# AUC Maximization in Imbalanced Lifelong Learning

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## Abstract

Imbalanced data is ubiquitous in machine learning, such as medical or fine-grained image datasets. The existing continual learning methods employ various techniques such as balanced sampling to improve classification accuracy in this setting. However, classification accuracy is not a suitable metric for imbalanced data, and hence these methods may not obtain a good classifier as measured by other metrics (e.g., Area under the ROC Curve). In this paper, we propose a solution to enable efficient imbalanced continual learning by designing an algorithm to effectively maximize one widely used metric in an imbalanced data setting: Area Under the ROC Curve (AUC). We find that simply replacing accuracy with AUC will cause *gradient interference problem* due to the imbalanced data distribution. To address this issue, we propose a new algorithm, namely DIANA, which performs a novel synthesis of model <u>D</u>ecouplIng <u>AN</u>d <u>A</u>lignment. In particular, the algorithm updates two models simultaneously: one focuses on learning the current knowledge while the other concentrates on reviewing previously-learned knowledge, and the two models gradually align during training. The results show that the proposed DIANA achieves state-of-the-art performance on all the imbalanced datasets compared with several competitive baselines. Code is available at `https://github.com/MingruiLiu-ML-Lab/Lifelong-AUC`.

## 1 INTRODUCTION

Models can achieve superior performance on a single task [Goodfellow et al., 2016]. However, they often lack the capability to mimic the continual learning ability of the human brain. Specifically, the model performance drops drastically on old tasks when being trained on a new task, also referred to as “catastrophic forgetting" [French, 1999, Kemker et al., 2018, Nguyen et al., 2019]. Current research on lifelong learning [Kirkpatrick et al., 2017, Lopez-Paz et al., 2017, Chaudhry et al., 2018b] mostly focuses on balanced datasets, while ignoring the more challenging imbalanced classification problem. This impedes applications of lifelong learning to online advertisement [Hu et al., 2022], satellite imagery [Tasar et al., 2019], or medical image classification [Irvin et al., 2019]. Moreover, it is not suitable to use classification accuracy to assess the model performance in these domains due to the data imbalanced issue. One of the popular metrics for measuring the performance of classifiers on the imbalanced task is the Area Under the Curve (AUC) [Hanley and McNeil, 1982, 1983]. For example, if the imbalanced data ratio is $99:1$, then a naive classifier that classifies every example to be positive has $99\%$ accuracy, but it is definitely not a good classifier. In this case, AUC is much more informative and we should directly optimize AUC rather than accuracy. However, the current lifelong learning methods are limited to maximize the classification accuracy [Kirkpatrick et al., 2017, Lopez-Paz et al., 2017, Chaudhry et al., 2018b] which makes them not suitable for imbalanced lifelong learning. Although one may tackle the imbalanced problem by class balanced sampling [Chrysakis and Moens, 2020, De Lange and Tuytelaars, 2021, Kim et al., 2020] to form balanced training batches, these approaches still may not be able to directly optimize metrics such as AUC.

Memory-based lifelong learning methods Lopez-Paz et al. [2017], Chaudhry et al. [2018b], Aljundi et al. [2019] achieve competitive performance across commonly used lifelong learning benchmarks. In memory-based lifelong learning methods, a replay buffer is used to store a subset of examples from old tasks for rehearsal. The gradient computed on the replay buffer [Lopez-Paz et al., 2017] is used as a reference to alter the direction of the gradient computed on the current task.

Proceeding from the memory-based lifelong learning methods for maximizing classification accuracy, one may think

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of applying the existing methods and replacing the metric of classification accuracy with AUC. One possible approach to directly maximize AUC for imbalanced lifelong learning is to employ the minimax reformulation of AUC as in the literature of online AUC maximization [Ying et al., 2016, Liu et al., 2020] . This minimax reformulation introduces a data-dependent decision threshold of the model to decouple the pairwise formulation of the AUC objective which facilitates model update in an online fashion. However, we find that maximizing AUC with memory-based lifelong learning introduces an issue called *gradient interference*. In particular, when the data stream is imbalanced, the gradients computed on the current task can interfere with the gradients computed on the replay buffer severely.

In this paper, we propose a novel algorithm, called DIANA, to address the *gradient interference problem* when replacing accuracy with AUC in the imbalanced lifelong learning setting. We first formulate the objective as a composite optimization problem as in works Lopez-Paz et al. [2017], Chaudhry et al. [2018b], Guo et al. [2020]. Similar to existing memory-based lifelong learning methods, we aim to maximize AUC on both the current task and the replay buffer to prevent catastrophic forgetting. Notably, DIANA is designed with two novel techniques to address the *gradient interference problem*: model decoupling and alignment. In particular, DIANA decouples the learning of previous tasks and current task into two models: one focuses on learning the current task while the other reviews previously-learned knowledge. The two models gradually align during training due to an alignment penalty. Since each model computes its own gradients, we can reduce interference between the learning of the current task and the reviewing of old tasks. As we will show, the introduction of the additional model greatly alleviates the gradient interference problem for maximizing AUC with an imbalanced data stream continually, while still being computationally efficient.

Our contributions can be summarized as follows:

- We advance imbalanced lifelong learning through a completely orthogonal approach to the traditional balanced sampling techniques, which enables the lifelong learning algorithm to directly maximize an important metric (AUC). We also identify the *gradient interference problem* under imbalanced setting when the existing memory-based lifelong learning methods are simply applied to maximize AUC.
- We design a new algorithm for maximizing AUC in imbalanced lifelong learning, DIANA, which decouples conflicting gradients into two models with an alignment penalty. We show that DIANA can alleviate the *gradient interference problem*.
- We verify the efficacy of DIANA on imbalanced lifelong learning benchmarks across natural images, medical images, and satellite images. We show that DIANA outperforms several state-of-the-art lifelong learning algorithms by a large margin (e.g., 6.5% AUC score on average over five benchmark datasets), including the approaches which purely use balanced sampling.
- We further expand the scope of our algorithm by considering maximizing AUC on balanced multi-class classification problems, which are standard benchmarks in lifelong learning literature.

# 2 RELATED WORK

## 2.1 LIFELONG LEARNING

Lifelong learning is an important topic in machine learning [Thrun and Mitchell, 1995] and is extensively studied in recent years [Parisi et al., 2019, Mai et al., 2022, Borsos et al., 2020, Ahrens et al., 2021]. The current methods tackle lifelong learning from multiple objectives.

