Science Focus 科言

Issue 026, 2024

School of Science 理學院

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THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Contents

Science Focus Issue 026, 2024

Message from the Editor-in-Chief 主編的話

Dear Readers,

Welcome to a bonus issue of *Science Focus*. Perhaps this will remind you of the joy of science, when you are kept busy by study trips, revision, or homework, during your Easter holiday.

We focus on biology and mathematics this time. Although most of us have uniform skin colors that change little, some animals are far better at changing theirs, at short notice. We explore the color-changing secrets of chameleon and octopus. For cat lovers, we discuss the physiology and genetics of cats through memes. We also delve into the science and ethics behind animal cloning. Have you ever watched the movie *Good Will Hunting*? We bring you an equally captivating Korean movie that is worth adding to your playlist. Finally, for those of you who are puzzled by various ways to tie shoelaces, you will find a mathematical guide in this issue.

We have just launched this year's "Science in Lyrics" Writing Competition. For details, please visit our website by scanning the QR code on the back cover.

Yours faithfully,
Prof. Ho Yi Mak
Editor-in-Chief

親愛的讀者：

歡迎閱讀額外出版的新一期《科言》！當你在復活節假期忙於參加遊學團，勤於溫習和做功課的時候，希望本刊能令你重拾對科學的喜悅。

今期我們主打生物和數學題材。雖然人類大多都擁有均勻而相對恆定的膚色，但也有動物擅長於轉瞬間變色，所以我們將揭開變色龍和八爪魚的變色秘密。各位貓癡也有福了，因為我們將從迷因解構貓咪的生理和遺傳學。我們亦會探討複製動物背後的科學和道德問題。你有看過電影《驕陽似我》嗎？今期我們會介紹另一套同樣值得你加入播放清單的韓國電影。最後，對於被眾多鞋帶綁法弄得頭昏腦脹的你，我們將會送上一份綁鞋帶的數學指南。

此外，今年的「歌詞與科學」寫作比賽現正接受報名，你可以掃瞄本刊背頁的二維碼參閱《科言》網頁上的詳情。

主編 麥晧怡教授
敬上

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What's Happening in Hong Kong? 香港科技活動

Fun in Spring Science Activities 春日科學好節目

Any plans for this Spring? Check out these activities!

計劃好這個春天的好去處了嗎？不妨考慮以下活動！

Science Fiction : Voyage to the Edge of Imagination 科幻旅航

Are you a science fiction lover? The Science Museum has launched a special exhibition called "Science Fiction: Voyage to the Edge of Imagination." This immersive experience takes visitors on a journey through iconic science fiction exhibits, featuring a variety of films, literature, and art. With six immersive zones, including "Planet Mission" and "Bio Lab", visitors can explore significant themes within science fiction, such as space exploration, cyborgs, and future habitats. This cross-disciplinary exhibition should prompt visitors to reflect on the societal implications of advancing technology, and consider the opportunities and challenges of today and tomorrow through science fiction.

Period: Now – May 29, 2024
Venue: Special Exhibition Hall, Hong Kong Science Museum
Admission fee: Extra entrance fee applies to this special exhibition. Please refer to the museum's website for details.

你喜歡科幻故事嗎？科學館精心籌備了名為「科幻旅航」的專題展覽，透過多件與科幻電影、文學和藝術作品相關的展品，帶大家投身一段奇妙的科幻旅程。展覽有「星際任務」和「生物研究室」等的六個沉浸式展區，大家可以從中探索多個科幻故事的熱門主題，例如太空探索、改造人（cyborgs）和未來居所等等。這個跨領域展覽希望促使大家思考先進科技為社會帶來的影響，以及從科幻作品察看現在和未來的機遇和挑戰。

展期： 現在至 2024 年 5 月 29 日
地點： 香港科學館特備展覽廳
入場費： 此專題展覽另設入場費，詳情請參閱科學館網頁。

Black Holes: The Information Barrier 黑洞 — 信息的盡頭

The special exhibition at the Space Museum delves into the mysteries of black holes, revealing their myths and truths. In addition to the technologies used to "see" and "hear" black holes, visitors can also learn about the paradoxes and possibilities of these mysterious cosmic bodies. Is a black hole really a time machine? What will happen if you fall into a black hole? While we did not recommend you to find the answers by falling into an actual one a few issues ago, you can now do it in the exhibition through virtual reality goggles!.

Period: Now – May 27, 2024
Venue: Foyer, Hong Kong Space Museum
Admission fee: Free admission

這個太空館的專題展覽介紹關於黑洞的流言和知識，解開其神秘面紗。大家除了可以認識科學家用於「看見」和「聽見」黑洞的科技外，還能了解更多關於黑洞的悖論和黑洞的各種可能性。黑洞真的是時光機嗎？掉進黑洞後會發生甚麼事？《科言》編採部之前曾勸告大家不要嘗試親身掉進黑洞找答案，但現在你可以在展覽中透過虛擬實境眼鏡實踐了！

時間： 現在至 2024 年 5 月 27 日
地點： 香港太空館大堂
入場費： 免費入場

MythBusters: Black Holes (Issue 019)
流言終結者：黑洞篇（第十九期）

Lazarus Rising: The Attempted Resurrection of the Gastric Brooding Frog

拉撒路復活：絕種胃育蛙重生記

By Sonia Choy 蔡蒨珩

The arrival of Dolly, the first cloned mammal in 1996, marked an important page in human history. It was the first time humans were able to create a mammal in a laboratory. Pictures of Dolly made headlines across the world. But did you know that scientists also tried to use the same technology to bring extinct frogs back to life?

The southern and northern gastric brooding frogs (*Rheobatrachus silus* and *Rheobatrachus vitellinus*) were discovered in 1972 and 1984 in Queensland, Australia respectively. Both being very small (about 30–54 mm for the southern and 55–80 mm for the northern species) and living in a limited rainforest area (less than 1000 km^2), they lived in streams of the Queensland mountain ranges [1, 2]. Like many native Australian species, the frogs were unique to the land down under, and possessed extremely unique characteristics.

One such thing that caught headlines at that time was how they reproduce [3]. The mother swallows a number of eggs and stops producing acid in its stomach to allow the eggs to hatch into tadpoles, and later into frogs. It was suggested that a substance called prostaglandin E2 secreted by the tadpoles can inhibit the acid secretion of the mother [4]. Then the mother does not eat for six weeks and stops breathing through the lungs but the skin, as its bloated stomach has squeezed the lungs to collapse. Eventually, the mother gives birth to fully formed baby frogs via "propulsive vomiting", much like a Russian doll. This dramatic method caught the attention of many, including zoologists who couldn't believe this was true…until they saw it for themselves. Sadly, these frogs did not live very long, and by 1981, the southern breed had gone extinct in the wild; the northern breed was also extinct in 1985, within a year after its discovery.

Mike Archer, a researcher from the University of New South Wales, had read about the frog and decided to do something no one had seriously attempted before – to bring it back to life [3]. His plan was to use somatic cell nuclear transfer (SCNT) to transfer a somatic cell nucleus obtained from the frozen frog sample to a fresh egg of a reasonably close but existing relative, the barred frog (*Mixophyes fasciolatus*). With a full set of the southern gastric brooding frog's DNA as the blueprint of life, they hoped the cell can divide and develop into a full-fledged individual.

SCNT is divided into a few main steps. The first step was to use ultraviolet radiation to destroy the nuclear DNA of a donor egg [5]. Into this "empty" egg, scientists inserted a nucleus collected from the long frozen gastric brooding frog's tissue sample. For the cell to divide and grow into an embryo, an electrical or chemical stimulation is usually given to activate the manipulated oocyte [6]. After many cell divisions, the embryo then underwent the crucial step of development, known as gastrulation, when cells in its exterior migrate internally – and there it stopped [3]. In the best-case scenario, the cell division and differentiation would continue further, to the point that we have a fully formed tadpole of the southern gastric brooding frog.

Current technology is very far from even creating a southern gastric brooding frog tadpole, but Mike Archer's great dream is to bring the frog back to life. This is partially for the potential medical benefits it may bring. As mentioned earlier in the article, female frogs temporarily stop the production of gastric acid after they swallow the egg and let it hatch. However, the species was extinct before scientists could further study them [7, 8]. If the cloning process is successful, this would lead to a better understanding of the southern gastric brooding frogs, and potentially help human patients suffering from too much or unwanted gastric acid. From acid reflux to ulcers and even more serious diseases, this could be a potential, permanent cure.

But Archer's goal seemed simpler. Quote, "If we were responsible for the extinction of the species, deliberately or inadvertently, we have a moral responsibility or imperative to undo that if we can [3]." Over the last century, many species have gone extinct from human activities. Widespread deforestation and global warming have destroyed the habitats of many species, especially very rare ones that are endemic to Australia. Some, including Archer, believe that our responsibility is to restore these endangered species to their natural habitats, and undo the damage we have caused in the past.

