ISSN 1870-4069

Evaluation of the Intervention Operator in Causal Bayesian Networks

Jenny B. Vázquez Aguirre, Nicandro Cruz Ramírez

Universidad Veracruzana, Centro de Investigación en Inteligencia Artificial, Mexico

jennybey13@gmail.com,ncruz@uv.mx

Abstract. Causal estimation is possible through the use of Bayesian Networks; an alternative is to use an intervention operator originally proposed by Pearl. This operator acts by manipulating a variable that is a candidate cause of another, eliminating from this potential cause any influence from other variables. This tool promises to be a powerful method for estimating causality; however, as far as we know, it does not have a validation that allows us to know its scope and limitations. This work presents the implementation of the intervention operator and its evaluation in different databases. This last one tries to measure the performance of the efficiency to determine causal routes using for it the estimation of the Causal Effects and the Bayes Factor. Our results allow us to identify operator improvements to be used in a general causal estimation scheme and not only in Bayesian Networks that meet certain characteristics.

Keywords: Bayesian networks, causality, intervention.

1 Introduction

The study of causality has its origins approximately 300 years ago with the works of Hume and Kant, who tried to explain how it is that causal knowledge is acquired naturally. This gave rise to various investigations that throughout history have tried to understand and replicate causality. Artificial Intelligence (AI) is an area interested, among other things, in the study of mental processes including causal learning. Intuitively, a causal relationship occurs when X causes Y. However, we cannot always identify such relationships, as they may be spurious or contain confounding factors that may be imperceptible to observations. Pearl argues that the best way to reproduce the causal inference is through the computer by first understanding the logic of causal thinking [1].

Pearl represents the natural causal process through a graphical representation called "the ladder of causation", that explains how the natural causal process is carried out and how it could be mapped artificially [1]. Below are described the ladder of causation levels:

- Level 1. Represents the identification of causal relationships (seen or observed).
- Level 2. Refers to the interventions and predictions of the effects of the deliberate intervention of the environment.
- Level 3. Is the ability to choose from the alterations that produce the best results.

The works of Judea Pearl propose an intervention operator for the study of causal relationships located on the second level. He introduced an intervention of variables to a Bayesian Network through an operator called “do or set”, with which a new probability resulting from the intervention of variables, is obtained [2].

There are currently no reports of work in which the functioning of the operator can be appreciated in real cases; in this evaluation proposal we try to find the ideal conditions under which the operator works, we evaluate how through the intervention it is possible to access a set of new probabilities that allow to express themselves in causal terms. In this work, we made use of a set of databases provided by experts in different areas; one inclusion criterion for these was the existence of causal relationships between some of their variables.

The results found as interventions were encouraging in terms of obtaining causal probabilities; however, they raised the imminent need to create a model that learns causal relationships, supporting the expert in creating Causal Bayesian Networks automatically.

2 Theoretical Framework

2.1 Causal Bayesian Networks

Bayesian Networks (BN) were developed and introduced by Judea Pearl in the early 1980s to facilitate the prediction and abduction of intelligent AI systems [7]. BN's are models that combine graph theory and Bayesian probability. They are represented by Directed Acyclic Graphs (DAG) that allow us to know the structure of the variables hierarchically, identifying parents and children in their structure and the existing relationship between them. The structure of a network provides information on the probabilistic dependence of variables or the conditional independence of one variable given to another (or set of them) [8]. The force of influence between the connections of a network is contained in the conditional probabilities and is represented by each node given the set of its parents. The joint probability of a BN can be obtained using equation 1:

$$P(x_1, \dots, x_n) = \prod_j P\left(x_j | pa_j\right), \tag{1}$$

where pa_j represents the parents of node x_j in the BN.

As of the structure of a BN, it is possible to carry out association consultations (Bayesian inference), which are supported by the criterion of d-separation to verify the conditional independence in the connections. This paper will not address details about BNs, such as the d-separation criterion and Markovian parents. If you are interested in consulting these issues, a more detailed review is given in [3].