**Regularization-based approaches:** Regularization-based approaches aim at preserving important weights for old tasks. Representative works include EWC [Kirkpatrick et al., 2017] which adopted Fisher information matrix, PI [Zenke et al., 2017] which introduced *intelligent synapses*, RWALK [Chaudhry et al., 2018a] which utilized a KL-divergence based regularization for preserving knowledge of old tasks, and MAS [Aljundi et al., 2018] in which the importance measure for each parameter was computed based on how sensitive the predicted output function is to a change in this parameter.

**Memory-based approaches:** Episodic memory based lifelong learning methods [Hayes et al., 2021, Verwimp et al., 2021, Jin et al., 2021] leverage a small episodic memory for storing examples from old tasks. In GEM [Lopez-Paz et al., 2017], A-GEM [Chaudhry et al., 2018b], MEGA [Guo et al., 2020] and OGD [Farajtabar et al., 2019], the direction of the current gradient is modified to overcome forgetting in lifelong learning. In MER [Riemer et al., 2018], meta-learning is employed as a subroutine for mitigating catastrophic forgetting. In iCARL [Rebuffi et al., 2017], *class exemplars* are stored for each class and used for classification in class incremental lifelong learning. In experience replay (ER) based methods [Chaudhry et al., 2019, Aljundi et al., 2019], the model is trained continuously with batch gradient descent by sampling examples from the current task and the episodic memory. CTN [Pham et al., 2020] exploits memory to store task features.

**Imbalanced lifelong learning:** Existing imbalanced lifelong learning methods mainly focus on maintaining a balanced memory. CBRS [Chrysakis and Moens, 2020] tackles imbalanced lifelong learning by using a Class-Balanced memory population strategy. Similar to CBRS, PRS [Kim et al., 2020], CoPE [De Lange and Tuytelaars, 2021] and Rainbow Memory [Bang et al., 2021] introduces different balanced sampling strategies. Zhou et al. [2022] discusses imbalanced lifelong learning in Reinforcement Learning. ROSE [Cano and Krawczyk, 2022] designs an online ensemble classifier to handle imbalanced data streams.

### 2.2 AUC OPTIMIZATION

Online AUC maximization aims to design algorithms to overcome the difficulty of sampling pairwise data due to the definition of AUC. Zhao et al. [2011] addressed this problem by maintaining a buffer and stored representative examples to construct the positive-negative label pair to calculate the gradient. Gao et al. [2013] maintained the mean and the covariance matrix for the streaming data and performed a gradient-based update. Ying et al. [2016] introduced the saddle point reformulation of AUC maximization with the squared loss and developed an algorithm that can update the model once receiving one data to maximize AUC. There are some future extensions of solving the saddle point formulation under different scenarios, including algorithms with fast rate under function growth condition [Liu et al., 2018], deep learning [Liu et al., 2020], proximal gradient methods [Lei and Ying, 2021], variance reduction [Dan and Sahoo, 2021]. However, none of them are directly applicable in continual learning setting, since they do not take into account the catastrophic forgetting.

## 3 PRELIMINARIES

**Lifelong Learning.** We closely follow the lifelong learning settings in Lopez-Paz et al. [2017], Chaudhry et al. [2018b], Guo et al. [2020]. Specifically, we consider *task-incremental lifelong learning* in which case the tasks are arriving sequentially. Suppose we have a total of $T$ tasks: $\{D_1, ..., D_T\}$. For each task $D_i$, we have a set of training examples $\{\mathbf{x}_j, y_j\}_{j=1}^{K}$. In this paper, we consider imbalanced classification, i.e., each class can have a different number of samples. The given model $f_{\mathbf{w}}$ is trained continuously on the tasks over a *single* pass of the samples. After training, the model is evaluated on the test datasets to assess its performance. The goal of task-incremental lifelong learning is to achieve high performance across all tasks. The crux of task-incremental lifelong learning is the catastrophic forgetting: the model tends to forget previously acquired knowledge while being trained on a new task.

In memory-based lifelong learning methods [Lopez-Paz et al., 2017, Chaudhry et al., 2018b, Guo et al., 2020], a replay buffer is used to store a subset of examples from old tasks. The central idea of Lopez-Paz et al. [2017], Chaudhry et al. [2018b], Guo et al. [2020] is to utilize the replay buffer for computing gradients which serves as the reference for modifying the direction of the gradient computed on the current task.

**Online AUC Optimization.** AUC is defined as the probability of the score of the positive sample being larger than the negative example. Denote $\mathbf{x} \in \mathbb{R}^d$ and $y \in \{+1, -1\}$ by feature and label respectively, and denote $\mathbf{z} = (\mathbf{x}, y)$ by the feature-label pair. We assume that $\mathbf{z}$ is sampled from an unknown distribution $\mathbb{P}$. Define $p = \Pr(y = 1) = \mathbb{E}_y \left[\mathbb{I}_{[y=1]}\right]$ as the likelihood of a random data being positive, where $\mathbb{I}(\cdot)$ is the indicator function. AUC for a general scoring function $h : \mathbb{R}^d \to \mathbb{R}$ is defined as

$$\mathrm{AUC}(h) = \Pr\left(h(\mathbf{w}; \mathbf{x}) \geq h(\mathbf{w}; \mathbf{x}') | y = 1, y' = -1\right), \tag{1}$$

where $\mathbf{w}$ is the model parameter, $\mathbf{z} = (\mathbf{x}, y)$ and $\mathbf{z}' = (\mathbf{x}', y')$ are drawn independently from $\mathbb{P}$, $h(\mathbf{w}; \mathbf{x})$ is the scoring function parameterized by $\mathbf{w}$. Following Gao et al. [2013], Ying et al. [2016], Liu et al. [2020], we use the squared function as a surrogate to replace the indication function and end up with the following loss function:

$$\min_{\mathbf{w} \in \mathbb{R}^d} \mathbb{E}_{\mathbf{z}, \mathbf{z}'} \left[(1 - h(\mathbf{w}; \mathbf{x}) + h(\mathbf{w}; \mathbf{x}'))^2 \middle| y = 1, y' = -1\right]. \tag{2}$$

The above formulation depends on pairwise data with both positive and negative labels, so it is hard to optimize in the online learning setting. It was shown in Ying et al. [2016] that the AUC maximization problem can be formulated as a minimax saddle point problem, and the stochastic gradient descent ascent algorithm can be employed to solve this saddle point problem. The saddle point reformulation is described in Proposition 1.

