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Appendix of 'Optimal ordering decision and information leakage preference under asymmetric forecast signal'

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A Proof of Lemma 1

Under the condition of no information leakage, the two retailers play a simultaneous-move game, and their expected profits are:

$$\mathrm{E}\left[\Pi_{is}^{\mathrm{NL}}(q_{is}^{\mathrm{NL}}|s)\right] = \left(\mathrm{E}\left[A|s\right] - q_{is}^{\mathrm{NL}} - q_{e}^{\mathrm{NL}}\right) q_{is}^{\mathrm{NL}} \tag{1}$$

and

$$\mathrm{E}\left[\Pi_{e}^{\mathrm{NL}}(q_{e}^{\mathrm{NL}})\right] = \sum \Pr(s)\left(\mathrm{E}[A|s] - q_{is}^{\mathrm{NL}} - q_{e}^{\mathrm{NL}}\right) q_{e}^{\mathrm{NL}}, \tag{2}$$

where $\mathrm{E}\left[A|s\right] = A_H \Pr(A_H|s) + A_L \Pr(A_L|s)$ and $s \in \{l, h\}$, specifically, $\mathrm{E}[A|h] = [1 + \delta(2\theta - 1)(1 - \rho) + \delta\rho]\bar{A}$ and $\mathrm{E}[A|l] = [1 + \delta(2\theta - 1)(1 - \rho) - \delta\rho]\bar{A}$. For simplify, we define $\tau = 1 + \delta(2\theta - 1)(1 - \rho)$ to represent the average of expected demand and $\Delta = \delta\rho$ to indicate the difference between the $\mathrm{E}[A|h]$ and $\mathrm{E}[A|l]$.

Obviously, their second orders are all negative, thus, we set the first orders equal to zero to get their optimal sourcing quantities: $q_{ih}^{\mathrm{NL}} = \frac{[\tau+\Delta+(1-\theta)\Delta]\bar{A}}{3}$, $q_{il}^{\mathrm{NL}} = \frac{(\tau-\Delta-\theta\Delta)\bar{A}}{3}$ and $q_{e}^{\mathrm{NL}} = \frac{(\tau-\Delta+2\theta\Delta)\bar{A}}{3}$. And their corresponding maximal expected profits are: $\mathrm{E}[\Pi_{ih}^{\mathrm{NL}}] = \frac{[\tau+\Delta+(1-\theta)\tau]^2\bar{A}^2}{9}$, $\mathrm{E}[\Pi_{il}^{\mathrm{NL}}] = \frac{(\tau-\Delta-\theta\Delta)^2\bar{A}^2}{9}$ and $\mathrm{E}[\Pi_{e}^{\mathrm{NL}}] = \frac{(\tau-\Delta+2\theta\Delta)^2\bar{A}^2}{9}$. Here $\Delta < \frac{\tau}{1+\theta}$ to ensure that all the sourcing quantities are non-negative.

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B Proof of Corollary 1

Under the condition of no information leakage, it is easy to find that $\mathrm{E}[\Pi_{ih}^{\mathrm{NL}}] > \mathrm{E}[\Pi_{e}^{\mathrm{NL}}] > \mathrm{E}[\Pi_{il}^{\mathrm{NL}}]$, so we omit it.

C Proof of Lemma 2 and Lemma 3

Under the condition of information leakage, the upstream manufacturer always leaks the incumbent's order information reflecting private forecast signal to the entrant, such that the entrant can make his sourcing decision according to the information he inferred. Under such a condition, the two retailers play a sequential-move game, where the incumbent moves firstly and the entrant moves later. We solve this sequential move game backwards, so we first solve the entrant's optimal sourcing quantity:

$$q_e^*(q_i) = \arg\max_{q_e} \left[(1 \cdot \mathrm{E}[A|s] - q_i - q_e)\, q_e\right] = \frac{\mathrm{E}[A|s] - q_i}{2}. \tag{3}$$

Considering the entrant's belief system, there exists a separating equilibrium if and only if the following constraints exist:

$$\max_{q_i} \mathrm{E}\left[\Pi_i\left(q_i > q_{il}^{\mathrm{LS}}|h\right)\right] \geq \max_{q_i} \mathrm{E}\left[\Pi_i\left(q_i \leq q_{il}^{\mathrm{LS}}|h\right)\right] \tag{4}$$

$$\max_{q_i} \mathrm{E}\left[\Pi_i\left(q_i \leq q_{il}^{\mathrm{LS}}|l\right)\right] \geq \max_{q_i} \mathrm{E}\left[\Pi_i\left(q_i > q_{il}^{\mathrm{LS}}|l\right)\right] \tag{5}$$

For simplify, we use $\mathrm{E}\left[\Pi_i\left(q_i > q_{il}^{\mathrm{LS}}|h\right)\right]$ to refer to the incumbent's expected profit with a h-type forecast signal under $q_i > q_{il}$. Similar notations are used in other conditions.

