Correlation Clustering via Strong Triadic Closure Labeling: Fast Approximation Algorithms and Practical Lower Bounds

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Abstract

Correlation clustering is a widely studied framework for clustering based on pairwise similarity and dissimilarity scores, but its best approximation algorithms rely on impractical linear programming relaxations. We present faster approximation algorithms that avoid these relaxations, for two well-studied special cases: cluster editing and cluster deletion. We accomplish this by drawing new connections to edge labeling problems related to the principle of strong triadic closure. This leads to faster and more practical linear programming algorithms, as well as extremely scalable combinatorial techniques, including the first combinatorial approximation algorithm for cluster deletion. In practice, our algorithms produce approximate solutions that nearly match the best algorithms in quality, while scaling to problems that are orders of magnitude larger.

1. Introduction

Correlation clustering is a framework for unsupervised learning that clusters items in a dataset based on pairwise similarity and dissimilarity scores, rather than based on explicit representations for data objects. The simplest version of the problem can be cast as a partitioning objective on a complete signed graph, where the goal is to form clusters of nodes in a way that avoids placing negative edges inside clusters or positive edges between clusters. Bansal, Blum, and Chawla (2004) introduced the problem, proved its NP-hardness, and presented the first approximation algorithms. Now, nearly two decades later, developing improved approximation algorithms and hardness results for different variants of the problem continues to be a very active area of research (Jafarov et al., 2020; 2021; Veldt et al., 2020; Bun et al., 2021; Cohen-Addad et al., 2021; Bonchi et al., 2022). In addition to extensive theoretical research, the problem has been applied in a wide variety of settings, including to image segmentation (Kim et al., 2011; Yarkony et al., 2012), community detection (Wang et al., 2013; Veldt et al., 2018; Chen et al., 2012), cross-lingual link detection (Van Gael & Zhu, 2007), cancer mutation analysis (Hou et al., 2016), and detecting co-regulated genes based on expression profiles (Bhattacharya & De, 2010; 2008; Ben-Dor et al., 1999). However, despite significant previous research on both theoretical and applied aspects of correlation clustering, there still exists a wide gap between the best approximation algorithms and the most practical tools for this task. Many fast heuristic methods come with no approximation guarantees (Shi et al., 2021; Beier et al., 2015; Bhattacharya & De, 2008; Levinkov et al., 2017), while rigorous approximation algorithms are often either impractically expensive, or only apply to the simplest version of the problem.

Linear programming algorithms. The best approximation factors for correlation clustering, both for the standard unweighted objective (Chawla et al., 2015) as well as more general weighted variants (van Zuylen & Williamson, 2009; Ailon et al., 2008; Puleo & Milenkovic, 2015; 2018; Veldt et al., 2018; Gleich et al., 2018; Li et al., 2019; Jafarov et al., 2020), are obtained by solving an expensive linear programming (LP) relaxation in order to obtain lower bounds for the objective. These lower bounds can useful for more than just designing approximation algorithms. In practice, linear programming lower bounds are often much closer to optimality than worst case theoretical results predict. If a good LP lower bound can be computed, the output of fast heuristic methods can be compared against the lower bound to obtain a posteriori approximation guarantees that are often very good in practice (Swoboda & Andres, 2017; Lange et al., 2018; Yarkony et al., 2012; Veldt et al., 2018). However, despite some recent work on specialized solvers (Veldt et al., 2019; Ruggles et al., 2020; Sonthalia & Gilbert, 2020), these lower bounds can only be computed for medium-sized instances at best (e.g., graphs with a few thousands nodes), and even then this can take a long time. There is therefore a need for faster approximation algorithms, as well as an even more basic need to efficiently compute good lower bounds for the NP-hard objective in practice.

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Pivot algorithms. PIVOT is a combinatorial algorithm that iteratively selects a random unclustered node and places it with all of its unclustered positive neighbors. This provides a randomized 3-approximation for complete unweighted correlation clustering (Ailon et al., 2008). This approach can be made very fast (Pan et al., 2015; Chierichetti et al., 2014; Cohen-Addad et al., 2021), but is not without its limitations. First, the approach is designed for the complete unweighted case and does not extend as easily as LP-based techniques to other variants of correlation clustering. As one example, *cluster deletion* is a simple variant that strictly prohibits clustering two nodes together if they share a negative edge. Although constant-factor linear programming algorithms have been designed for cluster deletion (Puleo & Milenkovic, 2015; Veldt et al., 2018; Charikar et al., 2005), the standard PIVOT technique does not even produce a feasible solution for this problem. Another limitation of PIVOT is that it only provides an expected approximation guarantee, and although derandomization techniques have been developed, these again rely on solving the expensive canonical LP relaxation (van Zuylen & Williamson, 2009). Finally, PIVOT does not produce explicit lower bounds for the NP-hard objective, so it provides no way to compute improved a posteriori approximation guarantees in practice.

The present work: practical lower bounds and faster algorithms via connections to strong triadic closure. We provide significant steps in bridging the theory-practice gap in correlation clustering by designing practical techniques for computing lower bounds, and faster corresponding approximation algorithms. We achieve our results by highlighting and exploiting a relationship between correlation clustering and edge-labeling problems related to the principle of *strong triadic closure* (Sintos & Tsaparas, 2014; Easley et al., 2010). Strong triadic closure (STC) posits that two people will often share at least a weak connection if they share strong connections to a mutual friend. An STC-labeling of a graph is a way to label edges as weak or strong in order to satisfy this principle. Similarities between clustering and STC-labeling problems have been noted in previous work (Konstantinidis et al., 2018; Grüttemeier & Komusiewicz, 2020). Our results significantly expand on these previously observed relationships, and provide new strategies for rounding lower bounds for STC-labeling problems into approximate solutions for clustering problems.

One of the central contributions of our paper is to develop *combinatorial* approximation algorithms for complete unweighted correlation clustering (also called cluster editing) and cluster deletion, which can also be made deterministic without LP relaxations. Our strategy works in three basic steps: *matching*, *flipping*, and *pivoting*. We first obtain lower bounds by computing maximal matchings in either an auxiliary graph (for cluster deletion) or an auxiliary 3-uniform hypergraph (for cluster editing). We use the results of our matching to determine edges to be flipped (i.e., added or deleted) to create a new graph. Finally we prove that applying a pivoting procedure on the resulting graph yields approximation guarantees for the original problem. In summary, we provide the following algorithms:

- We apply our *match-flip-pivot* approach to design the first combinatorial approximation algorithm for cluster deletion, which provides a 4-approximation guarantee.
- We use *match-flip-pivot* to design a combinatorial 6-approximation algorithm for complete unweighted correlation clustering. This can be made deterministic using purely combinatorial techniques, making it the best approximation guarantee achieved by any deterministic algorithm that does not depend on linear programming.
- We present additional approximation algorithms for both problems, each with a factor-4 approximation guarantee, by solving and rounding LPs with significantly fewer constraints than the canonical LP relaxations.

In proving these results, we show more generally that any α-approximation algorithm for the *minimum weakness strong triadic closure* problem (Sintos & Tsaparas, 2014) can be used to design a (2α)-approximation for cluster deletion. We show an analogous result for unweighted complete correlation clustering. Our results imply that an optimal solution to either of these clustering problems is always within a factor of 2 of the optimal solution to a corresponding edge labeling problem. This is especially significant for cluster deletion and minimum weakness strong triadic closure, as there are known examples where these objectives differ by up to a factor of $8/7$ (Grüttemeier & Komusiewicz, 2020). Thus, our upper bound on the difference between these two objectives is not far off of a known lower bound on their difference. Finally, we show that our algorithms are far more scalable in practice than algorithms that solve the canonical LP, producing approximate solutions that are within a small factor of optimality (factors ≈ 2) on graphs with millions of nodes, within a matter of seconds. Our fast algorithms for finding lower bounds can also be used to provide good a posteriori approximation guarantees for heuristics that previously came with no guarantees.

2. Preliminaries and Related Work

We begin with definitions, terminology, and notation. Although correlation clustering is often presented as a clustering objective on a signed graph, the variants we primarily consider can be cast as objectives on an unsigned and unweighted graph $G = (V, E)$. All objectives we consider in this paper are defined on undirected graphs.

Open wedges. The problems we consider rely on the notion of an *open wedge*. Given an unsigned graph $G = (V, E)$, a triplet of nodes (i, j, k) is an *open wedge centered at* k if $(i, k) \in E$ and $(j, k) \in E$ but $(i, j) \notin E$. Let $\mathcal{W}$ denote the set of triplets that define open wedges, and $\mathcal{W}_k \subseteq \mathcal{W}$ denote the set of open wedges centered at k.

2.1. Correlation Clustering Objectives

The most general weighted version of correlation clustering is defined by a node set V and two sets of nonnegative weights W^+ and W^- defined on pairs of nodes. Formally, each pair $(i, j) \in \binom{V}{2}$ is associated with an edge weight $w_{ij}^+ \in W^+$ and an edge weight $w_{ij}^- \in W^-$. In some applications these correspond to edge weights for positive and negative edges in a signed graph, but this does not always have to be the case. Broadly speaking, the nonnegative weights (w_{ij}^+, w_{ij}^-) indicate the level of attraction and repulsion, respectively, between nodes i and j in a clustering problem. The goal of the general weighted correlation clustering objective is to partition the nodes in a way that correlates as much as possible with these weights. This is accomplished by solving the following integer linear program (ILP), know as the *MinDisagree* objective:

$$\begin{array}{ll} \min & \displaystyle\sum_{(i,j)\in\binom{V}{2}} w_{ij}^+ x_{ij} + w_{ij}^-(1 - x_{ij}) \\ \text{such that} & x_{jk} + x_{ik} \geq x_{ij} \text{ for triplets } i, j, k \\ & x_{ij} \in \{0, 1\} \text{ for } (i, j) \in \binom{V}{2}. \end{array} \tag{1}$$

Here, $x_{ij} = 0$ if nodes i and j are clustered together and $x_{ij} = 1$ if they are separated. In other words, a penalty of w_{ij}^+ is applied for separating i and j, and a penalty of w_{ij}^- is applied if they are placed together. Note that there is a triangle inequality constraint $x_{ij} \leq x_{jk} + x_{ik}$ for each ordering of three distinct vertices $\{i, j, k\}$. An alternative objective for correlation clustering is to maximize the weight of agreements, which is the same at optimality but is different from the perspective of approximations. Throughout this paper, we focus on the *MinDisagree* objective (1).

Correlation clustering is NP-hard even for the simple unweighted case, but if the binary constraint $x_{ij} \in \{0, 1\}$ is relaxed to linear constraints $0 \leq x_{ij} \leq 1$, the result is the canonical linear programming relaxation of the problem, which can be solved in polynomial time. Many approximation algorithms rely on solving and rounding this LP. An $O(\log n)$ approximation can be obtained for the general case using LP rounding (Charikar et al., 2005; Demaine et al., 2006), and improved results exist for special weighted variants (Veldt et al., 2018; Puleo & Milenkovic, 2015; Jafarov et al., 2020; Ailon et al., 2008; Veldt et al., 2020).

Cluster editing. Minimizing disagreements in a complete unweighted signed graph (Bansal et al., 2004) is the widely-studied special case of objective (1) where $(w_{ij}^+, w_{ij}^-) \in \{(0, 1), (1, 0)\}$ for each node pair (i, j). This is equivalent to a problem called *cluster editing*: given a graph $G = (V, E)$, find the minimum number of edges to add or remove in order to convert G into a disjoint union of cliques (Shamir et al., 2004; Ben-Dor et al., 1999). Alternatively, this means clustering G in a way that minimizes the number of *mistakes* or *disagreements*: edges crossing between clusters or non-adjacent node pairs inside clusters.

