Quantification of the environmental structural risk with spoiling ties: Is randomization worthwhile?

SUPPLEMENTARY MATERIAL

Stochastic Environmental Research and Risk Assessment

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Δ_H	Δ_D	C^{Frk}	C^{Cly}	C^{Gmb}	C^{Nrm}	$C^{\text{t,4}}$
Independent randomization						
$\tau = 0.25$						
0.01	0.50	0.05124	0.79127	0.38351	0.13594	0.26872
0.10	1.00	0.04945	0.79477	0.39801	0.14384	0.27942
0.50	3.00	0.07014	0.85656	0.37201	0.21383	0.34402
$\tau = 0.5$						
0.01	0.50	0.05024	0.99835	0.79057	0.42781	0.70698
0.10	1.00	0.05244	0.99865	0.78977	0.45780	0.73238
0.50	3.00	0.13344	0.99985	0.78817	0.68108	0.88006
$\tau = 0.75$						
0.01	0.50	0.04695	0.99995	0.94936	0.87446	0.95415
0.10	1.00	0.05314	0.99995	0.95485	0.90776	0.96545
0.50	3.00	0.48510	0.99995	0.96105	0.98595	0.99805
Mixed randomization						
$\tau = 0.25$						
0.01	0.50	0.05624	0.78357	0.40301	0.14954	0.28982
0.10	1.00	0.04865	0.78467	0.39181	0.14284	0.27142
0.50	3.00	0.06284	0.80867	0.36471	0.17653	0.28032
$\tau = 0.5$						
0.01	0.50	0.05134	0.99745	0.78987	0.43161	0.70758
0.10	1.00	0.05074	0.99705	0.79227	0.43841	0.70018
0.50	3.00	0.08604	0.99775	0.77617	0.53160	0.71798
$\tau = 0.75$						
0.01	0.50	0.04665	0.99995	0.95155	0.87826	0.95105
0.10	1.00	0.05044	0.99995	0.95225	0.87116	0.94086
0.50	3.00	0.26952	0.99995	0.96675	0.94256	0.97885
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.50	0.05414	0.78937	0.38831	0.14044	0.27842
0.10	1.00	0.05454	0.77127	0.39981	0.13644	0.26362
0.50	3.00	0.11154	0.65968	0.51110	0.17803	0.23263
$\tau = 0.5$						
0.01	0.50	0.05104	0.99755	0.79537	0.43281	0.71168
0.10	1.00	0.05124	0.99655	0.79307	0.43211	0.68258
0.50	3.00	0.14354	0.98635	0.86526	0.50110	0.63939
$\tau = 0.75$						
0.01	0.50	0.04485	0.99995	0.94456	0.87906	0.95015
0.10	1.00	0.04625	0.99965	0.95305	0.86916	0.93116
0.50	3.00	0.37711	0.99995	0.98645	0.94596	0.97625

Table 1: **"Frank" case:** $N = 150$**,** $B = 10000$**.** Probability of rejection (nominal level 5%) of the Null hypothesis that the copula belongs, respectively, to the Frank (C^{Frk}), Clayton (C^{Cly}), Gumbel (C^{Gmb}), Normal (C^{Nrm}), and Student-t ($C^{\text{t,4}}$, with $\nu = 4$ degrees of freedom) family, for a random sample generated from a Frank copula, with $\tau = 0.25, 0.5, 0.75$ obtained from independent repetitions of the discretization and randomization procedures — see text.

Δ_H	Δ_D	C^{Frk}	C^{Cly}	C^{Gmb}	C^{Nrm}	$C^{\text{t,4}}$
Independent randomization						
$\tau = 0.25$						
0.01	0.50	0.04465	0.96075	0.72028	0.25282	0.58729
0.10	1.00	0.04515	0.96545	0.72578	0.25852	0.58699
0.50	3.00	0.07664	0.98395	0.71198	0.40311	0.69888
$\tau = 0.50$						
0.01	0.50	0.04155	0.99995	0.98875	0.79937	0.96845
0.10	1.00	0.04305	0.99995	0.98985	0.82347	0.97525
0.50	3.00	0.19843	0.99995	0.99185	0.95615	0.99695
$\tau = 0.75$						
0.01	0.50	0.03545	0.99995	0.99995	0.99725	0.99965
0.10	1.00	0.04265	0.99995	0.99985	0.99835	0.99975
0.50	3.00	0.78647	0.99995	0.99985	0.99995	0.99995
Mixed randomization						
$\tau = 0.25$						
0.01	0.50	0.04675	0.96225	0.72508	0.25832	0.58509
0.10	1.00	0.04595	0.95625	0.71688	0.26052	0.58359
0.50	3.00	0.06414	0.96565	0.69228	0.32792	0.59529
$\tau = 0.50$						
0.01	0.50	0.04275	0.99995	0.98915	0.80207	0.96975
0.10	1.00	0.04675	0.99995	0.99055	0.80187	0.96585
0.50	3.00	0.10064	0.99995	0.98765	0.87986	0.96985
$\tau = 0.75$						
0.01	0.50	0.03345	0.99995	0.99995	0.99535	0.99995
0.10	1.00	0.03945	0.99995	0.99965	0.99645	0.99935
0.50	3.00	0.46760	0.99995	0.99995	0.99955	0.99985
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.50	0.04485	0.96405	0.71368	0.25312	0.58819
0.10	1.00	0.04225	0.94796	0.71128	0.24493	0.55539
0.50	3.00	0.14684	0.87166	0.84107	0.31142	0.47770
$\tau = 0.50$						
0.01	0.50	0.04465	0.99995	0.98875	0.79737	0.96805
0.10	1.00	0.04775	0.99985	0.99025	0.78527	0.95415
0.50	3.00	0.24703	0.99995	0.99655	0.86776	0.93476
$\tau = 0.75$						
0.01	0.50	0.03445	0.99995	0.99985	0.99605	0.99985
0.10	1.00	0.04525	0.99995	0.99965	0.99475	0.99885
0.50	3.00	0.71128	0.99995	0.99995	0.99975	0.99995

Table 2: **"Frank" case:** $N = 300$**,** $B = 10000$**.** Same as Table 1.

Δ_H	Δ_D	C^{Gmb}	C^{Frk}	C^{Cly}	C^{Nrm}	$C^{\text{t,4}}$
Independent randomization						
$\tau = 0.25$						
0.01	0.5	0.03699	0.3469	0.9166	0.1949	0.2497
0.1	1	0.03699	0.3353	0.9086	0.1909	0.2381
0.5	3	0.04138	0.3828	0.9498	0.2821	0.3405
$\tau = 0.5$						
0.01	0.5	0.03659	0.6803	0.9998	0.378	0.41
0.1	1	0.03938	0.6771	0.9998	0.4044	0.4692
0.5	3	0.1098	0.7859	0.9998	0.7123	0.7691
$\tau = 0.75$						
0.01	0.5	0.03259	0.883	0.9998	0.4996	0.504
0.1	1	0.05858	0.8886	0.9998	0.6539	0.6551
0.5	3	0.5672	0.9806	0.9998	0.9842	0.987
Mixed randomization						
$\tau = 0.25$						
0.01	0.5	0.03739	0.3373	0.913	0.2033	0.2533
0.1	1	0.03619	0.3445	0.909	0.1929	0.2437
0.5	3	0.04098	0.3457	0.919	0.2277	0.2769
$\tau = 0.5$						
0.01	0.5	0.03778	0.6851	0.9998	0.3884	0.4348
0.1	1	0.03739	0.6831	0.9998	0.3964	0.4392
0.5	3	0.05738	0.7395	0.9998	0.56	0.5684
$\tau = 0.75$						
0.01	0.5	0.03059	0.8914	0.9998	0.4744	0.4868
0.1	1	0.03778	0.897	0.9998	0.55	0.52
0.5	3	0.2497	0.9702	0.9998	0.9042	0.8858
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.5	0.03619	0.3149	0.901	0.1849	0.2325
0.1	1	0.04098	0.3165	0.8998	0.1781	0.2181
0.5	3	0.07937	0.3501	0.8531	0.1741	0.1793
$\tau = 0.5$						
0.01	0.5	0.03459	0.6847	0.9998	0.3585	0.398
0.1	1	0.03898	0.6747	0.9998	0.3796	0.4028
0.5	3	0.08217	0.7715	0.9998	0.4532	0.4228
$\tau = 0.75$						
0.01	0.5	0.02779	0.8946	0.9998	0.4948	0.4972
0.1	1	0.04138	0.8874	0.9998	0.5344	0.498
0.5	3	0.2613	0.981	0.9998	0.8747	0.8455

Table 3: **"Gumbel" case: $N = 150$, $B = 2500$.** Probability of rejection (nominal level 5%) of the Null hypothesis that the copula belongs, respectively, to the Gumbel (C^{Gmb}), Frank (C^{Frk}), Clayton (C^{Cly}), Normal (C^{Nrm}), and Student-t ($C^{\text{t,4}}$, with $\nu = 4$ degrees of freedom) family, for a random sample generated from a Gumbel copula, with $\tau = 0.25, 0.5, 0.75$ obtained from independent repetitions of the discretization and randomization procedures — see text.

Δ_H	Δ_D	C^{Gmb}	C^{Frk}	C^{Cly}	C^{Nrm}	$C^{\text{t,4}}$
Independent randomization						
$\tau = 0.25$						
0.01	0.5	0.04218	0.5392	0.9906	0.3413	0.416
0.1	1	0.03739	0.5684	0.9958	0.3349	0.4288
0.5	3	0.04538	0.6188	0.9986	0.4808	0.6084
$\tau = 0.5$						
0.01	0.5	0.03898	0.9334	0.9998	0.6208	0.6779
0.1	1	0.05018	0.9262	0.9998	0.6683	0.7147
0.5	3	0.2181	0.9682	0.9998	0.9518	0.9718
$\tau = 0.75$						
0.01	0.5	0.03699	0.9966	0.9998	0.7347	0.7511
0.1	1	0.05218	0.993	0.9998	0.8966	0.8922
0.5	3	0.8858	0.9998	0.9998	0.9998	0.9998
Mixed randomization						
$\tau = 0.25$						
0.01	0.5	0.03778	0.5688	0.9938	0.3361	0.4308
0.1	1	0.04298	0.5524	0.9934	0.3329	0.4148
0.5	3	0.04778	0.5752	0.997	0.4064	0.4972
$\tau = 0.5$						
0.01	0.5	0.03379	0.9346	0.9998	0.6567	0.6883
0.1	1	0.03579	0.9342	0.9998	0.6539	0.6719
0.5	3	0.09776	0.9574	0.9998	0.8367	0.8423
$\tau = 0.75$						
0.01	0.5	0.03419	0.9966	0.9998	0.7223	0.7307
0.1	1	0.04338	0.9946	0.9998	0.7963	0.7487
0.5	3	0.4468	0.9998	0.9998	0.9922	0.991
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.5	0.04138	0.5484	0.991	0.3361	0.4212
0.1	1	0.04338	0.532	0.9922	0.3081	0.3701
0.5	3	0.1034	0.5924	0.9806	0.2673	0.2749
$\tau = 0.5$						
0.01	0.5	0.03299	0.9326	0.9998	0.6224	0.6591
0.1	1	0.04418	0.9238	0.9998	0.61	0.6339
0.5	3	0.1246	0.9662	0.9998	0.7283	0.6555
$\tau = 0.75$						
0.01	0.5	0.03059	0.9946	0.9998	0.7347	0.7351
0.1	1	0.04178	0.9962	0.9998	0.7763	0.7183
0.5	3	0.48	0.9998	0.9998	0.9882	0.9794

Table 4: **"Gumbel" case:** $N = 300$**,** $B = 2500$**.** Same as Table 3.

Δ_H	Δ_D	C^{Cly}	C^{Frk}	C^{Gmb}	C^{Nrm}	$C^{\text{t},4}$
Independent randomization						
$\tau = 0.25$						
0.01	0.5	0.07377	0.4364	0.8283	0.3049	0.3856
0.1	1	0.111	0.4092	0.8195	0.3037	0.3984
0.5	3	0.4704	0.1573	0.7327	0.2661	0.3988
$\tau = 0.5$						
0.01	0.5	0.09696	0.9058	0.9994	0.8171	0.8443
0.1	1	0.2829	0.8595	0.9978	0.8335	0.883
0.5	3	0.9638	0.2689	0.991	0.7511	0.921
$\tau = 0.75$						
0.01	0.5	0.1673	0.955	0.9998	0.9802	0.9654
0.1	1	0.7407	0.8111	0.9998	0.9798	0.9822
0.5	3	0.9998	0.1365	0.9946	0.9842	0.9962
Mixed randomization						
$\tau = 0.25$						
0.01	0.5	0.07257	0.4336	0.8091	0.3029	0.38
0.1	1	0.09216	0.4176	0.8179	0.3137	0.3988
0.5	3	0.3836	0.1821	0.7307	0.2613	0.3609
$\tau = 0.5$						
0.01	0.5	0.09536	0.9146	0.9994	0.8215	0.8375
0.1	1	0.1881	0.8886	0.9982	0.8263	0.8651
0.5	3	0.8762	0.4188	0.9974	0.7499	0.8802
$\tau = 0.75$						
0.01	0.5	0.1549	0.9614	0.9998	0.9778	0.9578
0.1	1	0.4336	0.9138	0.9998	0.977	0.9726
0.5	3	0.997	0.2709	0.9994	0.975	0.993
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.5	0.06657	0.452	0.8343	0.3201	0.4092
0.1	1	0.08337	0.44	0.8343	0.3249	0.3996
0.5	3	0.2225	0.4792	0.8671	0.4524	0.4912
$\tau = 0.5$						
0.01	0.5	0.08816	0.9082	0.9982	0.8303	0.8419
0.1	1	0.1317	0.9114	0.999	0.8331	0.8531
0.5	3	0.6711	0.8087	0.9998	0.8834	0.9326
$\tau = 0.75$						
0.01	0.5	0.1429	0.9558	0.9998	0.9818	0.9614
0.1	1	0.3257	0.945	0.9998	0.983	0.975
0.5	3	0.9842	0.6291	0.9994	0.991	0.9974

Table 5: **"Clayton" case:** $N = 150$**,** $B = 2500$**.** Probability of rejection (nominal level 5%) of the Null hypothesis that the copula belongs, respectively, to the Clayton (C^{Cly}), Frank (C^{Frk}), Gumbel (C^{Gmb}), Normal (C^{Nrm}), and Student-t ($C^{\text{t},4}$, with $\nu = 4$ degrees of freedom) family, for a random sample generated from a Clayton copula, with $\tau = 0.25, 0.5, 0.75$ obtained from independent repetitions of the discretization and randomization procedures — see text.