It is easy to prove that the second constraint of above does not bind, such that only the high-type incumbent has an intention to mimic a low-state one. Since $\mathrm{E}\left[\Pi_i(q_i > q_{il}^{\mathrm{LS}}|h)\right] = \frac{\mathrm{E}[A|h]-q_i}{2} q_i$ and $\mathrm{E}\left[\Pi_i(q_i \leq q_{il}^{\mathrm{LS}}|h)\right] = \frac{2\mathrm{E}[A|h]-\mathrm{E}[A|l]-q_i}{2} q_i$, in order to satisfy the constraints, the l-type sourcing quantity of the incumbent retailer needs to satisfy $q_{il}^1 \leq \frac{2\mathrm{E}[A|h]-\mathrm{E}[A|l]-\sqrt{(\mathrm{E}[A|h]-\mathrm{E}[A|l])(3\mathrm{E}[A|h]-\mathrm{E}[A|l])}}{2} = \frac{\tau-\Delta-2\left[\sqrt{(\tau+2\Delta)\Delta}-2\Delta\right]}{2} \bar{A}$. Meantime, it is plain to find that the incumbent retailer's optimal h-type sourcing quantity is $q_{ih}^{LS} = \frac{\mathrm{E}[A|h]}{2} = \frac{\tau+\Delta}{2}\bar{A}$, and the relevant maximal expected profit is $\max \mathrm{E}[\Pi_i(q_i > q_{il}^{\mathrm{LS}}|h)] = \frac{(\tau+\Delta)^2}{8}\bar{A}^2$. Notably, $\mathrm{E}[\Pi_i(q_i > q_{il}^{\mathrm{LS}}|h)] = \frac{2\mathrm{E}[A|l]-\mathrm{E}[A|h]-q_i}{2} q_i$ and $\mathrm{E}[\Pi_i(q_i \leq q_{il}^{\mathrm{LS}}|h)] = \frac{\mathrm{E}[A|l]-q_i}{2} q_i$, the first-best sourcing quantity for the l-type incumbent is $q_{il}^2 = \frac{\mathrm{E}[A|l]}{2} = \frac{\tau-\Delta}{2}\bar{A}$.

If $q_{il}^1 < q_{il}^2$ (i.e., $0 < \Delta < \frac{\tau}{2}$ where $0 < \frac{\tau}{2} < 1$), we can perceive that the secondary constraint will be more likely to be satisfied as the q_i increase, thus, we get the incumbent retailer's optimal equilibrium sourcing quantity which is $q_{il}^{\mathrm{LS}} = q_{il}^1 = \frac{\tau-\Delta-2\left[\sqrt{(\tau+2\Delta)\Delta}-2\Delta\right]}{2}\bar{A}$. Whereas, if $q_{il}^1 \geq q_{il}^2$ (i.e., $\frac{\tau}{2} \leq \Delta < 1$, alike, $0 < \frac{\tau}{2} < 1$), we can directly let $q_{il}^{\mathrm{LS}} = q_{il}^2 = \frac{\tau-\Delta}{2}\bar{A}$ be the l-type incumbent's optimal sourcing quantity, which satisfies the constraints too.

Therefore, the two retailers' most profitable separating equilibrium of sourcing quantities can be given as below:

$$q_i^{\mathrm{LS}} = \begin{cases} \frac{\tau+\varDelta}{2}\bar{A} & \text{if } s=h, \\ \frac{\tau-\varDelta}{2}(1-\mathcal{D})\bar{A} & \text{if } s=l \text{ and } 0<\varDelta<\frac{\tau}{2}, \\ \frac{\tau-\varDelta}{2}\bar{A} & \text{if } s=l \text{ and } \frac{\tau}{2}\le\varDelta<1. \end{cases}$$

$$q_e^{\mathrm{LS}} = \begin{cases} \frac{\tau+\varDelta}{4}\bar{A} & \text{if } s=h, \\ \frac{\tau-\varDelta}{4}(1+\mathcal{D})\bar{A} & \text{if } s=l \text{ and } 0<\varDelta<\frac{\tau}{2}, \\ \frac{\tau-\varDelta}{4}\bar{A} & \text{if } s=l \text{ and } \frac{\tau}{2}\le\varDelta<1. \end{cases}$$