A Causal Bayesian Network (CBN) can be understood as a BN, with the property that the parents of each node represent a direct cause of it. From equation (1), we can assume that the parents of the variable X_j, are its direct causes. Otherwise, if there are no parents, the marginal probability $P(X_j)$ must be used.

Definition 1. Causal Bayesian Network [3]

Let $P(v)$ be a probability distribution on a set V of variables, and let $P_x(v)$ denote the distribution resulting from the intervention $do(X = x)$ that sets a subset X of variables to constants x. Denote by P_* the set of all interventional distributions $P_x(v)$, $X \subseteq V$, including $P(v)$, which represents no intervention (i.e., $X = 0$). A DAG G is said to be a causal Bayesian network compatible with P_* if and only if the following three conditions hold for every $P_x \in P_*$:

1. The probability distribution P_x is Markov relative with the DAG G [6].
2. The probability of all the variables that are part of an intervention is equal to 1 for the value established in: $P_x(v_i) = 1$ for all $V_i \in X$ provided that $V_i = v_i$ is consistent with $X = x$ [6].
3. The probability of all the remaining variables that are not established in the intervention is equal to the original probability (the variable given by their parents). $P_x(v_i|pa_i) = P(v_i \mid pa_i)$ for all $V_i \notin X$ provided that pa_i is consistent with $X = x$ [6].

From Definition 1, the truncated factorization $P_x(v)$ can be calculated for any intervention $do(X = x)$. Formally remaining as the equation (2):

$$P_x(v) = \prod_{\{i|V_i \notin X\}} P(v_i|pa_i), \qquad (2)$$

for everything v consistent with x.

According to Pearl, the construction of causal DAG has several advantages. First, the judgments required for the construction of the models are more significant and accessible. In addition, the causal models indicate how these probabilities would change when performing external interventions [3]. The formal construction of these models is based on the assumption that parent-child relationships represent autonomous mechanisms, so it is possible to make changes in those relationships without changing or affecting the other existing relationships within the network.

The $do(x)$ operator simulates physical interventions in the network, eliminating some functions of the model and replacing it with constants $X = x$ while keeping the rest of the model unchanged. Due to the assumption of autonomy, the manipulated distribution of the intervened variable is independent of the rest of the network, so a pruning process can be applied, which implies the elimination of all the arcs (parents) received by the intervening variable [4].

The difference between observing and intervening is deduced from the last statement. For example, if we wanted to observe the effect of $B = b_0$ for a model of BN $P(a,b,c) = P(a)\,P(b \mid a)\,P(c \mid b)$, the probability would be obtained from of $P(A,C \mid B = b_0)$. However, by applying the assumptions of autonomy and pruning, the connection between variables B and C are eliminated, obtaining:

$$P(a, do(b), c) = P(a)P(c|do(b) = b_0). \qquad (3)$$

From the new probability expression, it is possible to calculate the influence of the intervened variables on their effects. The inference rules necessary for the calculation of causal probability expressions may be consulted in detail in [6].

Whenever a feasible reduction is detected for $P(y|\hat{x})$the effect of X on Y is said to be identifiable.

Definition 2.8 Identificability [2]

The causal effect of X on Y is said to be identifiable is the quantity $P(y|\hat{x})$ can be computed uniquely from the joint distribution of the observed variables. Identifiability means that $P(y|\hat{x})$ can be estimated consistently from an arbitrarily large sample randomly draw from the joint distribution.

Then, the causal effect of a variable X on another variable Y is:

Definition 2.9 Causal Effects [3]

Given two disjoint sets of variables, X and Y, the causal effect of X on Y, denoted either as $P(y|\hat{x})$ or as $P(y|do(x))$, is a function from X to the space of probability distributions on Y. For each realization x of X, $P(y|\hat{x})$ gives the probability of Y = y induced by deleting from the model of (3) all equations corresponding to variables in X and substituting X = x in the remaining equations.

Finally, the calculation of the causal effect (or average causal effect) that one variable has on another, can be calculated from equation (4):

$$EC = P\big(Y = y \big| do(\hat{x})\big) - P\big(Y = y \big| do(\hat{x}')\big), \qquad (4)$$

where:

$Y = y$ is a specific value of the effect.