Cluster deletion. Cluster deletion (Shamir et al., 2004) seeks to convert a graph $G = (V, E)$ into a disjoint union of cliques by deleting the smallest number of edges. This can be viewed as an instance of general weighted correlation clustering (1) where $(w_{ij}^+, w_{ij}^-) = (1, 0)$ if i and j are adjacent in G, and $(w_{ij}^+, w_{ij}^-) = (0, \infty)$ if these nodes are not adjacent. In this special case, the problem permits an integer programming formulation with fewer constraints, since $x_{ij} = 1$ when $(i, j) \notin E$:

$$\begin{array}{lll} \min & \sum_{(i,j)\in E} x_{ij} & \\ \text{s.t.} & x_{ij} \in \{0, 1\} & \text{for all } (i, j) \in E \\ & x_{jk} + x_{ik} \geq 1 & \text{if } (i, j, k) \in \mathcal{W}_k \\ & \left.\begin{array}{l} x_{jk} + x_{ik} \geq x_{ij} \\ x_{jk} + x_{ij} \geq x_{ik} \\ x_{ik} + x_{ij} \geq x_{jk} \end{array}\right\} & \text{if } i, j, k \text{ is a triangle.} \end{array} \tag{2}$$

The best approximation factor for cluster deletion is 2 (Veldt et al., 2018), which relies on rounding the linear programming relaxation of objective (2).

2.2. Strong Triadic Closure Labeling Objectives

Strong triadic closure (Easley et al., 2010; Granovetter, 1973) posits that two people are likely to share at least a weak connection if they both share strong connections to a mutual friend. This is used as a guiding principle for social network analysis, and is the foundation for certain edge labeling problems (Sintos & Tsaparas, 2014; Grüttemeier & Komusiewicz, 2020; Konstantinidis et al., 2018).

Min-weakness strong triadic closure. If every edge in a graph $G = (V, E)$ is labeled as either *weak* or as *strong*, we say this is a *strong triadic closure labeling* if at least one of the edges in each open wedge is *weak*. The rationale is that if both edges in a wedge are strong ties, we would expect the wedge to be closed because of strong triadic closure. The *minimum weakness strong triadic closure* (MINSTC) problem (Sintos & Tsaparas, 2014) seeks a strong triadic closure labeling with the minimum number of weak edges. We can cast this as an integer program where a binary variable z_{uv} equals 1 if $(u, v) \in E$ is labeled as a weak connection:

$$\begin{array}{lll} \min & \sum_{(i,j)\in E} z_{ij} & \\ \text{s.t.} & z_{jk} + z_{ik} \geq 1 & \text{if } (i, j, k) \in \mathcal{W}_k \\ & z_{ij} \in \{0, 1\} & \text{for all } (i, j) \in E. \end{array} \tag{3}$$

Strong triadic closure with edge additions. Minimum-weakness strong triadic closure with edge additions (MINSTC+) allows one to satisfy strong triadic closure by also *adding* weak edges between non-adjacent nodes in the graph $G = (V, E)$. This is equivalent to viewing certain *non-edges* as weak connections that were not observed. The integer program formulation for this problem is:

$$\begin{array}{lll} \min & \sum_{(i,j)\in\binom{V}{2}} z_{ij} & \\ \text{s.t.} & z_{jk} + z_{ik} + z_{ij} \geq 1 & \text{if } (i,j,k) \in \mathcal{W} \\ & z_{ij} \in \{0,1\} & \text{for } (i,j) \in \binom{V}{2}. \end{array} \tag{4}$$

If $(i, j) \in E$ and $z_{ij} = 1$, this again corresponds to labeling the edge as weak. If $(i, j) \notin E$ and $z_{ij} = 1$, this means we *add* a new edge between i and j. A feasible solution to MINSTC+ is therefore a set of new edges E' and a subset of edges $E_W \subseteq E$ that we will label as *weak*. The goal is to minimize $|E'| + |E_W|$. We assume all new edges E' are weak, to ensure we do not introduce new open wedges that violate strong triadic closure. We refer to a feasible solution (E', E_L) for MINSTC+ as an STC+ labeling.

2.3. Correlation Clustering and STC Connections

The cluster deletion (2) integer program can be obtained by taking the integer program for MINSTC (3) and adding the constraints $z_{ij} + z_{ik} \geq z_{jk}$ for all permutations of nodes i, j, k when (i, j, k) is a triangle in G. Thus, a feasible solution for cluster deletion will produce a feasible solution for MINSTC by labeling all the deleted edges as weak. Similarly, the constraints in the integer program for MINSTC+ (4) can be seen as a subset of the triangle inequality constraints in the integer program for cluster editing, once a change of variables is applied. The fact that MINSTC lower bounds cluster deletion, and MINSTC+ lower bounds cluster editing, has already been noted in previous work (Grüttemeier & Komusiewicz, 2020; Grüttemeier & Morawietz, 2020; Konstantinidis et al., 2018). The optimal solutions to cluster deletion and MINSTC are known to coincide for co-graphs (Konstantinidis et al., 2018), though there exist concrete examples to confirm that in general they are not the same problem (Grüttemeier & Komusiewicz, 2020). There also exist approximation algorithms for correlation clustering that rely on lower bounds from open wedge packings (also called *bad triangle packings*) (Ailon et al., 2008; Bansal et al., 2004). These can also be viewed implicitly as lower bounds for related STC-labeling problems, though this connection is not explicitly addressed in these works. Our paper expands on the known relationship between correlation clustering and STC-labeling problems, and provides new ways to round lower bounds for labeling problems into approximate solutions for clustering problems.

3. Faster Linear Programming Algorithms

We first show how to round LP relaxations for STC-labeling problems to develop 4-approximation algorithms for cluster deletion and cluster editing. While improved guarantees can be obtained by rounding tighter relaxations (Chawla et al., 2015; Veldt et al., 2018), our approach provides a useful tradeoff in runtime and approximation guarantee. In our experiments, we find that these less expensive LP relaxations are faster, while typically performing just as well as algorithms based on canonical relaxations. To prove our results, we first review a general pivoting strategy for correlation clustering, which we build on in our work.

3.1. Algorithmic Pivoting Primitive

PIVOT provides an expected 3-approximation for cluster editing (Ailon et al., 2008), but applying it directly to a weighted graph typically yields poor results. For cluster deletion, it does not even produce a feasible solution. However, pivoting can be successfully used as a step in more sophisticated algorithms for correlation clustering. We extract a general algorithmic strategy from the work of van Zuylen and Williamson (2009). This method takes in a weighted instance of correlation clustering (V, W^+, W^-), a set of "budgets" $\{b_{ij} : (i, j) \in \binom{V}{2}\}$, and a derived graph $\hat{G} = (V, \hat{E})$. Theorem 3.1 provides conditions for bounding the output solution from running PIVOT on $\hat{G}$.

Theorem 3.1. *(Thm 3.1, van Zuylen & Williamson (2009).) Let (V, W^+, W^-) define a weighted instance of correlation clustering (1), and b_{ij} define the* budget *for node pair $(i, j) \in \binom{V}{2}$. Assume that for some $\alpha > 0$, there is a graph $\hat{G} = (V, \hat{E})$ satisfying the following two properties:*

1. *For all $(i, j) \in \hat{E}$, we have $w^-_{ij} \leq \alpha b_{ij}$, and for all $(i, j) \notin \hat{E}$, we have $w^+_{ij} \leq \alpha b_{ij}$.*
2. *If (i, j, k) is an open wedge centered at j in $\hat{G}$, we have $w^+_{ij} + w^+_{jk} + w^-_{ik} \leq \alpha\,(b_{ij} + b_{jk} + b_{ik})$.*

Then applying PIVOT *to $\hat{G} = (V, \hat{E})$ with uniform random pivots will return a clustering with expected weight of disagreements bounded by $\alpha \sum_{i<j} b_{ij}$. There also exists a deterministic pivoting strategy that returns a clustering with disagreements bounded above by $\alpha \sum_{i<j} b_{ij}$.*

Appendix B provides details for the deterministic strategy. By setting budgets b_{ij} to be the contribution of a node pair (i, j) to the LP relaxation of (1), i.e., $b_{ij} = w^+_{ij} x_{ij} + w^-_{ij}(1 - x_{ij})$, one can obtain a derandomized 3-approximate PIVOT method for complete unweighted correlation clustering (van Zuylen & Williamson, 2009). However, the bottleneck is solving the LP relaxation.

3.2. Faster LP Algorithms for Clustering

We first present a new approximation algorithm for cluster editing by rounding the LP relaxation of MINSTC+ (4), obtained by replacing $z_{ij} \in \{0, 1\}$ with linear constraints

Algorithm 1 Rounding the MINSTC+ LP relaxation.

```
Input: Graph $G = (V, E)$
Output: Clustering of $G$.
Solve LP (5)
Set $\hat{E} \leftarrow \{(i, j) \in \binom{V}{2} : x_{ij} < 1/2\}$
Return PIVOT($\hat{G} = (V, \hat{E})$)
```

Algorithm 2 Rounding the MINSTC LP relaxation.

```
Input: Graph $G = (V, E)$
Output: Feasible cluster deletion clustering of $G$.
Solve LP relaxation of (3)
Set $\hat{E} \leftarrow \{(i, j) \in E : z_{ij} < 1/2\}$
Return PIVOT($\hat{G} = (V, \hat{E})$)
```

$0 \leq z_{ij} \leq 1$. We apply a convenient change of variables: set $x_{ij} = z_{ij}$ if $(i, j) \in E$, and $x_{ij} = 1 - z_{ij}$ otherwise. This leads to a linear program that amounts to the canonical cluster editing LP relaxation but with fewer constraints:

$$\begin{array}{ll} \min & \sum_{(i,j)\in E} x_{ij} + \sum_{(i,j)\notin E}(1 - x_{ij}) \\ \text{s.t.} & x_{ij} \leq x_{jk} + x_{ik} \text{ if } (i, j, k) \in \mathcal{W}_k \\ & 0 \leq x_{ij} \leq 1 \text{ for all } (i, j) \in \binom{V}{2}. \end{array} \tag{5}$$

The number of constraints in this LP is $O(|V|^2 + |\mathcal{W}|)$. Assuming a connected graph, this is bounded above by $O(|E||V|)$ and for many common graph classes will be smaller than the $O(|V|^3)$ constraints required for the canonical relaxation. Algorithm 1 is a 4-approximation for cluster editing. To prove its approximation guarantee, we show how to round the output of LP (5) to produce a graph satisfying the conditions of Theorem 3.1 with $\alpha = 4$.

Theorem 3.2. *Algorithm 1 is a randomized 4-approximation algorithm for cluster editing.*

Algorithm 2 rounds the MINSTC LP relaxation, obtained by replacing $x_{ij} \in \{0, 1\}$ with $0 \leq x_{ij} \leq 1$ in (3).

Theorem 3.3. *Algorithm 2 is a randomized 4-approximation algorithm for cluster deletion.*

The appendix provides full proofs for Theorems 3.2 and 3.3, as well as full details for how to derandomize both algorithms. Charikar, Guruswami, and Wirth (2005) previously showed that the canonical LP relaxations of cluster editing and cluster deletion can be rounded to produce 4-approximation algorithms for both problems. Theorems 3.2 and 3.3 show that the same approximation guarantee is possible using LP relaxations with only a subset of constraints. For many natural graph classes (e.g., sparse graphs), the number of constraints is asymptotically smaller. The *best* approximation algorithms known for these problems—a 2.06-approximation for cluster editing (Chawla et al., 2015) and a 2-approximation for cluster deletion (Veldt et al., 2018)—still use the canonical LP relaxations. However, these come as a significant increase in computational cost.

4. Match-Flip-Pivot Algorithms

We now present a *combinatorial* approach for obtaining lower bounds and approximation algorithms for cluster editing and cluster deletion. We begin by reviewing combinatorial strategies for lower bounding MINSTC and MINSTC+, which we round in new ways for clustering problems.

4.1. Lower Bounds via Maximal Matchings

The MINSTC problem (3) on a graph $G = (V, E)$ is equivalent to solving vertex cover on the Gallai graph $\mathcal{G}$ of G. The Gallai graph (Le, 1996) has a node v_{ij} for each edge $(i, j) \in E$, and an edge between two nodes v_{jk} and v_{ik} if (i, j, k) is an open wedge centered at k in G. The edges in $\mathcal{G}$ are in one-to-one correspondence with the open wedges of G. Placing node v_{ij} in the cover can be viewed as labeling the edge $(i, j) \in E$ as *weak*. Since all edges in $\mathcal{G}$ are adjacent to at least one node in any vertex cover, this means that all open wedges in G will have at least on weak edge.