**Proposition 1** *[Ying et al., 2016] The optimization problem (2) is equivalent to*

$$\min_{\mathbf{w} \in \mathbb{R}^d, (a,b) \in \mathbb{R}^2} \max_{\alpha \in \mathbb{R}} f\left(\mathbf{w}, a, b, \alpha\right) := \mathbb{E}_{\mathbf{z}}\left[F\left(\mathbf{w}, a, b, \alpha; \mathbf{z}\right)\right], \tag{3}$$

*where* $\mathbf{z} = (\mathbf{x}, y) \sim \mathbb{P}$*, and*

$$\begin{aligned} &F(\mathbf{w}, a, b, \alpha, \mathbf{z}) = (1 - p)\left(h(\mathbf{w}; \mathbf{x}) - a\right)^2 \mathbb{I}_{[y=1]} \\ &+ p(h(\mathbf{w}; \mathbf{x}) - b)^2 \mathbb{I}_{[y=-1]} - p(1 - p)\alpha^2 \\ &+ 2\left(1 + \alpha\right)\left(p h(\mathbf{w}; \mathbf{x}) \mathbb{I}_{[y=-1]} - (1 - p) h(\mathbf{w}; \mathbf{x}) \mathbb{I}_{[y=1]}\right), \end{aligned} \tag{4}$$

Intuitively speaking, the Proposition 1 allows us to update the model parameter $\mathbf{w}$ and the decision threshold $\alpha$ simultaneously to effectively improve AUC each time receiving a new individual sample. This mechanism avoids the requirement of updating model with pairwise data which is typically infeasible in online learning. It is worth mentioning that DIANA is also built upon the saddle point reformulation. The main difference between our work and previous works is that our work focuses on developing memory-based lifelong learning to maximize AUC in the imbalanced continual learning setting (to alleviate catastrophic forgetting).

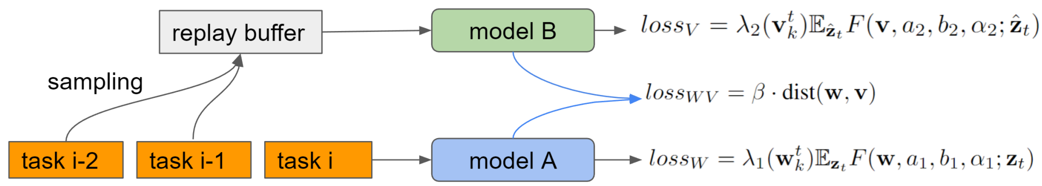



Figure 1: Illustration of DIANA. The loss function is based on the Equation 9. $loss_W$ aims to maximize AUC of model A on the current task, while $loss_V$ aims to optimize model B on replay buffer. The alignment of two models is implemented by $loss_{WV}$.

## 4 DIANA

In this section, we introduce a novel algorithmic framework for optimizing AUC in the lifelong learning setting with an imbalanced data stream. The proposed algorithmic framework is built upon the memory-based lifelong learning methods which leverage a replay buffer for rehearsal. The essence of the algorithmic framework is to maximize the AUC score both on the current task and replay buffer as a composite optimization problem. We circumvent the requirement of a pair of samples for computing AUC score based on the literature of online AUC optimization [Ying et al., 2016, Liu et al., 2018]. By decoupling conflict gradients into two models and aligning gradually, we effectively overcome the gradient interference problem caused by imbalanced data.

### 4.1 IMBALANCED LIFELONG LEARNING BY MAXIMIZING AUC

In lifelong learning, the model $f_{\mathbf{w}}$ is trained sequentially over $T$ tasks. On each task $t$, the samples are arriving in a batch-wise fashion. Let $\mathbf{w}_t^k$ denote the model parameter on the $k$-th minibatch of the $t$-th task. Define $\mathbf{z}_t$ and $\hat{\mathbf{z}}_t$ as random variables following the distribution of the $t$-th task's data and the replay buffer's data upon $t$-th task respectively. To balance the current task and the replay buffer, similar to Guo et al. [2020], we define $\lambda_1(\cdot), \lambda_2(\cdot) : \mathbb{R}^d \mapsto \mathbb{R}_+$ as real-valued functions which depend on the state of the model. On the $k$-th minibatch of the $t$-th task, we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \lambda_1(\mathbf{w}_k^t) \cdot \mathrm{AUC}_t(\mathbf{w}) + \lambda_2(\mathbf{w}_k^t) \cdot \mathrm{AUC}_{\mathrm{ref}}(\mathbf{w}) := \\ & \lambda_1(\mathbf{w}_k^t) \cdot \mathbb{E}_{\mathbf{z}_+,\mathbf{z}_-}\left[\mathrm{AUC}_t(\mathbf{w}; \mathbf{z}_+, \mathbf{z}_-)\right] \\ & + \lambda_2(\mathbf{w}_k^t) \cdot \mathbb{E}_{\hat{\mathbf{z}}_+,\hat{\mathbf{z}}_-}\left[\mathrm{AUC}_{\mathrm{ref}}(\mathbf{w}; \hat{\mathbf{z}}_+, \hat{\mathbf{z}}_-)\right], \end{aligned} \tag{5}$$

where $\mathbf{w}$ is the model parameter, $\mathrm{AUC}_t(\mathbf{w})$ denotes the population AUC at the $t$-th task, $\mathrm{AUC}_{\mathrm{ref}}(\mathbf{w})$ denotes the population AUC of the replay buffer, $\mathbf{z}_+$ ($\mathbf{z}_-$) and $\hat{\mathbf{z}}_+$ ($\hat{\mathbf{z}}_-$) denote random samples with positive (negative) labels on current task and replay buffer respectively, $\lambda_1(\mathbf{w}_k^t)$ and $\lambda_2(\mathbf{w}_k^t)$ characterize the scaling factors of the two AUC values on current task and replay buffer respectively on the $k$-th minibatch at the $t$-th task. The choice of the scaling factors determines the degree of prioritizing the current task or replay buffer. Inspired by Guo et al. [2020], we choose $\lambda_1 = \mathrm{AUC}_{\mathrm{ref}}(\mathbf{w})/\mathrm{AUC}_t(\mathbf{w})$ and $\lambda_2 = 1$ based on the model performance. If AUC score on replay buffer is worse compared with current task, then $\lambda_1 < 1 = \lambda_2$ and hence our algorithm puts more weights on replay buffer. Observing from our experiments, the weight $\lambda_1$ varies from 0.1 to 10 most of the time.

