Competition Rules and Objective Functions for the 10th DIMACS Implementation Challenge on Graph Partitioning and Graph Clustering

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Summary: There will be six (sub)challenges or competitions in total (two in each of three categories). For (nearly) balanced graph partitioning, there will be one category. It combines two measures (edge cut and communication volume) with equal importance. For graph clustering it was taken into consideration that solver results do not always match the expectation of the underlying applications, for instance depending on the optimization criterion. Thus, there will be two graph clustering categories. One is solely based on the established measure modularity, the other one combines several other measures, see Section 2.2.2. Within each category there will be two challenges, one purely based on quality and one that takes both the quality of a solution and the amount of work to compute it into account.

Description

In the following, let $G = (V, E, \omega)$ be the input graph with the edge weight function ω.

1 Graph Partitioning (GP)

There will be two challenges on graph partitioning, one *Quality Challenge* and one *Pareto Challenge*. The rationale of the Pareto Challenge is to take the work into account an algorithm requires to compute a solution. Hence, the two dimensions considered here are *quality* and *work*. Work will be normalized with respect to the machine performance, measured by a graph-based benchmark, see Section 3.

We describe the optimization objectives and the scoring rules in detail below.

1.1 Objective Functions

1.1.1 Minimizing the Balanced Edge Cut (EC)

- **Output**: Compute a partition Π of V into k parts of size at most $(1+\epsilon)\lceil \frac{|V|}{k} \rceil$, where k can be any power of 2 between 2 and 1024 and $\epsilon = 0.03$ (we might include more values for ϵ in the future, for example $\epsilon = 0$ and $\epsilon = 0.1$; if so, this will be announced before June 15, 2011). Note that the final competition does not necessarily include all powers of 2 in the range given above.
- **Objective/evaluation**: Smallest total weight of the set of cut edges $C = \{\{u, v\} \in E \mid \Pi(u) \neq \Pi(v)\}$. If no weights are used, then total size of C.

1.1.2 Minimizing the Maximum Communication Volume (CV)

- **Output**: As in Section 1.1.1 (edge cut minimization).
- **Objective/evaluation**: Let w denote a possible vertex weight function. If no vertex weights are used, $w(v) = 1$ for all $v \in V$ by definition. The communication volume for part π_p, $1 \leq p \leq k$, is defined as $comm(\pi_p) := \sum_{v \in \pi_p} w(v) f(v)$, where $f(v)$ denotes the number of different parts in which v has a neighbor vertex, excluding π_p. The partition $\Pi = \bigcup_{p=1}^{k} \pi_p$ is evaluated based on $\max_p comm(\pi_p)$.[1]

1.2 Quality Challenge Scoring

We anticipate the following scoring rules, which might have to be adapted slightly to the number of solver submissions.

For each challenge instance result (EC and CV results are counted as one instance *each*), points are given to the best ranks based on the Formula 1 scoring rules used between 1991 and 2002.[2] This means that the first six ranks receive a descending number of points (10, 6, 4, 3, 2, 1). The solver with the highest total number of points wins the Quality Challenge.

1.3 Pareto Challenge Scoring

The Formula 1 scoring scheme is also used in the Pareto Challenge. However, it might become necessary to adapt this scheme based on the number of participating solvers. In that process, the Pareto scoring scheme might become (slightly) different from the scoring scheme in the Quality Challenge.

Recall that the two dimensions considered here are quality and work. For each challenge instance result, each algorithm gets a Pareto dominance count, which expresses by how many other algorithms it has been Pareto-dominated; then algorithms are ranked by this number (lower count = better) and receive points according to the F1 scoring scheme (until further notice).

2 Graph Clustering (GC)

There will be two categories (modularity optimization, mix challenge) with two challenges each (one *Quality Challenge* and one *Pareto Challenge*) on graph clustering to account for the fact that the quality of a clustering can differ significantly depending on the underlying application. Again, the rationale of the Pareto Challenge is to take the work into account an algorithm requires to compute a solution and the two dimensions considered in the Pareto Challenges are *quality* and *work*. Work will be normalized with respect to the machine performance, measured by a graph-based benchmark.

We describe the optimization objectives and the scoring rules in detail below. Note that the quality measure definitions can be found in Appendix A.

2.1 Objective Functions

2.1.1 GC Objective 1: Modularity

- **Output:** Compute a partition $\mathcal{C}$ of V.
- **Objective/evaluation:** Highest modularity of $\mathcal{C}$. If edge weights are used, then they are also used to compute the modularity value; otherwise all edge weights are defined to be 1.

[1] Also see the Metis user guide, `http://glaros.dtc.umn.edu/gkhome/fetch/sw/metis/manual.pdf`, page 19, where the total communication volume is mentioned.

[2] See `https://secure.wikimedia.org/wikipedia/en/wiki/List_of_Formula_One_World_Championship_points_scoring_systems`.

2.1.2 GC Objective 2: Mix

- **Output:** Compute a partition $\mathcal{C}$ of V such that $\text{cov}(\mathcal{C}) \geq 0.5$ and $\overline{\text{cov}}(C) \geq 0.5$.
- **Objectives:** Optimize performance, average isolated inter-cluster conductance, average isolated inter-cluster expansion, and minimum intra-cluster density at the same time. If edge weights are used, then they are also used to compute the function values; otherwise all edge weights are defined to be 1.

2.2 Quality Challenge Scoring

2.2.1 Modularity Quality Challenge

The scoring scheme used for this Challenge is analogous to the one described in Section 1.2.