However, there are ethical issues surrounding this. First of all, extinction is part of nature's cycle; species regularly fall on and off the earth all the time. By cloning the southern gastric brooding frog and bringing it back to life, we are upsetting Mother Nature. By Archer's logic, if species died out by human behavior, then we should help restore those species back to the wild. The problem is that species go extinct due to a myriad of reasons; the Australian government, for example, lists the pathogenic fungus *Batrachochytrium dendrobatidis* as a possible extinction reason of the gastric brooding frogs [1, 2]. This is not to underplay the role of humans in destroying nature over the past century, but what is to say that the species definitely died only by human influence? In this case, why shouldn't we just let nature take its course?

Also, the world has changed since the southern gastric brooding frog went extinct. The current Queensland mountains are no doubt different from the ranges where the frog last thrived over 50 years ago. By sending them back into the wild after hatching in the laboratory, we may be sending them to a second death, as it is very likely that they will not survive for an extended period of time. In this case, what is the point of doing this? More fundamentally, which species should be brought back from extinction? Should humans be playing God? At least, most countries across the world agree that cloning should not be done in humans, making generating alternative organs from individuals currently impossible.

Cloning technologies no doubt leave us with many unanswered questions. It is a tool that can be used, and misused; the very thin line between the two will continue to be discussed for some time.

在 1996 年，首隻複製哺乳類動物「多莉」的誕生標誌著人類歷史上重要的一章，這是人類首次在實驗室合成哺乳類動物。多莉的照片佔據了全世界報紙的頭條，但你知道科學家亦曾經以同樣手法嘗試讓已絕種的青蛙重生嗎？

南部及北部胃育蛙（又稱胃育溪蟾，學名分別為 *Rheobatrachus silus* 和 *Rheobatrachus vitellinus*）分別於 1972 及 1984 年在澳洲昆士蘭被首次發現。牠們體型細小（南部品種身長只有 30–54 毫米，而北部品種也只有 55–80 毫米），居住在昆士蘭山脈的小溪中，出沒範圍僅限在一片少於 1000 km^2 的熱帶雨林中 [1, 2]。像很多澳洲本土物種一樣，胃育蛙僅在澳洲出現，而且擁有許多獨有的特徵。

其中，引起傳媒廣泛報導的是胃育蛙的繁殖方式 [3]。母蛙會吞下自己的卵，然後停止分泌胃酸，讓卵在胃裡孵化成蝌蚪，再發育成青蛙。科學家認為由蝌蚪分泌的前列腺素 E2 可以抑制母蛙的胃酸分泌 [4]。在接下來的六週裡母蛙不會進食，亦會停止用肺呼吸，轉而使用皮膚透氣，因為此時母蛙鼓起的胃已把肺部擠壓得塌陷。最後，就像俄羅斯套娃一樣，母蛙會透過嘔吐的方式誕下一群完全成形的小蛙。這種戲劇性的產子方式引起了許多人注意，當中包括不少親眼看見才相信的動物學家。遺憾的是，這些胃育蛙並沒有活很久：南部胃育蛙於 1981 年在野外滅絕，而北部胃育蛙也在首次發現後不足一年的 1985 年絕種。

新南威爾士大學研究員 Mike Archer 讀到關於胃育蛙的資料後，決定做一件未有人認真嘗試過的事情：使胃育蛙起死回生 [3]。他的計劃是使用體細胞核移植（somatic cell nuclear transfer / SCNT）技術，把從冷藏胃育蛙樣本獲得的體細胞核轉移至其尚存近親 — 橫斑蟾屬青蛙 *Mixophyes fasciolatus* 的活卵子內。研究團隊希望這顆含有完整胃育蛙 DNA 的細胞能按遺傳物質上早已寫好的生命藍圖，分裂並發育成發展完全的胃育蛙個體。

體細胞核移植可分為幾個主要步驟。第一步是用紫外線破壞卵子的細胞核 DNA [5]。然後科學家從冷藏的胃育蛙組織樣本取得細胞核，把其加入已去核的卵子中。為了讓細胞分裂並發育成胚胎，研究人員通常還需要施以電擊或化學刺激，以啟動被改造的卵細胞 [6]。經過多次細胞分裂後，胚胎經歷早期發育裡被稱為原腸胚形成（gastrulation）的關鍵步驟，這時胚胎外圍細胞移動至內層，然而實驗中的胚胎發育就止步於此，未能再下一城 [3]。最理想的結局當然是細胞分裂和分化繼續進行，直到發展出一隻完全成形的南方胃育蛙蝌蚪。

雖然目前技術離能夠複製出南方胃育蛙蝌蚪還差很遠，但 Mike Archer 的夢想是使胃育蛙重生。部分原因是為著可能帶來的醫療益處。如前文所述，雌性胃育蛙能在吞下卵後會暫停製造胃酸，讓卵孵化。然而，胃育蛙在科學家能進一步研究之前就已經滅絕 [7, 8]，如果能成功複製牠們，科學家將能對南方胃育蛙有更深入的瞭解，並有可能從中找到治療方法幫助過度分泌胃酸的病人。從胃酸倒流到消化性潰瘍，甚至是更嚴重的疾病，研究都有可能會為相關疾病提供根治方法。

然而 Archer 的動機也許更為簡單，如他所說：「如果我們有意無意地使一個物種滅絕，在力所能及的情況下，我們就有道德責任和義務將其復原 [3]。」在過去這個世紀，人類活動使許多物種滅絕，大規模砍林和全球暖化破壞了許

多物種的棲息地，尤其影響了許多澳洲獨有的罕見物種，因此包括 Archer 在內有些人認為我們有責任拯救這些瀕危物種，將牠們帶回原來的棲息地，並盡可能彌補我們過去對環境造成的破壞。

但這無疑會引申出一連串道德問題。首先，物種衰亡是自然更替的一部分，地球上不斷會有新物種誕生，也會有現存物種滅亡，而複製南方胃育蛙使牠其重生某程度上也是在擾亂大自然生態。按照 Archer 的思路，如果物種因人類行為滅絕，那我們就應該協助這些物種重返大自然。問題是使物種滅絕的原因往往不止一個，例如澳洲政府就將致病真菌 *Batrachochytrium dendrobatidis* 列入胃育蛙滅絕的可能原因之一 [1, 2]。這裡不是想低估人類在過去一世紀對大自然造成的破壞，但我們又怎能肯定一個物種是純粹因為人類影響而滅絕呢？在這種情況下，順其自然又有何不可？

此外，自南方胃育蛙滅絕以來，環境已經變了很多。現時的昆士蘭山脈無疑與 50 多年前胃育蛙茁壯成長的棲息地不同。在實驗室孵化後將牠們送到野外，我們可能只是將牠們送向第二次死亡，因為牠們很大機會無法在野外長時間存活。在這種情況下，這樣做有甚麼意義嗎？更根本的是，哪些物種應該被拯救，哪些不應？人類應該扮演上帝嗎？至少，世界上大多數國家都同意不應該複製人類，使現在我們不能透過製造複製人生產後備器官。

複製技術無疑為我們帶來了許多未能解答的道德問題；它是可以一件被善用或誤用的工具，這一線之差將在未來一段時間引起更多討論。

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The Science Behind Cat Memes: Can Your Cat Haz Cheezburgerz?

貓咪迷因大解構：主子「可摸耳」吃芝士漢堡？

By Aastha Shreeharsh

Since the dawn of the internet, cats have taken over our explore pages in the form of funny video compilations, heartwarming videos and of course: memes. Meme culture has guaranteed that cats can gain the same amount of worship and reverence now as they did in ancient Egypt. Some of these memes raised trivial yet interesting questions:

Can Cats Have Cheeseburgers?

This may be a seemingly silly question with a seemingly obvious answer; however, it may surprise you to know that it can prove beneficial to occasionally feed your cat hamburgers. As carnivorous animals, cats can have a variety of meats — making plain hamburgers a safe and welcome treat for them [1]. Feeding your cats burgers comes with its own set of "terms and conditions" though.

Unfortunately, your cat cannot "haz cheezburgerz" (footnote 1) specifically. Contrary to popular belief that cats love milk, most adult cats are lactose intolerant [1] because they can no longer produce enough lactase to break down lactose after weaning. Therefore, it's probably wise to skip the cheese and seasonings we add in our own hamburgers when you feed cats. Toppings such as onions and garlic can also harm their gastrointestinal tracts and damage red blood cells if consumed in large amounts. The oxidative compounds in onions can denature hemoglobin, causing anemia [1, 2].

