Online Supplement: An expectation conditional maximization algorithm for the skew-normal based stochastic frontier model

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Supplementary Material

This supplementary material includes the detailed information about the simulation study in Section 4 of main text.

The following six simulation scenarios with moderate skewness $\lambda = -5, -2$, and relative large skewness $\lambda = -10$ with small and moderate sample sizes $n = 50$, 100, and 200, are considered:

Scenario 1. ECM v.s. NM: $\beta = 0.5$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$, $\lambda = -5$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = -1$ for cost function.

Scenario 2. ECM v.s. NM: $\beta = 1$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$, $\lambda = -5$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = 1$ for production function.

Scenario 3. ECM v.s. NM: $\beta = 0.5$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$,

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$\lambda = -2$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = -1$.

Scenario 4. ECM v.s. NM: $\beta = 1$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$, $\lambda = -2$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = 1$.

Scenario 5. ECM v.s. NM: $\beta = 0.5$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$, $\lambda = -10$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = -1$.

Scenario 6. ECM v.s. NM: $\beta = 1$, $V \sim SN(0, \sigma_v^2, \lambda)$ with $\sigma_v = 1$, $\lambda = -10$, and $U \sim HN(0, \sigma_u^2)$ with $\sigma_u = 0.5$ and $s = 1$.

Table 1 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under simulation Scenario 1 for 1000 simulated data sets.

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n=50$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.520	0.141	0.520	0.142	0.522	0.151	0.520	0.142	0.520	0.142	0.511	0.137
$\hat{\sigma}_v$	1.0	0.962	0.085	0.962	0.085	0.964	0.085	0.960	0.084	0.960	0.084	0.968	0.080
$\hat{\sigma}_u$	0.5	0.417	0.167	0.417	0.168	0.420	0.169	0.413	0.163	0.412	0.163	0.439	0.149
$\hat{\lambda}$	-5.0	-1256.804	4712.939	-1108.443	4233.623	-2818.076	12027.745	-5.142	4.490	-5.088	4.426	-6.728	5.033
$\hat{\lambda}_w$	-5.0	-1066.343	3263.589	-853.427	2796.913	-2163.537	6768.167	-5.014	3.671	-4.965	3.577	-6.703	4.952
$\hat{\lambda}_T$	-5.0	-475.388	1599.549	-313.641	1151.119	-907.158	3132.608	-4.714	3.097	-4.676	3.023	-6.473	4.422

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n=100$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.484	0.194	0.482	0.197	0.476	0.196	0.483	0.192	0.484	0.192	0.473	0.186
$\hat{\sigma}_v$	1.0	0.943	0.112	0.941	0.115	0.952	0.105	0.942	0.110	0.941	0.111	0.956	0.100
$\hat{\sigma}_u$	0.5	0.416	0.184	0.411	0.189	0.420	0.167	0.415	0.173	0.412	0.175	0.457	0.151
$\hat{\lambda}$	-5.0	-2299.816	9054.092	-2730.388	9102.090	-9033.763	28666.134	-5.234	4.379	-5.174	4.283	-7.649	6.224
$\hat{\lambda}_w$	-5.0	-1763.478	5375.246	-2470.002	7527.073	-7516.779	19808.067	-5.153	4.079	-5.099	4.004	-7.540	5.779
$\hat{\lambda}_T$	-5.0	-780.719	2585.598	-1093.253	3535.225	-4430.423	12527.677	-4.799	3.383	-4.767	3.356	-7.290	5.150

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n=200$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.455	0.270	0.445	0.279	0.446	0.270	0.453	0.267	0.454	0.268	0.443	0.261
$\hat{\sigma}_v$	1.0	0.915	0.151	0.907	0.162	0.925	0.147	0.915	0.149	0.913	0.150	0.929	0.140
$\hat{\sigma}_u$	0.5	0.403	0.212	0.387	0.226	0.437	0.197	0.409	0.192	0.403	0.196	0.451	0.175
$\hat{\lambda}$	-5.0	-6147.461	18154.508	-6532.214	21056.548	-19346.642	47032.341	-5.715	5.769	-5.679	5.810	-8.112	7.294
$\hat{\lambda}_w$	-5.0	-5410.104	14195.274	-5695.146	16304.836	-18453.644	43239.918	-5.534	4.998	-5.500	5.065	-7.933	6.672
$\hat{\lambda}_T$	-5.0	-3136.300	8530.876	-2790.288	7967.408	-12212.398	28855.814	-5.119	4.229	-5.069	4.272	-7.635	5.957

Table 2 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under Scenario 2 for 1000 simulated data sets.