Δ_H	Δ_D	C^{Cly}	C^{Frk}	C^{Gmb}	C^{Nrm}	$C^{\text{t,4}}$
Independent randomization						
$\tau = 0.25$						
0.01	0.5	0.05978	0.7875	0.9874	0.6387	0.7567
0.1	1	0.111	0.7659	0.9874	0.6547	0.7839
0.5	3	0.7395	0.3389	0.9646	0.5596	0.7959
$\tau = 0.5$						
0.01	0.5	0.09456	0.9982	0.9998	0.9954	0.997
0.1	1	0.4408	0.9982	0.9998	0.9946	0.9978
0.5	3	0.9998	0.6208	0.9998	0.9894	0.9994
$\tau = 0.75$						
0.01	0.5	0.2061	0.9998	0.9998	0.9994	0.9994
0.1	1	0.9638	0.9986	0.9998	0.9998	0.9998
0.5	3	0.9998	0.4144	0.9998	0.9998	0.9998
Mixed randomization						
$\tau = 0.25$						
0.01	0.5	0.06897	0.7799	0.9882	0.6411	0.7439
0.1	1	0.1066	0.7603	0.9862	0.6275	0.7667
0.5	3	0.6124	0.3832	0.9646	0.5392	0.7331
$\tau = 0.5$						
0.01	0.5	0.101	0.999	0.9998	0.9934	0.9954
0.1	1	0.2721	0.9978	0.9998	0.995	0.9966
0.5	3	0.9822	0.8231	0.9998	0.9906	0.9966
$\tau = 0.75$						
0.01	0.5	0.1641	0.9998	0.9998	0.9998	0.9998
0.1	1	0.5924	0.9998	0.9998	0.9998	0.9998
0.5	3	0.9998	0.7571	0.9998	0.9998	0.9998
Co-monotone randomization						
$\tau = 0.25$						
0.01	0.5	0.06457	0.7875	0.9866	0.6319	0.7515
0.1	1	0.08297	0.7803	0.993	0.6467	0.7627
0.5	3	0.3221	0.7931	0.995	0.7775	0.8383
$\tau = 0.5$						
0.01	0.5	0.08617	0.9994	0.9998	0.9978	0.9982
0.1	1	0.1517	0.9978	0.9998	0.9962	0.997
0.5	3	0.8902	0.9886	0.9998	0.9978	0.9994
$\tau = 0.75$						
0.01	0.5	0.1525	0.9998	0.9998	0.9998	0.9998
0.1	1	0.4464	0.9998	0.9998	0.9998	0.9998
0.5	3	0.9998	0.9886	0.9998	0.9998	0.9998

Table 6: **"Clayton" case:** $N = 300$**,** $B = 2500$**.** Same as Table 5.

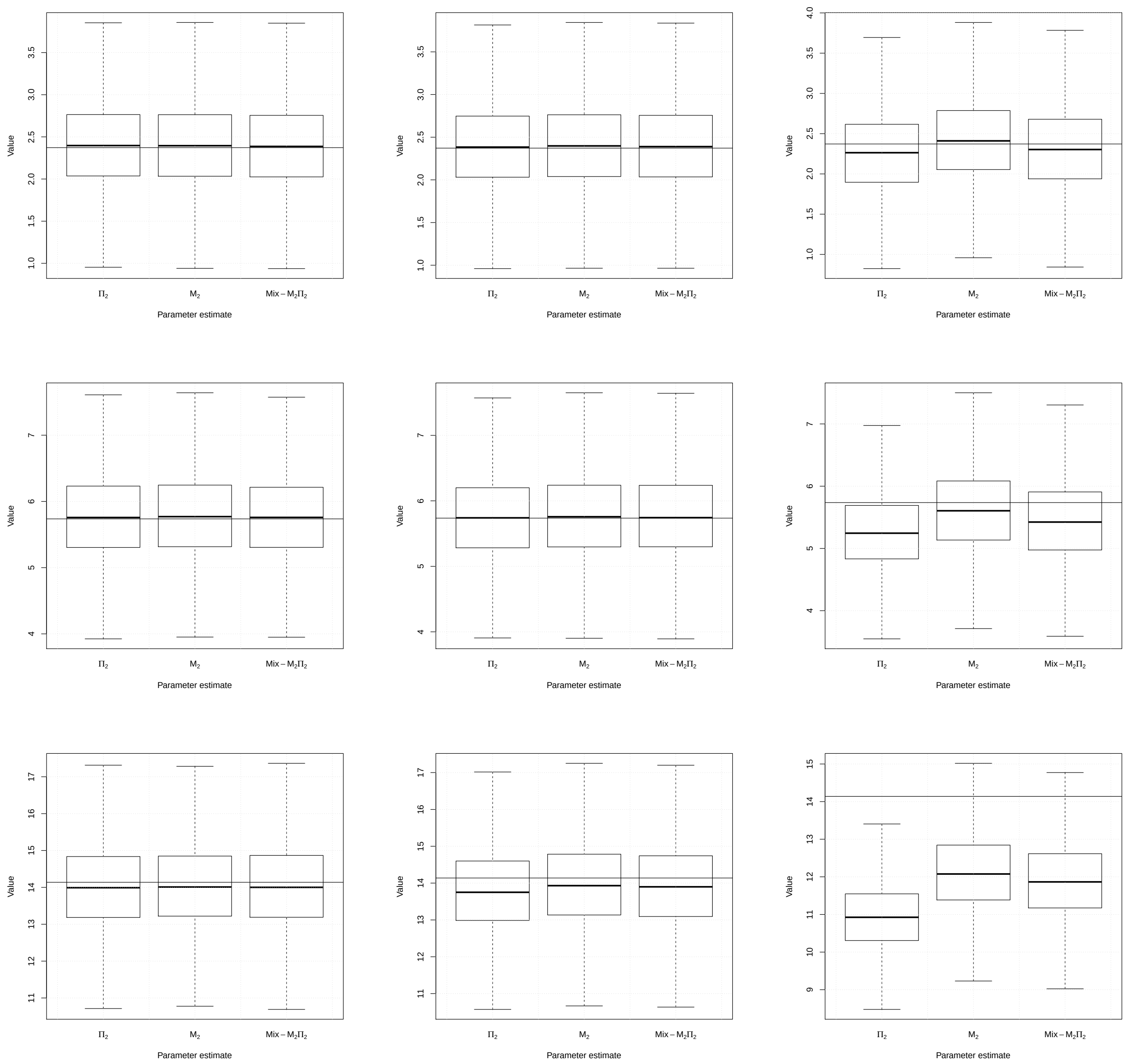

Figure 1: **"Frank" case:** $N = 150$**,** $B = 10000$**.** Boxplots of the copula parameter estimates: the horizontal thick lines indicate the "true" values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall's $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of Height and Duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels "$\Pi_2$", "$M_2$", and "Mix-$M_2\Pi_2$" denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

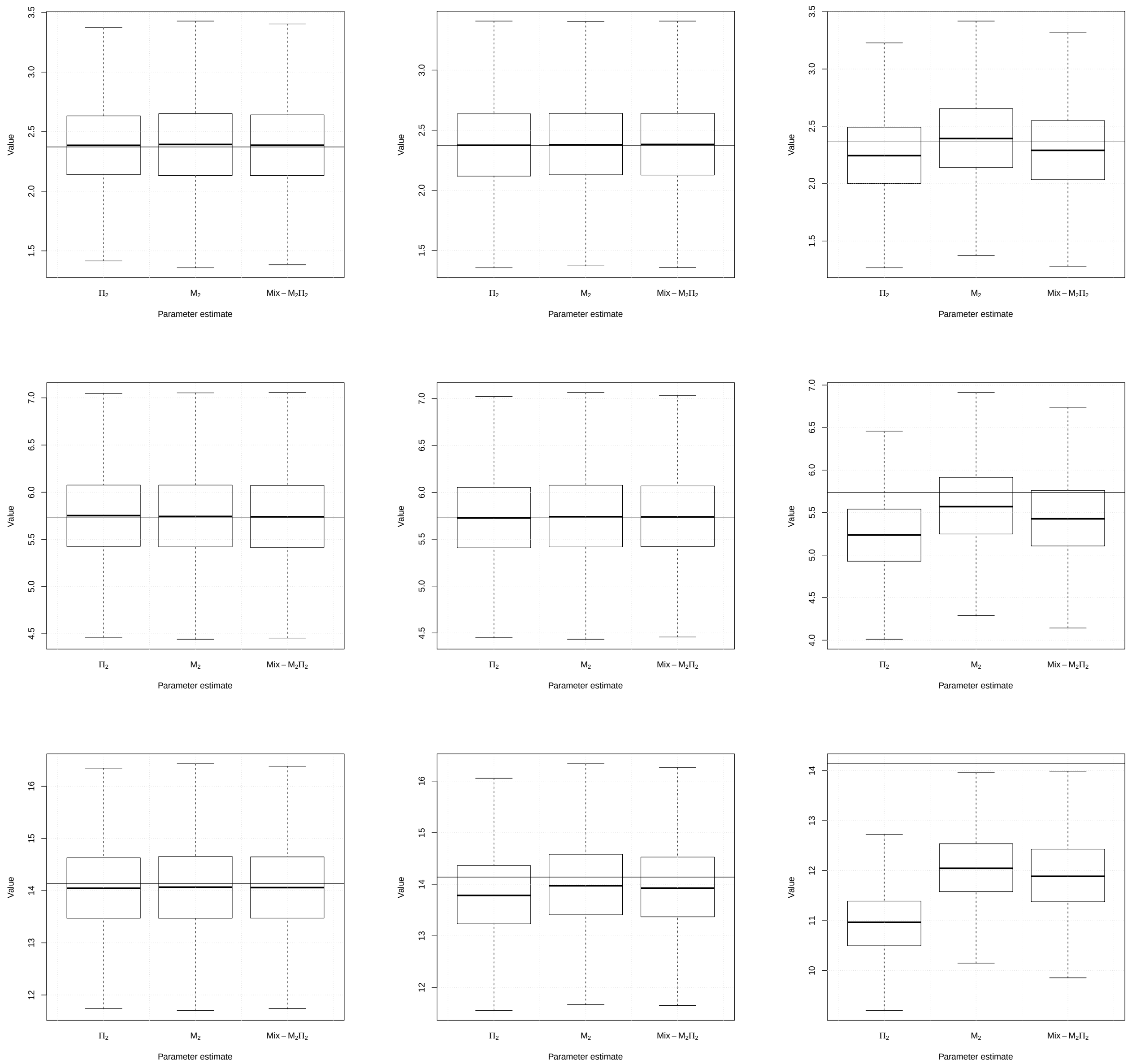

Figure 2: **"Frank" case:** $N = 300$**,** $B = 10000$**.** Same as Fig. 1.

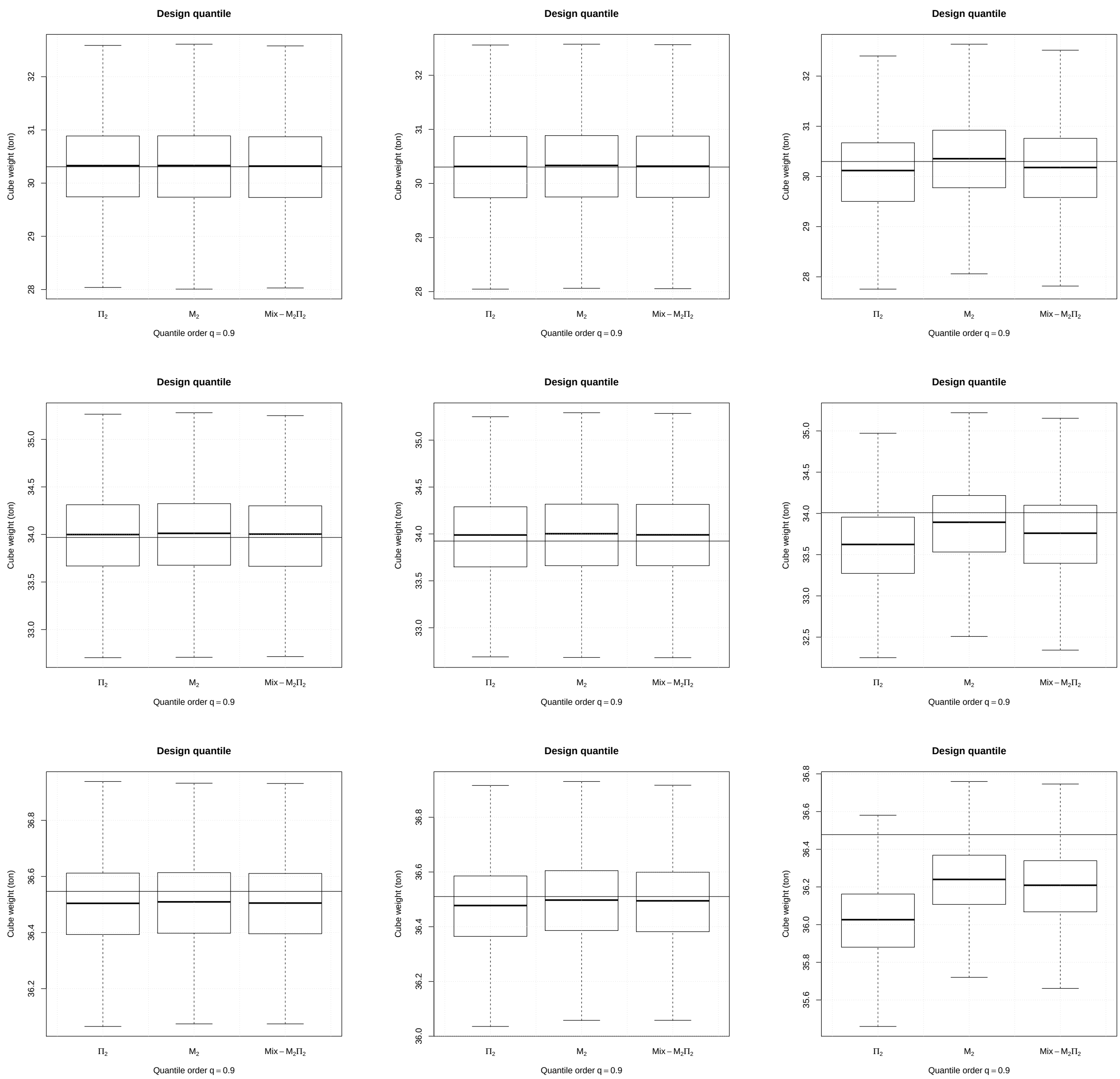

Figure 3: **“Frank” case:** $N = 150$**,** $B = 10000$**,** $q = 0.90$**.** Boxplots of the Cube Weight design quantiles estimates: the horizontal thick lines indicate the “true” values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall’s $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of height and duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels “$\Pi_2$”, “$M_2$”, and “Mix-$M_2\Pi_2$” denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