...Or Maybe a Little Salami?

As per the meme, a cat may as well have typed this out and, technically, it wouldn't be wrong. While cats can have a little bit of salami, it shouldn't become the staple of their diet. As a processed meat with high sodium content, salami may exacerbate health risks and conditions relating to the heart and kidneys [1, 3]. As all uncooked meats do, salami too poses risks of *Salmonella* infection and trichinellosis (a disease caused by infection with *Trichinella* parasites). Kittens and pregnant cats should not be fed salami, as their immune systems are vulnerable to toxins and bacteria [3].

Are They Really Liquid Enough to "Sits" Where They "Fits"?

We often ponder over unusual, inconsequential ideas and their meanings. Often, scientists are among those people pondering these questions. If you have ever wondered whether cats are truly liquid as you see yet another cat weasel its way into narrow crevices and hilariously small containers, you're in luck. Researcher Marc-Antoine Fardin is one such scientist, who was awarded the Ig Nobel Prize in Physics (footnote 2) for answering this absurd question. Using rheology, the study of deformations and flows of matter, he found that, under certain conditions, cats may fit the definition of a liquid [4].

To determine the state of a material over a given time frame, rheologists look at the relationship between two time periods: the relaxation time and the experimental time. Aptly named, the relaxation time is the time taken by a material to modify its form to fit in a container; in this case, it would be the time taken by a cat to "fits" where it "sits". Experimental time refers to the time elapsed since the deformation is observed; in this case, this could be the time the cat is observed since it first puts its paws in to "sits" in the fruit bowl. Finally, rheologists look at the ratio between the relaxation time and the experimental time — the Deborah number D_e:

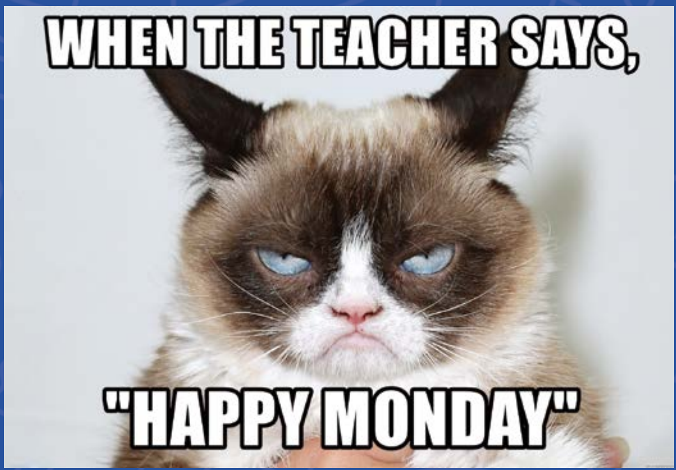
$$D_e = \frac{t_c}{t_p}$$

(where t_c is the relaxation time and t_p is the experimental time)

When the Deborah number is less than 1, i.e. $t_p > t_c$, the material is considered relatively liquid. If you think carefully, by this definition, whether a material is perceived to be liquid or solid all depends on whether you observe it long enough for the deformation to take place. As Fardin puts it, "If you take a time-lapse of a glacier on several years you will unmistakably see it flow down the mountain [5]." Similarly, a

waterfall can also appear "frozen" in a photo taken by a high speed camera. "For cats, the same principle holds," Fardin remarked. "If you are observing a cat on a time longer than its relaxation time, it will be soft and adapt to its container, like a liquid would [5]."

Why Is Grumpy Cat So Grumpy?



Signalling the end of the weekend, Mondays typically serve as the herald of yet another week filled with classes, tests, and assignments galore; which is why this cat's displeasure towards Monday seems relatable to us rather than unusual. However, if you were to look up Grumpy Cat memes on the internet, you'd very quickly see what earned her this nickname. With her piercing blue eyes forever narrowed in bemusement and a permanent scowl marring her face, Tardar Sauce, the star of the infamous Grumpy Cat meme, rose to fame as the poster child of grumpiness.

Contrary to popular belief, Grumpy Cat's surly expression is not caused by a morose disposition; rather, it is the direct result of an underbite due to a type of feline dwarfism known as achondroplasia [6]. Mutations in the *UDP-glucose 6-dehydrogenase (UGDH)* gene were suggested to cause this genetic disease [7, 8], leading to abnormal bone and cartilage development. This condition was speculated to have an autosomal dominant mode of inheritance. Homozygous mutants were not observed, likely because of its lethality at an early embryonic stage [7–9]. Although the resultant physical deformities may seem cute to us, they are also associated with spinal and degenerative joint problems [9], and a shortened lifespan [6]; yet breeders continue to breed cats afflicted with feline dwarfism to meet the high demand for unique-looking cats. Take this article as a sign: Adopt, don't shop!

1 Editor's note: The phrase "haz cheezburgerz" comes from LOLspeak – the vernacular associated with the LOLcats Internet memes that first became popular in 2007 [10]. LOLspeak embodies what made the LOLcats memes so popular – intentionally misspelled words that phonetically resemble the grammatically correct word ("cheezburger" for cheeseburger) and "purr-fect" puns that play on the fact that a cat is supposedly saying these phrases.

2 Ig Nobel Prize: An award that celebrates scientific research that "first makes people laugh, then think."

自互聯網誕生以來，貓以層出不窮的方式佔據了我們社交媒體的探索頁，有時是搞笑影片合輯，有時是窩心感人短片，但不得不提的是：迷因！迷因文化確保貓受到的讚頌和崇拜不亞於牠們在古埃及時期受到的膜拜，然而當中有些迷因卻使我們不禁反思一些有趣小問題：

貓能吃芝士漢堡嗎？

這看似是答案非常明顯的白痴問題，但你可能不知道偶爾餵一下主子吃漢堡包其實是有益的。作為肉食性動物，貓能吃不同種類的肉，所以「簡單」的漢堡包對貓來說是安全而受歡迎的食物 [1]；之所以加上「簡單」是因為有一些「注意事項」我們不能忽略。

遺憾的是，你的主子並不能吃芝士漢堡（註一）。大眾對貓的印象可能是認為牠們喜愛喝牛奶，然而牠們在戒奶後並不能製造乳糖酶分解乳糖，所以成年貓大多是乳糖不耐的 [1]，因此我們不應將芝士和醬料加到給貓吃的漢堡中。大量進食洋蔥、蒜頭等配料亦會對貓的消化道和紅血球造成損害，尤其洋蔥中具氧化性的化合物能使血紅蛋白變性，引致貧血 [1, 2]。

那少許莎樂美辣肉腸可以吧？

如這迷因所示，有主子可能曾因一時口腹之慾而在電腦前偷偷回覆「可以」，技術上這不能說是錯的。儘管貓可

以吃少許辣肉腸，但這不應成為其主食。辣肉腸這類高鈉的加工肉類可能增加心臟和腎臟的健康風險及使健康惡化 [1, 3]；像其他生肉一樣，進食辣肉腸會帶來感染沙門氏菌和旋毛蟲症（由旋毛形線蟲屬寄生蟲感染導致的疾病）的風險，因此小貓和懷孕貓隻不宜進食辣肉腸，因為牠們的免疫系統未必足以抵禦毒素和細菌 [3]。

貓真的能鑽進所有狹小容器裡嗎？

我們也許經常為不尋常，但無關痛癢的小問題陷入沉思，科學家通常都是其中一分子，而且沉思著類似的問題。當你看見再有一隻貓把身軀鑽進狹縫或是細小容器的時候，你是否也懷疑過貓到底是不是液體呢？如果你有這樣想過的話，你走運了！研究員 Marc-Antoine Fardin 正是其中一員，他因解答了這條可笑的問題而獲得搞笑諾貝爾物理獎(註二）。流變學（rheology）是研究物體變形和流動的一個範

睛，從這個角度著手，他發現在特定條件下，貓也能符合液體的定義 [4]。

要決定物料在特定時間裡的狀態，流變學家會觀察兩段時間之間的關係：鬆弛時間（relaxation time）和實驗時間（experimental time）。正如字面上的意思，鬆弛時間是物料改變形狀以填滿容器的所需時間，在我們的例子裡就是貓塞滿容器的所需時間。實驗時間則是指由開始觀察起計的時間，這可能是從我們最先看到貓爪佔據水果盤一刻計起。最後流變學家會計算鬆弛時間和實驗時間之間的比，即是底波拉數（Deborah number / De）：

$$D_e = \frac{t_c}{t_p}$$

（t_c 是鬆弛時間，而 t_p 是實驗時間。）

當底波拉數小於 1，即是 $t_p > t_c$，物料就會相對地被視為液態。如果你細心想想，根據這個定義，物料被我們認為是液態還是固態，完全取決於觀察時間是否長得使我們能看見變形過程。正如 Fardin 的解釋一樣：「如果你拍一條歷時數年的冰川縮時短片，毫無疑問地你會看見冰川往山下流動 [5]。」同樣地，在高速相機拍攝的照片裡，瀑布也能顯得像被凝住了一樣。「套用在貓上也是同樣原理。」Fardin 評論道，「如果你觀察貓的時間比其鬆弛時間長，貓就會像液體一樣柔軟，亦能填滿容器 [5]。」