		NM						ECM					
$n{=}50$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.926	0.254	0.913	0.263	0.918	0.257	0.901	0.260	0.899	0.261	0.922	0.254
$\hat{\sigma}_v$	1.0	0.617	0.450	0.568	0.489	0.691	0.410	0.499	0.535	0.505	0.530	0.870	0.249
$\hat{\sigma}_u$	0.5	0.849	0.439	0.900	0.455	0.768	0.400	0.960	0.498	0.955	0.493	0.586	0.268
$\hat{\lambda}$	-5.0	-781.804	1407.608	-767.642	1439.057	-883.959	1643.532	-6.071	5.825	-5.816	5.448	-9.653	9.952
$\hat{\lambda}_w$	-5.0	-751.868	1304.391	-740.578	1342.937	-854.714	1536.978	-5.979	5.463	-5.762	5.247	-9.505	9.337
$\hat{\lambda}_T$	-5.0	-640.016	1097.549	-617.339	1115.798	-708.910	1257.050	-5.601	4.667	-5.396	4.494	-8.598	7.021

		NM						ECM					
$n{=}100$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.956	0.180	0.955	0.180	0.954	0.180	0.942	0.178	0.942	0.178	0.956	0.176
$\hat{\sigma}_v$	1.0	0.605	0.460	0.556	0.488	0.712	0.398	0.503	0.529	0.505	0.525	0.896	0.240
$\hat{\sigma}_u$	0.5	0.882	0.454	0.934	0.473	0.768	0.397	0.980	0.506	0.977	0.503	0.579	0.264
$\hat{\lambda}$	-5.0	-548.573	1189.182	-544.971	1181.034	-584.392	1327.995	-4.912	4.788	-4.764	4.746	-8.542	10.251
$\hat{\lambda}_w$	-5.0	-527.741	1107.578	-529.323	1125.141	-551.467	1192.616	-4.808	4.397	-4.645	4.168	-8.115	7.270
$\hat{\lambda}_T$	-5.0	-398.682	853.134	-397.235	866.301	-402.491	884.657	-4.405	3.653	-4.273	3.500	-7.407	5.462

		NM						ECM					
$n{=}200$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.993	0.127	0.992	0.127	0.992	0.127	0.984	0.123	0.984	0.123	0.991	0.124
$\hat{\sigma}_v$	1.0	0.592	0.451	0.569	0.465	0.714	0.379	0.530	0.494	0.532	0.491	0.890	0.237
$\hat{\sigma}_u$	0.5	0.917	0.459	0.942	0.468	0.790	0.392	0.976	0.493	0.975	0.491	0.607	0.261
$\hat{\lambda}$	-5.0	-251.163	825.093	-236.497	772.072	-280.149	936.637	-3.966	3.685	-3.880	3.425	-6.704	5.775
$\hat{\lambda}_w$	-5.0	-223.175	681.274	-213.727	657.830	-245.003	752.653	-3.856	3.210	-3.806	3.155	-6.498	4.739
$\hat{\lambda}_T$	-5.0	-103.788	366.422	-97.737	353.608	-111.180	400.114	-3.524	2.676	-3.483	2.641	-6.010	3.451

Table 3 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under Scenario 3 for 1000 simulated data sets.

