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Alternating minimization for generalized rank one matrix sensing: Sharp predictions from a random initialization

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Abstract

We consider the problem of estimating the factors of a rank-1 matrix with i.i.d. Gaussian, rank-1 measurements that are nonlinearly transformed and corrupted by noise. Considering two prototypical choices for the nonlinearity, we study the convergence properties of a natural alternating update rule for this nonconvex optimization problem starting from a random initialization. We show sharp convergence guarantees for a sample-split version of the algorithm by deriving a deterministic recursion that is accurate even in high-dimensional problems. Notably, while the infinite-sample population update is uninformative and suggests exact recovery in a single step, the algorithm—and our deterministic prediction—converges geometrically fast from a random initialization. Our sharp, non-asymptotic analysis also exposes several other fine-grained properties of this problem, including how the nonlinearity and noise level affect convergence behavior.

On a technical level, our results are enabled by showing that the empirical error recursion can be predicted by our deterministic sequence within fluctuations of the order $n^{-1/2}$ when each iteration is run with n observations. Our technique leverages leave-one-out tools originating in the literature on high-dimensional M-estimation and provides an avenue for sharply analyzing complex iterative algorithms from a random initialization in other high-dimensional optimization problems with random data.

Keywords: nonconvex optimization; convergence rate; matrix sensing; alternating minimization.

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1. Extended abstract. Full version appears as [Verchand et al., 2022].
2. Kabir Aladin Verchand and Mengqi Lou contributed equally to this work.

References

Kabir Aladin Verchand, Mengqi Lou, and Ashwin Pananjady. Alternating minimization for generalized rank one matrix sensing: Sharp predictions from a random initialization. *arXiv preprint arXiv:2207.09660*, 2022.