為甚麼不爽貓這麼不爽？

星期一標誌著美好週末的終結，面前等著大家的又是沒完沒了的課堂、測驗和功課，這正是不爽貓（Grumpy Cat）

對星期一不爽的原因，相信大家都會有很深的共鳴。如果你嘗試上網搜尋不爽貓迷因，很快你就會明白她為何得到這個稱號。在半點疑惑下半開的銳利藍眼睛，怒瞪的殺氣劃破面孔 — 不爽貓迷因裡的主角他他醬（Tardar Sauce）憑這副惡形惡相成為互聯網上不爽的代表。

有別於大眾的理解，不爽貓的怒相並不是源自陰沉的性格，而是由貓侏儒症導致的下顎前突所致，病名又稱為軟骨發育不全（achondroplasia）[6]。尿苷二磷酸 – 葡萄糖 6– 脫氫酶（*UDP-glucose 6-dehydrogenase / UGDH*）基因上的突變被認為是導致這個遺傳病的元兇 [7, 8]，導致不正常的骨骼及軟骨生長。科學家推測這個疾病是以常染色體顯性模式(autosomal dominant mode of inheritance）遺傳；此外科學家沒有觀察到顯性純合的個體，因為這些個

體大概在胚胎發育初期就已經死亡 [7–9]。雖然疾病導致的殘缺外表看來非常可愛，但疾病有可能為個體帶來脊柱和關節退化問題 [9]，亦可能縮短個體壽命 [6]，然而商人仍然繼續透過配種繁殖受疾病折磨的侏儒貓，以應對市場對長相獨特貓隻的需求。所以，這篇文章亦希望呼籲大家：領養代替購買。

1 編按：此處英語原文為源自 LOL 語（LOLspeak）的「haz cheezburgerz」，貓咪迷因（LOLcat memes）由 2007 年開始在網上爆紅 [10]，LOL 語是貓咪迷因上的調皮文字，亦是貓咪迷因爆紅的原因，當中包括發音相近但故意串錯的詞彙（以「cheezburger」代替「cheeseburger」），也有模仿貓說話的雙關語（例如「purr-fect」，「purr」是貓獨特的低沉咕嚕聲）。

2 搞笑諾貝爾獎：一個獎項以表揚「首先令人發笑，繼而發人深省」的科學研究。

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MOVIE REVIEW: IN OUR PRIME (2022)

影評：奇怪的數學家 (2022)

By Peace Foo 胡適之

Directed by Park Dong-hoon, the movie opens with high school student Han Ji-woo (Kim Dong-hwi) and his struggles with mathematics. He crosses paths with security guard Lee Hak-sung (Choi Min-sik), a North Korean defector and reclusive mathematician whose story is gradually revealed throughout the movie, and ends up asking him for help in preparation for a tough exam. Hak-sung agrees to teach him – to appreciate math, rather than to cram. The movie then tracks their growing relationship (which the actors portray very well), the involvement of Ji-woo's friend Park Bo-ram (Jo Yun-Seo), the impact of the exam and Hak-sung's proof of the Riemann hypothesis (in reality still unsolved to this day). On the whole its setup has been compared to the 1997 film *Good Will Hunting*, which also features an anonymous but mathematically gifted university worker, but it has a strong story of its own and avoids rehashing the same plot. Without spoiling the movie, let's see whether its plot makes sense in reality!

Who Was the Old Man in Hak-sung's Flashback?

At the end of Hak-sung's first lesson he calls Ji-woo an "epsilon". The memory that explains this is centered on Hak-sung's childhood meeting with Paul Erdős, one of the most productive and eccentric mathematicians of the 20th century, who also independently proved the prime number theorem [1]. Erdős traveled around the world with only one suitcase of clothes, collaborating with so many different colleagues that mathematicians still use the "Erdős number" to track the network of people he worked with. He was also famous for his odd vocabulary, where he called women "bosses", men "slaves", lectures "sermons", and non-mathematicians "trivial beings" [2]. (Hak-sung also calls Ji-woo a "trivial being" in the same scene.) Children were called "epsilons" because in calculus an epsilon represents a positive but very small number.

Erdős met many "epsilons" throughout his life. According to his biographer, Paul Hoffman, Erdős "made it his mission to seek out child prodigies all over the world" and encourage them with further problems [3]. He met the 10-year-old Terence Tao at the 1985 International Mathematical Olympiad (IMO), very similar to Hak-sung's memory of Erdős meeting him at the 1982 IMO.

Erdős had already been teaching children gifted in mathematics as early as the 1960s:

"[László] Lovász only started doing serious mathematics late in life, said Erdős, 'at the ripe old age of nearly seventeen. When Lovász was still an epsilon, in the first year of high school, he and ... a fellow mathematician courted the same boss-child, also a mathematician and not a bad one as bosses go. The two slave children asked her to choose. She chose Lovász,' and they got married. But the love story could be improved upon, Erdős noted, by having the boss-child answer: 'I will choose the one who proves the Riemann Hypothesis.' [3]"

Did North Korea Actually Participate in the IMO?

In the segment announcing the proposed proof of the Riemann hypothesis, the mathematician Oh Jung-nam tells viewers that he once represented South Korea at the IMO alongside Hak-sung, who represented North Korea.

In reality, North Korea has participated in the IMO, but they did not begin doing so until 1990 and did not participate between 1993 and 2006. They were disqualified for suspected cheating in 1991 and 2010 [4, 5]. South Korea began participating in 1988 and has attended every IMO since then [6]. So neither Jung-nam nor Hak-sung could have gone to the IMO in 1982 as the movie shows. But it is quite likely that both would have done well based on the overall performance of the Koreas at the IMO: South Korea has the fourth most gold medals and North Korea has the 23rd, out of 120 participating countries in IMO history [7], although it is worth mentioning that North Korea has not participated as often as South Korea and the two countries often perform similarly well in individual tournaments.

What About the Nobel Prize?

Another newscaster heard that Hak-sung could be awarded a Nobel Prize for his proof of the Riemann hypothesis. This is not accurate. There is no Nobel Prize for mathematics; the most prestigious prize is the Fields Medal, which is only awarded to mathematicians 40 years old or under. The best estimate for Hak-sung's age is 45–50 years old, so he would not get the prize. However, he would certainly be awarded the Abel Prize, established in 2001 and often described as a mathematics' equivalent to the Nobel Prize, as well as receiving the Clay Institute's prize of one million US dollars for a successful solution to the Riemann hypothesis, one of their Millennium Problems. (Out of the seven problems, only one has been solved, the Poincaré conjecture, which we wrote about in Issue 019. Remarking that "Everybody understood that if the proof is correct then no other recognition is needed," Grigory Perelman, another "hermit mathematician" like Hak-sung, refused both the prize money, and the Fields Medal.)

Is the "π Song" Real?

A review claims that the highlight of the movie is the scene where Hak-sung plays the "π song" to "convince [Ji-woo] that math is of ethereal beauty" [8]. With the digits of π written out in front of him, Hak-sung says that he will "play π" by assigning the digit 1 to the note C, 2 to D, and so on. He tells Bo-ram to accompany the melody that this produces. This is not a new way to represent the digits of π and many examples can be found on YouTube [9, 10].

The opening of the song in the movie does correspond to the first 14 digits of π:

3.1415 92

65358 97

As the accompaniment begins, aside from repeating this block, Hak-sung also weaves extra notes into his part of the song that do not correspond to digits of π. Then after the first glissando, he plays a simple repeating pattern of the next 14 digits:

9323 846 2643 383

But he does not follow it exactly throughout. In fact his part corresponds to the sequence:

9323 846 2643 383

9323 846 5

The song dresses these 28 digits of π up with a piano accompaniment. It does not necessarily present many digits of π in order, and is more of an interpretative art piece, though a very beautiful one that matches the tone the director wants to communicate in the movie. By itself π would not make much of a song anyway, since there is perhaps nothing mathematically special about its pattern of decimal digits. What has turned this essentially random pattern into a proper piece of music is the application of the human imagination.

Nevertheless, it's a very fitting song for a movie that celebrates the advancement of mathematics. So much of this subject began as either a useless curiosity, like a dead butterfly pinned on a card, or a narrow solution to a narrow real-world problem; it is our imagination that propels individual parts of mathematics beyond either of these labels, and gives it the power and scope that makes it beautiful.