		NM						ECM					
$n{=}50$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.461	0.317	0.449	0.328	0.449	0.315	0.458	0.315	0.456	0.320	0.451	0.300
$\hat{\sigma}_v$	1.0	0.958	0.152	0.951	0.159	0.974	0.149	0.960	0.148	0.956	0.152	0.981	0.140
$\hat{\sigma}_u$	0.5	0.447	0.248	0.421	0.259	0.506	0.243	0.465	0.213	0.450	0.215	0.531	0.215
$\hat{\lambda}$	-2.0	-6076.404	20523.342	-5356.021	17641.647	-13158.913	35833.048	-4.648	6.114	-4.565	6.333	-7.192	8.365
$\hat{\lambda}_w$	-2.0	-5440.870	17130.449	-4885.812	15188.298	-12400.385	32517.653	-4.465	5.342	-4.347	5.321	-7.040	7.821
$\hat{\lambda}_T$	-2.0	-2222.813	7488.340	-2086.233	7116.580	-7230.291	19892.576	-4.004	4.398	-3.878	4.349	-6.695	7.097

		NM						ECM					
$n{=}100$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.492	0.233	0.489	0.233	0.481	0.231	0.491	0.230	0.491	0.231	0.478	0.225
$\hat{\sigma}_v$	1.0	0.981	0.111	0.977	0.115	0.995	0.109	0.983	0.106	0.981	0.108	1.001	0.103
$\hat{\sigma}_u$	0.5	0.459	0.223	0.444	0.233	0.509	0.220	0.467	0.198	0.460	0.200	0.529	0.202
$\hat{\lambda}$	-2.0	-2148.864	10034.275	-1501.571	6672.304	-4390.167	17703.191	-3.619	4.245	-3.595	4.248	-5.658	6.240
$\hat{\lambda}_w$	-2.0	-1425.096	5057.101	-1141.308	4230.792	-3234.480	10455.356	-3.506	3.632	-3.489	3.662	-5.563	5.864
$\hat{\lambda}_T$	-2.0	-436.996	2084.803	-280.193	1377.344	-1299.585	4972.154	-3.149	2.863	-3.124	2.877	-5.287	5.319

		NM						ECM					
$n{=}200$		Initial 1		Initial 2		Initial 3		Initial 1		Initial 2		Initial 3	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.505	0.173	0.504	0.176	0.496	0.173	0.505	0.172	0.505	0.173	0.491	0.169
$\hat{\sigma}_v$	1.0	0.982	0.093	0.981	0.096	0.993	0.092	0.983	0.089	0.982	0.090	1.000	0.088
$\hat{\sigma}_u$	0.5	0.455	0.206	0.449	0.210	0.491	0.206	0.458	0.188	0.455	0.189	0.515	0.195
$\hat{\lambda}$	-2.0	-583.078	3531.315	-610.449	3169.393	-1077.198	5625.668	-3.059	3.121	-3.047	3.156	-5.149	5.499
$\hat{\lambda}_w$	-2.0	-316.455	1363.255	-403.314	1738.878	-683.662	2809.003	-2.994	2.814	-2.973	2.791	-5.107	5.365
$\hat{\lambda}_T$	-2.0	-26.297	173.693	-32.151	218.556	-99.543	588.296	-2.657	1.980	-2.642	1.976	-4.841	4.861

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Table 4 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under Scenario 4 for 1000 simulated data sets.

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 50$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	1.003	0.309	0.993	0.326	0.990	0.340	0.978	0.344	0.977	0.344	0.995	0.332
$\hat{\sigma}_v$	1.0	0.826	0.295	0.687	0.386	0.816	0.305	0.630	0.418	0.630	0.418	0.945	0.191
$\hat{\sigma}_u$	0.5	0.664	0.368	0.839	0.432	0.687	0.366	0.913	0.462	0.915	0.463	0.537	0.264
$\hat{\lambda}$	-2.0	-218.235	740.675	-35.587	3247.246	263.982	4757.037	-2.212	4.121	-2.100	3.628	-3.664	4.829
$\hat{\lambda}_w$	-2.0	-187.905	603.922	-158.990	589.366	-168.102	551.023	-2.249	3.230	-2.233	3.182	-3.730	4.587
$\hat{\lambda}_T$	-2.0	-85.283	361.476	-41.156	221.938	-70.937	309.750	-1.760	1.896	-1.765	1.967	-3.090	3.010