$\hat{x}$ is a specific value of the intervened variable.

$\hat{x}$’ is another value of the intervened variable for the same value of y.

This last equation is the one that allows extracting the real estimation of the intervention of variables in the network, marking the difference with the simple observation.

3 Materials and Methods

3.1 Materials

The selection of the experimental units was made up of a set of "causal" databases. According to the expert opinions, these contained causal relationships in some of the variables.

We used a total of 5 datasets. The first called Ecological Integrity is a database of 23 variables (22 quantitative and one qualitative) with 290,687 cases; 4 variables as possible causes and 4 effects. This dataset contains information on ecological integrity in Mexico. Two other sets contain data related to Breast Cancer. One is a prospective sample and the other a retrospective sample, 3 variables were considered as possible causes and one as an effect of each of them. These databases contain 12 variables (11 quantitative and one qualitative) with 322 and 692 cases respectively.

Another dataset contained information on Gene Expression Levels, with a total of 12 quantitative variables (3 causes and one effect) and 31 cases. Finally, the Synthetic-data-BayesiaLab database was obtained from the BayesiaLab software to validate the results of this implementation. The database has 3 variables (one cause and one effect) and a total of 1000 cases. Causes and effects were determined by the experts who provided the data. Each variable was an experimental unit and the total was 34, in each run one value the variable intervened was fixed.

3.2 Methods

The pre-processing strategy started discretizing the quantitative variables in the datasets; for this we used Weka software, and the used methods were: Discretize and CAIM. The implementation was carried out in R, the algorithms and metrics used to BN's construction were Hill-Climbing with the BIC, and K2 metrics using the maximum likelihood estimator; the programing language used to implement the causal routes search and the new probabilities estimation is R.

To carry out the validation of the results, the causal effects were obtained by equation 4, and additionally, calculated the Bayes factor was used. The Bayes Factor (BF) is the relationship between the probability of one hypothesis and another. It can be interpreted as a measure of force in favor of a hypothesis (model) of two competing hypotheses and is denoted by equation 5 [5]:

$$BF = \frac{P(D|H_1)}{P(D|H_0)}. \quad (5)$$

The BF can take any positive value, and a way of interpreting it is given by what is indicated in Table 1.

Table 1. Interpretation of BF.

Bayes Factor	Interpretation
>100	Extreme evidence for H1
30-100	Very strong evidence for H1
10-30	Strong evidence for H1
3-10	Moderate evidence for H1
1-3	Anecdotal evidence for H1
1	No evidence
1-0.33	Anecdotal evidence for H0
0.33 – 0.1	Moderate evidence H0
0.1 – 0.03	Strong evidence for H0
0.03 – 0.01	Very strong evidence for H0
<0.01	Extreme evidence for H0

4 Methodology

A single treatment was designed for the experiment and applied to all experimental units. The treatment has only one level and consists of the intervention of a variable in the network, fixing for each run a specific value (do(x)) of the variable intervened.

Each experimental run was carried out in three stages. The first consisted of the construction of the BR; the second the search for the possible causal route and the third, the estimation of the causal probability.

Phase 1. (Construction and validation of BN's)

A BN is built from the data set using the bnlearn R library.

Once the BN has been obtained, and before the intervention, the causal relationships are validated by the expert.

If the relationships in the network do not reflect a causal match with the expert's knowledge, the parameters with which the BN is constructed can adjust - such as the metric - or indicate the permitted or restricted causal relationships that must be respected in this one.

Phase 2. (Search for Causal Routes)

From the BN's validated in Phase 2, the cause variable and the value to be intervened must be indicated, as well as the effect variable.

With the support of the causal.effect library, the search for the Causal Route (CR) in the BN is carried out.