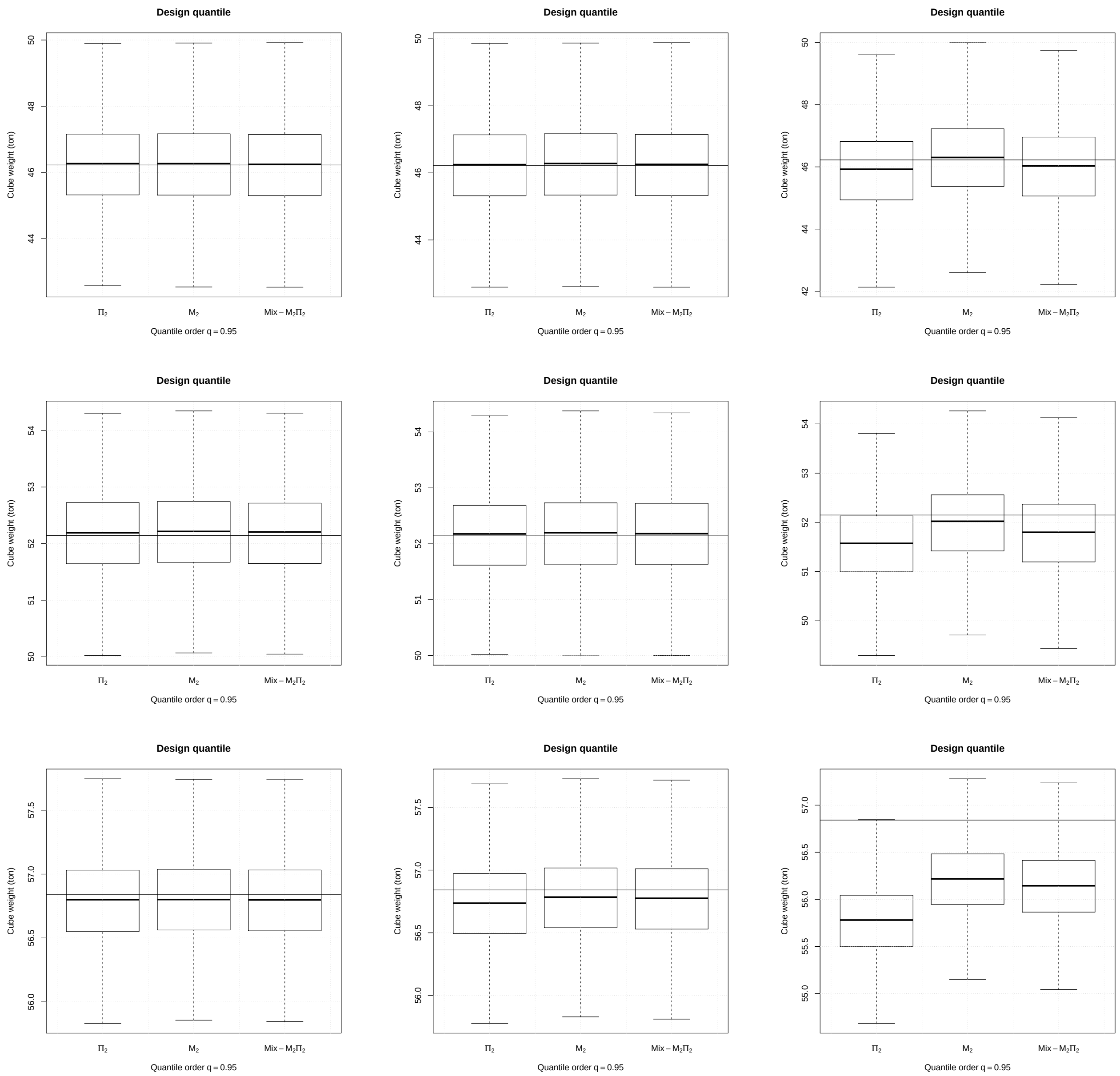

Figure 4: **“Frank” case:** $N = 150$**,** $B = 10000$**,** $q = 0.95$**.** Same as Fig. 3.

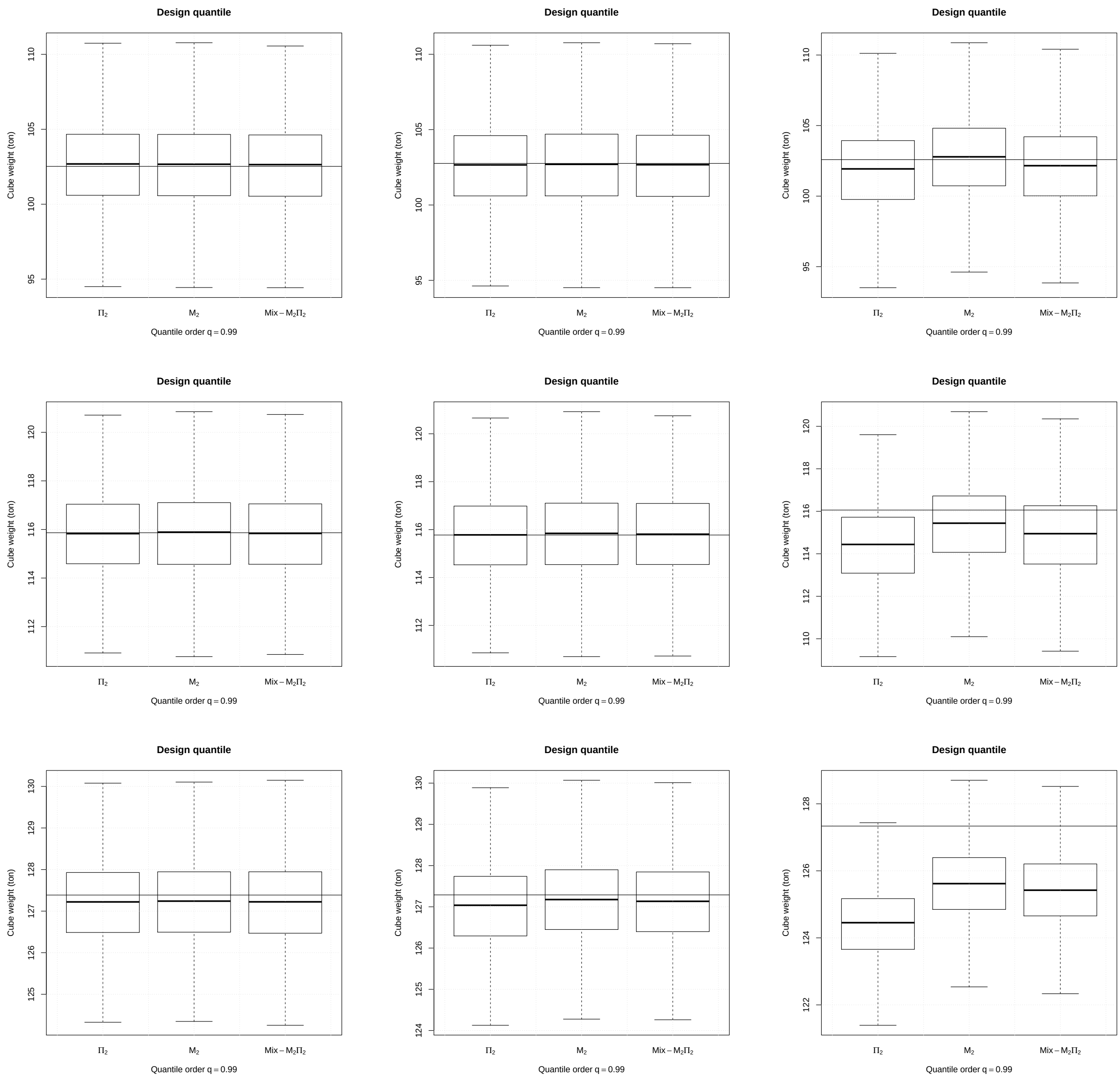

Figure 5: **“Frank” case:** $N = 150$**,** $B = 10000$**,** $q = 0.99$**.** Same as Fig. 3.

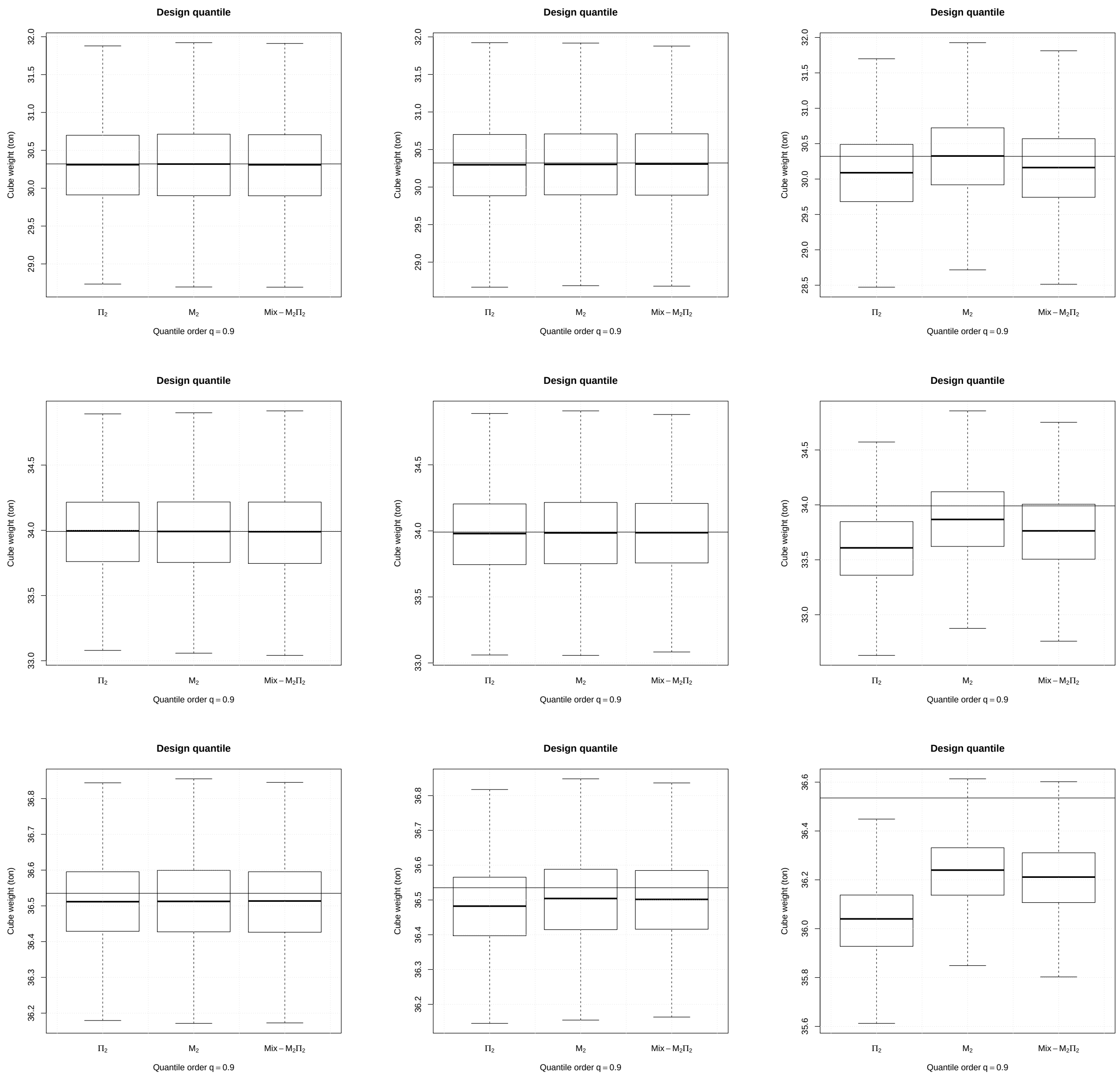

Figure 6: **“Frank” case:** $N = 300$**,** $B = 10000$**,** $q = 0.90$**.** Same as Fig. 3.