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 100$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.994	0.246	0.993	0.247	0.993	0.247	0.987	0.251	0.986	0.251	0.993	0.246
$\hat{\sigma}_v$	1.0	0.856	0.271	0.771	0.323	0.883	0.253	0.651	0.391	0.650	0.391	0.957	0.193
$\hat{\sigma}_u$	0.5	0.645	0.346	0.750	0.395	0.622	0.321	0.905	0.448	0.906	0.448	0.533	0.256
$\hat{\lambda}$	-2.0	-54.220	574.959	-40.580	314.681	-44.914	346.186	-1.459	1.804	-1.455	1.829	-2.572	2.581
$\hat{\lambda}_w$	-2.0	-1.993	0.966	-1.729	1.011	-2.131	1.230	-1.322	1.129	-1.313	1.125	-2.382	1.282
$\hat{\lambda}_T$	-2.0	-1.904	0.700	-1.642	0.787	-2.023	0.802	-1.252	0.974	-1.247	0.979	-2.248	0.817

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 200$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.989	0.168	0.989	0.168	0.989	0.168	0.986	0.169	0.986	0.169	0.990	0.167
$\hat{\sigma}_v$	1.0	0.842	0.259	0.797	0.286	0.894	0.226	0.669	0.361	0.668	0.361	0.951	0.172
$\hat{\sigma}_u$	0.5	0.670	0.321	0.724	0.353	0.612	0.279	0.885	0.417	0.886	0.417	0.544	0.220
$\hat{\lambda}$	-2.0	-1.725	0.745	-1.592	0.776	-1.892	0.795	-1.201	0.976	-1.199	0.978	-2.057	0.738
$\hat{\lambda}_w$	-2.0	-1.712	0.700	-1.582	0.738	-1.879	0.725	-1.191	0.945	-1.190	0.947	-2.044	0.663
$\hat{\lambda}_T$	-2.0	-1.681	0.620	-1.551	0.676	-1.857	0.590	-1.183	0.908	-1.182	0.910	-2.024	0.534

Table 5 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under Scenario 5 for 1000 simulated data sets.

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 50$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.428	0.258	0.422	0.270	0.417	0.256	0.426	0.257	0.426	0.258	0.416	0.248
$\hat{\sigma}_v$	1.0	0.911	0.160	0.904	0.170	0.918	0.156	0.910	0.159	0.908	0.161	0.924	0.149
$\hat{\sigma}_u$	0.5	0.415	0.191	0.397	0.209	0.442	0.176	0.417	0.177	0.411	0.181	0.457	0.153
$\hat{\lambda}$	-10.0	-8601.269	24526.916	-5990.448	17265.942	-22659.560	53112.283	-6.987	7.292	-6.879	7.397	-9.194	6.751
$\hat{\lambda}_w$	-10.0	-7703.213	19904.368	-5269.966	13433.061	-21729.484	49287.254	-6.767	6.634	-6.665	6.739	-9.079	6.395
$\hat{\lambda}_T$	-10.0	-4460.271	11484.351	-3142.453	8128.122	-14860.178	33759.385	-6.317	6.153	-6.180	6.217	-8.798	5.719

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 100$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.477	0.175	0.474	0.178	0.471	0.176	0.477	0.174	0.477	0.174	0.467	0.167
$\hat{\sigma}_v$	1.0	0.941	0.110	0.940	0.112	0.947	0.107	0.939	0.109	0.938	0.110	0.952	0.101
$\hat{\sigma}_u$	0.5	0.424	0.163	0.423	0.167	0.443	0.150	0.421	0.155	0.419	0.158	0.455	0.131
$\hat{\lambda}$	-10.0	-3390.862	9974.942	-3638.435	11324.990	-9326.790	26039.591	-6.243	6.064	-6.225	6.093	-8.457	5.682
$\hat{\lambda}_w$	-10.0	-2973.815	7843.929	-3194.042	8909.673	-8401.687	21231.074	-6.175	5.925	-6.152	5.951	-8.383	5.480
$\hat{\lambda}_T$	-10.0	-1712.983	4720.394	-1628.615	4543.551	-5143.284	13428.606	-5.895	5.730	-5.877	5.760	-8.194	5.007