However, by the pairwise formulation of AUC, to solve the problem (5), one needs to sample a pair of positive and negative examples ($\mathbf{z}_+$ and $\mathbf{z}_-$, $\hat{\mathbf{z}}_+$ and $\hat{\mathbf{z}}_-$) at every iteration, which is not feasible for lifelong learning. A natural idea to address this issue is to employ the minimax reformulation (Proposition 1) of AUC [Ying et al., 2016, Liu et al., 2020], which ends up with the following problem:

$$\begin{aligned} \min_{\mathbf{w}} [\lambda_1(\mathbf{w}_k^t) & \min_{(a,b)\in\mathbb{R}^2} \max_{\alpha\in\mathbb{R}} \mathbb{E}_{\mathbf{z}_t} F(\mathbf{w}, a, b, \alpha; \mathbf{z}_t) \\ + \lambda_2(\mathbf{w}_k^t) & \min_{(a,b)\in\mathbb{R}^2} \max_{\alpha\in\mathbb{R}} \mathbb{E}_{\hat{\mathbf{z}}_t} F(\mathbf{w}, a, b, \alpha; \hat{\mathbf{z}}_t)], \end{aligned} \tag{6}$$

where $F$ is defined in Equation (4). This is equivalent to the following formulation,

$$\begin{aligned} \min_{\mathbf{w}, a_1, a_2, b_1, b_2} \max_{\alpha_1, \alpha_2} & [\lambda_1(\mathbf{w}_k^t) \mathbb{E}_{\mathbf{z}_t} F(\mathbf{w}, a_1, b_1, \alpha_1; \mathbf{z}_t) \\ & + \lambda_2(\mathbf{w}_k^t) \mathbb{E}_{\hat{\mathbf{z}}_t} F(\mathbf{w}, a_2, b_2, \alpha_2; \hat{\mathbf{z}}_t)]. \end{aligned} \tag{7}$$

We can solve the problem (7) by stochastic gradient descent on variables $\mathbf{w}, a_1, a_2, b_1, b_2$ and stochastic gradient ascent on variables $\alpha_1, \alpha_2$. The stochastic gradient w.r.t. $\mathbf{w}$ is $\lambda_1(\mathbf{w}_k^t)\nabla F_{\mathbf{w}}(\mathbf{w}, a_1, b_1, \alpha_1; \mathbf{z}_t) + \lambda_2(\mathbf{w}_k^t)\nabla F_{\mathbf{w}}(\mathbf{w}, a_2, b_2, \alpha_2; \hat{\mathbf{z}}_t)$ which consists of the gradients on the current task and the gradients on the replay buffer, we refer to the gradients as the current gradient and reference gradient respectively.

### 4.2 GRADIENT INTERFERENCE PROBLEM

Imbalanced datasets are ubiquitous [Ramyachitra and Manikandan, 2014] but are largely overlooked by current research efforts on lifelong learning. We find that the imbalanced nature of the data stream poses severe challenges for optimization. Specifically, the gradients on the current task and the gradients on the replay buffer may *interfere* with each other during training due to a mismatch of the data distribution of the current task and the replay buffer. This resembles the observations made in GEM [Lopez-Paz et al., 2017] when the gradient is calculated based on the standard loss function: the angle between gradient of current task (the replay buffer) is used to evaluate whether the gradient update would harm previous tasks. When the angle is acute, the gradient is unlikely to increase the loss at previous tasks and vice versa. It proposes gradient projection to align gradients with obtuse angles and get remarkable improvement on performance. Similarly, we use the angle between the gradients as the metric of gradient interference.

In the following, we empirically demonstrate this phenomenon on a real-world dataset. We consider two settings: imbalanced data distribution and balanced data distribution. We construct imbalanced data from the medical dataset ISIC2019 [Gutman et al., 2016] which consists of 25331 images and 8 classes. These classes are divided into 4 disjoint tasks representing positive and negative samples, respectively. We reduce the number of positive samples to be 5% of the negative ones. The setting of balanced data is to keep the number of samples of different classes equal.

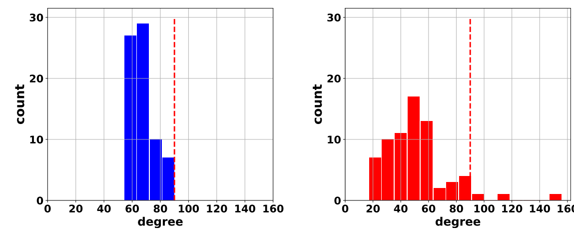
We train the models based on the formulation of Equation (7) on balanced and imbalanced data respectively. We compute the current gradient and reference gradient in each mini-batch. The angle distribution is shown in Figure 2. In the balanced case, the angles between the current gradient and the reference gradient are almost acute angles. In the imbalanced case, there appear to be more obtuse angles. That means, the gradients computed on the current task interfere with the gradients computed on the replay buffer, severely affecting the training of the model.

To investigate this further, we analyzed the gradient angles and present more results in Appendix Figures 5 and 6. They illustrates that the imbalanced case has a larger variance and show that after making the dataset imbalanced, the proportion of obtuse angles increased from 1.05% to 3.16% on Split-CUB200 and from 0.13% to 9.68% on Split-CIFAR100. Please notice on Split-CIFAR, the number of obtuse angles increase by around 75 times.

### 4.3 MODEL DECOUPLING AND ALIGNMENT

To address the gradient interference problem, we introduce our algorithm DIANA. The high-level idea is to use a relaxation of Equation (7) such that we can still learn useful information even if the gradients at the current task and the replay buffer are conflicting. In particular, we first note the equivalent formulation of (7):

$$\min_{\mathbf{w},\mathbf{v},a_1,a_2,b_1,b_2} \max_{\alpha_1,\alpha_2} [\lambda_1(\mathbf{w}_k^t)\mathbb{E}_{\mathbf{z}_t} F(\mathbf{w}, a_1, b_1, \alpha_1; \mathbf{z}_t) + \lambda_2(\mathbf{v}_k^t)\mathbb{E}_{\hat{\mathbf{z}}_t} F(\mathbf{v}, a_2, b_2, \alpha_2; \hat{\mathbf{z}}_t)], \text{s.t.} \quad \mathbf{w} = \mathbf{v}. \tag{8}$$




(a) Balanced ISIC2019 (b) Imbalanced ISIC2019

Figure 2: The distributions of the angles between the current gradient and reference gradient.

