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FINITE ALGEBRAS FROM FREGEAN VARIETIES: DECOMPOSITION AND POLYNOMIALS

This is an abstract of a talk presented at the Workshop on Algebra & Substructural Logic held at JAIST, 10–17 November 1999. It is based on [1], a joint work with P. M. Idziak and A. Wroński.

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A variety $\mathcal{V}$ of algebras with a distinguished constant $\mathbf{0}$ is called *Fregean* if it is $\mathbf{0}$-regular and $\Theta_{\mathbf{A}}(\mathbf{0}, a) = \Theta_{\mathbf{A}}(\mathbf{0}, b)$ implies $a = b$ for all $a, b \in \mathbf{A}$, $\mathbf{A} \in \mathcal{V}$. These properties allow us to introduce a natural partial order $\leq$ on the universe A of $\mathbf{A}$. We study the *decompositions* of elements of finite algebras from congruence permutable Fregean varieties and the *clones of polynomials* in such algebras.

Every congruence permutable Fregean variety has a binary term that turns every of its members into an equivalential algebra. This fact allows us to generalize the idea of decomposition from equivalential algebras. We introduce the notion of *irreducible elements* in an algebra $\mathbf{A}$ and define the equivalence relation $\sim$ in the set of all irreducible elements $I(\mathbf{A})$. Every element $x \in A$ can be uniquely decomposed as the equivalence of an

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$\leq$-antichain $R_{\mathbf{A}}(x)$ contained in $I(\mathbf{A})$ and such that every two elements from $R_{\mathbf{A}}(x)$ are not related with respect to the relation $\sim$. As both the set $I(\mathbf{A})$ and the relation $\sim$ are determined by the congruence lattice, we can recover the universe A from $\mathsf{Con}(\mathbf{A})$. Also the equivalence operation on A can be recovered from this structure.

We next indicate two bunches of polynomials of $\mathbf{A}$ which are constructed only with the aid of the congruence lattice and the commutator operation and generate (together with the equivalence operation) the whole clone of polynomials. Thus, the clone of polynomials of every finite algebra $\mathbf{A}$ from a congruence permutable Fregean variety is uniquely determined by the congruence lattice of $\mathbf{A}$ and the commutator operation. Actually such an algebra $\mathbf{A}$ itself can be recovered (up to polynomial equivalence) from its congruence lattice expanded by the commutator, i.e., from the structure $\mathsf{Concom}(\mathbf{A}) = (\mathsf{Con}(\mathbf{A}); \wedge, \vee, [\cdot,\cdot])$.

The construction presented above leads to the notion of *Fregean frames*. We show that there is a one-to-one correspondence between Fregean frames and finite algebras from congruence permutable Fregean varieties.

Reference

[1] Idziak, P.M., Słomczyńska, K. and Wroński, A. Equivalential algebras: A study of Fregean Varieties, in preparation.

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