Their corresponding profits show as the following:

$$\mathrm{E}[\varPi_i^{\mathrm{LS}}] = \begin{cases} \frac{(\tau+\varDelta)^2}{8}\bar{A}^2 & \text{if } s=h, \\ \frac{(\tau-\varDelta)^2}{8}(1-\mathcal{D}^2)\bar{A}^2 & \text{if } s=l \text{ and } 0<\varDelta<\frac{\tau}{2}, \\ \frac{(\tau-\varDelta)^2}{8}\bar{A}^2 & \text{if } s=l \text{ and } \frac{\tau}{2}\le\varDelta<1. \end{cases}$$

$$\mathrm{E}[\varPi_e^{\mathrm{LS}}] = \begin{cases} \frac{(\tau+\varDelta)^2}{16}\bar{A}^2 & \text{if } s=h, \\ \frac{(\tau-\varDelta)^2}{16}(1+\mathcal{D})^2\bar{A}^2 & \text{if } s=l \text{ and } 0<\varDelta<\frac{\tau}{2}, \\ \frac{(\tau-\varDelta)^2}{16}\bar{A}^2 & \text{if } s=l \text{ and } \frac{\tau}{2}\le\varDelta<1. \end{cases}$$

Where $\mathcal{D} = \frac{2\left[\sqrt{(\tau+2\varDelta)\varDelta}-2\varDelta\right]}{\tau-\varDelta} \in (0,1)$ can be viewed as the prediction information distorting degree through signalling approach.

D Proof of Lemma 4

When the incumbent orders a same quantity for any s and the upstream manufacturer always discloses her order information to the entrant, under such a condition, the entrant's optimal sourcing quantity is $q_e(q_i^{\mathrm{LP}}) = \arg\max_{q_e} \mathrm{E}[\varPi_e(q_e|q_i \le q_i^{\mathrm{LP}})] = \frac{\theta\mathrm{E}[A|h]+(1-\theta)\mathrm{E}[A|l]-q_i^{\mathrm{LP}}}{2} = \frac{(\tau-\varDelta+2\theta\varDelta)\bar{A}-q_i^{\mathrm{LP}}}{2}$. Therefore, considering the entrant's belief structure, there indeed exists the pooling equilibrium if and only if the following constraints are satisfied:

$$\begin{cases} \max_{q_i} \mathrm{E}[\varPi_i(q_i \le q_i^{\mathrm{LP}}|h)] = \mathrm{E}[\varPi_i(q_i = q_i^{\mathrm{LP}}|h)], \\ \max_{q_i} \mathrm{E}[\varPi_i(q_i \le q_i^{\mathrm{LP}}|l)] = \mathrm{E}[\varPi_i(q_i = q_i^{\mathrm{LP}}|l)], \\ \max_{q_i} \mathrm{E}[\varPi_i(q_i \le q_i^{\mathrm{LP}}|h)] \ge \max_{q_i} \mathrm{E}[\varPi_i(q_i > q_i^{\mathrm{LP}}|h)], \\ \max_{q_i} \mathrm{E}[\varPi_i(q_i \le q_i^{\mathrm{LP}}|l)] \ge \max_{q_i} \mathrm{E}[\varPi_i(q_i > q_i^{\mathrm{LP}}|l)]. \end{cases} \quad (6)$$

Where $\mathrm{E}[\varPi_i(q_i \le q_i^{\mathrm{LP}}|h)]$ represents the incumbent's expected profit when she sources $q_i \le q_i^{\mathrm{LP}}$ and $s=h$, similarly, we use the same notation for other situations too.

For a given signal s, there are $\mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|h)] = \frac{[\tau+\Delta+2(1-\theta)\Delta]\bar{A}-q_i^{\mathrm{LP}}}{2}q_i^{\mathrm{LP}}$ and $\mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|l)] = \frac{(\tau-\Delta-2\theta\Delta)\bar{A}-q_i^{LP}}{2}q_i^{\mathrm{LP}}$. In order to satisfy the constraints of the first two above, the incumbent must set her orders $q_i^{\mathrm{LP}} \le \frac{\tau-\Delta-2\theta\Delta}{2}\bar{A}$, where $0 < \Delta \le \frac{\tau}{1+2\theta}$ to ensure that the retailer's sourcing quantity is non negative. It is not opaque to observe that both $\mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|h)]$ and $\mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|l)]$ are strictly concave increasing functions of q_i when $q_i \le \frac{\tau-\Delta-2\theta\Delta}{2}\bar{A}$, thus, we can set $q_i^{\mathrm{LP}} = \frac{\tau-\Delta-2\theta\Delta}{2}\bar{A}$ to realize the most profitable outcome: $\max \mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|h)] = \mathrm{E}[\Pi_i(q_i = q_i^{\mathrm{P}}|h)] = \frac{(\tau-\Delta-2\theta\Delta)^2+8\Delta(\tau-\Delta-2\theta\Delta)}{8}\bar{A}^2$ and $\max \mathrm{E}[\Pi_i(q_i \le q_i^{\mathrm{LP}}|l)] = \mathrm{E}[\Pi_i(q_i = q_i^{\mathrm{LP}}|l)] = \frac{(\tau-\Delta-2\theta\Delta)^2}{8}\bar{A}^2$ respectively.