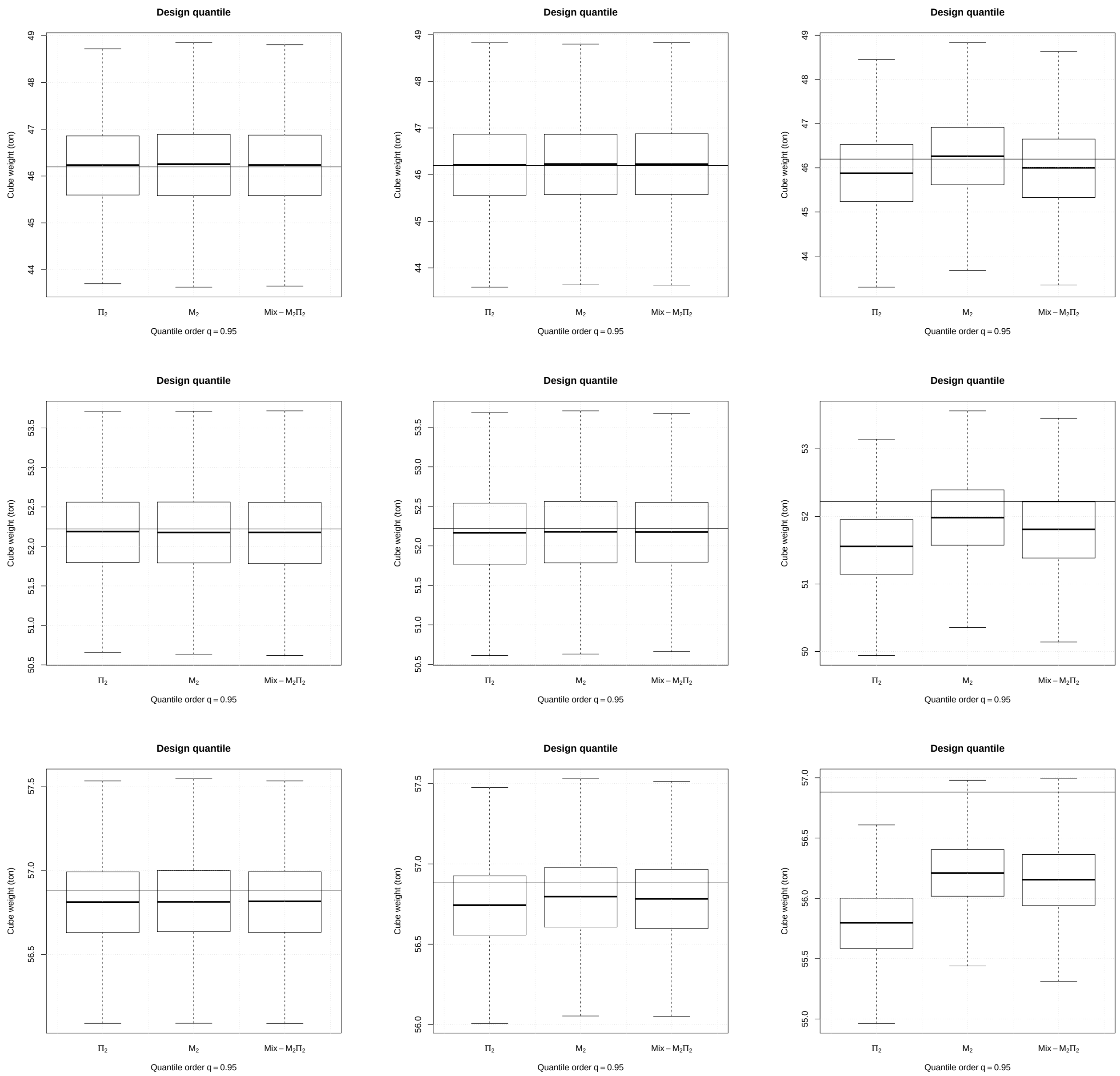

Figure 7: **“Frank” case:** $N = 300$**,** $B = 10000$**,** $q = 0.95$**.** Same as Fig. 4.

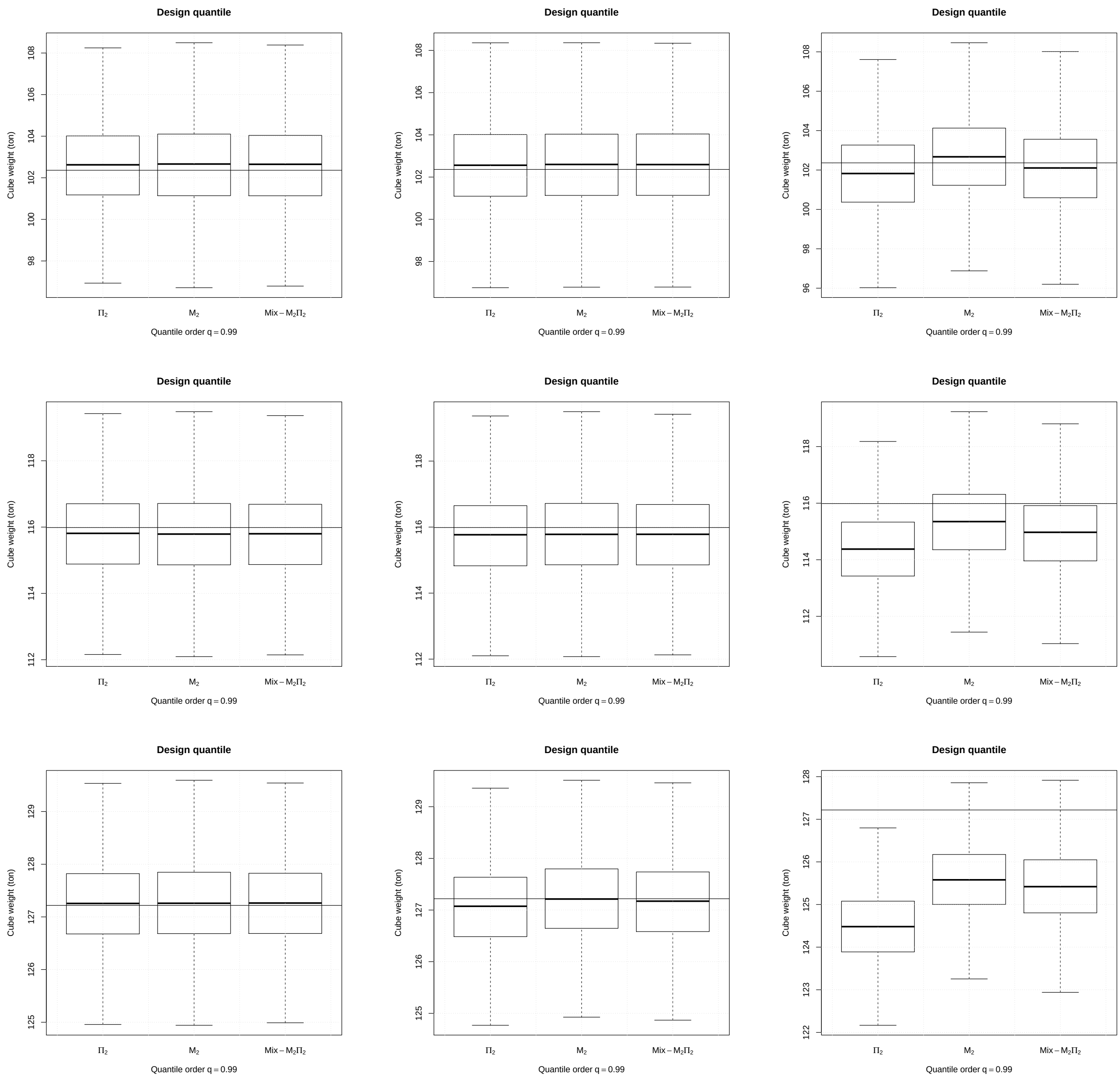

Figure 8: **“Frank” case:** $N = 300$**,** $B = 10000$**,** $q = 0.99$**.** Same as Fig. 5.

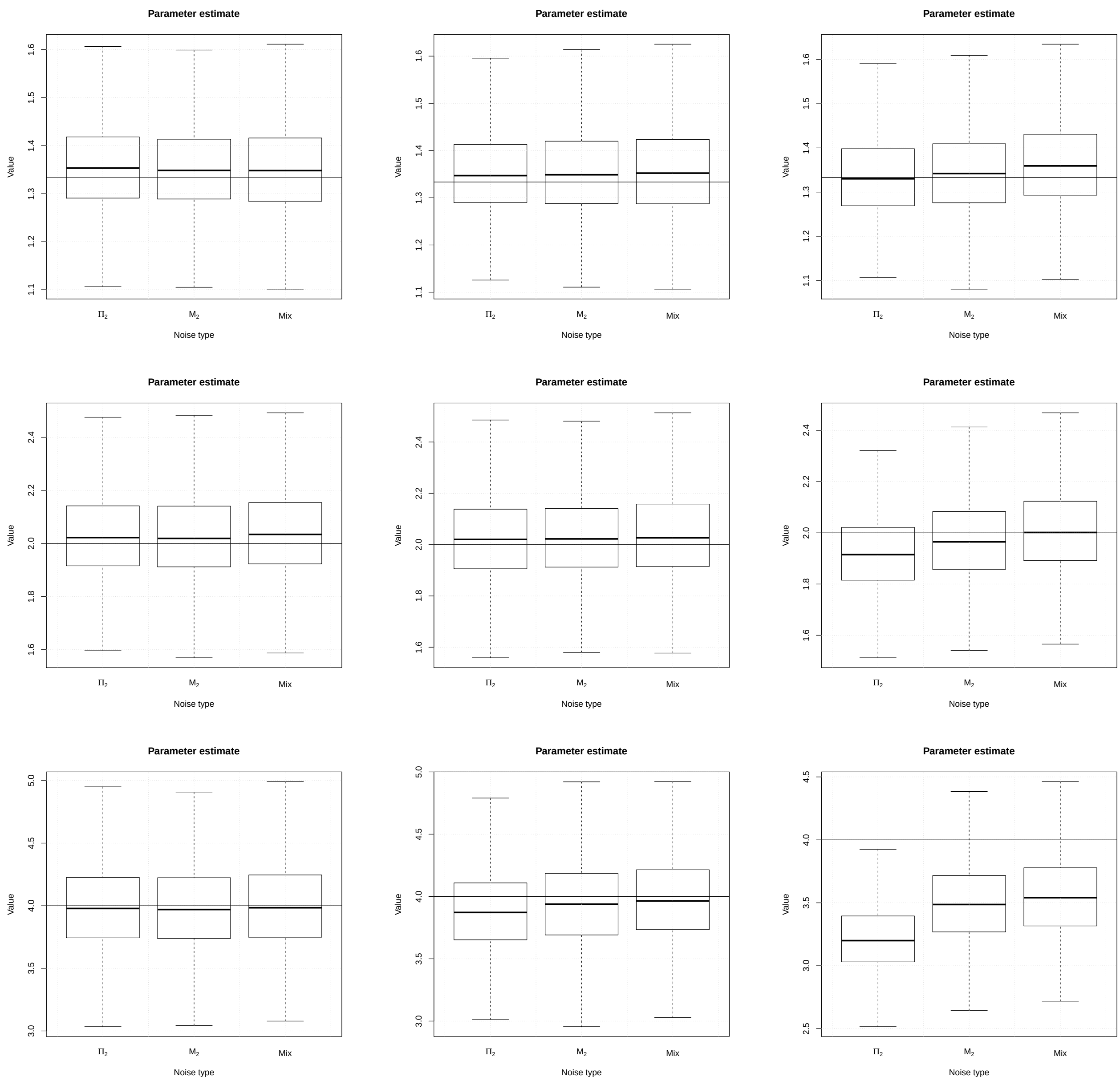

Figure 9: **“Gumbel” case:** $N = 150$**,** $B = 2500$**.** Boxplots of the copula parameter estimates: the horizontal thick lines indicate the “true” values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall’s $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of Height and Duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels “$\Pi_2$”, “$M_2$”, and “Mix” denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

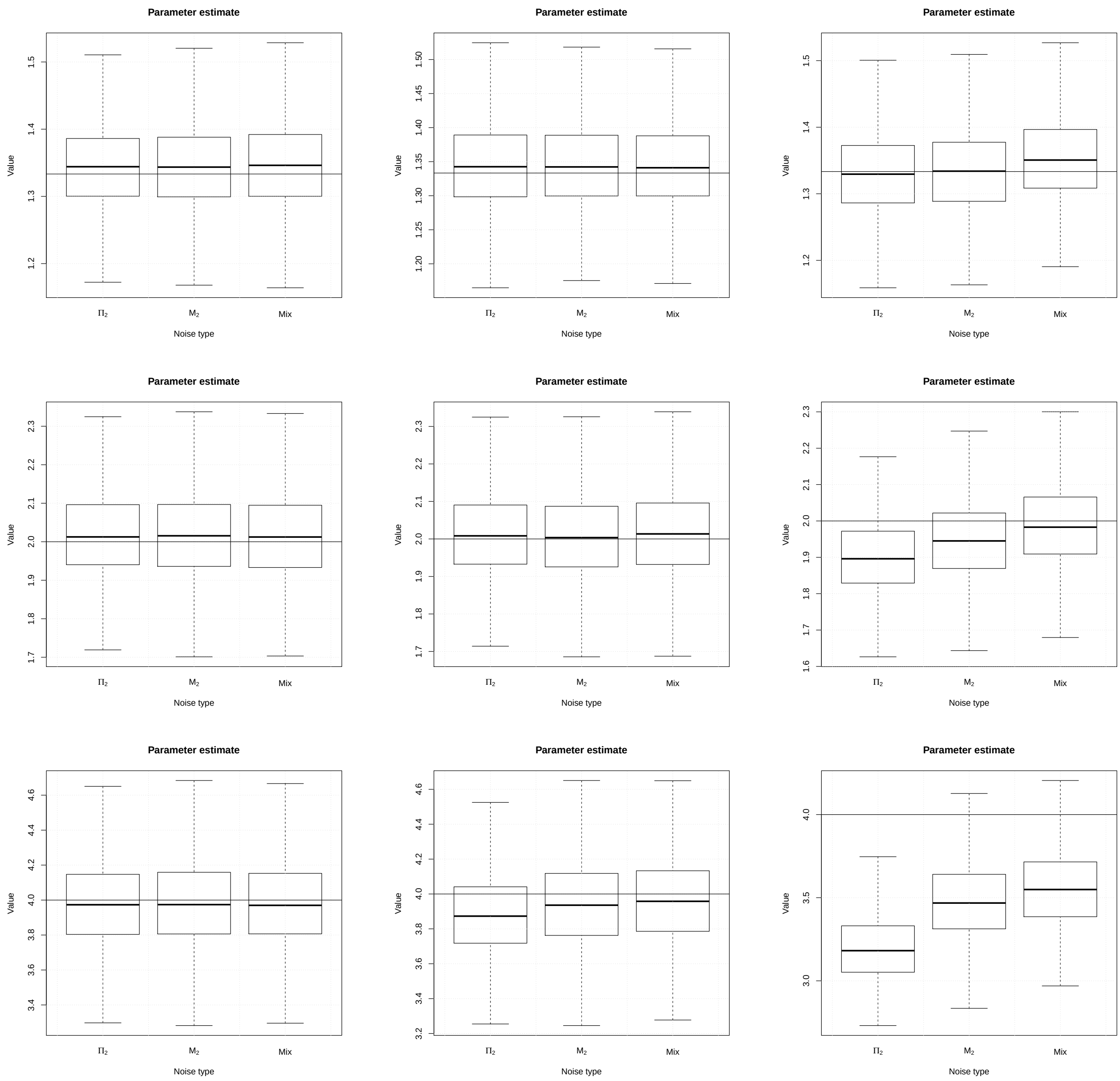

Figure 10: **"Gumbel" case:** $N = 300$**,** $B = 2500$**.** Same as Fig. 9.

