseline	proximal baseline	SerialRank similarity	0	SpringRank	1	
Faculty: Business	DIMPA proximal dist	proximal baseline	SerialRank similarity	0	SerialRank	1	
Faculty: CS	DIMPA proximal baseline	proximal baseline	dist	1	PageRank	0	
Faculty: History	DIMPA proximal baseline	proximal baseline	dist	1	BTL	0	
Basketball(1985)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1986)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1987)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1988)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Basketball(1989)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1990)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Basketball(1991)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1992)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SVD_NRS	0	
Basketball(1993)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Basketball(1994)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1995)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1996)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1997)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(1998)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0	
Basketball(1999)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2000)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2001)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2002)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2003)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2004)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2005)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2006)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1	
Basketball(2007)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	0	
Basketball(2008)	ib proximal baseline	proximal baseline	dist	0	SyncRank	1	
Basketball(2009)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Basketball(2010)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2011)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2012)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Basketball(2013)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball(2014)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Basketball finer(1985)	ib proximal baseline	dist	–	0	SpringRank	1	
Basketball finer(1986)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1987)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1988)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1989)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1990)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1991)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(1992)	ib proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1993)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1994)	ib proximal baseline	dist	–	1	SpringRank	0	
Basketball finer(1995)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1996)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(1997)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(1998)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(1999)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(2000)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(2001)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(2002)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2003)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2004)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2005)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2006)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2007)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2008)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2009)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2010)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2011)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2012)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Basketball finer(2013)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Basketball finer(2014)	DIMPA proximal baseline	innerproduct	–	0	SpringRank	1	
Football(2009)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Football(2010)	DIMPA proximal baseline	proximal innerproduct	dist	1	SpringRank	1	
Football(2011)	ib proximal baseline	proximal baseline	dist	1	SyncRank	0	
Football(2012)	ib proximal baseline	proximal baseline	SerialRank similarity	1	SyncRank	1	
Football(2013)	ib proximal baseline	proximal baseline	SerialRank similarity	0	SyncRank	1	
Football(2014)	ib proximal baseline	proximal baseline	dist	1	SyncRank	0	
Football finer(2009)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Football finer(2010)	DIMPA proximal baseline	dist	–	1	SpringRank	1	
Football finer(2011)	DIMPA proximal baseline	proximal innerproduct	dist	1	SpringRank	1	
Football finer(2012)	DIMPA proximal baseline	proximal innerproduct	dist	0	SpringRank	1	
Football finer(2013)	DIMPA proximal baseline	proximal innerproduct	dist	0	SpringRank	1	
Football finer(2014)	DIMPA proximal dist	dist	–	1	–	1	