If there is a CR, the system delivers the new probability equation, which shows the causal probabilities must be calculated. An example of the form of the equation is presented below:

$$\text{“}sum_{\{x_i,x_j\}P}\left(y|x_k,x_i,x_j\right)P\left(x_j|x_k\right)P(x_i|x_k)\text{”}, \quad (6)$$

where x_i, x_j, x_k denote CBN variables, which are not the effects. The effect is represented by the variable y.

Phase 3. (Calculation of causal probabilities)

To calculate the causal probability tables, the expression is broken down of equation 3, separating the conditional probabilities and the sum about which the calculation will run.

Calculate - from the data - the conditional probability tables for each element of the causal expression (equation 3). Carry out the normalization of each table, calculate the causal probabilities.

Once the probability tables are obtained, calculate the Causal Effects and Bayes Factor to find potential causes.

Finally, the probability values are compared before and after the intervention, the library "querygrain" make queries in the BN using probability propagation.

5 Results

Once the BN's were obtained, the search for possible causal routes was carried out. Figure 1 shows the BN (left) and CBN (right) for the Ecological Integrity base; the pink nodes shown in the CBN represent the set of variables that are part of the Causal Route, and that were used to obtain the causal probabilities. The variable used for this example was Landscape Transformation (landtrnas), and the variable on which its effect was calculated was Ecological Integrity (eiclas).

The new expression of probability resulting from the intervention, obtained through the inference rules, is presented below:

"sum{divfun, resistenci}P(eiclas|cropland, rangeland, irrigation, land-trans, divfun, resistenci)P(resistenci|cropland, rangeland, irrigation, landtrans, divfun)P(divfun|cropland, rangeland, irrigation, landtrans)" (7)

The causal probabilities were calculated from equation 6. This exercise allows us to appreciate the differences between intervention and observation. Table 2 shows the results of the intervention to a cause variable (do(landtrans = (0,2 − 0,4))) and the observation, this last one calculated through the propagation of probabilities in the BN.

From the results of Table 2, it may be thought, that the intervened variable, is a potential cause of the effect. However, this cannot be proven until calculating the Causal Effects (CE) and the Bayes Factor (BF).

All datasets were analyzed in the same way, from the generation of BN's to the obtaining of the CE and BF. Table 3 shows the results of the search of these routes, illustrating the low percentage of possible variables to intervene, from networks created with traditional AI algorithms.

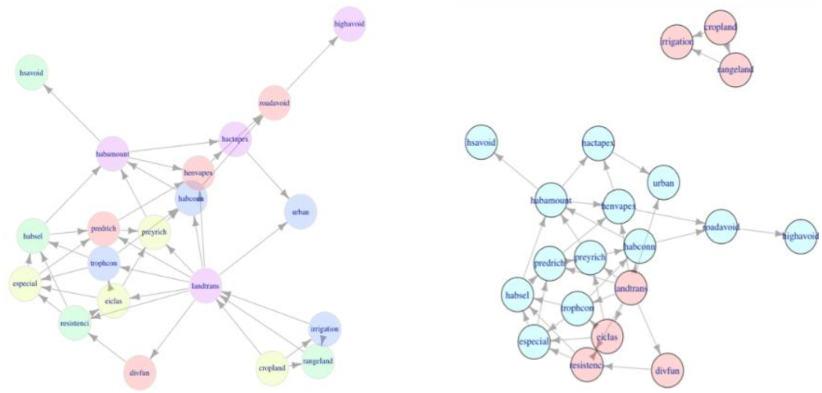

Fig. 1. Bayesian Networks (left) Causal Bayesian Network (right).

Table 2. Observed and causal probabilities for the cause variable landtrans = (0.2 - 0.4) and the eiclass effect.

Effect = eiclas	***Probability BN***	***Probability CBN***
High	0.04	0.02
Low	0.10	0.06
Medium	0.07	0.03
Transformed	0.79	0.89

Table 3. Proven databases for intervention and causal routes found in each.

