Research Group in Mathematical Inequalities and Applications

$$v(G) > \sum_{m \,\in\, G} v(m)$$

Problem Corner

Problem 1, (2010)

Ovidiu Furdui
Campia Turzii, 405100
Cluj, Romania
Email: ofurdui@yahoo.com

Open Problem. Let $(y_k)_{k \geq 1}$ be the sequence defined by

$$y_k = \frac{1}{2k}^{\frac{1}{2k}^{\cdot^{\cdot^{\cdot^{\frac{1}{2k}}}}}},$$

where the fraction $\frac{1}{2k}$ appears exactly $2k$ times in the definition of y_k. For example

$$y_1 = \frac{1}{2}^{\frac{1}{2}}, \quad y_2 = \frac{1}{4}^{\frac{1}{4}^{\frac{1}{4}^{\frac{1}{4}}}}, \quad y_3 = \frac{1}{6}^{\frac{1}{6}^{\frac{1}{6}^{\frac{1}{6}^{\frac{1}{6}^{\frac{1}{6}}}}}}.$$

Prove or disprove that, for $k \geq 10$, the following inequality holds

$$1 - \frac{1}{\ln(2k)} < y_k.$$