The MINSTC+ objective (4) can instead be viewed as a vertex cover problem in a three-uniform hypergraph $\mathcal{H} = (V_\mathcal{H}, E_\mathcal{H})$. This hypergraph has a node $v_{ij} \in V_E$ for every *pair* of distinct nodes $(i, j) \in \binom{V}{2}$ in the original graph $G = (V, E)$, and a hyperedge $w_{ijk} = \{v_{ij}, v_{ik}, v_{jk}\} \in E_\mathcal{H}$ whenever (i, j, k) is an open wedge in G. We will refer to $\mathcal{H}$ as the *open wedge hypergraph*.

Sintos and Tsaparas (2014) showed that a 2-approximation for MINSTC can be obtained by applying the 2-approximation for vertex cover (Vazirani, 2001) to the Gallai graph. A similar 3-approximation for MINSTC+ is obtained by approximating vertex cover on $\mathcal{H}$ (Grüttemeier & Morawietz, 2020). These approximate solutions are obtained by first finding a maximal matching in $\mathcal{G}$ or $\mathcal{H}$ respectively. For our purposes, a maximal matching in $\mathcal{G}$ corresponds to an edge-disjoint set of open wedges in G, and lower bounds the cluster deletion objective. Similarly, a maximal matching in $\mathcal{H}$ is a node-pair disjoint set of open wedges in G, and lower bounds cluster editing.

4.2. Math-Flip-Pivot Algorithm for Cluster Editing

A vertex cover in the Gallai graph corresponds to a feasible STC+ labeling, and hence is a feasible solution for the MINSTC+ integer program (4). We now show how to round these approximately optimal solutions to MINSTC+ to approximate cluster editing. We start with a generic approach that rounds any STC+ labeling (E', E_W) into a solution for cluster editing. This algorithms first *flips* edges in the original graph $G = (V, E)$, meaning that we convert some

Algorithm 3 MATCHFLIPPIVOTCE(G)

```
Input: Graph $G = (V, E)$
Output: Clustering of $G$.
Reduce: Build open wedge hypergraph $\mathcal{H} = (V_\mathcal{H}, E_\mathcal{H})$
Match: Find maximal matching $\mathcal{M} \subseteq E_\mathcal{H}$
Vertex Cover:
    $C = \{v_{ij} \in V_\mathcal{H} : v_{ij} \in w \text{ for some } w \in \mathcal{M}\}$
STC+ Labeling:
    $E' = \{(i, j) \notin E : v_{ij} \in C\}$
    $E_W = \{(i, j) \in E : v_{ij} \in C\}$
Construct $\hat{G} = (V, \hat{E})$ where $\hat{E} = E' \cup (E - E_W)$
Return PIVOT($\hat{G}$)
```

non-adjacent node pairs into edges E', and we delete edges E_W that were previously in E. We then run PIVOT on the new graph. The number of mistakes made can be bounded in terms of the number of flipped edges.

Theorem 4.1. *Let (E', E_w) be an STC+ labeling for graph $G = (V, E)$. If $\hat{E} = E' \cup (E - E_W)$, applying* PIVOT *to derived graph $\hat{G} = (V, \hat{E})$ returns a cluster editing solution for G with at most $2(|E'| + |E_W|)$ mistakes in expectation.*

We prove this result in the appendix and also provide a derandomized version. We can then use any approximation algorithm for MINSTC+ to approximate cluster editing.

Corollary 4.2. *If $\mathcal{A}$ is an α-approximation algorithm for* MINSTC+*, applying the algorithm in Theorem 4.1 on an* STC+ *labeling (E', E_W) returned by $\mathcal{A}$ produces a (2α)-approximation for cluster editing. If OPT^+ and OPT^{CE} are optimal solutions to* STC+ *and cluster editing, then $OPT^+ \leq OPT^{CE} \leq 2OPT^+$.*

Combining Corollary 4.2 with the 3-approximation for MINSTC+ produces a fast algorithm for cluster editing.

Corollary 4.3. *Algorithm 3 is a randomized 6-approximation for cluster editing.*

Recall that standard PIVOT produces a better expected approximation factor of 3 and is also faster, as it requires neither *match* nor *flip* steps. Nevertheless, Algorithm 3 provides new benefits and advantages in both theory and practice. First of all, this algorithm can be derandomized using a completely combinatorial approach (Appendix B). Previous approaches for derandomizing PIVOT require solving the impractical canonical LP (van Zuylen & Williamson, 2009). Our approximation factor of 6 is therefore the best approximation guarantee for complete unweighted correlation clustering among methods that are both combinatorial and deterministic. Furthermore, the *match* step of our algorithm is extremely useful in practice, as it provides explicit lower bounds and a posteriori approximation guarantees that are typically much better than 3. In our experimental results, we find that using our lower bounds in conjunction with standard PIVOT yields a very fast method that produces a posteriori approximations that are much better than the 3-approximate a priori guarantee for PIVOT.

Algorithm 4 MATCHFLIPPIVOTCD(G)

```
Input: Graph $G = (V, E)$
Output: Feasible cluster deletion clustering of $G$.
Reduce: Build Gallai graph $\mathcal{G} = (V_\mathcal{G}, E_\mathcal{G})$ (Section 4.1)
Match: Find maximal matching $\mathcal{M} \subseteq E_\mathcal{G}$
Cover: $\mathcal{C} = \{v_{ij} \in V_\mathcal{G} : v_{ij} \in w \text{ for some } w \in \mathcal{M}\}$
STC Labeling: $E_W = \{(i, j) \in E : v_{ij} \in \mathcal{C}\}$
Construct graph $\hat{G} = (V, \hat{E})$ where $\hat{E} = (E - E_W)$
Return PIVOT($\hat{G}$)
```

4.3. Match-Flip-Pivot Algorithm for Cluster Deletion

We analogously round an approximate feasible solution to MINSTC to approximate cluster deletion. Recall that a feasible solution to MINSTC for a graph $G = (V, E)$ is a set of edges E_W to label as weak to ensure all open wedges in G have at least one weak edge. We prove that applying PIVOT to G after deleting all of the edges in E_W will produce a feasible cluster deletion solution with a bound on the number of deleted edges. Proofs and derandomized algorithms are given in the appendix.

Theorem 4.4. *Let E_W be an STC label set for graph $G = (V, E)$. Applying* PIVOT *to graph $\hat{G} = (V, E - E_W)$ returns a cluster deletion solution for G with at most $2|E_W|$ deleted edges in expectation.*

We obtain the following corollary on the relationship between MINSTC and cluster deletion.

Corollary 4.5. *If $\mathcal{A}$ is an α-approximation algorithm for* MINSTC*, running the algorithm in Theorem 4.4 with an* STC *label set returned by $\mathcal{A}$ produces a (2α)-approximation for cluster deletion. If OPT^{STC} and OPT^{CD} are optimal solutions to* MINSTC *and cluster deletion, then $OPT^{STC} \leq OPT^{CD} \leq 2OPT^{STC}$.*

We can also use Theorem 4.4 to obtain the first combinatorial approximation algorithm for cluster deletion.

Corollary 4.6. *Algorithm 4 is a randomized 4-approximation algorithm for cluster deletion.*

A derandomized (and still combinatorial) version of the algorithm is provided in Appendix B.

5. Experiments

In practice, our algorithms are far more scalable than the best LP relaxation algorithms, and can be run on graphs that are orders of magnitude larger, at a very small loss in approximation guarantee. We demonstrate this by computing lower bounds and approximate solutions for cluster deletion and cluster editing problems on a range of different types of public graph datasets from the Suitesparse matrix collection (Davis & Hu, 2011), the SNAP graph collection (Leskovec & Krevl, 2014), and the Facebook100 collection (Traud et al., 2012). We also run experiments on the 2021 PACE Challenge benchmark graphs for cluster editing (Kellerhals et al., 2021). Cluster editing and cluster deletion were first motivated by applications to clustering biological networks (Ben-Dor et al., 1999; Shamir et al., 2004). These problems can also be viewed as special cases of more general frameworks for community detection in unsigned graphs (Veldt et al., 2018), which is why we consider solving them on graphs from a range of different application domains (e.g., social networks, biological networks, email networks, collaboration networks). Our experiments are run on a MacBook Air with 16GB of RAM and an Apple M1 chip. All of our algorithms are implemented in the Julia programming language, and we use Gurobi software to solve linear programming relaxations. Code for all of our algorithms and experimental results are available at `github.com/nveldt/FastCC-via-STC`.

5.1. Cluster Deletion Approximation Algorithms

For cluster deletion, we run our strong triadic closure LP rounding algorithm (LP-STC, Algorithm 2), our match-flip-pivot technique (MFP-CD, Algorithm 4), and compare these against solving and rounding the canonical LP relaxation (LP-CD). All previous approximation algorithms for cluster deletion rely on the canonical LP (Charikar et al., 2005; Puleo & Milenkovic, 2015; Veldt et al., 2018). We specifically compare against the rounding procedure that provides the best-case 2-approximation (Veldt et al., 2018). All the lower bounds we consider can be rounded with either a deterministic or randomized pivoting procedure on some type of derived graph. The bottleneck in all runtimes is computing the lower bound, so for these experiments we apply randomized PIVOT 100 times on each derived graph and take the best result as this is simple, fast, and effective.

Table 1 shows results on 4 of the larger graphs we consider. In the appendix, we show full results for a wider range of graphs that vary in size. We find overall that **LP-STC is roughly twice as fast as LP-CD, while matching it in solution quality**. This method always returns the *same lower bound* as LP-CD. In some cases it outputs a solution that is also feasible for the canonical relaxation. In other cases, the solution is not feasible for the canonical relaxation but still has a matching objective score. The differences in rounded solutions for these methods is negligible (they trade off in performance) and is likely due to slight variations in the randomized rounding pivot procedure rather than the lower bound itself. Meanwhile, **MFP-CD is 2-3 *orders of magnitude* faster than LP-CD, with very minor loss to approximation factor**. A posteriori approximation guarantees are always at or below 2.12, and are usually below 2. The lower bound is only slightly worse than the bounds from the LP methods in all cases. In 21 out of 24 cases, MFP-CD in fact returns a better rounded solution (see appendix).

Table 1. Lower bounds (LB), rounded objective scores (UB), approximation ratios (Ratio) and runtimes (Run) for MFP-CD, LP-STC, and the existing 2-approximation for cluster deletion (LP-CD, Veldt et al. (2018)). Larger LB is better; smaller UB is better. The lower bound for LP-STC is guaranteed to be bounded above by the bound for LP-CD. In practice, the former is twice as fast, while outputting a matching lower bound in all cases. MFP-CD is theoretically guaranteed to produce even looser lower bounds, but these are still very close in practice and the method is 2-3 orders of magnitude faster. In many cases, MFP-CD actually produces better rounded solutions (best shown in bold), even if the approximation ratio is slightly higher because of a looser lower bound.