**Algorithm 1** Lifelong AUC Maximization with Model Decoupling and Alignment (DIANA)

Buffer $\leftarrow \{\}$
**for** $t \leftarrow 1$ to $T$ **do**
  **for** $k \leftarrow 1$ to $|D_t^{tr}|$ **do**
    **if** Buffer $\neq \{\}$ **then**
      $\zeta_k^t \leftarrow$ SAMPLE(Buffer)
      Compute $\lambda_1(\mathbf{w}_k^t)$ and $\lambda_2(\mathbf{v}_k^t)$
    **else**
      Set $\lambda_1(\mathbf{w}_k^t) = 1$ and $\lambda_2(\mathbf{v}_k^t) = 0$.
    **end if**
    Update $\mathbf{w}, \mathbf{v}, a_1, a_2, b_1, b_2$ by one step of stochastic gradient descent w.r.t. the objective function defined in (9)
    Update $\alpha_1, \alpha_2$ by one step of stochastic gradient ascent w.r.t. the objective function defined in (9) .
    Buffer $\leftarrow$ Buffer $\bigcup(\xi_k^t)$
    Remove samples if the Buffer is full.
  **end for**
**end for**

Since using the same model for data distribution of different tasks would lead to the gradient interference problem, we propose a model decoupling and alignment technique to address the issue. In particular, we propose to solve the following problem (9) to relax the equality constraint in (8),

$$\min_{\mathbf{w},\mathbf{v},a_1,a_2,b_1,b_2} \max_{\alpha_1,\alpha_2} [\lambda_1(\mathbf{w}_k^t)\mathbb{E}_{\mathbf{z}_t} F(\mathbf{w}, a_1, b_1, \alpha_1; \mathbf{z}_t) + \lambda_2(\mathbf{v}_k^t)\mathbb{E}_{\hat{\mathbf{z}}_t} F(\mathbf{v}, a_2, b_2, \alpha_2; \hat{\mathbf{z}}_t)] + \beta \cdot \text{dist}(\mathbf{w}, \mathbf{v}), \tag{9}$$

where $\text{dist}(\cdot, \cdot)$ denotes a distance function between two models, and $\beta > 0$ is a penalty parameter. The typical choice of dist can be squared loss, distillation loss [Hinton et al., 2015], etc. The term $\beta \cdot \text{dist}(\mathbf{w}, \mathbf{v})$ is referred to as the alignment penalty.

In view of Equation (9), we know that we are decoupling one model as in (8) into two models $\mathbf{w}$ and $\mathbf{v}$ under the coordination of an alignment penalty, which has the following two benefits. First, the formulation is a standard minimax optimization, where stochastic gradient descent ascent suffices to solve it efficiently. Second, it can partially alleviate the gradient interference problem since conflicting gradients are decoupled. For example, in DIANA, the gradient w.r.t. $\mathbf{w}$ is $\lambda_1(\mathbf{w}_k^t)\nabla_{\mathbf{w}} F(\mathbf{w}, a_1, b_1, \alpha_1; \mathbf{z}_t) + \beta\nabla_{\mathbf{w}}\text{dist}(\mathbf{w}, \mathbf{v})$. The gradient w.r.t. $\mathbf{w}$ consists of two terms, the first term represents the current gradient which depends on the current data, while the second term characterizes the gradient of the distance function between two models and it is independent of data. Essentially, we undermine the impact of some noisy gradients on the model update. By pulling the current model close to the model on the replay buffer *in each step*, the current model can essentially *learn* from the model on the replay buffer to retain performance on old tasks.

For implementation, since we cannot solve (9) exactly, we use one step of stochastic gradient descent ascent as an approximate solution. This is consistent with the literature of memory-based lifelong learning [Chaudhry et al., 2018b, Guo et al., 2020]. Please refer to Algorithm 1 and Figure 1 for details.

# 5 EXPERIMENTS

## 5.1 EXPERIMENTAL SETUPS

**Datasets.** We perform experiments on popular lifelong learning benchmarks, Split-CIFAR [Zenke et al., 2017], Split-CUB and Split-AWA2 [Chaudhry et al., 2018b]. Moreover, to further explore the application of lifelong AUC Maximization in industry, a medical dataset ISIC2019 [Gutman et al., 2016] and a satellite dataset EuroSat [Helber et al., 2019] are also introduced as new benchmarks. The medical dataset ISIC2019 consists of 25331 medical images and 8 different diagnostic categories. While satellite dataset EuroSat covers 13 spectral bands and consists of 10 classes with in total of 27,000 labeled and geo-referenced images, We believe it is important to measure performance in real industrial scenarios, especially for the medical scenario, which is naturally imbalanced and prefers AUC as the criterion rather than accuracy. A false positive causes severe consequences, such as fault diagnosis of cancers. Split-CIFAR and Split-CUB consist of 20 tasks, while Split-AWA2 has 25 tasks, ISIC2019 has 4 tasks and EuroSat has 5 tasks. For ISIC2019, every two classes constitute a task (4 tasks in total since ISIC2019 has 8 classes). Similar to ISIC2019, we construct one task with data from two classes (5 tasks in total).