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 200$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	0.5	0.502	0.131	0.502	0.131	0.499	0.138	0.504	0.130	0.504	0.130	0.494	0.125
$\hat{\sigma}_v$	1.0	0.960	0.084	0.960	0.084	0.963	0.081	0.957	0.084	0.957	0.084	0.966	0.076
$\hat{\sigma}_u$	0.5	0.436	0.143	0.436	0.143	0.446	0.137	0.429	0.141	0.428	0.141	0.457	0.120
$\hat{\lambda}$	-10.0	-1339.100	4043.510	-1451.094	4674.754	-4086.857	12170.311	-6.555	5.773	-6.561	5.827	-8.545	5.389
$\hat{\lambda}_w$	-10.0	-1204.519	3338.669	-1235.850	3409.059	-3564.778	9409.214	-6.427	5.376	-6.425	5.380	-8.382	4.757
$\hat{\lambda}_T$	-10.0	-625.787	1749.751	-650.695	1831.476	-2066.752	5775.747	-6.225	5.159	-6.215	5.156	-8.307	4.342

Table 6 The sample mean and RMSE of the MLE for parameters SN-SFM based on ECM-algorithms and NM algorithm given for sample sizes $n = 50, 100$ and 200, under Scenario 6 for 1000 simulated data sets.

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 50$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.944	0.248	0.937	0.253	0.937	0.252	0.921	0.250	0.920	0.251	0.940	0.247
$\hat{\sigma}_v$	1.0	0.630	0.438	0.581	0.474	0.700	0.400	0.499	0.536	0.506	0.528	0.869	0.254
$\hat{\sigma}_u$	0.5	0.841	0.434	0.893	0.451	0.766	0.391	0.964	0.502	0.958	0.495	0.591	0.269
$\hat{\lambda}$	-10.0	-807.511	1449.260	-757.650	1394.262	-895.028	1636.819	-6.670	9.500	-6.103	6.979	-9.877	9.398
$\hat{\lambda}_w$	-10.0	-784.386	1367.575	-732.876	1307.826	-869.291	1548.397	-6.271	6.725	-5.992	6.720	-9.574	7.920
$\hat{\lambda}_T$	-10.0	-658.231	1122.281	-607.542	1063.042	-722.832	1265.452	-5.886	6.340	-5.625	6.401	-8.768	6.138

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 100$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.974	0.180	0.972	0.180	0.972	0.181	0.958	0.176	0.959	0.177	0.971	0.177
$\hat{\sigma}_v$	1.0	0.607	0.453	0.575	0.473	0.697	0.402	0.507	0.525	0.512	0.519	0.885	0.252
$\hat{\sigma}_u$	0.5	0.888	0.456	0.923	0.468	0.794	0.408	0.981	0.509	0.978	0.505	0.600	0.272
$\hat{\lambda}$	-10.0	-581.222	1240.422	-573.587	1253.052	-671.273	1490.272	-5.455	12.356	-4.953	6.832	-8.914	9.705
$\hat{\lambda}_w$	-10.0	-551.028	1104.009	-541.470	1117.422	-633.888	1336.367	-4.990	6.612	-4.825	6.603	-8.569	7.638
$\hat{\lambda}_T$	-10.0	-432.123	878.845	-410.406	854.241	-465.962	980.535	-4.631	6.466	-4.483	6.486	-7.648	5.706

| | | NM | | | | | | ECM | | | | | |
| | | Initial 1 | | Initial 2 | | Initial 3 | | Initial 1 | | Initial 2 | | Initial 3 | |
$n = 200$	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
$\hat{\beta}$	1.0	0.986	0.131	0.986	0.131	0.986	0.131	0.977	0.127	0.977	0.128	0.985	0.128
$\hat{\sigma}_v$	1.0	0.604	0.441	0.574	0.459	0.727	0.368	0.534	0.488	0.534	0.488	0.875	0.248
$\hat{\sigma}_u$	0.5	0.902	0.448	0.935	0.461	0.774	0.379	0.969	0.486	0.969	0.485	0.619	0.265
$\hat{\lambda}$	-10.0	-234.826	752.702	-246.996	789.136	-294.477	943.125	-3.920	6.983	-3.897	6.959	-6.374	6.221
$\hat{\lambda}_w$	-10.0	-211.294	638.747	-226.343	683.618	-269.227	814.364	-3.845	6.876	-3.832	6.876	-6.194	5.634
$\hat{\lambda}_T$	-10.0	-99.159	348.801	-108.575	386.531	-126.444	440.133	-3.483	6.893	-3.479	6.898	-5.725	5.171