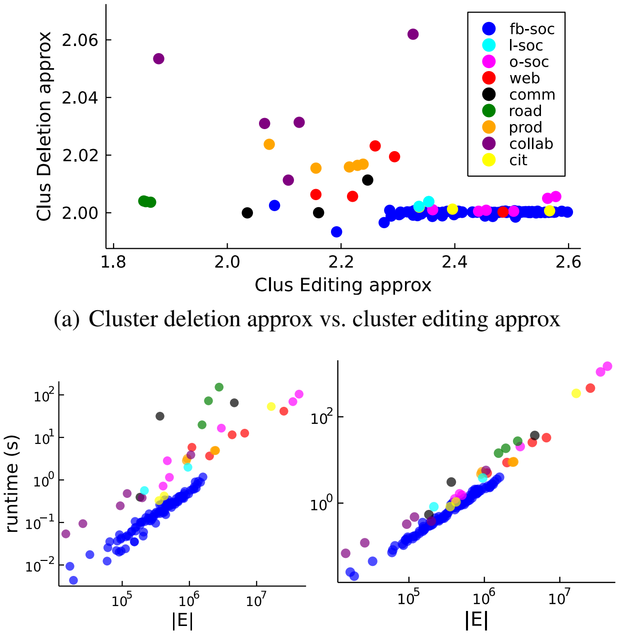
Graph		MFP-CD	LP-STC	LP-CD
EmailEnron	LB	84385	87861.0	87861.0
	UB	**169793**	173936.0	174035.0
$n = 36692$	Ratio	2.012	1.98	1.981
$m = 183831$	Run	0.398	243.0	391.0
condmat05	LB	72428	79287.5	79287.5
	UB	**147826**	152791.0	153446.0
$n = 36458$	Ratio	2.041	1.927	1.935
$m = 171734$	Run	0.433	39.5	72.5
caAstroPh	LB	87563	91188.0	91188.0
	UB	178278	174918.0	**174802.0**
$n = 17903$	Ratio	2.036	1.918	1.917
$m = 196972$	Run	0.367	78.4	376.0
loc-Brightkite	LB	101924	106429.0	106429.0
	UB	**204104**	211219.0	211240.0
$n = 58228$	Ratio	2.003	1.985	1.985
$m = 214078$	Run	0.632	151.0	241.0

5.2. Cluster Editing Approximation Algorithms

We run a similar set of experiments for cluster editing (CE), i.e., complete unweighted correlation clustering, with the same overall findings. Table 2 summarizes results for a few graphs. Our method for rounding the MINSTC+ LP relaxation (LP-STC+, Algorithm 1), is noticeably faster than solving and rounding the canonical LP-relaxation (LP-CE, Chawla et al. (2015)), and produces nearly identical approximation results. As a bonus, LP-STC+ can be used as a first step of a more efficient approach for solving the full canonical relaxation for cluster editing. In fact, we are not even able to solve the canonical relaxation on graphs with a few hundred nodes without using a "lazy constraints" method that involves first applying LP-STC+ and then iteratively updating constraints (see the appendix for further details). If we tried forming the entire constraint matrix for

Table 2. Lower bounds (LB), rounded solution scores (UB), approximation ratios (Ratio) and runtimes (Run) for MFP-CE, LP-STC+, and the 2.06 approximation for cluster editing (LP-CE, Chawla et al. (2015)). To even run LP-CE, we first solve the STC+ relaxation and iteratively add violated constraints. This often (but not always) makes it possible to run LP-CE almost as quickly as LP-STC+. MFP-CE obtains good approximations in under a second for problems that are so large the LP methods run out of memory (indicated by a dash).

Graph		MFP-CE	LP-STC+	LP-CE
caHepTh	LB	10609	11289.8	11290.5
	UB	29049	21625	20597
$n = 8638$	Ratio	2.738	1.915	1.824
$m = 24806$	Run	0.049	62.6	562.1
Simmons81	LB	16402	16490.0	16490.0
	UB	38856	32977	34391
$n = 1518$	Ratio	2.369	2.0	2.086
$m = 32988$	Run	0.0645	235.8	236.9
caAstroPh	LB	86369	–	–
	UB	225943	–	–
$n = 17903$	Ratio	2.616	–	–
$m = 196972$	Run	0.488	–	–
loc	LB	101544	–	–
-Brightkite	UB	267453	–	–
$n = 58228$	Ratio	2.634	–	–
$m = 214078$	Run	0.559	–	–



(a) Cluster deletion approx vs. cluster editing approx

(b) MFP-CD runtimes

(c) MFP-CE runtimes

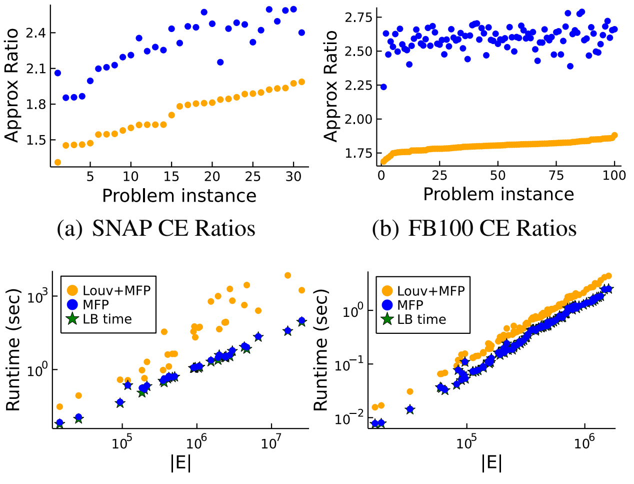
Figure 1. (a) Our algorithms allow us to observe different patterns in how easy or hard it is to approximate cluster editing and cluster deletion in different graph classes. Runtimes for our match-flip-pivot algorithms for cluster deletion (b) and cluster editing (c) are extremely fast, and scale roughly linearly in term of $|E|$.

the canonical LP relaxation, LP-CE would not be able to run on the graphs in Table 2. In comparison with LP-based methods, our match-flip-pivot method (MFP-CE, Algorithm 3) is orders of magnitude faster than the LP methods. **Even for problems where LP-based methods run out of memory, MFP-CE obtains good results in under a second**. The appendix includes results on more graphs, and provides additional details and results for various alternative rounding schemes. For example, we find that combining the lower bounds from MFP-CE with the clusterings obtained by running standard Pivot leads to significantly improved a posteriori approximation guarantees, with no increase in runtime.

5.3. Match-Flip-Pivot on Large Graphs

Next we test the scalability of our match-flip-pivot techniques on graphs with up to millions of nodes and billions of edges. We run MFP-CE and MFP-CD on all graphs from the Facebook100 dataset (Traud et al., 2012), and on a wide range of SNAP graphs (Leskovec & Krevl, 2014). Figure 1(a) reports our a posteriori approximation guarantees for cluster deletion vs. our approximations for cluster editing, on graphs from 8 different classes (e.g., Facebook social networks, other social networks, road networks; see appendix for more details). For MFP-CE, we report the approximation factor obtained by combining our lower bounds with the clusterings obtained by standard Pivot, as this is just as fast and tends to produce better results. Our results allow us to highlight and detect interesting patterns that arise for different classes of graphs when it comes to solving these edge modification problems. For example, the three road networks (green) are unique in that cluster editing approximations for these graphs are significantly better than cluster deletion approximations. Meanwhile, the collaboration networks (purple) exhibit worse cluster deletion approximation results than other graphs in general. Finally, social networks are characterized by better cluster deletion approximations, and poorer cluster editing approximations.

These results and observations about graph classes would not be possible if our algorithms were not extremely scalable. Figure 1 shows that our methods scale roughly linearly in terms of the number of edges. For most graphs, our methods take a few seconds or less, and only take a few minutes for large graphs with millions of nodes and edges. The largest graph we consider (soc-Livejournal1) has 4.2 million nodes and 4.7 billion edges. We find a 2.01 approximation for cluster deletion in 100 seconds, and a 2.56 approximation for cluster editing in 24 minutes. By comparison, standard LP solvers run out of memory for problems that take us less than a second. Specialized solvers for correlation clustering



(a) SNAP CE Ratios (b) FB100 CE Ratios

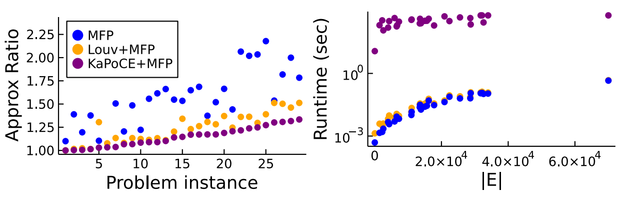
(c) SNAP CE Runtimes (d) FB100 CE Runtimes

Figure 2. Approximation ratios and runtimes for MFP-CE, and for combining lower bounds from MFP-CE with Louvain-based heuristics (Louv-MFP), on SNAP and Facebook100 graphs. Runtimes for both methods include the time it takes to obtain the MFP lower bound. We separately show the time it takes to compute this lower bound (LB time). The time it takes MFP-CE to round this lower bound in negligible, but the Louvain approach is slower.

relaxations have also been designed (Ruggles et al., 2020; Sonthalia & Gilbert, 2020; Veldt et al., 2019). These are more memory efficient than standard black-box optimization methods and can be applied to more general problems, but can be very slow in practice. We tried these methods as well but found they were not competitive on the problems we consider. See appendix for details.

5.4. A Posteriori Guarantees for Fast Heuristics

The lower bounds computed by our match-flip-pivot techniques can be used to provide a posteriori approximation guarantees for fast heuristic algorithms that come with no approximation guarantees of their own. To illustrate this, we run a heuristic method for correlation clustering called LAMBDALOUVAIN (Veldt et al., 2018) based on the popular Louvain method for graph clustering (Blondel et al., 2008). Figure 2 displays the approximation guarantees we obtain for the cluster editing objective (CE) by combining our MFP lower bounds with LAMBDALOUVAIN. Overall this leads to significantly improved a posteriori approximation ratios in comparison with running MFP by itself, at the expense of slower runtimes. Viewed from another perspective, this shows that we can obtain lower bounds to certify that heuristic methods provide approximately optimal solutions, in significantly less time than it takes to actually run these heuristic methods. The appendix provides additional details.



(a) PACE Graphs Ratios (b) PACE Graphs Runtimes

Figure 3. Comparison against KaPoCE on PACE graphs.

5.5. Comparisons on PACE Graphs

Finally, we illustrate the performance of our methods on graphs from the 2021 Parameterized Algorithms and Computational Experiments (PACE) challenge on algorithms for solving the cluster editing objective (Kellerhals et al., 2021). We compare against the winning method KaPoCE, which comes with an exact version for finding optimal solutions and a heuristic version with no approximation guarantees. The exact version times out on even on some problems with under 100 nodes, and even the heuristic approach does not scale to large graphs. Figure 3 shows results for MFP-CE, compared with results for combining MFP-CE lower bounds with LAMBDALOUVAIN and the heuristic KaPoCE algorithm. KaPoCE finds high quality solutions on small graphs, but comes with no approximation guarantees of its own and is orders of magnitude slower than MFP-CE.

6. Discussion and Open Questions

We have presented new approximation algorithms for cluster editing and cluster deletion, based on new ways to round lower bounds for related edge labeling problems. We proved that cluster deletion and MINSTC are always within a factor of 2; previous work has shown cases where their objectives differ by a factor of $8/7$ (Grüttemeier & Komusiewicz, 2020). One open question is whether we can tighten these bounds in either direction, or tighten the corresponding bounds between MINSTC+ and cluster editing. Our research also motivates further work on improved lower bounds and approximation algorithms for correlation clustering that do not rely on LP relaxations. Is it possible to obtain even better approximation guarantees using alternative rounding schemes or simply alternative analyses? One particularly compelling open question is to see whether we can obtain a deterministic combinatorial PIVOT method that is a 3-approximation, rather than a 6-approximation, or a combinatorial 2-approximation for cluster deletion. Finally, perhaps the most interesting and meaningful direction for future work is to see whether our fast and practical match-flip-pivot lower bounds can be used to obtain faster approximation algorithms for weighted variants of correlation clustering whose only approximation algorithms currently rely on LP relaxations (Jafarov et al., 2020; Veldt et al., 2018; Puleo & Milenkovic, 2015).