What is the Riemann Hypothesis?

If you are a serious math lover who wants to know more about the Riemann hypothesis and analytic number theory, please find the additional content on our website.

劇情簡介

這套電影由朴東勳執導，以高中生韓智宇（金東輝 飾）學習數學的煩惱揭開序幕。韓智宇遇上當保安糊口的脫北退隱數學家李學成（崔岷植 飾），後者的身世隨故事的推進逐步揭曉，韓智宇最後請求李學成教他應付艱深的數學考試。李學成同意，但只答應教他欣賞數學，而不會操練試題。故事之後描述二人逐漸變得熟絡的關係（演員對此演繹得恰到好處！）、韓智宇朋友朴寶藍（趙尹瑞 飾）的加入、那場考試帶來的影響，以及李學成解開黎曼猜想（Riemann hypothesis）的證明（在現實上黎曼猜想尚未被證實）。有人把故事劇情與在 1997 年上映的電影《驕陽似我》（*Good Will Hunting*）作對比，皆因兩者同樣涉及默默無名但極具數學天份的大學工友，但《奇怪的數學家》本身就是獨樹一格的原創故事，因此並不會令觀眾有舊調重彈之感。就讓我們在避免透露劇情的情況下，看看劇情在現實世界是否合理吧！

李學成回憶中的老人是誰？

李學成在第一堂課結束時稱韓智宇為「epsilon」，這關乎李學成在童年時與 Paul Erdős 會面的一段回憶。Paul Erdős 是 20 世紀其中一個最多產的古怪數學家，曾獨立證明質數定理（prime number theorem）[1]。Erdős 帶著一箱衣服就環遊世界跟不同數學家合作，協作者多至數學界現在還用「Erdős 數」來追蹤任何一個數學家與他的淵源。使 Erdős 聞名的還有他自創的奇怪詞彙，例如他稱女性為「老闆」，男性為「奴隸」，講課為「講道」，以及非數學家為「凡人」[2]。（李學成在同一幕也稱韓智宇為「凡人」。）小孩被稱為「epsilon」的原因是因為 epsilon 在微積分裡代表一個非常小的正數。

Erdős 一生遇過不少「epsilon」。傳記作者 Paul Hoffman 寫到 Erdős「致力尋找世界各地的天才兒童」，並以超越他們程度的數學問題激勵他們 [3]。他在 1985 年國際數學奧林匹克（IMO）就遇上當時只有十歲的陶哲軒，這呼應李學成回憶中 Erdős 在 1982 年 IMO 跟他會面的情景。

其實 Erdős 早於 1960 年代就開始教導數學資優兒童：

「[László] Lovász 在較遲的時候才開始認真接觸數學，Erdős 形容為『在十七歲左右的成熟之年。當 Lovász 仍是個 epsilon，讀第一年高中的時候，他與……一位熱愛數學的同輩追求著同一個小老闆，那老闆亦是數學同好，以老闆來說算是學得不錯。兩個小奴隸著她在兩人中選一個，她選了 Lovász。』然後兩人就結婚了。但 Erdős 認為他們的愛情故事可以錦上添花，例如小老闆可以回答：『我會選能證明黎曼猜想的那位。』[3]」

北韓事實上有參與過 IMO 嗎？

在公佈黎曼猜想證明擬定稿的一段，數學家吳正南告訴觀眾他曾代表南韓，與北韓代表李學成在 IMO 碰頭。

現實上北韓的確有參加過 IMO，他們由 1990 年起才開始參加，其後在 1993 至 2006 年間均缺席比賽；另外他們在 1991 和 2010 年因懷疑作弊被取消資格 [4, 5]。另一方面南韓由 1988 年起到現在依然是 IMO 從未缺席的常客 [6]。由此可見，電影中無論是吳正南或李學成都不可能出現在 1982 年的 IMO 裡，但按兩韓的整體往績推論，兩者如果參賽的話也很可能會有不錯的表現：南韓是總金牌數目排名第四的國家，而北韓則在 IMO 歷史上 120 個參賽國家中排行第 23 [7]，雖然值得留意的是北韓的參賽次數並不如南韓多，而兩國在單次對賽中通常都旗鼓相當。

那諾貝爾獎呢？

另一個主播聽說李學成可能會因成功證明黎曼猜想而獲頒諾貝爾獎，然而這是不可能的。諾貝爾並不設數學獎，數學界最頂尖的獎項是菲爾茲獎（Fields Medal），但那只頒予 40 歲或以下數學家，李學成怎樣看也有 45 至 50 歲，因此他也不會獲頒菲爾茲獎。但他應該會獲頒在 2001 年創立的阿貝爾獎（Abel Prize），那亦是一個經常被形容為相當於諾貝爾獎的數學獎項；亦會因成功解決黎曼猜想而獲得克雷研究所（Clay Institute）的 100 萬美元獎金，因為那是研究所其中一道懸賞的千禧年難題。（在他們

列出的七道難題中只有一道被成功解開，那就是我們在第十九期提及過的龐加萊猜想（Poincaré conjecture）。成功解決難題的 Grigory Perelman 跟李學成一樣都是極其低調的退隱數學家，他堅拒接受獎金和菲爾茲獎，並解釋道：「所有人都明白如果證明正確就不需要額外的嘉許。」）

「π 歌」是真的嗎？

其中一個影評認為整套電影最精彩的部分是李學成演奏「π 歌」來「說服[韓智宇]數學擁有非凡的美」的一幕 [8]。李學成把 π 的值詳列在面前，說自己會把數字 1 彈作音符 C，2 作 D 並如此類推地演奏 π，又著朴寶藍奏出伴奏旋律。這其實並不是甚麼新鮮事，在 YouTube 上也能找到不少例子 [9, 10]。

電影中歌曲的開首的確對應 π 的頭 14 位數字：

3.1415 92

65358 97

然而隨著伴奏開始，李學成除了重覆以上部分還加入了與 π 數值無關的額外音符。在第一個滑音之後，他重覆彈奏之後的 14 位數字：

9323 846 2643 383

但他也沒有完全依照數值來彈，事實上他彈的是：

9323 846 2643 383

9323 846 5

歌曲基本上就是把 π 的 28 位數字以鋼琴伴奏潤飾，並沒有順序演奏出 π 的值，反而更像一闋經重新詮釋的藝術品，當中美麗的旋律正與導演想在戲中表達的格調吻合；但無論如何 π 本身也不太可能構成一首歌的主體，因為小數點後的值在數學上也許並沒有任何特別之處，能把這串基本上是隨機的數字轉化成像樣的音樂多多少少只是我們活用了自己的想像而已。

儘管如此，這音樂卻能奏出電影想帶出的訊息：到底是甚麼驅使數學發展呢？不少數學始於無用的好奇，一開始只像把一隻垂死蝴蝶釘在卡紙上孤芳自賞的一門藝術，又或是解決現實上某個專門問題的專門答案，但正是我們的想像使數學突破界限，在各個層面上融會貫通，為蘸上數學更美麗的色彩。

甚麼是黎曼猜想？

如果你是數學狂迷，想更深入了解黎曼猜想和解析數論（analytic number theory），請參閱《科言》網頁上的附加內容。

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Masters of Transformation: COLOR-CHANGING

偽裝大師：變色動物

Introduction

Amidst the myriad of phenomena that animals exhibit – from mimicry, hibernation, to bioluminescence – one of the most fascinating is the ability of some animals to change their colors in the blink of an eye. As a result, they can hide from predators, communicate with peers, intimidate when threatened, and regulate body temperature.

How Do Animals Change Color?

The process by which animals change color involves specialized cells called chromatophores. Located in the skin, they are colored cells that contain pigments or photonic structures [1]. Pigments absorb specific wavelengths of visible light and reflect those that are not absorbed, resulting in the colors we see. There are various types of pigment-containing chromatophores, named according to their color: melanophores for black or dark brown, cyanophores for blue, xanthophores for yellow chromatophores, and so on. In addition to pigmentary colors, there are structural colors created by a different mechanism.

Iridophores in fish, amphibians and reptiles are made up of transparent guanine nanocrystals [2] that function to interfere with the light reflected off the skin at certain frequencies [3]. Iridophores can also produce iridescence, in which the color appears to change depending on the viewing angle [1].

Chameleons

Chameleons are commonly known for being the representatives of color-changing animals. While most animals can change their color by controlling the dispersion and aggregation of chromatophores, chameleons have an additional level of control with two overlaying sets of iridophores [2]. The upper layer contains regularly arranged guanine nanocrystals. For panther chameleons, they can change their skin color by increasing the distance between these tiny crystals to manipulate the interference of light, leading to the reflection of longer wavelengths from blue to red. In the background of yellow xanthophores, the skin color of a male panther chameleon can thus change from green to yellow or orange when it is excited in male contests or courtship.