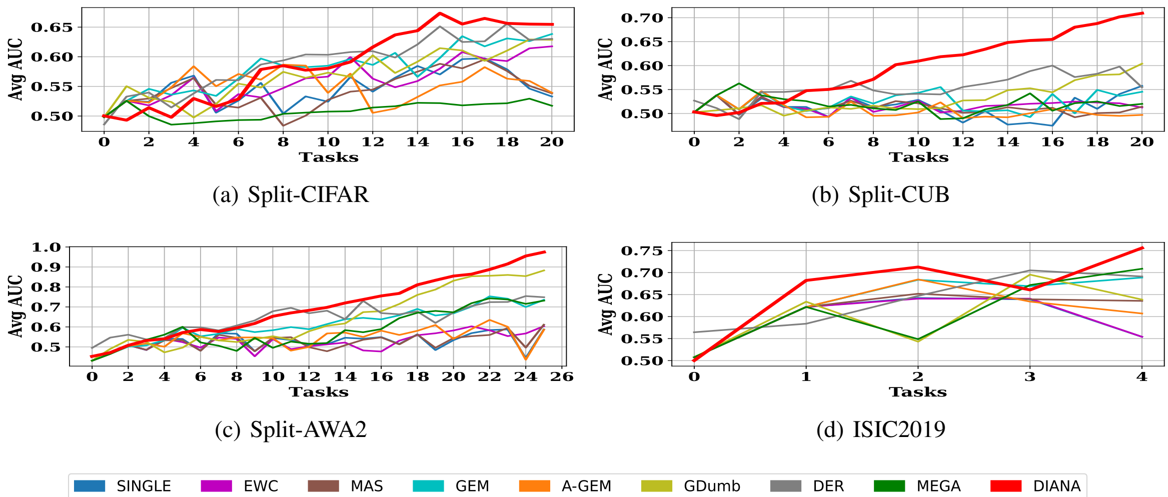
**Make Imbalanced.** Following Yuan et al. [2021], we make training set imbalanced with a pre-defined imbalanced ratio (**imratio**) and leave validation set and test set unchanged. According to chosen imbalanced ratio, positive samples are randomly discarded, until the ratio of positive samples to all samples equals the imbalanced ratio. We set imratio=0.05 in all experiments, which means that only 5% data are positive. In addition, an ablation study of different imbalanced ratios is proposed in Appendix, which conducts experiments with proportions from 0.01 to 0.1. If a task has multiple classes, half of the classes are regarded as negative and the rest are regarded as positive. For example, in Split-CIFAR, the negative class is defined as classes $\{0+i*n \sim 3+i*n\}_{i=1}^T$ in the original CIFAR100 dataset, where $T$ is the total number of tasks, $n$ is the number of classes in each task in the original Split CIFAR-10 dataset. The rest of classes are all defined as positive. To clarify, Split-CIFAR is separated into 20 disjoint tasks, each task contains 5 classes, classes 0-3 in each task are regarded as negative, and classes 4-5 are positive.

**Metrics.** Since our purpose is to maximize the AUC score, AUC ($AUC_T$) is used as the primary metric. $AUC_T$ represents the averaged AUC value when finishing training the $T$-th task, where $T$ is the total number of tasks. To be consistent with previous works [Kamp et al., 2018], Accuracy ($ACC_T$) and Forgetting ($FGT_T$) are also reported. $ACC_T$ is the average accuracy tested on all tasks after finishing training $T$-th task. $FGT_T$ measures the drop of AUC on past tasks after training on the $T$-th task. It's defined as $FGT_T = \frac{1}{T-1}\sum_{j=1}^{T-1}(\max_{l\in\{1,\cdots,T-1\}} AUC_{l,j} - AUC_{T,j})$. where $AUC_{l,j}$ is the AUC score tested on the $j$-th task after training on the $l$-th task.

**Implementation Details.** For Split-CIFAR, ISIC2019 and EuroSAT, we use a reduced ResNet18 [Chaudhry et al., 2018b] to handle small input resolution. For Split-CUB and Split-AWA2, we use a standard ResNet18 pre-trained model on ImageNet. We set batch size as 64 for Split-CIFAR, Split CUB, and Split-AWA2, while batch size as 128 for ISIC2019 and EuroSAT. The learning rate is 0.1 across different datasets and methods. As to memory size for each task, it's fixed to 64 for Split-CIFAR, Split CUB, and Split-AWA. Fixed to 128 for ISIC2019 and EuroSAT.

**Baselines.** We consider EWC [Kirkpatrick et al., 2017] , MAS [Aljundi et al., 2018], GEM [Lopez-Paz et al., 2017], A-GEM [Chaudhry et al., 2018b], MEGA [Guo et al., 2020], DER [Buzzega et al., 2020] and GDumb [Prabhu et al., 2020] as baselines. EWC and MAS are regularization-based approaches. GEM, A-GEM, MEGA, DER, and GDumb are built upon episodic memory. We also consider a simple baseline that uses stochastic gradient descent to train these tasks sequentially without any memory or regularization. It's marked as SINGLE in our experiments. All baselines are task-incremental and only require one-pass, so class-incremental[Shim et al., 2021, Mai et al., 2021] and multiple-passes methods [Ebrahimi et al., 2020, Mallya and Lazebnik, 2018] are not considered. To clarify, SINGLE, EWC, and MAS are only trained on current task. GEM, A-GEM, MEGA, DER are trained on the combination of current task and replay buffer. GDumb is only trained on replay buffer.

**Sampling.** For a fair comparison, we use the same memory size and regular reservoir sampling [Chaudhry et al.,




(a) Split-CIFAR

(b) Split-CUB

(c) Split-AWA2

(d) ISIC2019

Figure 3: Evolution of average AUC during the lifelong learning process. For the results on EuroSat, please see Appendix.

Table 1: Comparison of ONE-MODEL and TWO-MODEL (DIANA)

| Method | **Split-CIFAR** *AUC* (↑) | **Split-CUB** *AUC* (↑) | **Split-AWA2** *AUC* (↑) | **ISIC2019** *AUC* (↑) | **EuroSat** *AUC* (↑) |
|---|---|---|---|---|---|
| ONE-MODEL | $58.7 \pm 1.7$ | $\mathbf{73.5 \pm 2.3}$ | $79.1 \pm 2.7$ | $71.3 \pm 9.2$ | $72.4 \pm 6.5$ |
| TWO-MODEL | $\mathbf{68.4 \pm 2.0}$ | $70.4 \pm 0.7$ | $\mathbf{98.2 \pm 1.1}$ | $\mathbf{73.6 \pm 4.8}$ | $\mathbf{86.7 \pm 1.5}$ |

2019] for all the memory-based methods, including GEM, A-GEM, DER, MEGA, and DIANA. Since the benchmark datasets are imbalanced, to be fair, GDumb is also implemented with reservoir sampling [Chaudhry et al., 2019] instead of class-balanced sampling.Data from the current task and replay buffer are both imbalanced. The sampling size is the same as the batch size used on the current task similar to existing continual learning methods Chaudhry et al. [2018b], namely 64 on Split-CIFAR100, Split-CUB200, and Split-AWA2. To clarify, when running on Split-CIFAR100, the replay buffer feeds 64 samples and the current data stream feeds 64 samples. We also compare our method with Rainbow-memory (RM) [Bang et al., 2021] and CBRS [Chrysakis and Moens, 2020], which use class-balanced sampling. Thus, the train batches from the replay buffer are collected in a balanced form, while data from the current task stay imbalanced. The results are presented in Appendix A. We show that our method and class-balanced sampling can be combined to further improve performance.