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A. Proofs for Approximation Algorithms

A.1. Proof of Theorem 3.2

Proof. We must prove that Theorem 3.1 is satisfied with $\alpha = 4$ for an appropriate choice of correlation clustering weights and budgets. For cluster editing, the weights are given by

$$(w_{ij}^+, w_{ij}^-) = \begin{cases} (1,0) & \text{if } (i,j) \in E \\ (0,1) & \text{if } (i,j) \notin E, \end{cases} \tag{6}$$

and the budgets defined by the LP relaxation are given by

$$b_{ij} = \begin{cases} x_{ij} & \text{if } (i,j) \in E \\ 1 - x_{ij} & \text{if } (i,j) \notin E. \end{cases} \tag{7}$$

Considering the way $\hat{G}$ is constructed in Algorithm 1, the conditions in Theorem 3.1 translate to the following:

1. If $x_{ij} < 1/2$, we have $w_{ij}^- \leq 4b_{ij}$, and if $x_{ij} \geq 1/2$, then $w_{ij}^+ \leq 4b_{ij}$.
2. If $x_{ij} < 1/2$ and $x_{jk} < 1/2$ but $x_{ik} \geq 1/2$, then

$$w_{ij}^+ + w_{jk}^+ + w_{ik}^- \leq 4(b_{ij} + b_{jk} + b_{ik}). \tag{8}$$

Condition 1 is straightforward to check by considering the definitions of edge weights (6) and budgets (7). We can prove the second condition by case analysis, considering separately whether each node pair (i,j), (i,k), and (j,k) is an edge or not in original graph $G = (V,E)$. Regardless of the case, we have the following bounds, based on the assumption that (i,j,k) is an open wedge centered at j in $\hat{G}$:

$$1 - x_{ij} > 1/2, \qquad 1 - x_{jk} > 1/2, \qquad x_{ik} \geq 1/2. \tag{9}$$

We summarize all of the cases in succinct tabular format, where we state whether each edge is in E or not, and then give lower bounds on the right hand side of inequality (8) to show it is greater than the left hand side in each case. We have ordered cases so that moving from one row to the next changes the edge status of only one node pair at a time, making it easy to quickly see changes in the left and right hand sides of the inequality (8) for the corresponding budgets and weights. Several of the bounds we list for the right hand side of (8) could be tightened further, but this would not lead to an improved overall approximation guarantee.

Is the edge in E?			Right side of (8)	Left side of (8)	Explanation Note
(i,j)	(j,k)	(i,k)	$4(b_{ij} + b_{jk} + b_{ik})$	$w_{ij}^+ + w_{jk}^+ + w_{ik}^-$	
Y	Y	Y	$4(x_{ij} + x_{jk} + x_{ik}) \geq 4x_{ik} \geq 2$	2 = 1+ 1+ 0	$x_{ik} \geq 1/2$
Y	Y	N	$4(x_{ij} + x_{jk} + 1 - x_{ik}) \geq 4$	3 = 1+ 1+ 1	$x_{ij} + x_{jk} - x_{ik} \geq 0$ (LP constraint)
Y	N	N	$4(x_{ij} + 1 - x_{jk} + 1 - x_{ik}) > 2$	2 = 1 + 0 + 1	$1 - x_{jk} > 1/2$
Y	N	Y	$4(x_{ij} + 1 - x_{jk} + x_{ik}) \geq 2$	1 = 1 + 0 + 0	$x_{ik} \geq 1/2$
N	N	Y	$4(1 - x_{ij} + 1 - x_{jk} + x_{ik}) \geq 0$	0 = 0 + 0 + 0	zero left side
N	Y	Y	$4(1 - x_{ij} + x_{jk} + x_{ik}) \geq 2$	1 = 0 + 1 + 0	$x_{ik} \geq 1/2$
N	Y	N	$4(1 - x_{ij} + x_{jk} + 1 - x_{ik}) > 2$	2 = 0 + 1 + 1	$1 - x_{ij} > 1/2$
N	N	N	$4(1 - x_{ij} + 1 - x_{jk} + 1 - x_{ik}) > 2$	1 = 0 + 0 + 1	$1 - x_{ij} > 1/2$

□

A.2. Proof of Theorem 3.3

Proof. First of all, note that applying PIVOT to the derived graph $\hat{G} = (V, \hat{E})$, using any order of pivot choices, will produce a feasible instance for cluster deletion. To see why, observe that if k is the pivot node and i and j are two of its neighbors in $\hat{G}$, then $z_{ki} < 1/2$ and $z_{kj} < 1/2$, which implies that $(i, j) \in E$. If (i, j) were not an edge, then (i, j, k) would be an open wedge and the LP relaxation would include the constraint $z_{ki} + z_{kj} \geq 1$. It remains to check that Theorem 3.1 is satisfied with $\alpha = 4$, for the right choice of budgets and weights. The weights in this case are

$$(w_{ij}^+, w_{ij}^-) = \begin{cases} (1, 0) & \text{if } (i, j) \in E \\ (0, \infty) & \text{if } (i, j) \notin E, \end{cases} \tag{10}$$

since we are solving cluster deletion. We set our budgets to be the contributions to the LP objective: $b_{ij} = z_{ij}$ if $(i, j) \in E$, and $b_{ij} = 0$ otherwise. These conditions we need to satisfy are:

1. If $(i, j) \in \hat{E}$, we have $w_{ij}^- \leq 4b_{ij}$, and if $(i, j) \notin \hat{E}$, then $w_{ij}^+ \leq 4b_{ij}$.
2. If $(i, j) \in \hat{E}$ and $(j, k) \in \hat{E}$ and $(i, k) \notin \hat{E}$, then

$$w_{ij}^+ + w_{jk}^+ + w_{ik}^- \leq 4(b_{ij} + b_{jk} + b_{ik}). \tag{11}$$

Checking condition 1. Observe that $(i, j) \in \hat{E} \implies (i, j) \in E \implies w_{ij}^- = 0 \leq 4b_{ij}$. Similarly, if $(i, j) \notin \hat{E}$ and $(i, j) \notin E$, then $w_{ij}^+ = b_{ij} = 0$. If $(i, j) \notin \hat{E}$ but $(i, j) \in E$, then $z_{ij} = b_{ij} \geq 1/2$ and so $w_{ij}^+ = 1 < 4b_{ij}$.

Checking condition 2. For condition 2, note that $(i, j) \in \hat{E} \subseteq E$ and $(j, k) \in \hat{E} \subseteq E$ imply that $z_{ij} + z_{jk} < 1$ and therefore (i, j, k) is not an open wedge in G and so $(i, k) \in E$. Since $(i, k) \notin \hat{E}$, we know $b_{ik} = z_{ik} \geq 1/2$. Overall, we have that

$$w_{ij}^+ + w_{jk}^+ + w_{ik}^- = 2 = 4 \cdot \frac{1}{2} \leq 4b_{ik} \leq 4(b_{ij} + b_{jk} + b_{ik}).$$

□

A.3. Proof of Theorem 4.1

Proof. One convenient way to prove Theorem 4.1 is to show that it satisfies the conditions of Theorem 3.1 for an appropriate choice of weights, budgets, and parameter α. For cluster editing, recall that the correlation clustering weights are

$$(w_{ij}^+, w_{ij}^-) = \begin{cases} (1, 0) & \text{if } (i, j) \in E \\ (0, 1) & \text{if } (i, j) \notin E. \end{cases}$$

For this theorem, we do not choose budgets to correspond to a lower bound for a labeling or clustering problems. Instead, the budgets are defined in terms of the set of flipped edges:

$$b_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E_W \cup E' \\ 0 & \text{otherwise} \end{cases}.$$

The sum of budgets is exactly $\sum_{i<j} b_{ij} = |E'| + |E_W|$. Note then that the result holds if we can prove that the conditions of Theorem 3.1 are satisfied with $\alpha = 2$. We first need to check that

$$(i, j) \in \hat{E} \implies w_{ij}^- \leq 2b_{ij} \text{ and} \tag{12}$$

$$(i, j) \notin \hat{E} \implies w_{ij}^+ \leq 2b_{ij}. \tag{13}$$

Checking (12): If $(i, j) \in \hat{E} \cap E$ then $w_{ij}^- = 0 = b_{ij}$, and if $(i, j) \in \hat{E}$ but $(i, j) \notin E$, then $b_{ij} = w_{ij}^- = 1$ since (i, j) is a non-edge ($w_{ij}^- = 1$) that was flipped ($b_{ij} = 1$).

Checking (13): If $(i, j) \notin \hat{E}$ and $(i, j) \notin E$ then we have $w_{ij}^+ = b_{ij} = 0$. If $(i, j) \notin \hat{E}$ and $(i, j) \in E$, then $w_{ij}^+ = 1 = b_{ij}$.

Next we confirm that if (i, j, k) is an open wedge centered at j in $\hat{G} = (V, \hat{E})$, then

$$w_{ij}^{+} + w_{jk}^{+} + w_{ik}^{-} \leq 2\left(b_{ij} + b_{jk} + b_{ik}\right). \tag{14}$$

Regardless of the edge structure of (i, j, k) in the original graph $G = (V, E)$, we must have

$$b_{ij} + b_{jk} + b_{ik} + w_{ij}^{+} + w_{jk}^{+} + w_{ik}^{-} = 3. \tag{15}$$

To see why, observe first of all that $(i, j) \in \hat{E}$, $(j, k) \in \hat{E}$, and $(i, k) \notin \hat{E}$, by our assumption that (i, j, k) is an open wedge centered at j in $\hat{G}$. Consider node pair (i, j): either this pair is an edge $(i, j) \in E$ (meaning $w_{ij}^{+} = 1$) *or* it was flipped (meaning $b_{ij} = 1$) but not both. Therefore, $b_{ij} + w_{ij}^{+} = 1$, and by the same argument we can show $b_{jk} + w_{jk}^{+} = b_{ik} + w_{ik}^{-} = 1$. This yields (15).

A key step in the proof is to realize that

$$b_{ij} + b_{jk} + b_{ik} \geq 1. \tag{16}$$

If instead we assume $b_{ij} + b_{jk} + b_{ik} = 0$, this means that none of the edges were flipped, so (i, j, k) is also an open wedge in the original graph $G = (V, E)$. This *contradicts the fact that* (E', E_W) *is a strong triadic closure labeling*. A strong triadic closure labeling would either add (i, k) to the new edge set E', or label one of the edges as weak, which would subsequently lead to one node pair being flipped. Combining (16) and (15), we can see that

$$w_{ij}^{+} + w_{jk}^{+} + w_{ik}^{-} \leq 2 = 2(1) \leq 2(b_{ij} + b_{jk} + b_{ik}).$$

□

A.4. Proof of Corollary 4.2

Proof. We have previously established that $OPT^{+} \leq OPT^{CE}$. If (E', E_W) is the α-approximate STC+ labeling returned by $\mathcal{A}$ and $B = |E'| + |E_W|$, then

$$B \leq \alpha OPT^{+} \leq \alpha OPT^{CE} \implies \frac{B}{\alpha} \leq OPT^{CE},$$

which provides a lower bound on the optimal cluster editing solution. Using Theorem 4.1, we can find a cluster editing solution that makes at most $2B$ mistakes, which is within 2α of the lower bound. If we solve MINSTC+ optimally, this mean $\alpha = 1$, which shows that $OPT^{+} \leq OPT^{CE} \leq 2OPT^{+}$. □

We also observe in passing that an α-approximation algorithm for vertex cover would imply a (2α)-approximation for cluster editing, since MINSTC+ can be reduced to vertex cover in an approximation preserving way.

A.5. Proof of Corollary 4.3

Proof. By construction, the minimum vertex cover in the 3-uniform hypergraph $\mathcal{H} = (V_{\mathcal{H}}, E_{\mathcal{H}})$ is equivalent to MINSTC+ on $G = (V, E)$. The algorithm performs the standard steps to obtain a 3-approximation: find a maximal matching, and place all nodes from the matched edges in the vertex cover. This can be converted to an STC+ labeling that is a 3-approximation for MINSTC+, which can be fed to FLIPPIVOT to produce a $2 \cdot 3 = 6$ approximation for cluster editing. □

A.6. Proof of Theorem 4.4

Proof. We must first confirm that this approach produces a feasible solution to cluster deletion, meaning that all clusters returned are cliques in the original graph $G = (V, E)$. Consider pivoting on any node j in the derived graph $\hat{G} = (V, \hat{E})$. If $(j, k) \in \hat{E}$ and $(i, j) \in \hat{E}$, this means neither of these edges were labeled weak, and so we must have $(i, k) \in E$ or else strong triadic closure would be violated. Thus, pivoting on any node produces cliques.

The rest of the theorem follows by showing that Theorem 3.1 holds with $\alpha = 2$ if we choose budgets $b_{ij} = 1$ if $(i, j) \in E_W$ and $b_{ij} = 0$ otherwise, and use weights (w_{ij}^{+}, w_{ij}^{-}) that corresponding to cluster deletion (see (10)). In this case, the sum of budgets in is $\sum_{i<j} b_{ij} = |E_W|$. The conditions we must check are:

1. For all $(i, j) \in \hat{E}$, we have $w_{ij}^{-} \leq 2b_{ij}$, and for all $(i, j) \notin \hat{E}$, we have $w_{ij}^{+} \leq 2b_{ij}$.