The lower layer contains disordered guanine crystals of high reflectivity in the near-infrared region (700–1,400 nm). It provides passive thermal protection to chameleons by reflecting direct and indirect "heat radiations" from the sun back into the environment, thus lowering their body temperature in the dry and sunny habitat.

Mimic Octopus

While chameleons have been the fascination of scientists for decades, the mimic octopus takes the art of adaptation to complex new heights. Among the diverse species of octopus, *Thaumoctopus mimicus* stands out for its remarkable mimicry skills, through a combination of sophisticated body movements and dynamic skin patterning [4]. The color-changing properties of the mimic octopus is enabled by the fine control over its chromatophores and iridophores [1]. Surrounded by radial muscles, their chromatophores

ANIMALS

By Roshni Printer

expand upon muscle contraction, increasing the area of exposure of the pigment, hence displaying a more intense skin color. Similar to chameleons, cephalopods also possess iridophores to modify their skin color through optical interference. By modifying the arrangement and molecular structure of reflectin proteins and expulsing water out of the cell, iridophores can be turned "on" and "off" as both actions can change their optical properties [1]. By selectively activating different groups of chromatophores and iridophores to mix different colors, the mimic octopus can accurately mimic the patterns of its surroundings, and change its morphology to imitate a wide variety of marine animals like flatfish, lionfish, and banded sea-snakes [4].

In addition to chromatophores, the mimic octopus also utilizes other specialized skin cells called papillae to enhance its mimicry [5]. These papillae are small muscular structures that can change the three-dimensional texture of the octopus's skin, providing an added layer of deception.

Conclusion

The study of color-changing abilities in animals such as chameleons and octopuses, highlights the intricate mechanisms and adaptations in nature. Through studying these processes, we gain insight into the strategies of animal survival and communication, enhancing our knowledge and awareness of the value of biodiversity.

簡介

在動物展示的多種現象中 — 由擬態、冬眠，到生物發光 — 其中一種最吸引的是一些動物在眨眼間改變身體顏色的能力。藉此牠們能避過捕食者的法眼，與同伴溝通，在受威脅時恐嚇對方，以及調節身體溫度。

動物如何改變顏色？

動物變色的過程涉及名為色素細胞的特化細胞。這些帶顏色的細胞位於皮膚內，含有色素或光子結構 [1]。色素吸收可見光中特定波長的光，並反射那些沒有吸收到的波長，結果產生我們所見的顏色。含色素的色素細胞以其顏色命名，包括黑色素細胞（黑色或深棕色）、藍色素細胞、黃色素細胞等等。除了由色素構成的顏色外，還有透過另一種原理構成的結構色（structural colors）。在魚類、兩棲類和爬蟲類找到的晶體細胞含有透明的鳥嘌呤奈米晶體 [2]，它能干涉（interfere）從皮膚表面反射特定頻率的光 [3]。晶體細胞亦能產生顏色隨觀察角度改變的虹彩 [1]。

變色龍

變色龍眾所周知是變色動物的代表。其他動物透過控制色素細胞的聚散變色，而變色龍就有兩層晶體細胞加強對身體顏色的控制 [2]。上層包含排列整齊的鳥嘌呤奈米晶體。七彩變色龍（panther chameleons）可以透過增加細小晶體間的距離控制光的干涉以改變顏色，使皮膚由原本反射藍光變成反射波長較長的紅光。雄性七彩變色龍會在雄性競爭或求偶中被觸發，在黃色素細胞提供背景色下，將身體顏色由整體的綠色變成黃或橙色。

下層包含對近紅外線（700–1,400 nm）有高反射性的雜亂鳥嘌呤晶體。

它們能把直接和間接來自太陽的「熱輻射」反射至環境，為變色龍提供被動的隔熱功能，使牠們在乾燥和烈日下的生境中體溫亦得以下降。

擬態章魚

變色龍無疑是使科學家多年以來一直著迷的研究對象，但擬態章魚的偽裝技術也不相伯仲，甚至有過之而無不及。在眾多品種的章魚中，*Thaumoctopus mimicus* 的偽裝技術最受人矚目，它能透過複雜的身體動作和瞬息萬變地使皮膚變色來模仿目標 [4]。擬態章魚對其色素細胞和晶體細胞的精密控制使牠能隨意變色 [1]。牠們的色素細胞被輻射肌環繞，肌肉收縮時色素細胞得以展開，增加色素展示的面積，令皮膚顏色變深。與變色龍相似，頭足類也擁有使牠們可以透過光學干涉改變顏色的晶體細胞。通過改變反射素（reflectin）蛋白的排列和分子結構，以及把水排出細胞外，晶體細胞可以被隨意「開關」，因為以上兩個程序皆能改變細胞的光學特性 [1]。透過選擇性啟動不同色素細胞和晶體細胞組合來調配不同顏色，擬態章魚可以準確地模仿四圍環境的樣式，又能變成不同形狀扮演多種海洋生物，例如比目魚、魔鬼簑鮋（獅子魚）和黃唇青斑海蛇 [4]。

除了色素細胞外，擬態章魚也會用名為乳突（papillae）的特化皮膚細胞使擬態變得更為逼真 [5]。這些乳突是細小肌肉組織，可以改變章魚皮膚立體質感，為本來已經非常像真的偽裝點龍點睛。

結語

對於變色龍、章魚等動物的變色能力所作的研究展現了生物體內錯綜複雜的運作機制及物種對自然環境的適應性。通過研究這些機制，我們可以了解動物生存和溝通的策略，增進我們相關知識和對生物多樣性的關注。

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THE SHOELACE PROBLEM – WHAT'S THE BEST WAY TO LACE YOUR SHOES?

鞋帶問題 — 怎樣穿鞋帶才是最佳方法？

By Peace Foo 胡適之

What is the best way to lace your shoes?

This isn't a math question, so let's change it slightly: What is the shortest way, and what is the strongest way to lace your shoes?

My guess is you've never had to think about these questions, but maybe your parents remember cutting off lengths of shoelace to use or testing the strength of their own shoelaces. You can see how relevant those questions might be. Nowadays we don't consider them because the shoe manufacturers do it for us; their costs and profits depend on it, and they may even use the same mathematics as this article describes to come up with answers.

Lacing a Shoe

Mathematically, a shoe has two columns of n eyelets each, which we can label $A_1, \ldots, A_n, B_1, \ldots, B_n$ in columns A and B (Figure 1). Without loss of generality, let the gap between columns A and B be g units, and the height between successive eyelets in a column be h units. A segment is formed every time the lace go through an eyelet from the previous one. A lacing is a way to pass a shoelace through every eyelet once each, starting at A_n and ending at B_n. By counting, we know that a lacing consists of $2n - 1$ segments.

怎樣穿鞋帶才是最佳方法？

這不算是個數學問題，所以讓我們換個問法：怎樣能用最少的鞋帶，以及怎樣能最穩妥地把鞋帶穿好？

相信你從來都不用擔心這些問題，但也許你的父母還記得以前剪下鞋帶來穿，然後確定鞋帶是否穿得穩妥的日子，在這種情況下就顯出這些問題的重要性了；現在我們之所以不用操心是因為製造商在銷售前就已經穿好鞋帶，而他們的成本和利潤全都取決於這些問題，他們或許也使用與本文相同的數學角度作出選擇。

關於穿鞋帶

從數學角度看，一隻鞋有兩行孔眼，每行各有 n 個孔眼，我們可以把 A 行和 B 行的孔眼以 A_1 到 A_n，及 B_1 到 B_n 表示（見圖一）。在不失一般性的情況下，設 A、B 行之間的間隙為 g 單位，同一行上下孔眼之間的高度則為 h 單位。由一個孔眼到另一個孔眼之間的一段鞋帶被稱為一個線段；而一個綁法是指把鞋帶由 A_n 穿起，中間穿過每個孔眼一次，最後穿過 B_n 作結。透過簡單數算，我們知道一個完整綁法裡會有 $2n - 1$ 個線段。

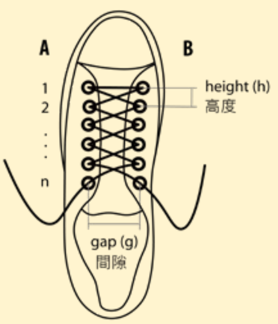


Figure 1 A shoe in a lacing which contains 2n – 1 segments.

圖一 以由 2n – 1 個線段組成的綁法穿好的鞋

Since the main purpose of lacing a shoe is to pull the two sides of the shoe together, we can add a constraint that three consecutive eyelets along a lacing can't all be in the same column [1]. In other words, we can't form a long vertical line along a single column.