Datasets	***Effects***	***Cause***	***Total experiments***	***CR - found***
Ecological integrity	4	6	24	4
Breast cancer - prospective	1	3	3	1
Breast cancer - retrospective	1	3	3	1
Genetic expression levels	1	3	3	1
Synthetic data BayesiaLab	1	1	1	1

It is important to mention that in the first experimental version (the one presented in this document), BN was not considered to be carried out manually, that is, only with the knowledge of the experts. In a second attempt, the networks were modified, which considerably increased the number of intervened variables, and causal routes found.

From the new expressions of probability, resulting from the intervention of variables in the BN's, it is possible to estimate the new probabilities that we call causal. This was shown in the previous example (Table 2). Subsequently, the BF and the CE were calculated. To obtain them, it was necessary to calculate the causal probability for two different values of the cause and the same value of the effect.

Table 4 presents the results of some of these values. The column called effect shows the name of the variables for this purpose and the value for which their causal

Table 4. Bayes Factor and Causal Effects results for intervention.

Datasets	***Effect***	***Cause***	***Interventions***	***BF***	***CE***
Ecological integrity	Eiclas *high*	landtrans	(-inf-0.2] (0.2-0.4]	25.00	0.47
Breast cancer - prospective	Outcome *Malignant*	Size	Present Absent	0.33	-0.37
Breast cancer - retrospective	Outcome *Malignant*	Nuclear.Size	Present Absent	7.91	0.72
Genetic expression levels	APOE *(319.22 - 387.64)*	BACE1	(166.94 - 221.33) (221.33 - 493.43)	3.4	0.40
Synthetic data BayesiaLab	Outcome *Patient Recovered*	Treatment	Yes No	1.12	0.08

probability was calculated. The column called interventions shows two values of the cause in which the causal probabilities were compared; columns BF and CE show the results of the calculation of these tests.

The BF can be used as a hypothesis test to contrast two models, making a comparison of this test with the CEs allowed testing the operator's consistencies in estimating causal probabilities and their interpretation of these as potential causes. According to the interpretation corresponding to the BF this is consistent with the CE, the values between 1 and 100 obtained with the BF must correspond to positive values of the CE, and that supports the evidence of potential causes for the intervened values found in the numerator of the BF or to the left of the CE.

6 Conclusions and Future Work

The variables intervention through the operator proved to be a good method of causal estimation if the conditions for the intervention are favorable. The tests carried out after the implementation meant that its effectiveness on estimating the causal probability could be confirmed.

This work not only explores the complex issue of causality but also provides an understanding of how to observe relationships, makes estimates based on observations and interprets them; it should not be a difficult task. However, finding the set of appropriate variables that could be probable causes, and carrying out interventions that provide information on the causal force of one value over another, does have a higher degree of difficulty. This is because estimating causality adequately requires much expert knowledge, and intuition, which cannot be reflected in the calculation of the causal probabilities.

Once it has been proven that it is possible to estimate the probability of the intervention, it becomes interesting to find a way to connect the first level of the ladder of causation with the second. To do this, it will be necessary to turn towards the areas that study the process in which the learning of the causal relations occurs naturally and look at these algorithms that allow the creation of a CBN that resembles causal learning, with the same precision that is achieved naturally. This will provide artificial entities with mechanisms that learn approximate causality to the same level of a human being.

References

1. Pearl, J.; Mackenzie, D.: The book of why: the new science of cause and effect. Basic Books (2018).
2. Pearl, J.: Causal diagrams for empirical research. Biometrika, 82(4), pp. 669–688 (1995).
3. Pearl, J.: Causality: models, reasoning and inference, 29, Springer (2000).
4. Pearl, J.; Glymour, M.; Jewell, N.P.: Causal inference in statistics: A primer. John Wiley & Sons (2016).
5. Kass, R.E.; Raftery, A.E.: Bayes factors. Journal of the American statistical association, 90(430), pp: 773–795 (1995).
6. Sucar, L.E.: Probabilistic graphical models. Springer. Reverend Bayes on inference engines: A distributed hierarchical approach. Cognitive Systems Laboratory, School of Engineering and Applied Science (2015).
7. Sucar, L.E.: Redes bayesianas. BS Araujo, Aprendizaje Automático: conceptos básicos y avanzados, pp. 77–100 (2006).