Multi-Objective Particle Swarm Optimization (MOPSO) based on Pareto Dominance Approach

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ABSTRACT
This paper presents a comprehensive review of a multi-objective particle swarm optimization (MOPSO) reported in the specialized literature. The success of the Particle Swarm Optimization (PSO) algorithm as a single-objective optimizer has motivated researchers to extend the use of bio-inspired technique to other areas. One of them is multi-objective optimization. Multi-objective optimization is a class of problems with solutions that can be evaluated along two or more incomparable or conflicting objectives. These types of problems differ from standard optimization problems in that the end result is not a single \best solution" but rather a set of alternatives, where for each member of the set, no other solution is completely better (the Pareto set). Multi-objective optimization problems occur in many different real-world domains such as automobile design and architecture. A multi-objective particle swarm optimization (MOPSO) method can be used to solve the problem of effective channel selection.

General Terms
MOPSO and PSO Algorithm

Keywords
Particle Swarm Optimization (PSO), Multi Objective Particle Swarm Optimization (MOPSO), Pareto Dominance.

1. INTRODUCTION
1.1 Multi Objective Particle Swarm Optimization (MOPSO)
Multi-objective optimization problems consist of several objectives that are necessary to be handled simultaneously. Such problems arise in many applications, where two or more, sometimes competing and/or incommensurable, objective functions have to be minimized concurrently.[1] Due to the multi-criteria nature of such problems, optimality of a solution has to be redefined, giving rise to the concept of Pareto optimality. In contrast to the single-objective optimization case, multi-objective problems are characterized by trade-offs and, thus, there is a multitude of Pareto optimal solutions, which correspond to different settings of the investigated multi-objective problem. For example, in shape optimization, different Pareto optimal solutions correspond to different structure configurations of equal fitness but different properties. Thus, the necessity of finding the largest allowed number of such solutions, with adequate variety of their corresponding properties, is highly desirable.

1.2 Particle Swarm Optimization (PSO)
Particle Swarm Optimization (PSO) is a swarm intelligence method that roughly models the social behavior of swarms (Kennedy & Eberhart, 2001). PSO shares many features with evolutionary algorithms that rendered its adaptation to the multi-objective context straightforward [1],[2]. Although several ideas can be adopted directly from evolutionary algorithms, the special characteristics that distinguish PSO from them, such as the directed mutation, population representation and operators must be taken into consideration in order to produce schemes that take full advantage of PSO's efficiency.

1.3 Dominance and Pareto Optimality
In a multi-objective optimization problem we seek to simultaneously extremise D objectives:

$y_i = f_i(x)$, where $i = 1, \ldots, D$ and where each objective depends upon a vector x of K parameters or decision variables [5], [6].

The parameters may also be subject to the J constraints:

$e_j(x) \geq 0$ for $j = 1, \ldots, J$.

Without loss of generality it is assumed that these objectives are to be minimized, as such the problem can be stated as:

$$\text{minimize } y = f(x) \equiv (f_1(x), f_2(x), \ldots, f_D(x)) \quad \ldots\ldots\ldots\ldots(1)$$

$$\text{subject to } e(x) \equiv (e_1(x), e_2(x), \ldots, e_J(x)) \geq 0 \quad \ldots\ldots\ldots\ldots(2)$$

A decision vector u is said to strictly dominate another v, if $f_i(u) \leq f_i(v)$ ¥$_i$ = 1, . . . ,D and $f_i(u) < f_i(v)$ for some i; less stringently u weakly dominates v, if $f_i(u) \leq f_i(v)$ for all i. A set of decision vectors is said to be a non-dominated set if no member of the set is dominated by any other member. The true Pareto front, Ƥ, is the non-dominated set of solutions which are not dominated by any feasible solution.

2. BASIC STEPS TO IMPLEMENT PSO AND MOPSO
The basic idea behind the algorithm is to use a collection of "particles" to explore the fitness landscape of a particular problem. Each particle is a vector that describes a candidate solution, and can be evaluated (in the multi-objective case) along several quality dimensions (or, equivalently, with several fitness functions) [3]. The algorithm is iterative, and at each iteration each particle "moves" through the fitness landscape according to it's current fitness values as well as those of nearby particles, and the swarm as a whole. The basic steps of PSO algorithm for the single-objective case are:

1. Initialize the swarm

2. For each particle in the swarm:
   - A. Select leader
   - B. Update velocity
   - C. Update position

3. Update global best

4. Repeat

When tackling multi-objective problems however, a few modifications must be made. First, the objective is to find not one "global best" solution, but a set of solutions comprising the Pareto Front. To do this, an archive of non-dominated solutions is kept, where all non-dominated solutions found at each iteration are stored. The MOPSO algorithm steps are:

1. Initialize the swarm & archive

2. For each particle in the swarm:

   A. Select leader from the archive

   B. Update velocity

   C. Update position

3. Update the archive of non-dominated solutions

4. Repeat

Since the front is usually continuous, some further criteria must be used to decide which non-dominated solutions to keep in the finite archive. Generally, the criteria enforces some diversity measures, the intuition being that a more diverse front will ensure good coverage, as opposed to clustering the reported solutions in one area. One technique for encouraging diversity is to use ϵ-dominance, where the area dominated by a given point is increased by a small constant. Since nearby points will be considered dominated under this new definition, it has the effect of spreading out those solutions kept in the archive.

3. ESTABLISHED MULTI-OBJECTIVE PSO APPROACH

In this section we review the Pareto Dominance concept for Multi-objective PSO. These approaches use the concept of Pareto dominance to determine the best positions (leaders) that will guide the swarm during search. several questions arise regarding the underlying schemes and rules for the selection of these positions among equally good solutions. For the imposition of additional criteria that take into consideration further issues (such as swarm diversity, Pareto front spread, etc.) is inevitable, the development of Pareto-based PSO approaches became a blossoming research area.

Coello and Salazar Lechuga (2002) proposed the Multi-objective PSO (MOPSO), one of the first Pareto-based PSO approaches (Coello, Toscano Pulido, & Salazar Lechuga, 2004). In MOPSO, the non-dominated solutions detected by the particles are stored in a repository. Also, the search space is divided in hypercubes. Each hypercube is assigned a fitness value that is inversely proportional to the number of particles it contains. Then, the classical roulette wheel selection is used to select a hypercube and a leader from it. Thus, the velocity update for the *i*-th particle becomes

$$v_{ij}(t+1) = w\, v_{ij}(t) + c_1 r_1 (p_{ij}(t) - x_{ij}(t)) + c_2 r_2 (R_h(t) - x_{ij}(t)),$$

where p_i is its best position and R_h is the selected leader from the repository. The best position p_i is updated at each iteration, based on the domination relation between the existing best position of the particle and its new position [8],[9].

Also, the repository has limited size and, if it is full, new solutions are inserted based on the retention criterion, that is, giving priority to solutions located in less crowded areas of the objective space. MOPSO was competitive against NSGA-II and PAES on typical benchmark problems, under common performance metrics, and it is currently considered one of the most typical multi-objective PSO approaches. A sensitivity analysis on the parameters of the algorithm, including the number of hypercubes used, can provide further useful information on this simple though efficient approach [10],[11].

Fieldsend and Singh (2002) proposed a multi-objective PSO scheme that addresses the inefficiencies caused by the truncation of limited archives of non-dominated solutions. For this purpose, a complex tree-like structure for unconstrained archiving maintenance, called the dominated tree, is used (Fieldsend, Everson, & Singh, 2003). The algorithm works similarly to MOPSO, except the repository, which is maintained through the aforementioned structures. An additional feature that works beneficially is the use of mutation, called craziness, on the particle velocity, in order to preserve diversity. The algorithm has shown to be competitive with PAES, although the authors underline the general deficiency of such approaches in cases where closeness in the objective space is loosely related to closeness in the parameter space

4. FUTURE RESEARCH PATHS

As we have seen, despite the fact that MOPSOs started to be developed less than ten years ago, the growth of this field has exceeded even the most optimistic expectations. By looking at the papers that we reviewed, the core of the work on MOPSOs has focused on algorithmic aspects, but there is much more to do in this area. Such as Emphasis on Efficiency, Self-Adaptation of Parameters in MOPSOs, Theoretical Developments and Application Work.

5. CONCLUSION

In this paper we have given an extensive review on the different approaches of MOPSO algorithm based on Pareto dominance schemes. One avenue for potential future work would be to analyze a larger set of test problems, as well as different mechanisms for promoting diversity. The concept of ϵ-dominance is presented as one of the best techniques in terms of an efficiency/quality trade off, but it requires some parameter tweaking to work well for a specific problem. Techniques such as fitness sharing, which scales the objective values of a point inversely with the number and nearness of its neighbors, might be able to achieve similar diversity results without any problem specific parameter tuning (at the expense of increased running time).