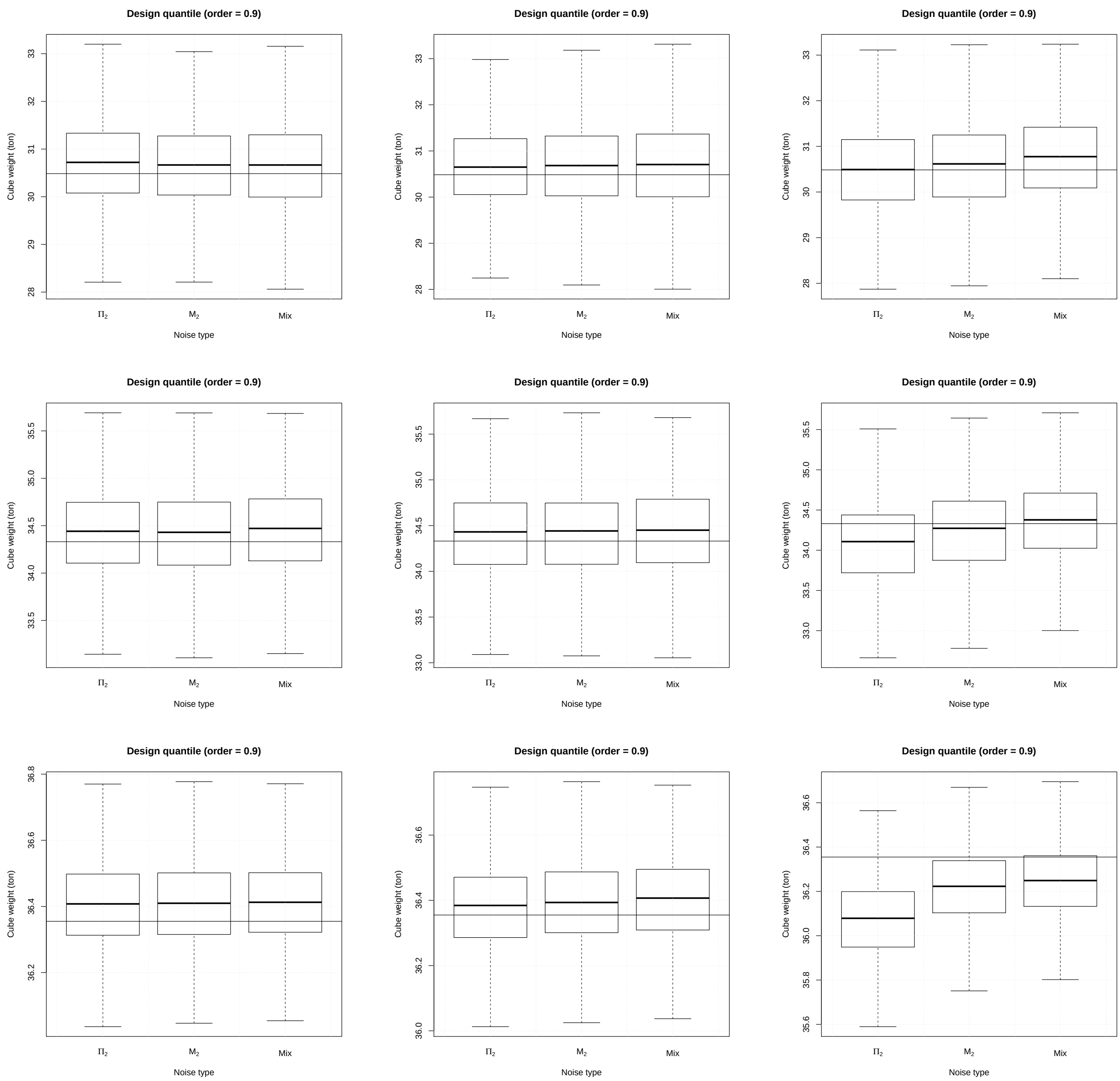

Figure 11: **"Gumbel" case:** $N = 150$**,** $B = 2500$**,** $q = 0.90$**.** Boxplots of the Cube Weight design quantiles estimates: the horizontal thick lines indicate the "true" values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall's $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of height and duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels "$\Pi_2$", "$M_2$", and "Mix" denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

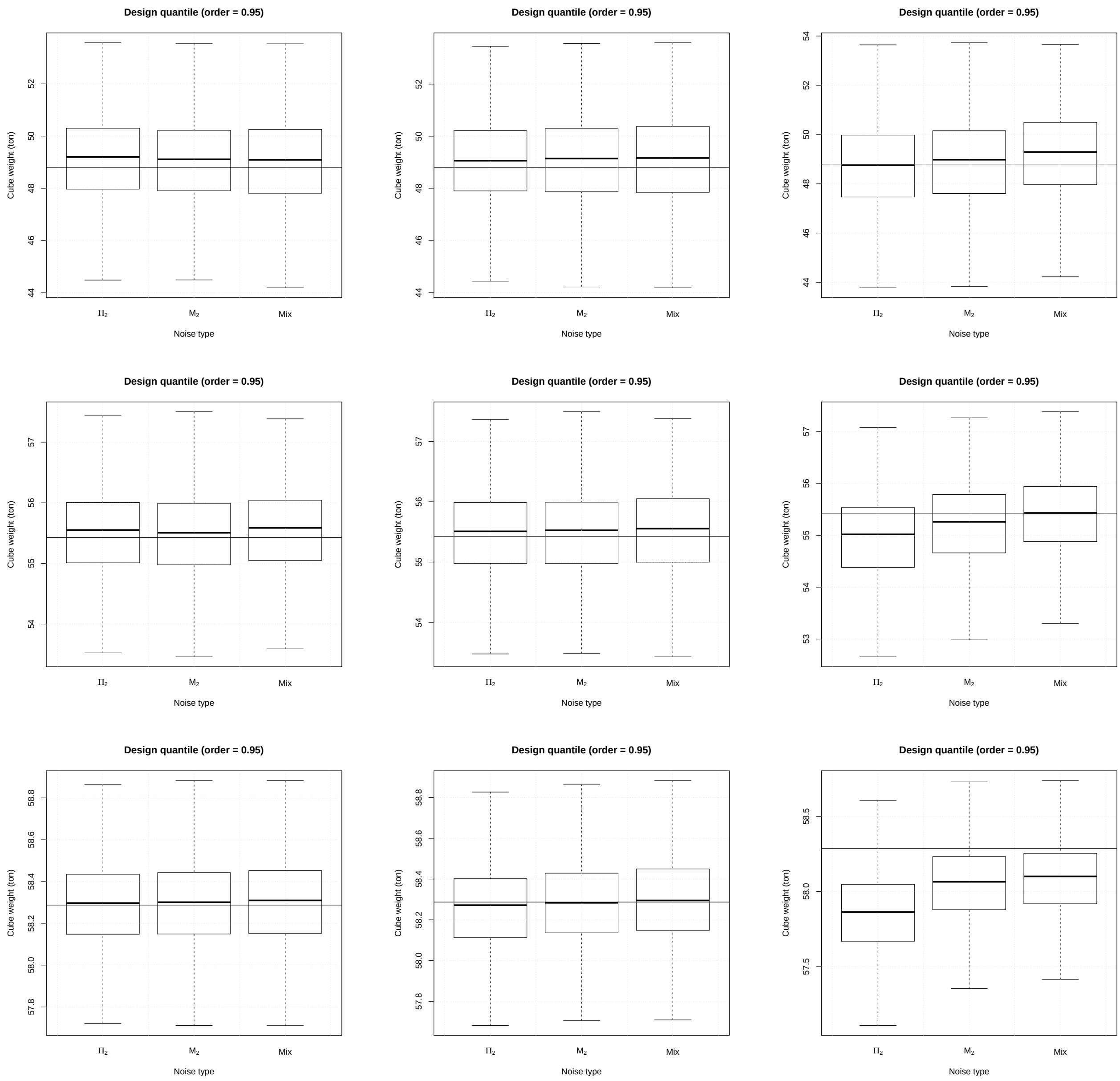

Figure 12: **“Gumbel” case:** $N = 150$**,** $B = 2500$**,** $q = 0.95$**.** Same as Fig. 11.

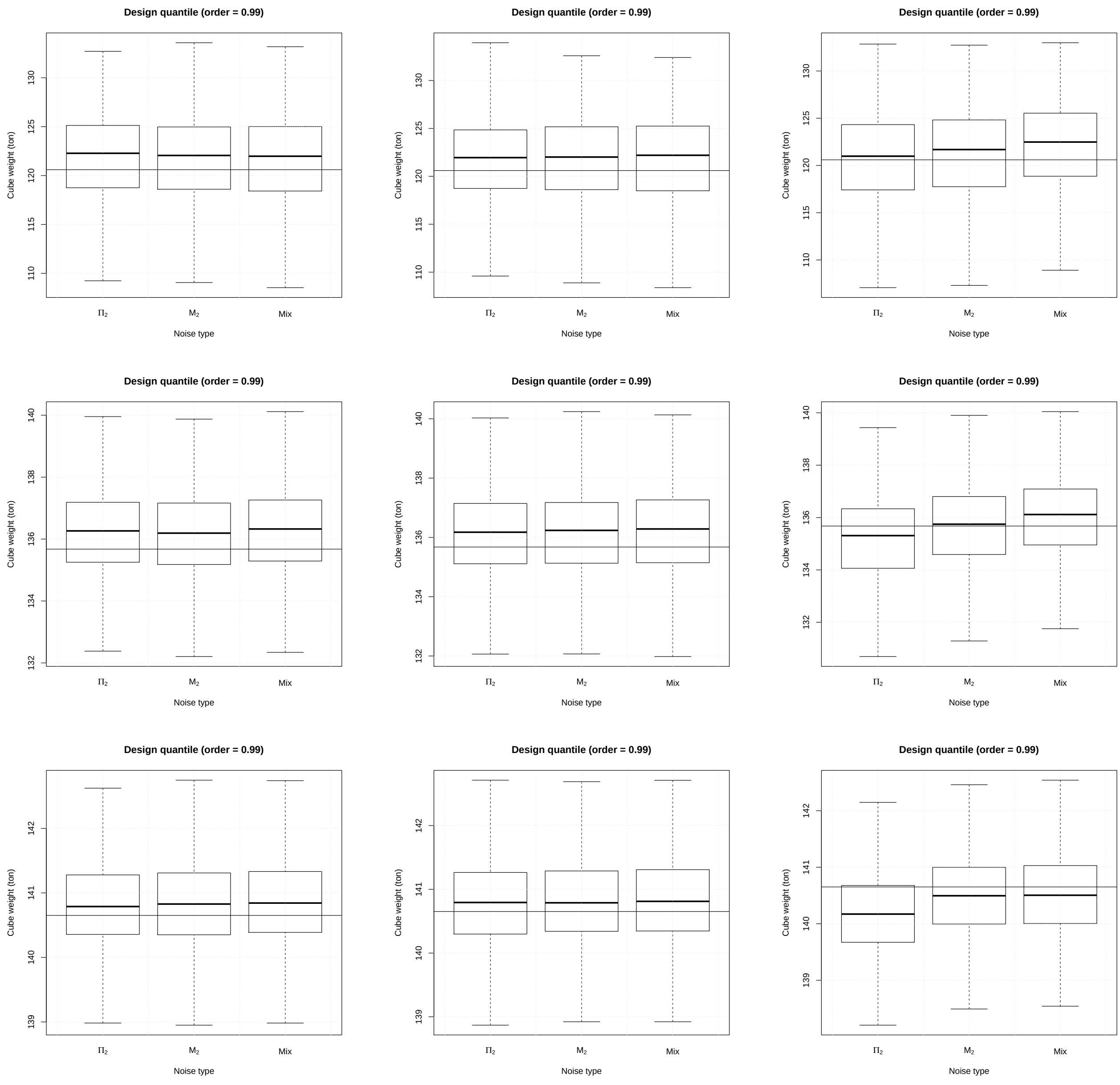

Figure 13: **"Gumbel" case:** $N = 150$**,** $B = 2500$**,** $q = 0.99$**.** Same as Fig. 11.