Algorithm 5 $\textsc{DetPivot}(V, W^+, W^-, \{b_{ij}\}, \hat{E})$

```
Input: Correlation clustering instance $(V, W^+, W^-)$, budgets $\{b_{ij}\}$, derived graph $\hat{G} = (V, \hat{E})$
Output: $\mathcal{C} = \textsc{DetPivot}(V, W^+, W^-, \{b_{ij}\}, \hat{E})$
for $k \in V$ do
    $T_k^+ = \{(i,j) \in \hat{E} \colon (j,k) \notin \hat{E}, (i,k) \in \hat{E}\}$
    $T_k^- = \{(i,j) \notin \hat{E} \colon (j,k) \in \hat{E}, (i,k) \in \hat{E}\}$
    $P_k = \frac{\sum_{(i,j) \in T_k^+} w_{ij}^+ + \sum_{(i,j) \in T_k^-} w_{ij}^-}{\sum_{(i,j) \in T_k^+ \cup T_k^-} b_{ij}}$
end for
$p = \text{argmin}_{k \in V} P_k$                                  // select pivot
$S = \{v \in V : (p,v) \in \hat{E}\}$                              // form cluster
$T = V \backslash S$                                               // update node set
$W_T^+ = \{w_{ij}^+ \colon i \in T, j \in T\}$             // update remaining weights, edges, and budgets
$W_T^- = \{w_{ij}^- \colon i \in T, j \in T\}$
$\hat{E}_T = \{(i,j) \in \hat{E} \colon i \in T, j \in T\}$
$\mathcal{B}_T = \{b_{ij} \colon i \in T, j \in T\}$
Return $\mathcal{C} = \{S, \textsc{DetPivot}(T, W_T^+, W_T^-, \mathcal{B}_T, \hat{E}_T)$
```

2. If (i,j,k) is an open wedge centered at j in $\hat{G}$, we have $w_{ij}^+ + w_{jk}^+ + w_{ik}^- \leq 2\,(b_{ij} + b_{jk} + b_{ik})$.

Checking condition 1: If $(i,j) \in \hat{E}$, then $(i,j) \in E$, so $w_{ij}^- = 0 \leq 2b_{ij}$. If $(i,j) \notin \hat{E}$ and $(i,j) \notin E$, then have $w_{ij}^+ = 0 \leq 2b_{ij}$. If $(i,j) \notin \hat{E}$ but $(i,j) \in E$, then $(i,j) \in E_W$ and so $b_{ij} = 1$, and thus $w_{ij}^+ = 1 = b_{ij}$.

Checking condition 2: If $(i,j) \in \hat{E}$ and $(j,k) \in \hat{E}$, then we must have $(i,k) \in E$ or else there would be a violation of strong triadic closure. Since we are assuming in condition 2 that (i,j,k) is an open wedge centered at j in $\hat{G}$, the edge $(i,k) \in E_W$, and so $b_{ik} = 1$. Thus, we have

$$w_{ij}^+ + w_{jk}^+ + w_{ik}^- = 2 = 2b_{ik}.$$

Therefore, the weight of mistakes (i.e, the number of deleted edges) resulting from running a pivoting procedure on $\hat{G} = (V, E - E_W)$ is at most $\alpha \sum_{i<j} b_{ij} = 2|E_W|$ in expectation. □

We omit proofs for Corollaries 4.5 and 4.6 as they follows the same arguments as Corollaries 4.2 and 4.3.

B. Deterministic Approximation Algorithms

Our main text focused on randomized algorithms with expected approximation guarantees that can be obtained by applying a standard pivot procedure to a derived graph $\hat{G} = (V, \hat{E})$. All of our algorithms can be made deterministic by applying the deterministic pivoting procedure of van Zuylen and Williamson (2009). Algorithm 5 provides pseudocode for this method, whose choice of pivot nodes is guided by the derived graph $\hat{G}$, the budgets $\{b_{ij}\}$, and the weights $\{w_{ij}^+, w_{ij}^-\}$ that define the correlation clustering instance. For completeness, we show how to set all parameters for our algorithms to obtain deterministic approximation guarantees.

Algorithms 6 and 7 are deterministic versions of Algorithms 1 and 2, and show explicitly how to set budgets and weights to guide the deterministic pivoting strategy (Algorithm 5). Similarly, Algorithm 8 is a deterministic counterpart to Algorithm 3, and provides a way to turn an STC+ labeling (E', E_W) into a cluster editing solution that is guaranteed to make at most $2(|E'| + |E_W|)$ mistakes (i.e., deleted and added edges). Finally, Algorithm 9 is a deterministic version of Algorithm 4, our combinatorial 4-approximation for cluster deletion.

Algorithm 6 Deterministic Rounding for the minSTC+ LP relaxation.

```
Input: Graph $G = (V, E)$
Output: Clustering of $G$.
Solve LP-relaxation of (5)
Set $\hat{E} \leftarrow \{(i,j) \in V \times V : x_{ij} < 1/2\}$
For $(i,j) \in E$, $(w_{ij}^+, w_{ij}^-) = (1, 0)$, and $b_{ij} = x_{ij}$
For $(i,j) \notin E$, $(w_{ij}^+, w_{ij}^-) = (0, 1)$, and $b_{ij} = 1 - x_{ij}$
Return DetPivot$(V, \{w_{ij}^+\}, \{w_{ij}^-\}, \{b_{ij}\}, \hat{E})$
```

Algorithm 7 Deterministic Rounding for the minSTC LP relaxation.

```
Input: Graph $G = (V, E)$
Output: Feasible cluster deletion clustering of $G$.
Solve LP relaxation of (3)
Set $\hat{E} \leftarrow \{(i,j) \in E : z_{ij} < 1/2\}$
For $(i,j) \in E$, $(w_{ij}^+, w_{ij}^-) = (1, 0)$, and $b_{ij} = z_{ij}$
For $(i,j) \notin E$, $(w_{ij}^+, w_{ij}^-) = (0, \infty)$ and $b_{ij} = 0$
Return DetPivot$(V, \{w_{ij}^+\}, \{w_{ij}^-\}, \{b_{ij}\}, \hat{E})$
```

Algorithm 8 DetMatchFlipPivotCE(G, E', E_W)

```
Input: $G = (V, E)$
Output: Clustering of $G$.
Reduce: Build open wedge hypergraph $\mathcal{H} = (V_\mathcal{H}, E_\mathcal{H})$ (Section 4.1)
Match: Find maximal matching $\mathcal{M} \subseteq E_\mathcal{H}$
Vertex Cover: $C = \{v_{ij} \in V_\mathcal{H} \colon v_{ij} \in w$ for some $w \in \mathcal{M}\}$
STC+ Labeling:
    $E' = \{(i,j) \notin E \colon v_{ij} \in C\}$
    $E_W = \{(i,j) \in E \colon v_{ij} \in C\}$
Construct $\hat{G} = (V, \hat{E})$ where $\hat{E} = E' \cup (E - E_W)$
Set budgets $(b_{ij})$ and weights $(w_{ij}^+, w_{ij}^-)$:
    $b_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E_W \cup E' \\ 0 & \text{otherwise} \end{cases}$
    $(w_{ij}^+, w_{ij}^-) = \begin{cases} (1,0) & \text{if } (i,j) \in E \\ (0,1) & \text{if } (i,j) \notin E \end{cases}$
Return DetPivot$(V, \{w_{ij}^+\}, \{w_{ij}^-\}, \{b_{ij}\}, \hat{E})$
```

Algorithm 9 DetMatchFlipPivotCD(G, E_W)

```
Input: Graph $G = (V, E)$ and STC label set $E_W$
Output: Feasible cluster deletion clustering of $G$.
Reduce: Build Gallai graph $\mathcal{G} = (V_\mathcal{G}, E_\mathcal{G})$ (Section 4.1)
Match: Find maximal matching $\mathcal{M} \subseteq E_\mathcal{G}$
Cover: $\mathcal{C} = \{v_{ij} \in V_\mathcal{G} \colon v_{ij} \in w$ for some $w \in \mathcal{M}\}$
STC Labeling: $E_W = \{(i,j) \in E \colon v_{ij} \in \mathcal{C}\}$
Construct graph $\hat{G} = (V, \hat{E})$ where $\hat{E} = (E - E_W)$
Set budgets $(b_{ij})$ and weights $(w_{ij}^+, w_{ij}^-)$:
    $b_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E_W \\ 0 & \text{otherwise} \end{cases}$
    $(w_{ij}^+, w_{ij}^-) = \begin{cases} (1,0) & \text{if } (i,j) \in E \\ (0,\infty) & \text{if } (i,j) \notin E \end{cases}$
Return DetPivot$(V, \{w_{ij}^+\}, \{w_{ij}^-\}, \{b_{ij}\}, \hat{E})$
```

C. Extended Experimental Results and Details

C.1. Graph details

Aside from PACE challenge graphs, all of the graphs we consider in our experimental results come from either the Facebook100 dataset (Traud et al., 2012), or are available on the Suitesparse Matrix Collection (Davis & Hu, 2011), or the SNAP large network repository (Leskovec & Krevl, 2014). These can be categorized into the following classes:

- **fb-social**: Facebook social networks (all graphs from the Facebook100 dataset)
- **loc-social**: Location-based social networks (loc-Gowalla, loc-Brightkite)
- **o-social**: Other social networks (com-LiveJournal, com-Youtube, soc-Epinions1, soc-LiveJournal1, soc-Slashdot0811, soc-Slashdot0902)
- **web**: web networks (web-BerkStan, web-Google, web-NotreDame, web-Stanford, wiki-topcats)
- **comm**: communication networks (email, email-Enron, email-EuAll, wiki-Talk)
- **road**: road networks (roadNet-CA, roadNet-PA, roadNet-TX)
- **prod**: product co-purchasing networks (amazon0302, amazon0312, amazon0505, amazon0601, com-Amazon)
- **collab**: collaboration networks (Erdos991, Netscience, ca-AstroPh, ca-CondMat, ca-GrQc, ca-HepTh, ca-HepPh, condmat2005, com-DBLP)
- **cit**: citation networks (SmaGri, cit-HepPh, cit-HepTh, cit-Patents)
- **bio**: biological networks (celegans-neural, celegans-metabolic)
- **other**: all other graphs (Harvard500, Roget, polblogs)

Some of these graphs have weights or directions. Before running our experiments, we standardize all graphs by removing edge directions and weights.

C.2. Results for Cluster Deletion Approximation Algorithms

In Tables 4 we show more results for running our approximation algorithms for cluster deletion on various graphs. Overall, our LP-STC algorithm is twice as fast as the canonical relaxation algorithm (LP-CD), while obtaining similar approximation results. Our match-flip-pivot technique (MFP-CD) is far more scalable, and comes with a minor loss in approximation guarantees.

C.3. Extended Results and Details for Cluster Editing Approximation Algorithms

Solving cluster editing on a graph G is equivalent to solving the complete unweighted correlation clustering objective on the signed graph obtained by treating edges of G as positive edges and non-edges in G as negative edges. Cluster editing is much more computationally expensive than cluster deletion, as it involves both deleting *and adding* edges, rather than just deleting edges. Approximation algorithms and LP-rounding schemes for this problem are more abundant than approximation algorithms for cluster deletion. Therefore, for this problem we provide additional details on techniques for scaling LP algorithms as much as possible, and we also provide additional details on different methods for rounding the lower bounds we consider. Interestingly, we find in practice that the best results are obtained by combining the lower bounds of our algorithms with alternative rounding schemes than the ones we theoretically analyze in the main text. This suggests that improved approximation results may also be obtained by analyzing other rounding strategies.

