Banded Waveguides on Circular Topologies and of Beating Modes: Tibetan Singing Bowls and Glass Harmonicas

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Abstract

Banded waveguides were originally introduced to allow for efficient physical modelling of bowed bar percussion instruments. Recently the method has been used to model instruments whose topology is more complex. In this paper we discuss the use of the method on circular topologies of conical, cylindrical and hemispherical shells, specifically the Tibetan singing bowl and the glass harmonica. The tibetan bowl poses a special challenge to modeling as its sound production is very resonant and also some mode-pairs lie close together to create beating modes. In this paper we discuss how the simulation can be achieved using banded waveguides.

1 Introduction

Though banded waveguides were initially introduced to model bar percussion instruments, that is instruments that are well-described by one-dimensional partial differential equations (Essl and Cook 1999), the idea can be extended to objects of higher dimensionality or more complex topology (Essl and Cook 2001; Essl and Cook 2002). Banded waveguides provide an efficient alternative to more general finite element methods (O'Brien, Cook, and Essl 2001), but to achieve proper spatial representation, the relation of the geometry to closed wavetrains has to be studied. In a purely modal synthesis approach the spatial information has to be aquired by measurement over the whole object (van den Doel, Kry, and Pai 2001). Some work in the direction of banded waveguides on higher-dimensional topologies has been made for two-dimensional circular objects like drums (Essl and Cook 2001) and cymbals (Serafin, Huang, and Smith 2001) and three-dimensional objects like rubbed wine glasses and glass harmonicas (Essl and Cook 2001).

In this paper we look at circularily symmetric three-dimensional structures. These are cylindrical shells, conical shells, hemispherical shells and the like, with the additional constraint that the dominating modes of oscillation have circular paths. In part this is an extension of the work initially reported in (Essl and Cook 2001) for wine glasses. We study also the so-called

"Tibetan singing bowl". The tibetan bowl provides additional challenges as it has more perceptually relevant resonant modes as well as modes which are close in frequency and yield a perceptual beating pattern. Finally the Tibetan bowl is very highly resonant, that is has very weak internal damping and hence will ring for a very long time.

2 Rubbing a Wine Glass
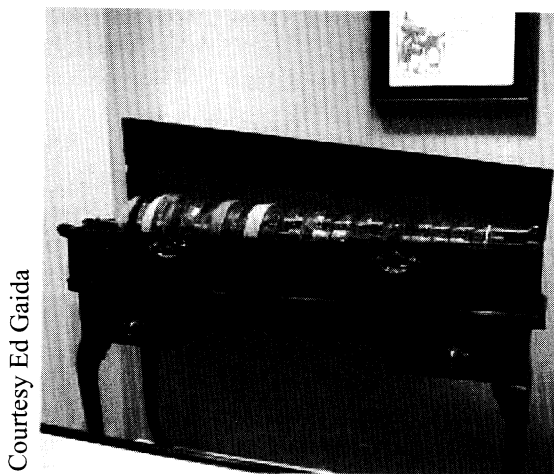
Figure 1: Benjamin Franklin's glass harmonica, which he called "armonica", as seen in the Franklin Institute Science Museum in Philadelphia.

Drinking glasses, in particular wine glasses, can be made to ring in many different ways. They can be excited by impact, by rubbing the top rim with a wet finger, or by radially bowing with a violin bow. While impact can easily be simulated using modal models, rubbing and bowing cannot. Geometrically, a wine glass is a three-dimensional object and disturbances travel along the object in all dimensions. The object is symmetrical, however, and the dominant modes are essentially two-dimensional (Rossing 2000). One is left with bending modes along the cylindrical axis, which can be excited by rubbing, plucking or bowing, but most of the energy really goes into flexual modes of the circumfence of the glass. This is a closed path — essentially a bar being bent into a circular shape, closing onto it-

self. Hence the path is quasi one-dimensional. The path traced along the wine glass can be seen in Figure 2.
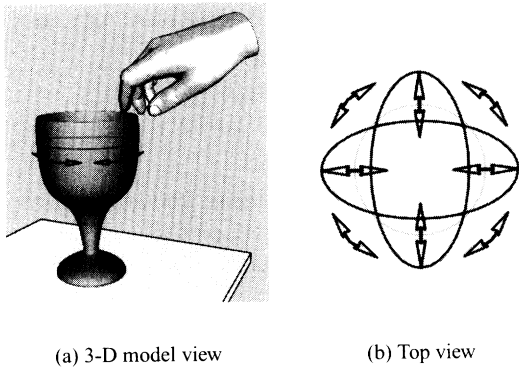

(a) 3-D model view   (b) Top view

Figure 2: The wavetrain closure on the rim of a wine glass and corresponding flexual waves as seen from the top (after Rossing 2000).