2.2.2 Mix Quality Challenge

A solution for a particular challenge instance is ranked as follows: Let r_1 be the rank of $\mathcal{C}$ among all competing solutions with respect to performance (e.g. if $r_1 = 2$, then there is only one competing solution with a performance higher than the performance of $\mathcal{C}$). Similarly, let r_2 be the rank of $\mathcal{C}$ with respect to $-$aixc, r_3 the rank with respect to mid and r_4 the rank with respect to $-$aixe. The mixed final rank r of the solution for the mix challenge is computed as $r = \frac{1}{4} \cdot (r_1 + r_2 + r_3 + r_4)$. Based on the mixed final rank r, Formula 1 points are assigned analogous to the description in Section 1.2.

2.3 Pareto Challenge Scoring

For each of the two clustering challenges (modularity and mix), we have one Pareto challenge each. Note that the Mix Pareto Challenge uses the combined rank r described in Section 2.2.2 as pseudo-quality function. The scoring of the Pareto tuples will be analogous to the scheme described in Section 1.3.

3 Speed Considerations

Since we do not anticipate that all submissions will run on the same (or at least similar) hardware, there should also be a measure to account for different execution speeds. We will provide a graph-based benchmark that is to be executed to measure the system performance. More details will be announced at a later date.

A Clustering Measures

Let $G = (V, E, \omega)$ be an undirected, weighted graph without parallel edges and with non-empty sets V and E. In the following, we assume $e = \{u, v\} \in E$ is a multiset, i.e., $u = v$ is allowed. We also assume the sum iterating over an empty set (for example $\sum_{v\in\emptyset} \omega(v)$) to be zero. **Addendum for aixc**: Clusters without edges contribute 0 to outer sum over $C \in \mathcal{C}$ (as if $0/0 = 0$).
Let $M = \max_{e\in E} \omega(e)$ be the maximum weight of an edge in the graph. Then, in a slight abuse of notation, let the vertex weight function be defined as:

$$\omega(v) = \begin{cases} \sum_{\{u,v\}\in E} \omega(e) & \text{if } \{v,v\} \notin E \\ \sum_{\{u,v\}\in E, u\neq v} \omega(e) + 2 \cdot \omega(\{v,v\}) & \text{if } \{v,v\} \in E \end{cases}$$

We define the following quality measures for a clustering $\mathcal{C}$:

- Modularity:

$$\text{modularity}(\mathcal{C}) := \frac{\displaystyle\sum_{C\in\mathcal{C}} \sum_{\substack{\{u,v\}\in E\\ u,v\in C}} \omega(\{u,v\})}{\displaystyle\sum_{e\in E}\omega(e)} - \frac{\displaystyle\sum_{C\in\mathcal{C}}\left(\sum_{v\in C}\omega(v)\right)^2}{4\left(\displaystyle\sum_{e\in E}\omega(e)\right)^2}$$

- Minimum intra-cluster density:

$$\text{mid}(\mathcal{C}) := \min_{C\in\mathcal{C}} \begin{cases} \dfrac{\displaystyle\sum_{\substack{\{u,v\}\in E\\ u\neq v\in C}} \omega(\{u,v\})}{\binom{|C|}{2}\cdot M} & \text{, if } |C|>1 \\ 1 & \text{, otherwise} \end{cases}$$

- Average isolated inter-cluster conductance:

$$\text{aixc}(\mathcal{C}) := \frac{1}{|\mathcal{C}|}\sum_{C\in\mathcal{C}} \frac{\displaystyle\sum_{\substack{\{u,v\}\in E\\ u\in C, v\notin C}} \omega(\{u,v\})}{\min\{\displaystyle\sum_{v\in C}\omega(v), \sum_{v\in V\setminus C}\omega(v)\}}$$

- Average isolated inter-cluster expansion:

$$\text{aixe}(\mathcal{C}) := \frac{1}{|\mathcal{C}|}\sum_{C\in\mathcal{C}} \frac{\displaystyle\sum_{\substack{\{u,v\}\in E\\ u\in C, v\notin C}} \omega(\{u,v\})}{\min\{|C|, |V\setminus C|\}}$$

- Performance: Let $\omega(\{u,v\})$ be defined as zero, if $\{u,v\}\notin E$.

$$\text{performance}(\mathcal{C}) := \frac{\displaystyle\sum_{C\in\mathcal{C}} \sum_{\substack{\{u,v\}\in E\\ u\neq v\in C}} \omega(\{u,v\}) + \sum_{C\neq D\in\mathcal{C}} \sum_{u\in C, v\in D} (M-\omega(\{u,v\}))}{\binom{|V|}{2}\cdot M}$$

- Coverage:

$$\text{cov}(\mathcal{C}) := \frac{\displaystyle\sum_{C\in\mathcal{C}} \sum_{\substack{\{u,v\}\in E\\ u,v\in C}} \omega(\{u,v\})}{\displaystyle\sum_{e\in E}\omega(e)}$$

- Mirror Coverage (Note: This equals one minus the coverage of $\mathcal{C}$ in the complement graph): Let $\omega(\{u,v\})$ be defined as zero, if $\{u,v\}\notin E$.

$$\overline{\text{cov}}(\mathcal{C}) := \frac{\displaystyle\sum_{C\neq D\in\mathcal{C}} \sum_{u\in C, v\in D} (M-\omega(\{u,v\}))}{\displaystyle\sum_{u\neq v\in V} (M-\omega(\{u,v\}))}$$