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7. REFERENCES

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyari- van. "A fast and elitist multiobjective genetic algorithm: NSGA-II. Evolutionary Computation", IEEE Transactions on, 6(2):182{197, 2002.

[2] M. Reyes-Sierra and C. Coello. "Multi-objective particle swarm optimizers: A survey of the state-of-the-art. International Journal of Computational Intelligence Research" , 2(3):287{308, 2006

[3] Zitzler, M. Laumanns, and L. Thiele . "SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multi-objective Optimization", In EUROGEN 2001. CIMNE, 2002.

[4] Alvarez-Benitez, J. E., Everson, R. M., & Fieldsend, J. E. "A MOPSO algorithm based exclusively on Pareto dominance concepts" Lecture notes in computer science Springer Verlag, (Vol. 3410, pp. 459-473), 2005.

[5] Srinivas, N., Deb, "Multi-objective Optimization Using Non-dominated Sorting in Genetic Algorithms", IEEE Transactions on Evolutionary Computation 2(3) (1995) 221–248

[6] Zitzler, E., Thiele, " Multi-objective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach", IEEE Transactions on Evolution- ary Computation 3(4) (1999) 257–271.

[7] Hussein A. Abbass. "The self-adaptive pareto differential evolution algorithm" In Congress on Evolutionary Computation (CEC'2002), volume 1, pages 831–836, Piscataway, New Jersey, IEEE Service Center, May 2002.

[8] Julio E. Alvarez-Benitez, Richard M. Everson, and Jonathan E. Fieldsend. "A MOPSO algorithm based exclusively on pareto dominance concepts", In Third International Conference on Evolutionary Multi-Criterion Optimization, EMO 2005., pages 459–473, Guanajuato, M´exico, 2005. LNCS 3410, Springer-Verlag.

[9] Peter J. Angeline, "Evolutionary optimization versus particle swarm optimization: Philosophy and performance differences" In V.W. Porto, N. Saravanan, D. Waagen, and A.E. Eiben, editors, Evolutionary Programming. VII. 7th International Conference, EP 98, pages 601 610. Springer. Lecture Notes in Computer Science Vol. 1447, San Diego, California, USA, March 1998.

[10] Richard Balling. "The maximin fitness function; multi-objective city and regional planning", In Carlos M. Fonseca, Peter J. Fleming, Eckart Zitzler, Kalyanmoy Deb, and Lothar Thiele, editors, Second International Conference on Evolutionary Multi-Criterion Optimization, EMO 2003, pages 1–15, Faro, Portugal, April 2003.

[11] Thomas Bartz-Beielstein, Philipp Limbourg, Konstantinos E. Parsopoulos, Michael N. Vrahatis, J¨orn Mehnen, and Karlheinz Schmitt. "Particle swarm optimizers for pareto optimization with enhanced archiving techniques", In Congress on Evolutionary Computation (CEC'2003), volume 3, pages 1780–1787, Canberra, Australia, IEEE Press, December 2003

[12] U. Baumgartner, Ch. Magele, and W. Renhart. Pareto optimality and particle swarm optimization. IEEE Transactions on Magnetics, 40(2):1172–1175, March 2004.

[13] Dirk B¨uche, Sibylle M¨uller, and Petro Koumoutsakos. "Self-adaptation for multi-objective evolutionary algorithms" Second International Conference on Evolutionary Multi-Criterion Optimization, EMO ,pages 267–281, Faro, Portugal, April 2003.

[14] Chi-kin Chow and Hung-tat Tsui. "Autonomous agent response learning by a multi-species particle swarm optimization", In Congress on Evolutionary Computation (CEC'2004), volume 1, pages 778–785, June 2004.

[15] Maurice Clerc and James Kennedy. "The particle swarm–explosion, stability, and convergence in a multidimensional complex space", IEEE Transactions on Evolutionary Computation, 6(1):58–73, February 2002.

[16] Carlos A. Coello Coello. "A comprehensive survey of evolutionary-based multiobjective optimization techniques. Knowledge and Information Systems", An International Journal, 1(3):269–308, August 1999.

[17] Carlos A. Coello Coello and Maximino Salazar Lechuga. "MOPSO: A proposal for multiple objective particle swarm optimization", In Congress on Evolutionary Computation (CEC'2002), volume 2, pages 1051– 1056, Piscataway, New Jersey, May 2002

[18] Carlos A. Coello Coello, Gregorio Toscano Pulido, and Maximino Salazar Lechuga. "Handling multiple objectives with particle swarm optimization", IEEE Transactions on Evolutionary Computation, 8(3):256–279, June 2004.