From the distance to be traveled along the rim, the circumfence of a circle $l = 2\pi r$, and the wave velocity of flexual waves on a uniform homogeneous bar we can derive the actual frequency and its connection to traveling length (Rossing 2000), which completely determines the wavetrain closure to be modeled:

$$\frac{1}{\omega} = \frac{l}{v} = \frac{2\pi r}{\sqrt{a\omega}}$$

$$\omega = \frac{a^*}{r^2}$$

(1)

Due to the circularity of the path, the banded waveguide system makes no reference to the actual position on the rim. In practice, however, the point of interaction provides this reference. If the glass is struck, the point of excitation is defined along the circle. The same is true for bowing, which usually happens at one point radially to the rim. Rubbing the rim is a peculiar case because the point of interaction is moving slowly along the path. In our model we make no difference between rubbing and bowing as the rubbing is a very slow motion compared to the wave traveling on the path and hence treating the rubbing interaction as stationary does not significantly alter the non-linear behavior. The effect of the slowly shifting interaction point is measurable, though not audible as it is too slow (Rossing 2000).

The results are presented in Table 1. The struck excitation was a quick sharp strike with a finger nail against the glass, which was modeled using a simple impulse. Bowing on the real glass was performed using a rosined violin bow. The rubbing was performed with a wet finger. With the violin bow it is possible to excite the second harmonic as the fundamental frequency of the bowing response. The same result can also easily

n	Struck measured	Struck sim.	Rubbed measured	Bowed measured	Bowed sim.
2	2.32	2.31	2.00	2.00	2.00
3	4.25	4.20	3.00	2.99	3.00
4	6.63	6.69		4.00	4.00
5	9.38	8.81	5.00	5.00	5.00

Table 1: Spectral frequencies of dominant partials of measured and simulated struck, rubbed and bowed wine glass given as $f_n : f_1$.

be achieved using the simulation model. The simulation model captures both the harmonic spectra as well as a other non-linear effects of the real interaction. We did not model a rubbed interaction separately as we assume it follows in principle the same mechanism as bowing (Fenny, Guran, Hinrichs, and Popp 1998).

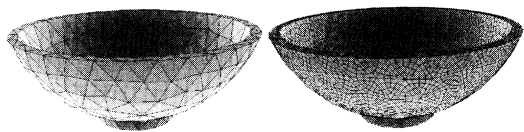## 3 Tibetan Singing Bowl

Courtesy James F. O'Brien

Figure 3: Mesh of simulated bowl.

The tibetan singing bowl are geometrically close to spherical segments. A discretized mesh version of the bowl can be seen in Figure 3). In typical performance the bowl is rubbed with a wooden stick wrapped in a thin sheet of leather along it's rim. Depending on the rubbing velocity and initial state of the bow (i.e. certain modes may be already ringing), various frequencies can be made to oscillate. The behavior is comparable to rubbing or bowing a wine glass in terms of dynamic envelope, mode locking, mode duplication and related phenomenon as a result of the non-linear interaction of the stick-slip-based rubbing.

If struck, the bowl will show a modal response of circular-symmetric form. The first few modal shapes are depicted in Figure 4 with exaggerated amplitudes. These shapes will oscillate around the circular rest position comparable to circular flexing motion of the wine glass depicted in Figure 2. The circularily repeating pattern is depicted in Figure 5. This picture also shows non-circular modes, which tend not to be excited by the circular rubbing motion.

The measured spectra of the struck bowl can be seen in Figure 6 for various impact positions. As can be seen, there are a number of higher modes which lie
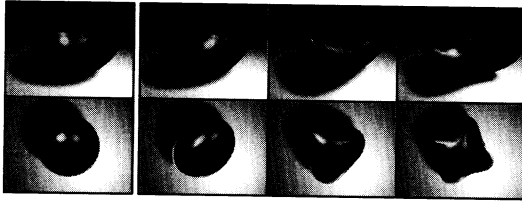
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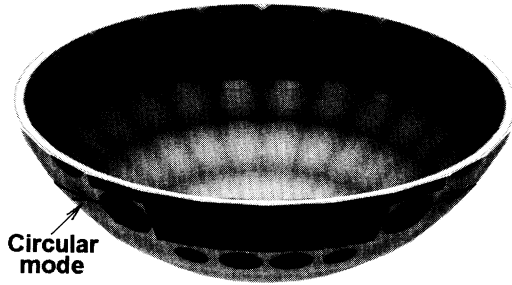Figure 4: Simulated mode shapes of the bowl.