Rounding for MFP-CE For MFP-CE, we compare our combinatorial lower bound against three types of pivoting strategies. The first is the standard approach of applying a random PIVOT procedure to the derived graph $\hat{G}$ in Algorithm 3. The second is a proof-of-concept implementation of the deterministic version of this algorithm (Algorithm 8), which significantly improves a posteriori approximation guarantees but is slower as our implementation of the deterministic pivoting procedure is not optimized. An optimized version of this code would be significantly faster, though we expect it to typically still be noticeably slower than randomized pivot node selections. Finally, we find that the best results can be obtained simply by running standard PIVOT on the original graph G multiple times and comparing the best result against the lower bound from MFP-CE. This last approach is extremely fast while producing results that are comparable to deterministic pivoting on the derived graph $\hat{G}$. In Tables 5 and 6, we show results for each rounding method (when applying randomized PIVOT, we take the best result from 50 different runs). In Table 2 in the main text, we have displayed results for the standard MFP-CE algorithm that applies pivoting on the derived graph $\hat{G}$, in order to show how the approximation algorithm performs when it is run exactly according to its theoretical design. In subsequent experiments in the main text, we display results for applying PIVOT to the original graph $G = (V, E)$, as this perform better typically and is just as fast.

Rounding for LP-STC+ For the canonical LP-relaxation (LP-CE), we use the rounding scheme with a 2.06-approximation guarantee, due to Chawla, Makarychev, Schramm, and Yaroslavtsev (2015), which we refer to as *CMSY* rounding. This involves a more careful randomized construction of a derived graph before running a pivot procedure; we perform this randomized construction ten times and take the best result. For LP-STC+, we use the more simplistic rounding scheme that gives us our 4-approximation (Algorithm 1). For our results in the appendix, we additionally apply the same *CMSY* rounding procedure, which is cheap in comparison with finding the lower bound. We find in practice that *CMSY* produces the best results.

Scalability and LP solvers. Linear programming relaxations for cluster deletion have $O(|E|)$ variables, whereas LPs for cluster editing involve $O(|V|^2)$ variables. Even more significantly, the canonical LP for cluster editing has $O(|V|^3)$ constraints. In our experiments, it becomes prohibitively expensive to even *form* the constraint matrix when $|V|$ equals a few hundred. In the runtimes listed in Table 2 and in the appendix, the runtimes for LP-STC+ and LP-CE appear very similar, but this is only because we apply a useful warm-start approach for solving the canonical LP relaxation. This approach first solves the LP relaxation for STC+ and then iteratively adds in constraints from the canonical cluster editing relaxation that were violated. This process continues adding in violated constraints and re-solving the problem until all of the canonical LP constraints are satisfied, even if they were not included explicitly. Often, the solution for the STC+ relaxation matches the solution for LP-CE and this procedure terminates quickly, while in many other cases only a few iterations are needed. This lazy constraints approach has previously been used to help scale up LP solvers for correlation clustering (Veldt et al., 2019), though without an explicit realization that this method actually begins by solving the STC+ relaxation.

There also exist specialized solvers for *approximately* solving the correlation clustering LP relaxation by applying memory-efficient projection methods (Ruggles et al., 2020; Veldt et al., 2019; Sonthalia & Gilbert, 2020). However, although these methods come with a smaller memory requirement and can be generalized to other weighted variants of the problem, they are still quite slow and were not competitive for the problems we considered. In Table 3 we show lower bounds and runtimes for using the projection method of Veldt et al. (2019) on a sample of graphs. When it comes to lower bounding the cluster editing relaxation, this method is much slower than LP-STC+ and LP-CE, and returns poorer lower bounds.

C.4. Details for Louvain Experiments and Cluster Deletion Results

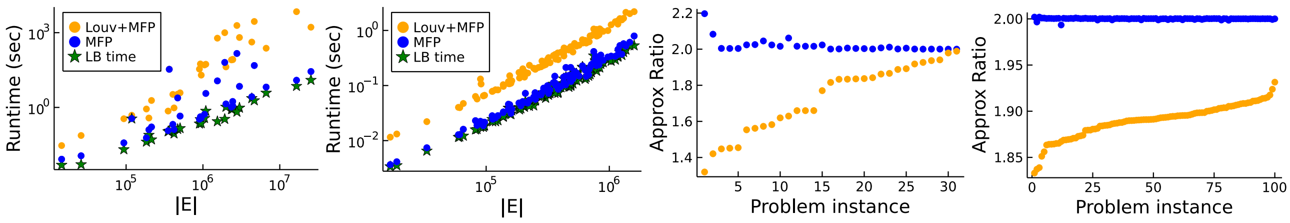
We used an implementation of the LAMBDALOUVAIN method from the author's previous work (Veldt et al., 2020) (available at `https://github.com/nveldt/ParamCC/blob/master/src/Graph_Louvain.jl`). Running this method with a parameter $\lambda = 1/2$ greedily optimizes the cluster editing objective, while λ close to 1 greedily optimizes cluster deletion (Veldt et al., 2018).

The performance and runtime of this methods depends on how long the greedy procedure is allowed to run when searching for improved clusterings. In more detail, the method selects a random ordering of nodes, places all nodes in singleton clusters to start, and then iteratively visits each node to move it to the adjacent cluster leading to the greatest improvement in the correlation clustering objective. In theory, this greedy moving can continue until no more improvement is possible, but usually the number of passes over the nodes is truncated. Once there is no more improvement from visiting nodes, or once the maximum number of passes over the nodes is reached, the algorithm enters a second phase where nodes in the same cluster are agglomerated into supernodes and the procedure is run again on a reduced graph. Often, the algorithm is run multiple times with different random node orderings, and the best result is returned.

Parameter settings and cluster deletion results In our experiments, we run LAMBDALOUVAIN with the fastest possible settings. For each graph we fix a single random ordering of the nodes and run the algorithm only for this ordering. When iteratively visiting nodes to perform greedy moves, we visit each node only once time, and then skipped the second

Table 3. Cluster editing lower bounds (LB) for three solvers, and runtimes (Run). Value n denotes number of nodes, m is the number of edges. Proj-CE solves a quadratic program that approximates the canonical cluster editing LP. This method is not competitive with LP-STC+ or LP-CE when applied to the cluster editing objective. Dashed lines indicate that Proj-CE did not converge after half an hour.

Graph			**LP-STC+**	**LP-CE**	**Proj-CE**
HARVARD500	$n = 500$	LB	727.0	727.0	696.3
	$m = 2043$	Run	0.376	0.447	33.4
ROGET	$n = 994$	LB	1819.5	1819.5	1736.6
	$m = 3640$	Run	0.622	0.893	218.8
SMAGRI	$n = 1024$	LB	2457.0	2457.0	2345.3
	$m = 4916$	Run	2.7	2.9	278.5
POLBLOGS	$n = 1222$	LB	8356.0	8356.0	7976.3
	$m = 16714$	Run	61.6	63.1	326.4
EMAIL	$n = 1133$	LB	2722.0	2722.0	2598.6
	$m = 5451$	Run	1.6	1.9	416.0
CA-GRQC	$n = 5242$	LB	4931.0	4931.0	–
	$m = 14484$	Run	16.2	80.9	–
CAHEPTH	$n = 8638$	LB	11289.8	11290.5	–
	$m = 24806$	Run	64.7	625.5	–



(a) SNAP CD Runtimes (b) FB100 CD Runtimes (c) SNAP CD Ratios (d) FB100 CD Ratios

Figure 4. Approximation ratios and runtimes when solving the cluster deletion objective.

phase where nodes are agglomerated into supernodes. Since we use the fastest settings for LambdaLouvain, we do the same for MFP-CE, and only perform one step of the pivot rounding procedure for this comparison. This provides the most straightforward comparison between the methods, as we are comparing the fastest settings for each method. More importantly, we run LambdaLouvain with the fastest settings as this provides most of the improvement in terms of solution quality (i.e., the a posteriori approximation guarantee), at a fraction of the runtime. Overall this provides the most favorable comparison for LambdaLouvain. Even so, this method can be quite a bit slower than MFP-CE. Our plots do not report results for running LambdaLouvain on the largest SNAP graph, (soc-Livejournal1), as it took too long even with the fastest parameter settings.

We also provide approximation ratios and runtimes for MFP-CD and for LambdaLouvain when λ is slightly less than one, which greedily optimizes the cluster deletion objective (Figure 4). MFP-CD tends to return solutions with an approximation guarantee around two for nearly every graph. LambdaLouvain again returns better solutions. This is often at the expense of longer runtimes, though for this objective MFP-CD is actually a little slower for some graphs.

Better but slower results with Louvain By visiting nodes more than once and performing the second phase of the Louvain method, we can obtain better results using LambdaLouvain, but this significantly increases the runtime. Meanwhile, for MFP-CE, increasing the number of pivot rounding steps has only a small increase in runtime since the main bottleneck for this method is the matching step for finding lower bounds. A more detailed comparison (specifically for the cluster editing objective) is provided in Figure 5. Subfigures (a) and (c) are the same approximation and runtime plots from the main text, showing results for the fastest settings for each method. Subfigures (b) and (d) show results for running LambdaLouvain when the number of passes over the nodes is increased to 10, and where we also apply the second phase of the algorithm. These plots also show results for running MFP-CE with 50 pivot steps for rounding. Running these algorithms longer leads

Table 4. Detailed cluster deletion results ($n = |V|$, and $m = |E|$). An asterisk next to the result of LP-STC indicates this LP relaxation returns an optimal solution for the canonical cluster deletion LP relaxation. Dashed lines indicate the method ran out of memory.

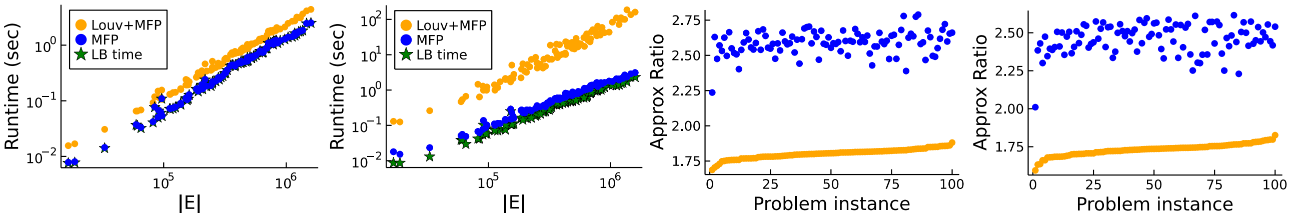
Graph		MFP-CD	LP-STC	LP-CD	**Graph**		MFP-CD	LP-STC	LP-CD
Netscience	LB	315	356.5*	356.5	ca-HepTh	LB	10673	11320.5	11320.5
	UB	669	597.0	605.0		UB	21508	21631.0	21712.0
$n = 379$	Ratio	2.124	1.675	1.697	$n = 8638$	Ratio	2.015	1.911	1.918
$m = 914$	Run	0.00365	0.0117	0.0231	$m = 24806$	Run	0.0848	1.42	2.38
Erdos991	LB	674	701.0*	701.0	Simmons	LB	16451	16490.0*	16490.0
	UB	1338	1372.0	1378.0		UB	32919	32976.0	32976.0
$n = 446$	Ratio	1.985	1.957	1.966	$n = 1518$	Ratio	2.001	2.0	2.0
$m = 1413$	Run	0.000419	0.0513	0.101	$m = 32988$	Run	0.02	11.5	18.5
celegans-	LB	966	996.5	996.5	Haverford	LB	29771	29794.5*	29794.5
metabolic	UB	1917	1961.0	1963.0		UB	59508	59588.0	59588.0
$n = 453$	Ratio	1.984	1.968	1.97	$n = 1446$	Ratio	1.999	2.0	2.0
$m = 2025$	Run	0.00058	0.15	0.257	$m = 59589$	Run	0.0153	38.0	110.0
Harvard500	LB	776	823.0*	823.0	Swarthmore	LB	30500	30524.0*	30524.0
	UB	1548	1584.0	1584.0		UB	61005	61048.0	61048.0
$n = 500$	Ratio	1.995	1.925	1.925	$n = 1659$	Ratio	2.0	2.0	2.0
$m = 2043$	Run	0.00424	0.092	0.252	$m = 61050$	Run	0.0292	36.0	68.4
celegans-	LB	1062	1074.0*	1074.0	Bowdoin	LB	42163	42192.5*	42192.5
neural	UB	2117	2148.0	2148.0		UB	84298	84382.0	84382.0
$n = 297$	Ratio	1.993	2.0	2.0	$n = 2252$	Ratio	1.999	2.0	2.0
$m = 2148$	Run	0.000474	0.107	0.196	$m = 84387$	Run	0.0216	55.2	114.0
Roget	LB	1788	1819.5*	1819.5	Amherst	LB	45450	45477.0*	45477.0
	UB	3571	3623.0	3624.0		UB	90887	90953.0	90953.0
$n = 994$	Ratio	1.997	1.991	1.992	$n = 2235$	Ratio	2.0	2.0	2.0
$m = 3640$	Run	0.00117	0.141	0.143	$m = 90954$	Run	0.0227	70.0	162.0
SmaGri	LB	2410	2457.0*	2457.0	condmat05	LB	72428	79287.5	79287.5
	UB	4811	4909.0	4910.0		UB	147826	152791.0	153446.0
$n = 1024$	Ratio	1.996	1.998	1.998	$n = 36458$	Ratio	2.041	1.927	1.935
$m = 4916$	Run	0.0015	0.435	0.58	$m = 171734$	Run	0.433	39.5	72.5
email	LB	2616	2722.0*	2722.0	EmailEnron	LB	84385	87861.0	87861.0
	UB	5169	5430.0	5432.0		UB	169793	173936.0	174035.0
$n = 1133$	Ratio	1.976	1.995	1.996	$n = 36692$	Ratio	2.012	1.98	1.981
$m = 5451$	Run	0.00248	0.234	0.478	$m = 183831$	Run	0.398	243.0	391.0
ca-GrQc	LB	4789	5196.0	5196.0	Rice31	LB	92342	92410.5*	–
	UB	10095	8598.0	8620.0		UB	184692	184814.0	–
$n = 5242$	Ratio	2.108	1.655	1.659	$n = 4087$	Ratio	2.0	2.0	–
$m = 14484$	Run	0.0674	0.559	2.2	$m = 184828$	Run	0.0952	227.0	–
Caltech36	LB	8295	8324.5*	8324.5	ca-AstroPh	LB	87563	91188.0	91188.0
	UB	16613	16648.0	16648.0		UB	178278	174918.0	174802.0
$n = 769$	Ratio	2.003	2.0	2.0	$n = 17903$	Ratio	2.036	1.918	1.917
$m = 16656$	Run	0.101	4.36	9.56	$m = 196972$	Run	0.367	78.4	376.0
polblogs	LB	8336	8356.0*	8356.0	Lehigh96	LB	99082	99172.5*	99172.5
	UB	16660	16705.0	16706.0		UB	198201	198345.0	198345.0
$n = 1222$	Ratio	1.999	1.999	1.999	$n = 5075$	Ratio	2.0	2.0	2.0
$m = 16714$	Run	0.00441	4.64	9.48	$m = 198347$	Run	0.102	210.0	1420.0
Reed98	LB	9369	9405.5*	9405.5	loc-	LB	101924	106429.0	106429.0
	UB	18676	18811.0	18811.0	Brightkite	UB	204104	211219.0	211240.0
$n = 962$	Ratio	1.993	2.0	2.0	$n = 58228$	Ratio	2.003	1.985	1.985
$m = 18812$	Run	0.00434	4.79	9.24	$m = 214078$	Run	0.632	151.0	241.0