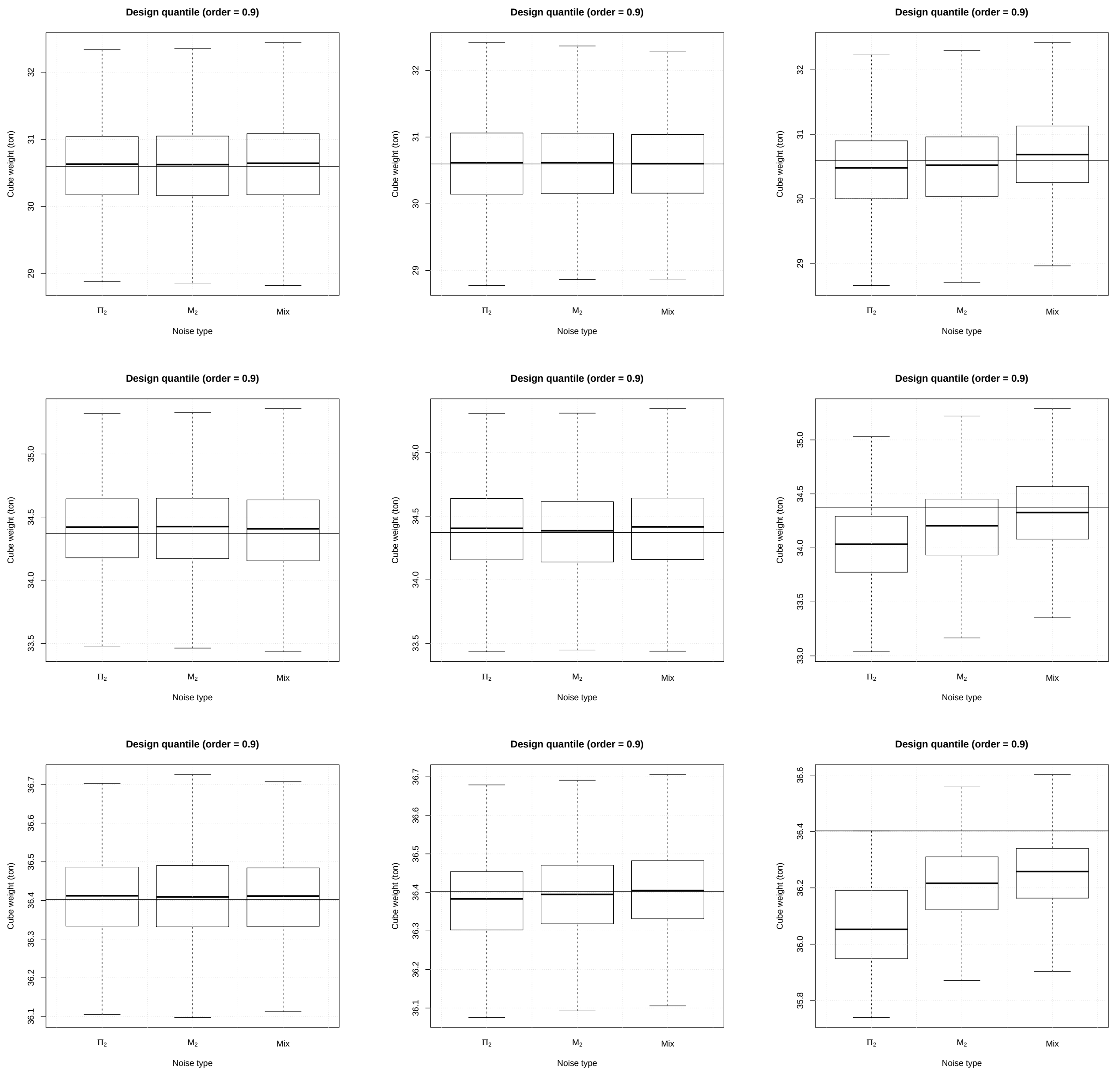

Figure 14: **"Gumbel" case:** $N = 300$**,** $B = 2500$**,** $q = 0.90$**.** Same as Fig. 11.

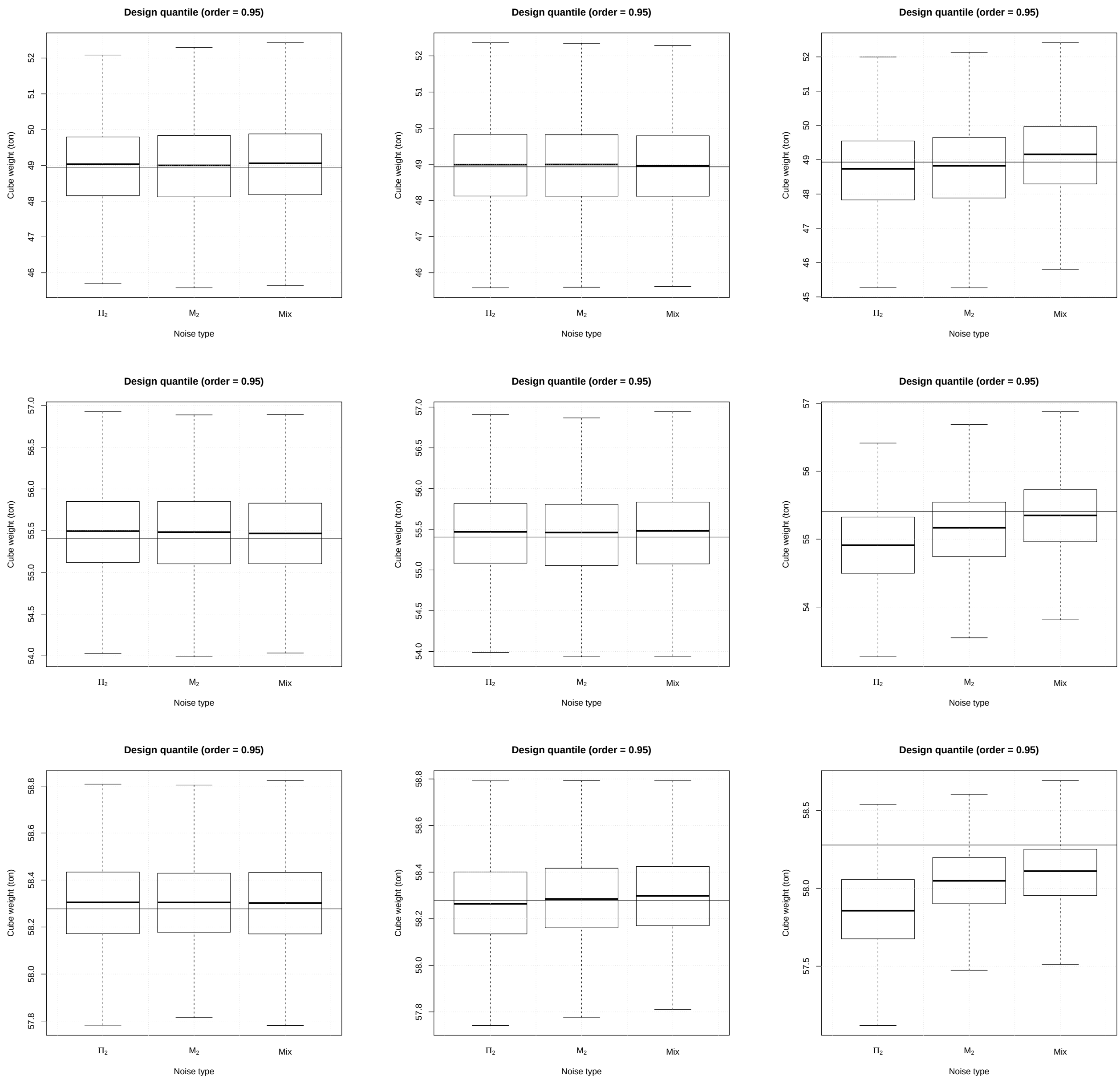

Figure 15: **“Gumbel” case:** $N = 300$**,** $B = 2500$**,** $q = 0.95$**.** Same as Fig. 12.

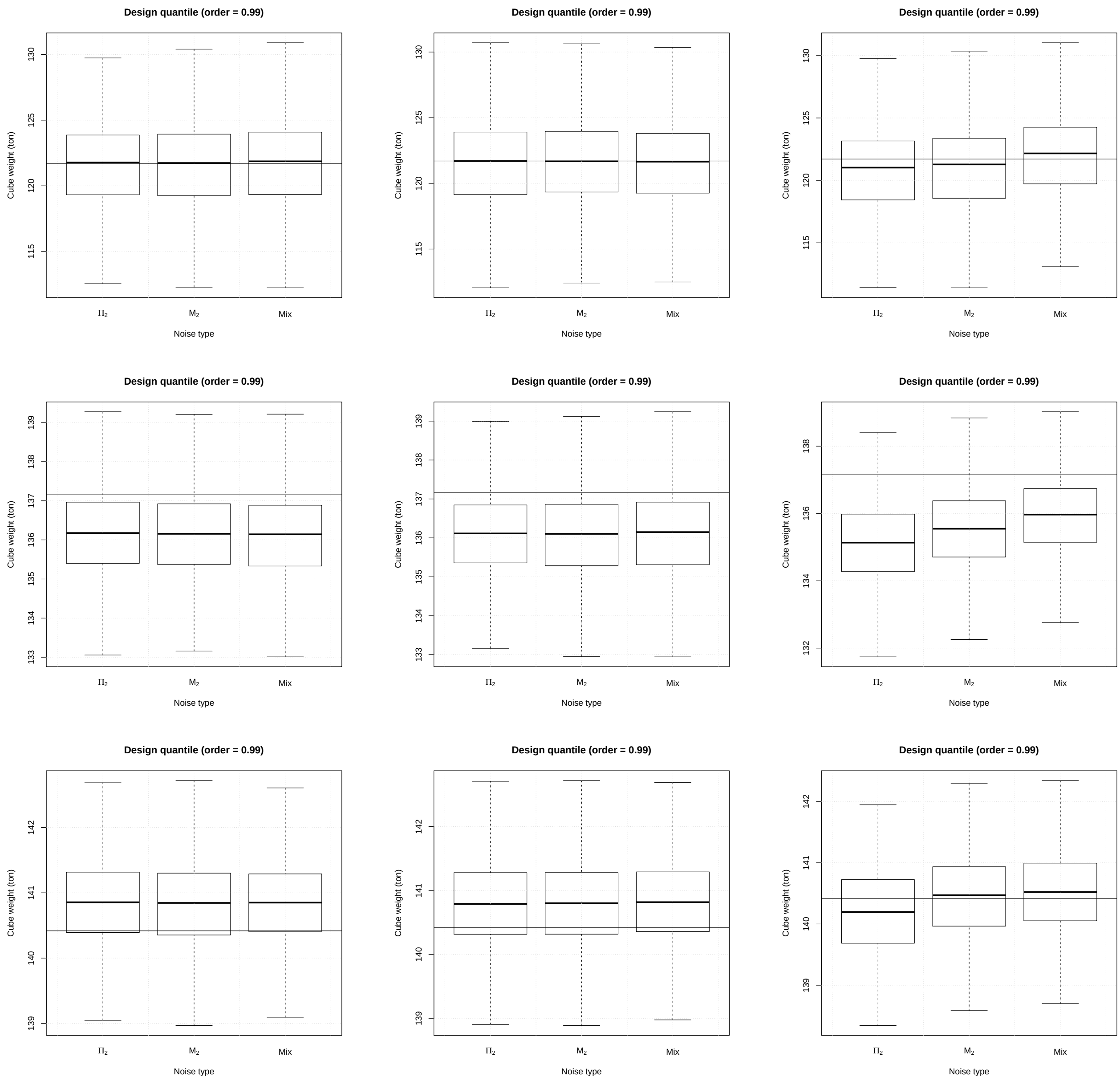

Figure 16: **“Gumbel” case:** $N = 300$**,** $B = 2500$**,** $q = 0.99$**.** Same as Fig. 13.

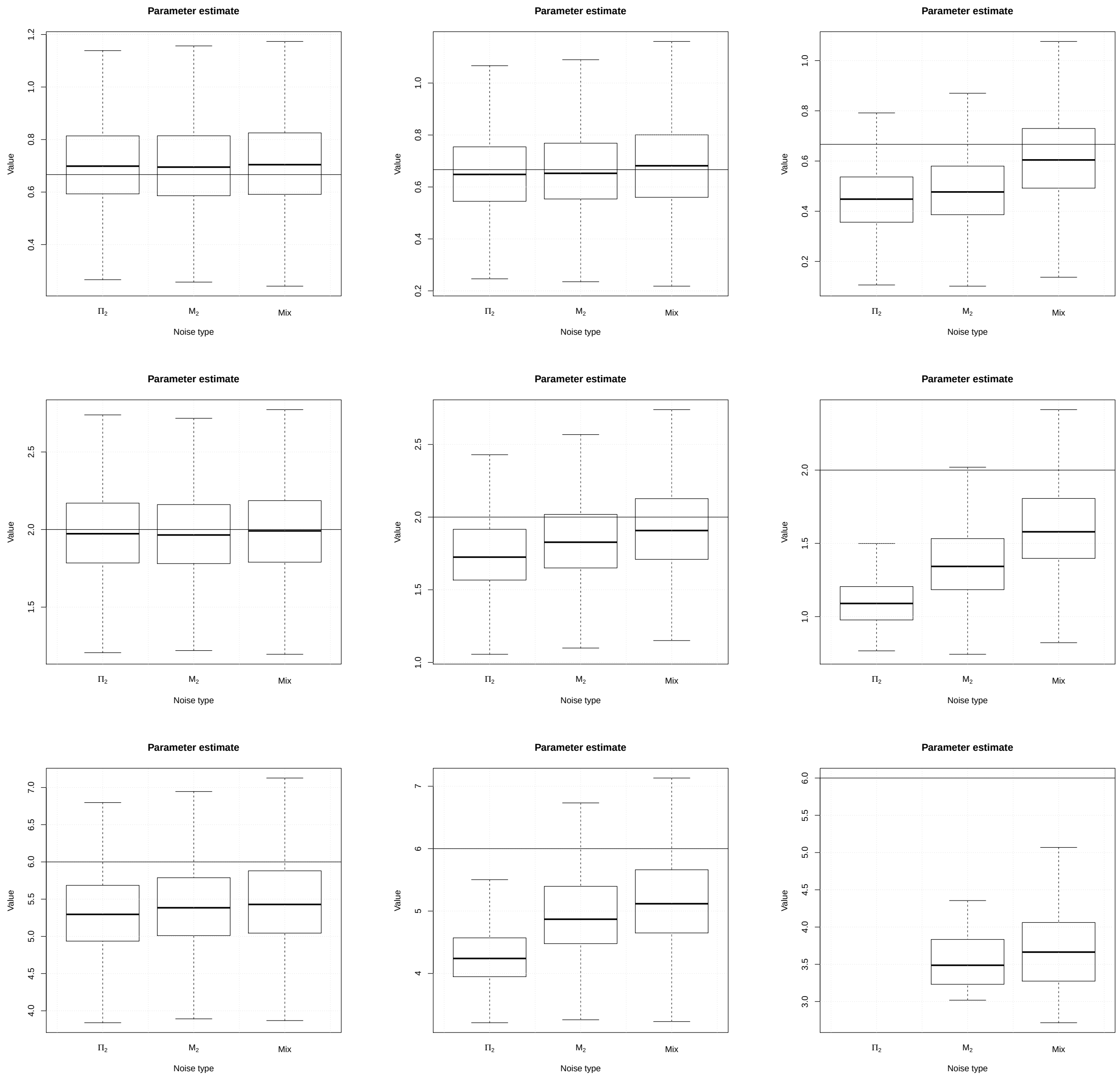

Figure 17: **"Clayton" case:** $N = 150$**,** $B = 2500$**.** Boxplots of the copula parameter estimates: the horizontal thick lines indicate the "true" values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall's $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of Height and Duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels "$\Pi_2$", "$M_2$", and "Mix" denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