"Dense" lacing methods have the extra condition that the lace always alternates between columns A and B; in other words, there are no vertical segments. The three most common dense methods are shown in Figure 2, which we'll call the crisscross, the European, and the shoe-store methods [2].

由於穿鞋帶的主要目的是把鞋的左右兩邊拉在一起，因此我們可以加一條規則，訂明鞋帶不能連續穿過三個或以上同一行的孔眼 [1]，換言之即是我們不能在同一行穿一條長直線。

「緊密」綁法還有多一個額外條件，就是鞋帶必須在 A、B 行之間交錯，不能包含垂直線段。圖二顯示三種最常見的緊密綁法，分別是交叉綁法（crisscross）、歐洲綁法（European）和鞋店綁法（shoe-store）[2]。

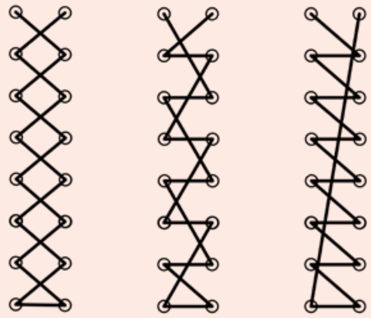


Figure 2 Common dense lacing methods: crisscross (left), European (middle), and shoe-store (right).

圖二 常見的緊密綁法：交叉綁法（左）、歐洲綁法（中）和鞋店綁法（右）

On the other hand, two common methods that use vertical segments are Canadian straitlacing and the bowtie methods (Figure 3). (Canadian straitlacing is named for its historical usage by Canada's armed forces, mainly because boots laced this way can be easily removed with one cut of a knife in case of emergencies [3].)

另一方面，包含垂直線段的兩個常見綁法是加拿大五線譜綁法（Canadian straitlacing）和領結綁法（bowtie）（圖三）。（前者是傳統上加拿大軍隊使用的綁法，優點是在危急時用一刀就能把軍靴的鞋帶切開，容易把鞋脫出 [3]。）

Figure 3 Common lacing methods which use vertical segments: Canadian straitlacing (left) and bowtie (right). Gray lines represent the segments that cannot be seen from outside.

圖三 包含垂直線段的常見綁法：加拿大五線譜綁法（左）和領結綁法（右）。灰色線代表藏在底下的鞋帶。

The Shortest Lacing

First let's test the dense lacings. Assuming the laces are pulled tight, we could use Pythagoras' theorem to find formulas for their lengths in terms of h, g and n [4]; interested readers may try to derive them by referring to the diagrams:

最短的綁法

首先讓我們探討緊密綁法。假設鞋帶被拉緊，沒有因鬆弛而耗用多餘長度，我們就能用畢氏定理，以 h、g 和 n 表示各種綁法所需的鞋帶長度 [4]，有興趣的讀者可以試試看著圖樣推導公式：

Crisscross 交叉綁法 : $g + 2(n-1)\sqrt{(h^2+g^2)}$

European 歐洲綁法 : $(n-1)g + 2\sqrt{(h^2+g^2)} + (n-2)\sqrt{(4h^2+g^2)}$

Shoe-store 鞋店綁法 : $(n-1)g + (n-1)\sqrt{(h^2+g^2)} + \sqrt{[(n-1)2h^2+g^2]}$

But the alternating property gives us a nice way to compare their lengths without doing any calculation. Say one end of the lacing lies on column A, at eyelet A_k. Imagine setting a mirror on column B so column A is reflected; then we can draw the reflected column A as a new column C with the eyelet A_k reflected onto C_k. Continue "unfolding" the lacing in this way [5]. Now instead of being laced between two columns of eyelets, our lacing moves horizontally across Figure 4 column by column, while reflecting the real length of each segment throughout the lacing.

By considering the orange triangle in Figure 4, we know that the crisscross lacing must be shorter than the European lacing, because the European lacing always runs along two sides of a triangle while the crisscross runs along the third side. By the triangle inequality (the sum of any two sides of a triangle is longer than the third side) the European is longer than the crisscross. To compare the European and the shoe-store method we eliminate the horizontal segments from both lacings (each of them has n – 1 horizontal segments) and

然而它們線段交錯的性質使我們不需進行計算就能比較三者長度，譬如鞋帶線段的一端位於 A 行的 A_k 孔眼，假設我們在 B 行設置一塊鏡把 A 行反射，鏡中反射的 A 行鏡像現在就可以被畫在圖表上，並被命名為新設的 C 行，而 A_k 的鏡像則被標示為 C_k。透過繼續重覆這個步驟「解開」鞋帶 [5]，我們的鞋帶就不再是交錯在兩行之間，而變成逐行向橫發展的圖四，當中每個線段都反映其真實長度。

透過考慮圖四中的橙色三角形，我們知道以交叉綁法穿的鞋帶一定短於歐洲綁法，因為歐洲綁法的鞋帶反覆地在圖表裡構成三角形的兩條邊，而交叉綁法的鞋帶則成為餘下的第三條邊，因此根據三角不等式（三角形任何兩條邊的長度總和必定比餘下一條長），歐洲綁法比交叉綁法長。至於要比較歐洲綁法和鞋店綁法，我們可以暫時忽略水平線段（兩者同樣有 n – 1 個水平線段）和相同斜度的線段，把它們移除後會得到圖五（左）以深色線表示的路徑。

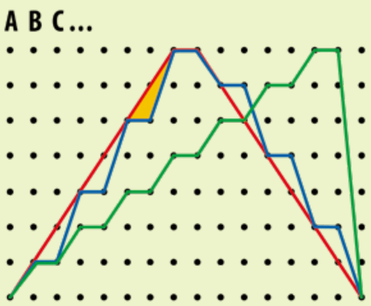


Figure 4 "Unfolded" lacing paths of the three alternating lacings: crisscross (red), European (blue), and shoe-store (green) [4]. The triangle shaded in orange can be used to compare the length of the crisscross and the European lacings.

圖四 三種交錯綁法經「拆解」後的路徑圖：交叉綁法（紅色）、歐洲綁法（藍色）和鞋店綁法（綠色）[4]。橙色三角形能被用於比較交叉綁法和歐洲綁法的長度。

現在如果我們在兩個 V 字的尖端放一塊水平方

the segments of matching slope. The remaining segments form the paths in darker colors in Figure 5 (left).

Now if we place a horizontal mirror at the tip of each V, the reflections in lighter colors show clearly that the shoe-store forms two sides of a triangle and the European forms the third (Figure 5 (right)). So again by the triangle inequality, the shoe-store is longer than the European, and of these three methods the crisscross is always the shortest. In fact, it can be shown that the crisscross is the shortest among all possible alternating lacings [2].

向的鏡，以淺色表示的反射影像清楚顯示鞋店綁法的線段成了三角形的兩條邊，而歐洲綁法的線段則構成第三條邊（圖五（右））。所以再一次根據三角不等式，鞋店綁法比歐洲綁法長；而三者之中交叉綁法必為最短。事實上，交叉綁法亦已被證實是所有可行交錯綁法中最短的一款 [2]。

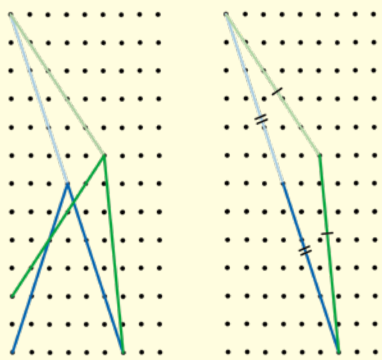


Figure 5 (Left) “Unfolded” lacing paths of the European (blue) and the shoe-store (green) lacings and their reflections (lighter colors) after subtracting common segments [3]. (Right) The triangle to determine the length of both lacing methods.

圖五 （左）減去共同線段後歐洲綁法（藍色）和鞋店綁法（綠色）經「拆解」後的路徑及其反射影像（以淺色表示）[3]；（右）判斷兩者長度需考慮的三角形。

To find the shortest lacing in general, consider that we must keep the two sides of the shoe pulled together. The shortest way to do this while also advancing vertically is to use crossings; notice that lacings with only horizontal and vertical parts, such as the Canadian straitlacing, use excessively long vertical segments. Hence it is necessary for the shortest lacing to contain crossings. There are many possible complex crossings with overlapping crosses, but the shortest is clearly a simple cross between the pairs of eyelets in two adjacent rows. So the shortest lacing must contain only simple crosses. But by the triangle

至於要找出所有綁法中最短的一款，記得穿鞋帶的目的是要把鞋的左右兩半拉在一起。打交叉是用最少鞋帶就能把鞋水平拉緊，而又能垂直地向下一行孔眼發展的方法。雖然包括加拿大五線譜綁法在內的一些綁法只用水平和垂直線段，但是這些方法都耗用極長的垂直線段，因此最短的綁法需要包含交叉。也有綁法包含由多個重疊交叉組合而成的複雜圖案，但最短的綁法明顯地只需在兩行孔眼之間打出簡單交叉，所以最短的綁法只能包含簡單交叉。然而根據三角不等式，兩條垂直線段長度之和總是短於任何交叉（見圖六），所以我們要儘可能增加垂直線段的數目，同時亦要減少簡單交叉。

inequality, the sum of two vertical segments is shorter in length than any crossing (as visualized in Figure 6), so we want to maximize the number of vertical segments and minimize the number of simple crosses.

