

The Predicate Modal Logic of Provability

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Introduction The propositional modal logic of provability GL and its arithmetical interpretations have been studied by various logicians: among others, Boolos, De Jongh, Magari and his school, including the author, Smorynski, Solovay, and Visser. As observed by Boolos in the introduction of [4], the arithmetical interpretation of GL can be extended to its extension to the language of modal predicate calculus, denoted by QGL . By this observation, one might reasonably expect that QGL can offer a more complete description of the logic of provability. Unfortunately, however, many desirable properties of GL do not extend to its predicate version; for example, in [1], Avron shows that the most natural sequential formulation of QGL does not admit cut-elimination (where a similar sequent calculus for GL does: see [8], [14], and [19]). In this paper, we show that other important results about GL fail to hold for QGL ; for example, QGL is not complete with respect to any class of Kripke frames; moreover, QGL is not arithmetically complete, and does not enjoy the fixed point property.

In spite of these negative results, we believe that many aspects of the predicate logic of provability are worthy of further investigation; in particular, since QGL is not arithmetically complete, one could try to find new significant provability principles which are arithmetically valid, but not provable in QGL . In any case, even if most important problems on QGL have a negative solution, there are also positive results: for example, in [1], Avron shows that QGL enjoys some closure properties, and that the notion of PA validity satisfies

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