## 5.2 RESULTS

In Figure 3, we present the evolution of average AUC during the lifelong learning process. We can observe that DIANA dominates the baselines in most cases. An interesting observation is that DIANA begins to gain advantages over baselines as the training proceeds. One plausible reason is that it is harder to learn useful features for maximizing the AUC objective in the initial training process. However, as shown in Figure 3, by optimizing the right objective as in the proposed DIANA, we can achieve a high average AUC score in the long run. This observation indicates that DIANA has the potential to handle large amounts of tasks.

We report results of $AUC_T$, $ACC_T$, and $FGT_T$ for all algorithms on the imbalanced benchmarks. Details can be found in Appendix: Figure 2, Table 3 and Table 4. In terms of AUC score, our DIANA outperforms other baselines with a large margin. In particular, compared with the best baseline, DIANA improves 2.9% on Split-CIFAR, 9.0% on Split-CUB, 13.0% on Split-AWA2, 1.4% on ISIC2019, and 6.2% on EuroSat. This shows that it is more effective to directly optimize AUC as in the proposed DIANA. It can also be observed that DIANA achieves the highest Accuracy $ACC_T$ on all the datasets except ISIC2019. this further shows the discriminativeness of the learned features.

One interesting observation is that GDumb generally achieves a high AUC than other baselines on the imbalanced benchmarks. However, it is worth noting that GDumb is trained *offline* on the replay buffer with *multiple* passes over the data. GDumb avoids the gradient interference problem by only leveraging the gradients on the replay buffer. This impedes the practical applicability of GDumb.

Table 2: Comparison of capacity on Split-CIFAR100

| Method | architecture | AUC(↑) % | memory | GFLOPS | params |
|---|---|---|---|---|---|
| ONE-MODEL | ResNet34 | 56.2 | 1280 | 0.11 | **2.50M** |
| TWO-MODEL | ResNet18(x2) | **68.4** | 1280 | **0.12** | 2.24M |

### 5.3 ABLATION

**One model vs. two models.** To verify the gradient interference problem discussed in Section 4.2, we conduct ablation on ONE-MODEL and TWO-MODEL. ONE-MODEL denotes the method of solving Equation (7) which is a naive combination of AUC maximization and memory-based lifelong learning. It would suffer from the gradient interference problem due to conflict gradients computed on the current task and the replay buffer. TWO-MODEL denotes the method of solving Equation (9) (a.k.a., DIANA). We denote the model $\mathbf{w}$ and $\mathbf{v}$ in Equation (9) as the current model and reference model respectively. The gradient w.r.t. current (reference) model is the summation over two parts: the gradient calculated based on current (reference) task data, and the gradient of the distance function between two models.

As shown in Table 1, TWO-MODEL outperforms ONE-MODEL in all benchmarks except for Split-CUB. In particular, TWO-MODEL is better than ONE-MODEL by $+10.3\%$, $+19.9\%$, $+2.3\%$, and $+14.3\%$ on Split-CIFAR, Split-AWA2, ISIC2019, and EuroSat respectively.

We analyze why TWO-MODEL is better than ONE-MODEL on Split-CIFAR but a bit worse on Split-CUB. We probe this problem by observing the angle between the current gradient and reference gradient in ONE-MODEL. Concretely, we follow GEM and A-GEM by firstly storing the gradients computed on the current mini-batch and the episodic memory, then calculating the angle between them. We repeat this in each iteration and show the distribution the angles on two standard datasets (balanced and imbalanced) with a histograms in Figure 6 in Appendix. Especially, we experimentally find that $9.68\%$ of the angles are obtuse in imbalanced Split-CIFAR; while only $3.16\%$ are obtuse in imbalanced Split-CUB. Fewer obtuse angles indicate less gradient interference, so ONE-MODEL is better on Split-CUB. One possible reason for the Split-CUB dataset to have fewer obtuse angles is that the images in Split-CUB dataset are fine-grained and different tasks have high similarity, so the gradients between tasks are more likely to be similar.

We have the following conjecture: when the data on different tasks has very different distributions, TWO-MODEL is preferred over ONE-MODEL and vice versa.

Furthermore, to eliminate the concern of unfair comparison, we compare the capacity in terms of episodic memory size, computation (GFLOPS, i.e., one billion ($10^9$) floating-point operations per second), and the parameters. We enlarged the one-model approach's model architecture to match the two-model capacity. Table 2 presents the comparison under same capacity. With nearly the same capacity, TWO-MODEL still outperforms ONE-MODEL. In conclusion, it's the decoupled and aligned mechanism itself that benefits imbalanced lifelong learning, not the extra model parameters.

**Multi-class Balanced Classification.** To further verify the effectiveness of our algorithm on general lifelong learning, we conduct experiments on standard multi-class classification benchmarks without making datasets imbalanced. Similar to most of the existing AUC maximization literature, our algorithm focuses on binary classification. In order to apply DIANA for general lifelong learning problems, we extend our method to accommodate multi-class AUC maximization following works Liu et al. [2020], Yang et al. [2021]. If there are $c$ classes, we have $c$ output scores from the network, one score for each class. If a sample belongs to $i$-th class, the corresponding score at $i$-th position is treated as positive and the rest scores negative, a binary AUC loss is calculated based on these scores. By iterating over $c$ classes, multi-class AUC loss accumulates. To obtain AUC metric in multi-class form, we calculate AUC score for each class pair (i.e., one versus all) and then perform the average.

Table 3 presents results on standard Split-CIFAR, Split-CUB200 and Split-AWA2. Because most of the baselines have reported the accuracy and tuned hyper-parameters on Split-CIFAR, we just follow their settings. As to Split-CUB200 and Split-AWA2, the setups follow Section 5.1. DIANA outperforms other baselines in terms of AUC, Accuracy, and forgetting. Particularly, on Split-CUB200, DIANA obtains a slightly lower accuracy than Gdumb but the highest on the AUC metric. The results show that our method is not only suitable for imbalanced scenarios but also works well under general lifelong learning settings such as multi-class classification on balanced datasets.