Circular mode

Figure 5: Path of circular mode on bowl (used with permission from (Cook 2002).)

close together yielding audible beating. The beating can be seen more clearly in Figure 7.
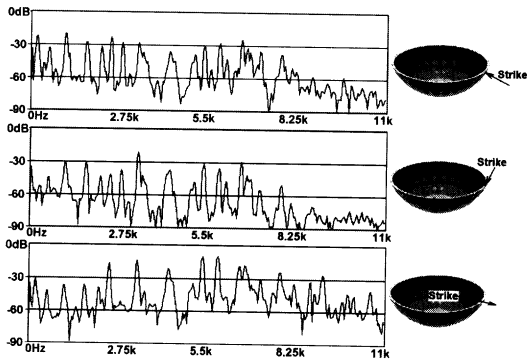

Figure 6: Spectra of different excitations (used with permission from (Cook 2002).)
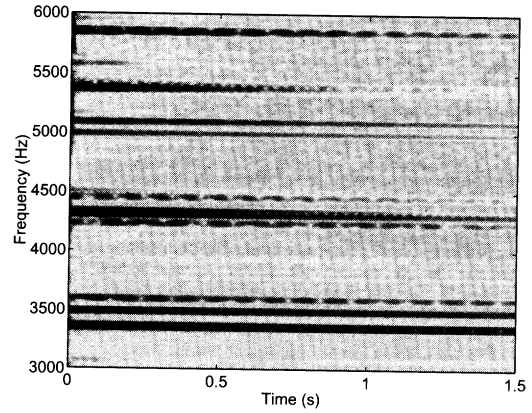

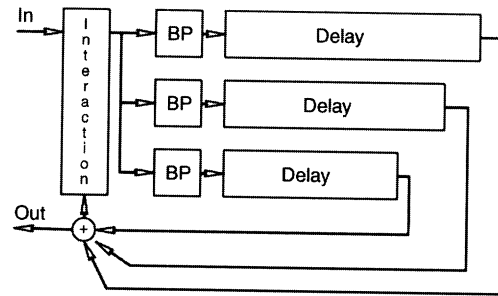Figure 7: Beating upper partials in spectrogram of a recorded Tibetan bowl.

Figure 8: A complete banded waveguide system.

spective banded wavepaths can be calculated from the maximum of the overlapping bandpass filter amplitude characteristics. This maximum has to be tuned to the desired gain and the respective gain of the bandpasses is adjusted by the weight of the overlap. The resulting simulation of an isolated beating mode pair can be seen in Figure 9. The relative ratio between the modes is $1 : 1.05$.

The beating modes following this construct, combined with plain modes then yields the complete simulation of the Tibetan bowl, which can be achieved with less than 20 banded wavepaths including beating mode-pairs.

3.1 Beating Banded Waveguides

The beating modes combined with the very weak damping poses the main challange for modeling the dynamics using banded waveguides (as depicted in Figure 8.)

For two neighboring banded wavepaths whose center frequencies get close, the respective frequency-bands start to overlap strongly. This means that energy will contribute to traveling waves in both bands simultaneously. To guarantee stability within the frequency region the sum gain of both waveguides can not exceed unity as both are summed together for interaction or feedback. More specifically the gain of the re-

4   Conclusion

Banded waveguide models on higher-dimensional objects need additional consideration to understand how quasi-one-dimensional traveling wave paths correspond to the higher-dimensional topologies. The current work shows how cases of cylindrical shells or related topologies of circular thin-layered symmetry can be seen as thin bars bent into circular shape and connected at its ends. With this observation, musical instruments like glass harmonicas or Tibetan singing bowls can be efficiently implemented while retaining the benefits of banded waveguide simulation with regards to non-linear
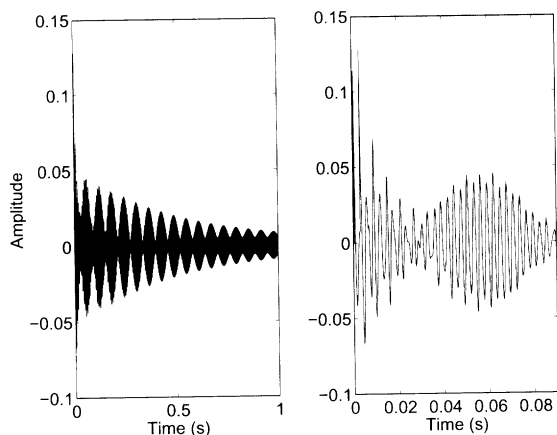
Figure 9: Left: Evolution of an isolated simulation of a beating mode pair. Right: Initial transient and the first beating period.

interactions like bowing, rubbing or other related stick-slip friction excitations.

Beating modes can be implemented by balancing the gain of the bandpass filters of two banded wavepaths.

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