Table 5. Cluster editing (complete unweighted correlation clustering) results for smaller graphs. An asterisk next to the result of LP-STC+ indicates this LP relaxation returns an optimal solution for the canonical cluster editing LP relaxation for the given graph. Value n denotes number of nodes, m is the number of edges, LB is the lower bound returned by the method, UB is the upper bound achieved by rounding the lower bound, and Ratio = LB/UB is the a posteriori approximation guarantee. Runtime is given is seconds. We apply three ways to round the lower bound of MFP-CE. The first two are our own randomized ($+piv(\hat{G})$) and deterministic ($det.$, Algorithm 8) rounding strategies, which apply randomized and deterministic pivoting methods to a derived graph $\hat{G}$. The third ($+piv(G)$) is simply running Pivot on the original graph. For LP-STC+, we apply both our own randomized pivoting procedure ($+piv(\hat{G})$), to a derived graph $\hat{G}$ that differs from the derived graph generated by MFP-CE, as well as the rounding procedure that is guaranteed to output a 2.06 approximation (*CMSY*) if applied to the cluster editing LP (Chawla et al., 2015). The deterministic rounding for MFP-CE is slow as it is a proof-of-concept implementation, and not optimized.

Graph		**MFP-CE**			**LP-STC+**		**LP-CE**
		$+piv(\hat{G})$	$+det.$	$+piv(G)$	$+piv(\hat{G})$	$+CMSY$	$+CMSY$
Harvard500	LB	687			727.0*		727.0
	UB	1832	1547	1331	1395	1303	1321
$n = 500$	Ratio	2.667	2.252	1.937	1.919	1.792	1.817
$m = 2043$	Run	0.00333	0.0368	0.00381	0.419	0.426	0.451
celegansneural	LB	1055			1074.0*		1074.0
	UB	2593	2341	2118	2148	2092	2100
$n = 297$	Ratio	2.458	2.219	2.008	2.0	1.948	1.955
$m = 2148$	Run	0.012	0.0299	0.00435	0.58	0.583	0.59
Roget	LB	1786			1819.5*		1819.5
	UB	4654	4061	3873	3621	3442	3413
$n = 994$	Ratio	2.606	2.274	2.169	1.99	1.892	1.876
$m = 3640$	Run	0.00663	0.174	0.00949	0.632	0.656	0.844
SmaGri	LB	2403			2457.0*		2457.0
	UB	6084	5330	5166	4905	4851	4817
$n = 1024$	Ratio	2.532	2.218	2.15	1.996	1.974	1.961
$m = 4916$	Run	0.00845	0.282	0.0103	2.6	2.6	2.8
email	LB	2606			2722.0*		2722.0
	UB	6729	5692	5859	5429	5269	5357
$n = 1133$	Ratio	2.582	2.184	2.248	1.994	1.936	1.968
$m = 5451$	Run	0.00997	0.302	0.0105	1.4	1.4	1.7
ca-GrQc	LB	4551			4931.0		4931.0
	UB	12414	9268	8610	8311	7595	7741
$n = 5242$	Ratio	2.728	2.036	1.892	1.685	1.54	1.57
$m = 14484$	Run	0.0239	7.5	0.0358	15.4	15.9	73.4
Caltech36	LB	8239			8324.5*		8324.5
	UB	18835	17204	17166	16648	17085	16891
$n = 769$	Ratio	2.286	2.088	2.084	2.0	2.052	2.029
$m = 16656$	Run	0.0277	2.2	0.0167	118.9	118.9	119.1
polblogs	LB	8309			8356.0*		8356.0
	UB	18869	17312	17664	16710	17212	17132
$n = 1222$	Ratio	2.271	2.084	2.126	2.0	2.06	2.05
$m = 16714$	Run	0.0299	4.5	0.0182	63.9	64.0	64.4
Reed98	LB	9338			9405.5*		9405.5
	UB	21805	19511	20386	18811	19723	19385
$n = 962$	Ratio	2.335	2.089	2.183	2.0	2.097	2.061
$m = 18812$	Run	0.0311	2.5	0.0186	67.3	67.4	67.7
caHepTh	LB	10609			11289.8		11290.5
	UB	29049	23442	22214	21625	20470	20597
$n = 8638$	Ratio	2.738	2.21	2.094	1.915	1.813	1.824
$m = 24806$	Run	0.049	12.0	0.0582	62.6	64.1	562.1

Table 6. Cluster editing (complete unweighted correlation clustering) results for larger graphs. Dashed lines indicate that the method ran out of memory. See caption of Table 5 for more information about each column and row meaning.

Graph		**MFP-CE**			**LP-STC+**		**LP-CE**
		$+piv(\hat{G})$	$+det.$	$+piv(G)$	$+piv(\hat{G})$	$+CMSY$	$+CMSY$
Simmons81	LB	16402			16490.0*		16490.0
	UB	38856	34424	37919	32977	34374	34391
$n = 1518$	Ratio	2.369	2.099	2.312	2.0	2.085	2.086
$m = 32988$	Run	0.0645	6.8	0.0263	235.8	235.9	236.9
Haverford76	LB	29676			–		–
	UB	68742	61494	67437	–	–	–
$n = 1446$	Ratio	2.316	2.072	2.272	–	–	–
$m = 59589$	Run	0.133	14.0	0.0611	–	–	–
Swarthmore42	LB	30411			–		–
	UB	70247	63138	69110	–	–	–
$n = 1659$	Ratio	2.31	2.076	2.273	–	–	–
$m = 61050$	Run	0.125	16.3	0.0475	–	–	–
Bowdoin47	LB	42077			–		–
	UB	99161	87449	98881	–	–	–
$n = 2252$	Ratio	2.357	2.078	2.35	–	–	–
$m = 84387$	Run	0.17	36.6	0.0668	–	–	–
Amherst41	LB	45346			–		–
	UB	105353	93857	103404	–	–	–
$n = 2235$	Ratio	2.323	2.07	2.28	–	–	–
$m = 90954$	Run	0.226	39.4	0.0967	–	–	–
condmat2005	LB	71658			–		–
	UB	200854	151017	161977	–	–	–
$n = 36458$	Ratio	2.803	2.107	2.26	–	–	–
$m = 171734$	Run	0.36	354.2	0.308	–	–	–
EmailEnron	LB	83991			–		–
	UB	221420	182653	184296	–	–	–
$n = 36692$	Ratio	2.636	2.175	2.194	–	–	–
$m = 183831$	Run	0.428	828.4	0.334	–	–	–
Rice31	LB	92199			–		–
	UB	219129	189818	218071	–	–	–
$n = 4087$	Ratio	2.377	2.059	2.365	–	–	–
$m = 184828$	Run	0.447	170.9	0.212	–	–	–
ca-AstroPh	LB	86369			–		–
	UB	225943	178356	186697	–	–	–
$n = 17903$	Ratio	2.616	2.065	2.162	–	–	–
$m = 196972$	Run	0.488	496.8	0.302	–	–	–
Lehigh96	LB	98929			–		–
	UB	237145	204838	241219	–	–	–
$n = 5075$	Ratio	2.397	2.071	2.438	–	–	–
$m = 198347$	Run	0.489	203.3	0.226	–	–	–
loc-Brightkite	LB	101544			–		–
	UB	267453	221993	239849	–	–	–
$n = 58228$	Ratio	2.634	2.186	2.362	–	–	–
$m = 214078$	Run	0.559	1287.9	0.493	–	–	–



(a) Alg Fastest (Runtime) (b) Alg Better Quality (Runtime) (c) Alg Fastest (Ratios) (d) Alg Better Quality (Ratios)

Figure 5. Approximation ratios and runtimes for MFP-CE and Louvain heuristics (Louv-MFP) on Facebook100 graphs. In (a) and (c), we use the fastest settings for each algorithm. For (b) and (d), we run the algorithms longer to obtain improved guarantees. Running the greedy Louvain heuristics longer leads to slightly improved guarantees at the expense of a significantly increased runtime.

to slight improvements to approximation ratios, though the results are qualitatively very similar. The runtime for MFP-CE is affected only slightly, but this increases the runtime for LambdaLouvain by an order of magnitude.

C.5. Additional Details for PACE Challenge Graphs

Details and results from the original PACE challenge are available online at https://pacechallenge.org/2021/tracks/. The KaPoCE algorithm was the winning method both for the exact and heuristic track of this challenge, so we focus on comparing against this method (https://github.com/kittobi1992/cluster_editing). The results displayed in the main text are for a subset of the benchmark graphs used for the heuristic track of the PACE challenge. In particular, we use odd-instance graphs from 1 to 57 (the even instance graphs were hidden and used as test cases for the challenge). These correspond to available benchmark graphs with fewer than 500 nodes. We capped the runtime of the heuristic method of KaPoCE to 10 minutes. Even for these small graphs, there were several cases where the algorithm reached this maximum runtime. The heuristic KaPoCE algorithm obtains high quality solutions in terms of the cluster editing objective, but comes with no approximation guarantees of its own and does not scale to large graphs. Overall, our experiments illustrate that PACE challenge algorithms are simply focused on an alternative task—finding high quality solutions for small problems, rather than obtaining approximate solutions at a large scale.