**Time and space complexity.** All the experiments are performed on a single GTX-1080Ti GPU. The running time is reported in Table 4. Our TWO-MODEL-based approach increases slightly computational cost but improves significantly performance. As for space complexity, DIANA uses the same memory size as those reply methods like A-GEM. Model decoupling with an alignment penalty can be computationally expensive compared with training one model because of maintaining two models. However, this approach can often yield better results and faster than using a single model with a complex function. According to Table 4, although our approach cost 34.5s, it is still faster than EWC,

Table 3: The multi-class results of average AUC, average ACC, and average Forgetting (FGT) of different methods on Split CIFAR100, Split CUB200, and Split AWA2.

| Method | Split-CIFAR | | | Split-CUB | | | Split-AWA2 | | |
|---|---|---|---|---|---|---|---|---|---|
| | *AUC* (↑)% | *ACC* (↑)% | *FGT* (↓)% | *AUC* (↑)% | *ACC* (↑)% | *FGT* (↓)% | *AUC* (↑)% | *ACC* (↑)% | *FGT* (↓)% |
| SINGLE | 75.8 ± 2.0 | 42.8 ± 3.4 | 12.7 ± 2.1 | 91.9 ± 2.4 | 48.1 ± 6.3 | 3.2 ± 2.3 | 51.0 ± 2.1 | 58.0 ± 0.7 | 28.4 ± 2.9 |
| EWC | 76.4 ± 1.8 | 43.9 ± 2.4 | 11.9 ± 1.4 | 91.5 ± 2.6 | 47.2 ± 6.3 | 3.8 ± 2.5 | 51.7 ± 1.3 | 57.9 ± 1.6 | 28.7 ± 3.8 |
| MAS | 78.2 ± 1.7 | 44.5 ± 3.3 | 8.1 ± 1.9 | 92.7 ± 2.1 | 49.9 ± 6.2 | 2.6 ± 2.1 | 50.1 ± 0.1 | 53.6 ± 6.2 | 28.5 ± 3.9 |
| A-GEM | 81.6 ± 0.9 | 54.5 ± 1.7 | 7.2 ± 0.9 | 92.8 ± 0.8 | 54.4 ± 7.5 | 1.1 ± 1.1 | 51.1 ± 3.2 | 59.6 ± 0.8 | 27.3 ± 1.6 |
| GDumb | 75.9 ± 0.9 | 49.4 ± 1.4 | 2.6 ± 0.4 | 92.8 ± 1.8 | **65.5 ± 2.3** | 0.3 ± s0.2 | 54.4 ± 3.4 | 60.8 ± 2.7 | 14.1 ± 6.1 |
| DER | 80.3 ± 1.2 | 40.7 ± 2.5 | 8.9 ± 1.2 | 92.6 ± 2.8 | 59.5 ± 9.0 | 1.5 ± 0.7 | 71.3 ± 16.0 | 63.1 ± 3.4 | 15.3 ± 9.2 |
| MEGA | 62.2 ± 2.7 | 32.3 ± 2.4 | 10.8 ± 2.2 | 89.6 ± 4.0 | 50.6 ± 7.3 | 4.3 ± 2.5 | 51.1 ± 1.0 | 59.6 ± 5.6 | 12.8 ± 2.9 |
| DIANA | **89.5 ± 0.3** | **65.5 ± 0.9** | **1.2 ± 1.2** | **93.4 ± 0.6** | 64.3 ± 1.8 | **0.2 ± 0.1** | **92.6 ± 1.4** | **82.5 ± 2.3** | **0.3 ± 0.2** |

MAS, and GEM, which cost 78.6s, 77.3s, and 109s respectively.

Table 4: Time and space complexity on Split-CIFAR

| Method | Training time (s) | memory |
|---|---|---|
| Single | 8.8 | 0 |
| EWC | 78.6 | 0 |
| MAS | 77.3 | 0 |
| GEM | 109 | 1280 |
| A-GEM | 17.1 | 1280 |
| GDumb | 8.8 | 1280 |
| DER | 20.0 | 1280 |
| MEGA | 15.7 | 1280 |
| DIANA | 34.5 | 1280 |

**Hyperparameters**. $\lambda$ and $\beta$ are two hyperparameters used in the alignment penalty. $\lambda$ is adaptively changed according to our algorithm so practitioners do not need to tune $\lambda$. Now we provide some results in varying $\beta$ because $\beta$ is a tuned hyperparameter that controls the tradeoff of maximizing AUC and alignment of two models, the result is presented in Table 5. We find that the best tradeoff is choosing $\beta$ to be $0.1$ in various benchmarks, so we suggest practitioners use $\beta = 0.1$ for their own tasks.

Table 5: Varing hyperparameter $\beta$

| $\beta$ | CIFAR100 | CUB200 | AWA2 |
|---|---|---|---|
| 0.1 | 68.4 | 70.4 | 98.2 |
| 0 | 66.09 | 72.03 | 95.8 |
| 0.01 | 65.8 | 69.6 | 93.7 |
| 1.0 | 63.6 | 51.11 | 54.1 |

**Combine AUC maximization**. To further validate the advantage of AUC maximization in imbalanced setting, we combine AUC maximization with previous methods, like EWC and DER. Table 6 shows when replacing the cross-entropy loss with AUC maximization, both EWC and DER have significant improvement. However, our proposed method achieve superior performance compared to these existing approaches, owing to the incorporation of an additional model decoupling mechanism. Our algorithm DIANA is slightly worse than DER-AUC on split-CIFAR100, but much better than all other baselines on datasets split-CUB200 and split-AWA2.

Table 6: Combine AUC maximization with Existing literature $\beta$

| method | CIFAR100 | CUB200 | AWA2 |
|---|---|---|---|
| EWC | 64.4 | 51.5 | 56.2 |
| EWC-auc | 62.4 | 69.7 | 97.1 |
| DER | 62.6 | 65.7 | 80.4 |
| DER-auc | 70.1 | 62.1 | 91.5 |
| DIANA | 68.4 | 70.4 | 98.2 |

**More ablations in Appendix.** Due to limited space, more ablations are studied in Appendix, including results with balanced sampling in Appendix A, imbalanced ratio in Appendix C, balanced vs. imbalanced in Appendix D.

# 6 CONCLUSION

We study AUC optimization under imbalanced continual learning settings. We propose a novel algorithm DIANA based on the minimax reformulation of the AUC objective. We systematically study the gradient interference problem on imbalanced data. We demonstrate that this problem can be alleviated by employing two models with model decoupling and alignment. We extend our algorithm to multi-class AUC maximization in general balanced lifelong learning. Compared to existing approaches, the proposed algorithm achieves a higher AUC score as well as less forgetting. One limitation is that our approach uses two models and slightly increases the computational cost.

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