This can be obtained from the crisscross by adding in the maximum number of vertical lines to replace some crosses without making consecutive vertical lines, which exactly describes the bowtie lacing! Since only the bowtie has these exact characteristics, it must be the shortest possible lacing [6].

在顧及不可連續綁出垂直線段的前提下，我們可以在交叉綁法中儘可能加入垂直線段代替部分交叉，而得出的綁法不就是領結嗎？因為只有領結綁法符合這些描述，所以它必定是最短的綁法 [6]。

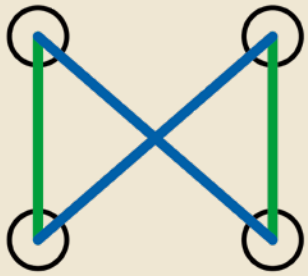


Figure 6 Comparison of the length of a simple cross (blue) and two vertical segments (green).

圖六 簡單交叉（藍色）和兩條垂直線段（綠色）之間的長度比較。

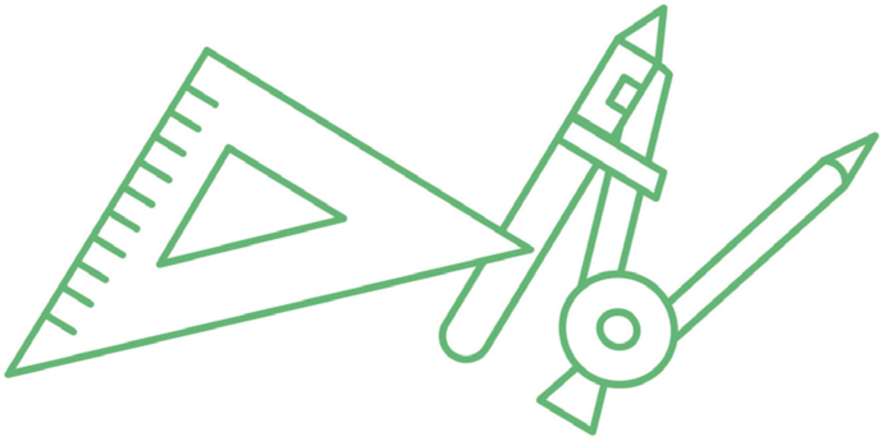
The Strongest Lacing

Now, what is the strongest lacing? When you pull on your shoelaces to tighten them, the lacing at each eyelet acts like a pulley, and the question can be restated in terms of finding the strongest pulleys. When tied, the tension along a shoelace is some constant force T. The important thing to measure is the horizontal tension T_h, the direction in which the two sides of the shoe are being pulled together by the lacing. For horizontal segments, $T_h = T$; for vertical segments, $T_h = 0$; and for diagonal segments, T_h is the horizontal component of T which can be calculated by trigonometry. Then the total sum of T_h across the eyelets of a

最穩妥的綁法

那麼，哪種才是最穩妥的綁法？當你索緊鞋帶時，孔眼和鞋帶的運作原理就像滑輪（pulley），因此問題可以改為如何打造最強的滑輪組。穿好鞋帶後，其張力為恆定的 T；而我們關注的是水平張力 T_h，因為水平是把鞋的左右兩邊拉在一起的方向。在水平線段中，$T_h = T$；在垂直線段中，$T_h = 0$；在斜向線段中，T_h 是 T 的水平分量（horizontal component），數值可以藉簡單的三角函數計算。一個綁法中每個孔眼間 T_h 的總和被稱為滑輪和（pulley sum），而最穩妥的綁法應擁有最大的滑輪和。

再進一步討論之前，讓我們設兩行孔眼之間的間隙 g 為 1 個單位。這能簡化之後的計算，因為



lacing is called its pulley sum, and the strongest lacing has the largest possible pulley sum.

Before we go further, let's set the gap g between columns to 1 unit. This will simplify any calculations because now h, the height between successive eyelets in the same column, can also be thought of as the ratio of {height between eyelets}/{gap between columns}.

There is a nice theorem that the strongest possible lacings are also the two most commonly used: crisscross and shoe-store [6]. To determine which of the two lacings is stronger in a given case, we need to consider the number of eyelets n, and the relative height between eyelets h.

For n > 2, let C(n, h) be the pulley sum of the crisscross lacing and S(n, h) be that of the shoe-store lacing. Then for a given number n of eyelets, there exists exactly one $h_n > 0$ such that $C(n, h_n) = S(n, h_n)$; and furthermore:

- when $h < h_n$, the crisscross lacing is strongest;
- when $h = h_n$, both lacings are strongest;
- when $h > h_n$, the shoe-store lacing is strongest.

In other words, the strongest lacing is the crisscross when the eyelets are close together in a column, and the shoe-store when they are farther apart.

The approximate values of h_n for small n are listed in the following table [6].

現在同一行上下孔眼之間的高度 h 亦可以被想作 { 上下孔眼間高度 } / { 左右孔眼間隙 } 之比。

已有理論證明最穩妥的綁法為最常見的交叉綁法和鞋店綁法 [6]。為了決定在某一情況下哪種才是較穩妥的綁法，我們需考慮孔眼數目 n 和上下孔眼間的相對高度 h。

在 n > 2 的情況下，設 C(n, h) 為交叉綁法的滑輪和，S(n, h) 為鞋店綁法的滑輪和。每個孔眼數目 n 都有一個對應的 $h_n > 0$ 使 $C(n, h_n) = S(n, h_n)$，而：

- 當 $h < h_n$，交叉綁法最為穩妥；
- 當 $h = h_n$，兩種綁法同樣穩妥；
- 當 $h > h_n$，鞋店綁法最為穩妥。

換言之，當孔眼上下高度比較接近，最穩妥的會是交叉綁法；上下高度比較遠時，最穩妥的則會是鞋店綁法。

對於數值較小的 n，h_n 的近似值可見於表一 [6]。

你可以自行量度鞋上高度間隙之比找出相對高度 h，再透過比對孔眼數目 n，在表上找出 h_n 的相應值。筆者寫這篇文章時穿著以交叉綁法穿的鞋，鞋子有 4 對孔眼，而 h 值約為 0.4，所以我在穿鞋帶時顯然做了正確的決定。當然，現在鞋店裡售賣的鞋都用上 h 值非常接近 h_n 的設計，所以無論你選擇哪款綁法都會非常穩妥 [6]。

n	3	4	5	6	7	8	9	10
h_n	0.9029	0.7412	0.6450	0.5794	0.5309	0.4931	0.4625	0.4372

Table 1 The approximate values of h_n for small n.
表一 n 在數值較小的情況下 h_n 的近似值

You can measure h as a ratio on shoes with n pairs of eyelets, and compare n with the corresponding value of h_n in the table. As I write this I'm wearing a pair of shoes laced by the crisscross method: They have 4 pairs of eyelets and h with value roughly 0.4, so obviously I made the right decision when I laced them. Of course, most shoes in stores today are made with h very close to h_n, so whichever lacing you use will be quite strong regardless [6].

So how should you lace your shoes? It depends on whether you prioritize shorter or stronger lacings. The one that best balances minimal length and maximal strength seems to be the crisscross, so without other information you should go for that one, although bowtie can give a reasonable strength that at least your shoes won't fall apart. But such information is always at your fingertips if you have a ruler, internet access (or a copy of *Science Focus*), and an inquiring mind. The last one is the most important of these – with that in hand, you can find answers to almost any question that comes your way.

所以我們應該怎樣穿鞋帶呢？這視乎你想選擇使用更少鞋帶的綁法，還是採取更穩妥的方法。如果要在最短的長度和最大的穩妥程度之間取個平衡，交叉綁法似乎是個最佳選擇，所以在沒有數值在手的情況下你應該選擇這種綁法；雖然領結綁法也能給你可接受的穩妥程度，它至少不會使你的鞋子分開兩截。然而這些數值其實也近在手邊，只要你有尺子、網絡（或《科言》實體書）和追根究底的「八卦」天性就足夠；最後一樣尤其重要，有了它，你就幾乎能解決任何迎面而來的問題。

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