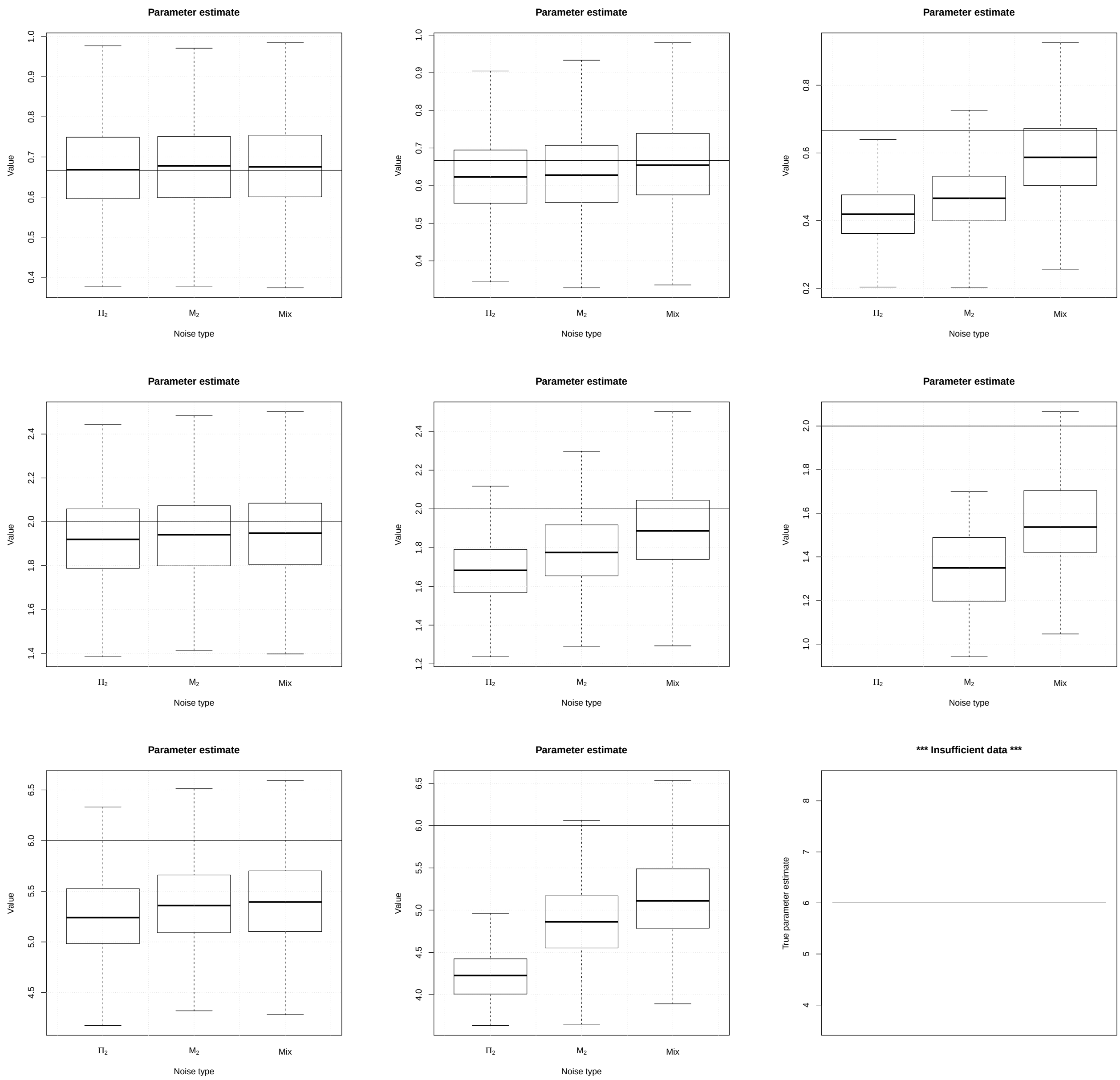

Figure 18: **“Clayton” case:** $N = 300$**,** $B = 2500$**.** Same as Fig. 17.

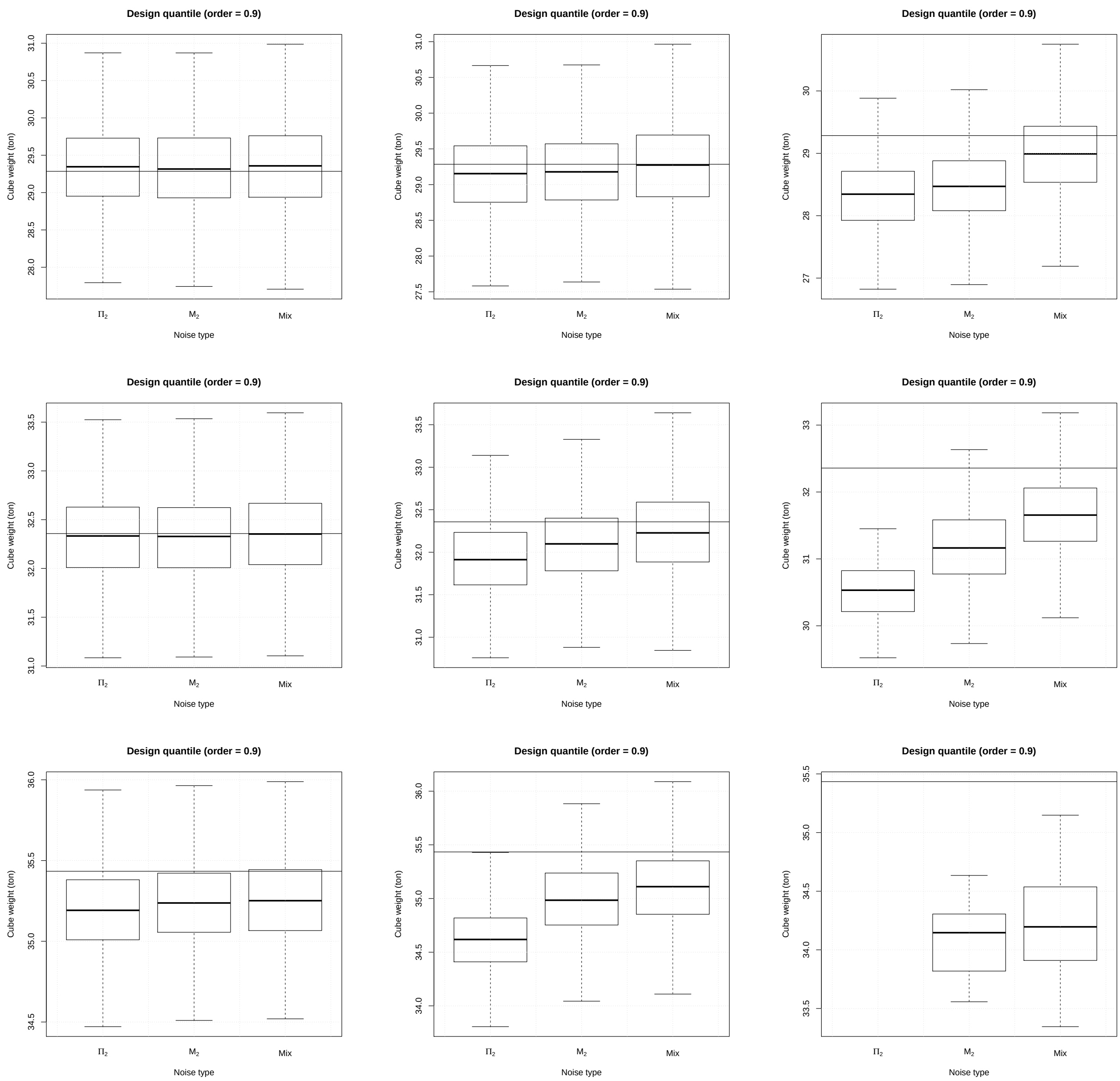

Figure 19: **“Clayton” case: $N = 150$, $B = 2500$, $q = 0.90$.** Boxplots of the Cube Weight design quantiles estimates: the horizontal thick lines indicate the “true” values — see text. From top to bottom, the rows correspond, respectively, to the values of the Kendall’s $\tau = 0.25, 0.5, 0.75$. From left to right, the columns correspond, respectively, to the following pairs of height and duration resolutions: $(\Delta_H, \Delta_D) \in \{(0.01, 0.5), (0.1, 1), (0.5, 3)\}$. The labels “$\Pi_2$”, “$M_2$”, and “Mix” denote the use of an independent, a co-monotone, and a mixed randomization, respectively.

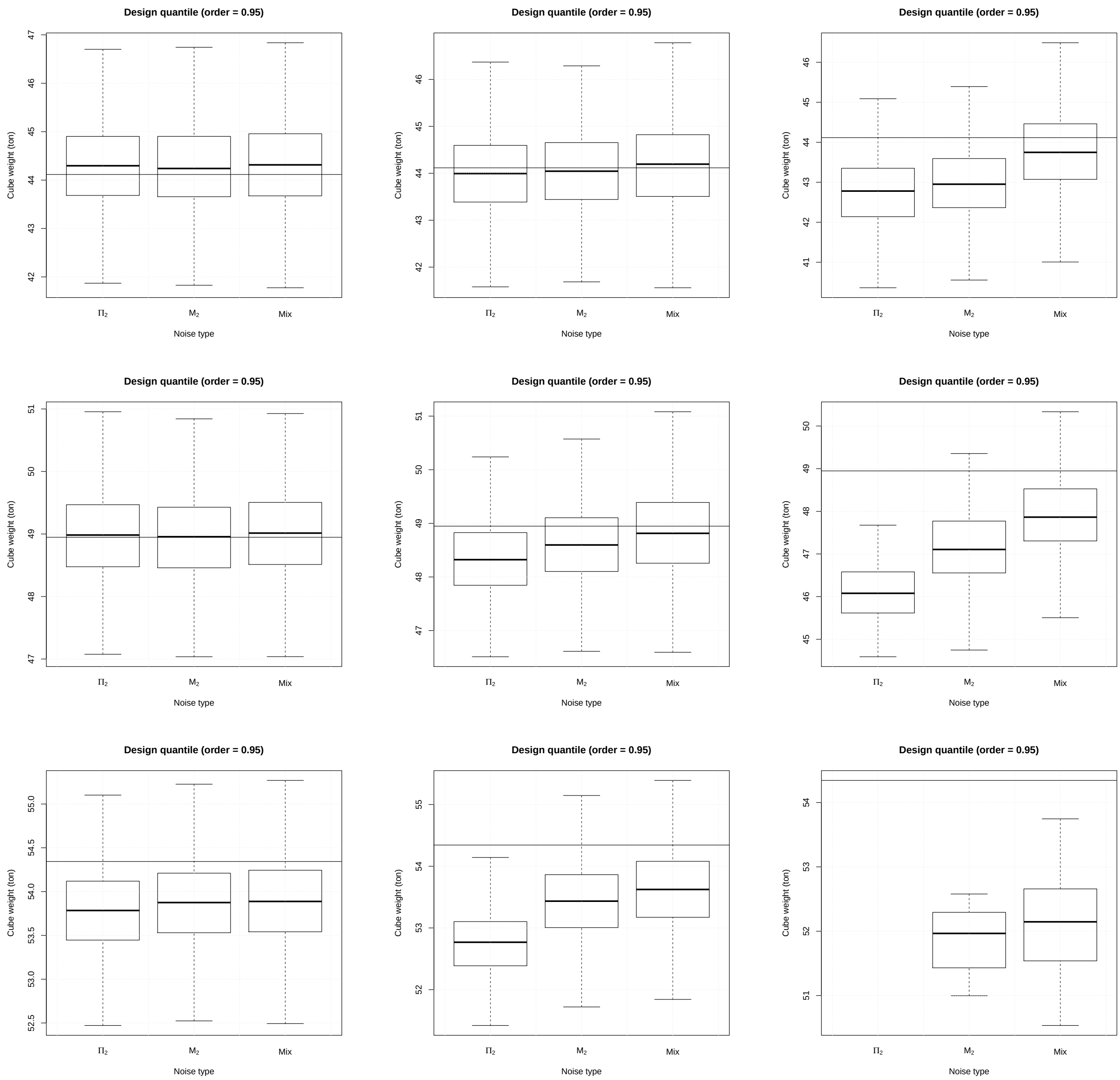

Figure 20: **"Clayton" case:** $N = 150$**,** $B = 2500$**,** $q = 0.95$**.** Same as Fig. 19.

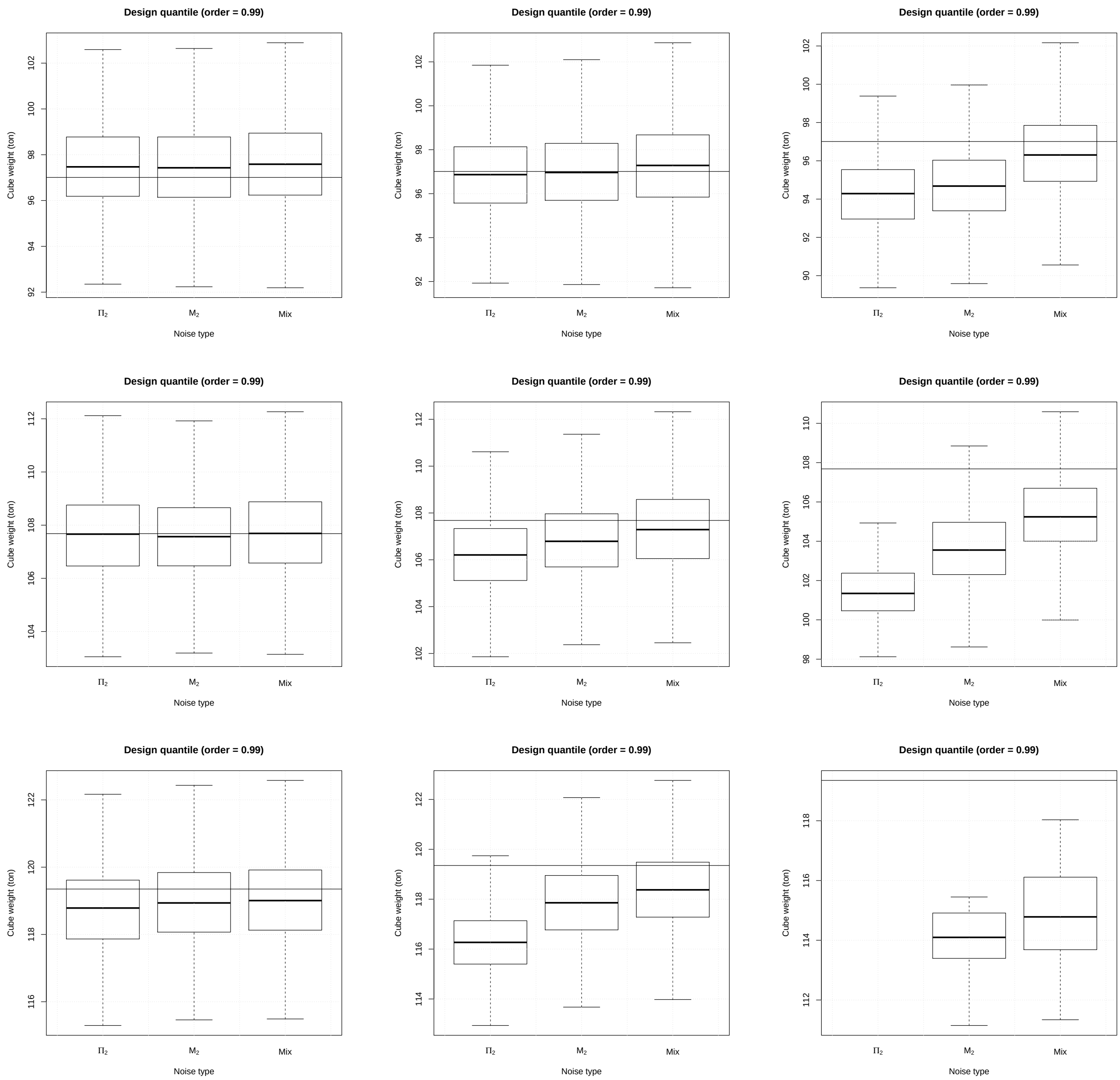

Figure 21: **"Clayton" case:** $N = 150$**,** $B = 2500$**,** $q = 0.99$**.** Same as Fig. 19.

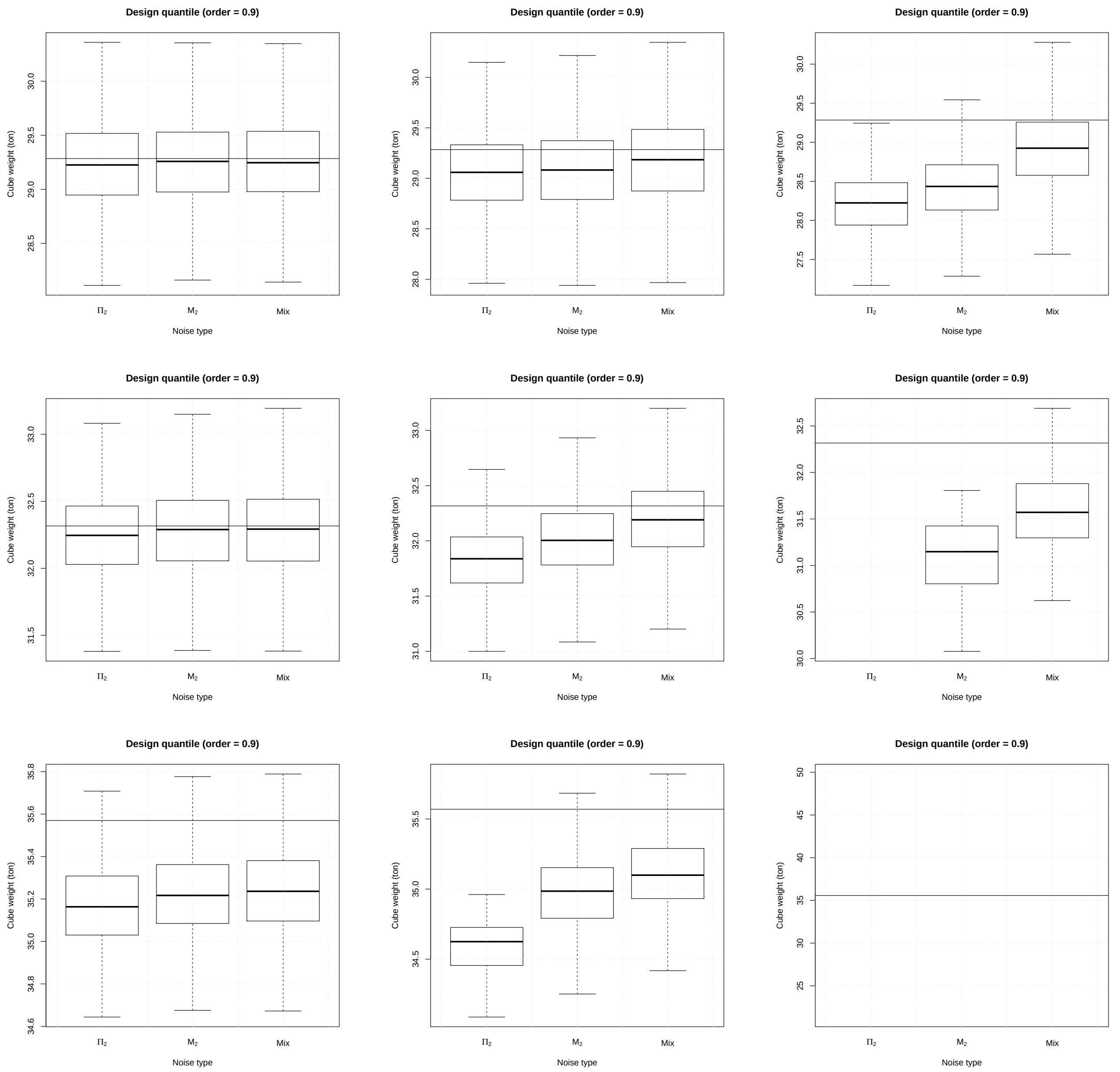

Figure 22: **“Clayton” case:** $N = 300$**,** $B = 2500$**,** $q = 0.90$**.** Same as Fig. 19.

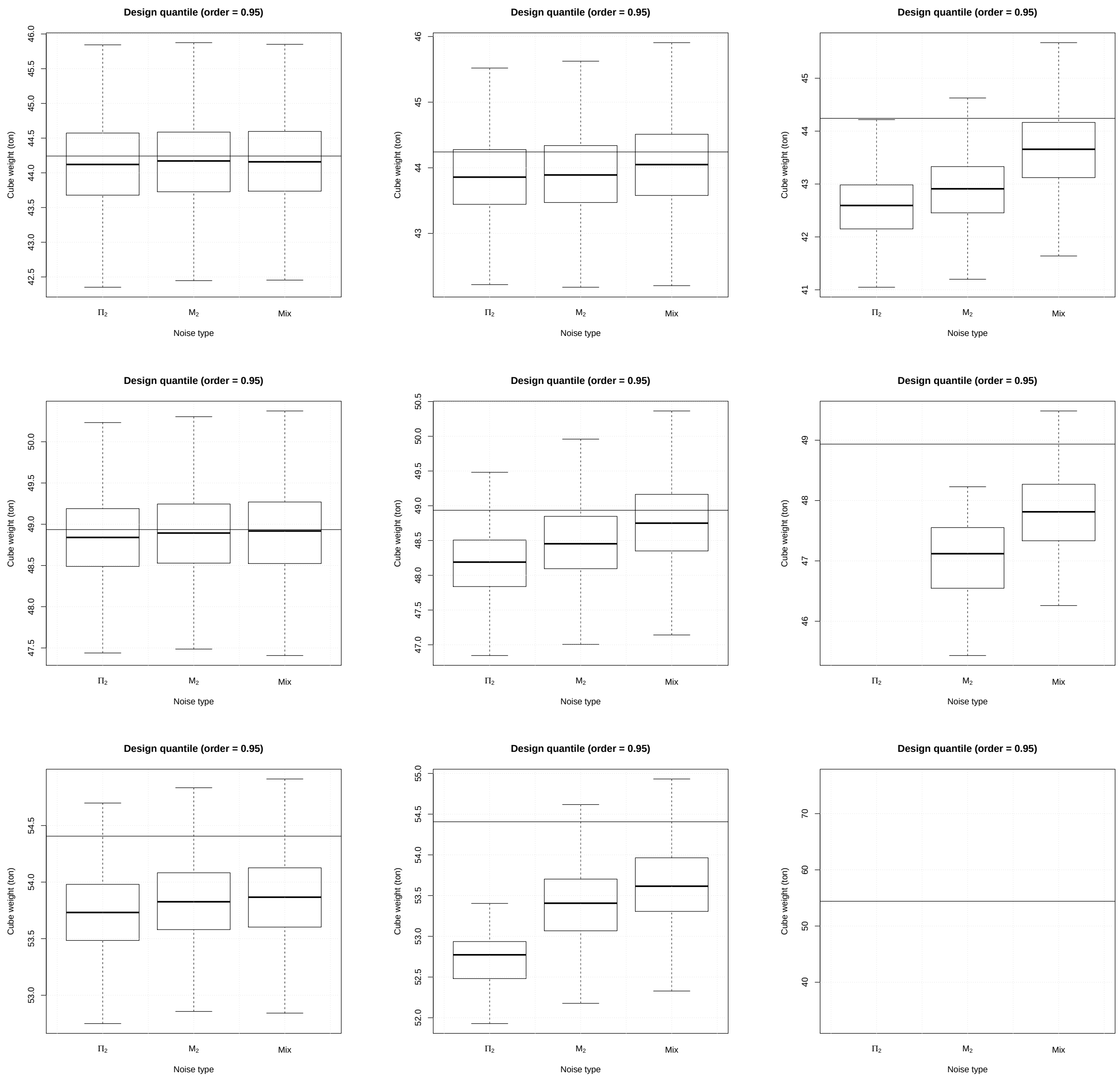

Figure 23: **"Clayton" case:** $N = 300$**,** $B = 2500$**,** $q = 0.95$**.** Same as Fig. 20.

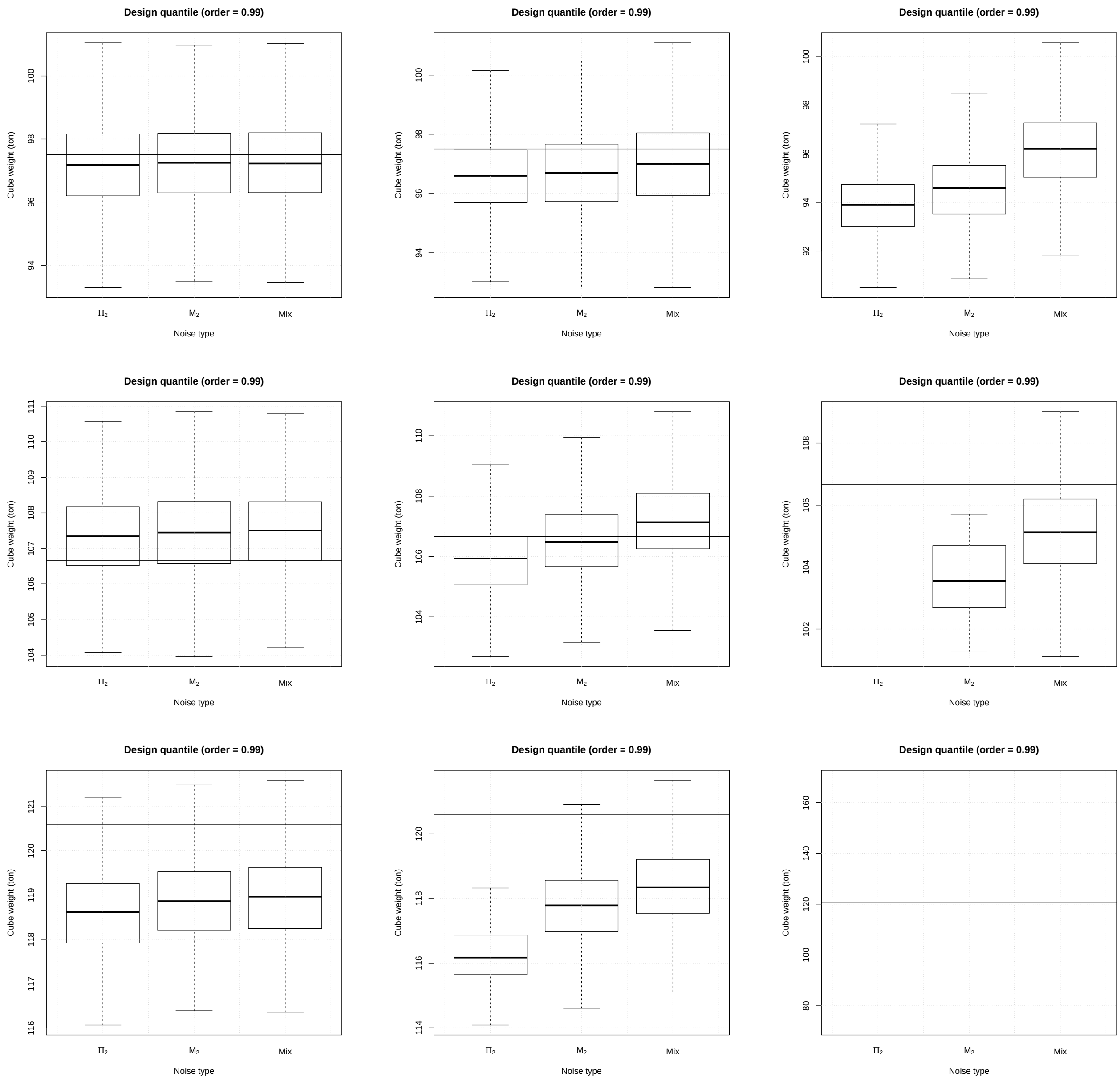

Figure 24: **“Clayton” case:** $N = 300$**,** $B = 2500$**,** $q = 0.99$**.** Same as Fig. 21.