Meantime, since $\mathrm{E}[\Pi_i(q_i > q_i^{\mathrm{LP}}|h)] = \frac{(\tau+\Delta)\bar{A}-q_i}{2}q_i$ and $\mathrm{E}[\Pi_i(q_i > q_i^{\mathrm{LP}}|l)] = \frac{(\tau-3\Delta)\bar{A}-q_i}{2}q_i$, in order to satisfy the constraints of the last two above, the following inequations must hold:

$$\begin{cases} \frac{(\tau+\Delta)\bar{A}-q_i}{2}q_i \le \frac{(\tau-\Delta-2\theta\Delta)^2+8\Delta(\tau-\Delta-2\theta\Delta)}{8}\bar{A}^2, \\ \frac{(\tau-3\Delta)\bar{A}-q_i}{2}q_i \le \frac{(\tau-\Delta-2\theta\Delta)^2}{8}\bar{A}^2. \end{cases} \tag{7}$$

Equivalently,

$$\begin{cases} \left(\frac{\tau+\Delta}{2}\right)^2 - 4(-\frac{1}{2})\left[-\frac{(\tau-\Delta-2\theta\Delta)^2+8\Delta(\tau-\Delta-2\theta\Delta)}{8}\right] < 0, \\ \left(\frac{\tau-3\Delta}{2}\right)^2 - 4(-\frac{1}{2})\left[-\frac{(\tau-\Delta-2\theta\Delta)^2}{8}\right] < 0. \end{cases} \tag{8}$$

Then, we have

$$\begin{cases} \Delta > \frac{(\theta-1)\tau}{\theta^2-3\theta-2}, \\ \Delta < \frac{\tau}{2+\theta}. \end{cases} \tag{9}$$

For any $\theta \in (0,1)$, there is $\frac{\tau}{1+2\theta} > \frac{\tau}{2+\theta} > \frac{(\theta-1)\tau}{\theta^2-3\theta-2}$, which means that there exists the most profitable pooling equilibrium to the incumbent if and only if $\frac{(\theta-1)\tau}{\theta^2-3\theta-2} < \Delta < \frac{\tau}{2+\theta}$, the results are summarized as follows: $q_i^{\mathrm{LP}} = \frac{\tau-\Delta-2\theta\Delta}{2}\bar{A}$ and $q_e^{\mathrm{LP}} = \frac{\tau-\Delta+6\theta\Delta}{4}\bar{A}$. The retailers' corresponding expected profits can be given as below:

$$\mathrm{E}[\Pi_i^{\mathrm{LP}}] = \begin{cases} \frac{(\tau-\Delta-2\theta\Delta)^2+8\Delta(\tau-\Delta-2\theta\Delta)}{8}\bar{A}^2 & \text{if } s = h, \\ \frac{(\tau-\Delta-2\theta\Delta)^2}{8}\bar{A}^2 & \text{if } s = l. \end{cases}$$

$$\mathrm{E}[\Pi_e^{\mathrm{LP}}] = \frac{(\tau-\Delta+6\theta\Delta)^2}{16}\bar{A}^2.$$

Where $\frac{(\theta-1)\tau}{\theta^2-3\theta-2} < \Delta < \frac{\tau}{2+\theta}$ to ensure that the pooling equilibrium above is the most profitable outcome.

E Proof of Proposition 1

As we can see, there may exist multiple perfect Bayesian equilibrium (PBE) in our model setting: the optimal separating equilibrium and the optimal pooling equilibrium, especially when $\frac{(1-\theta)\tau}{2-3\theta-\theta^2} < \Delta < \frac{\tau}{2+\theta}$. So we adopt the concept of *Lexicographically Maximum Sequential Equilibrium* ($LMSE$) to find the unique equilibrium outcome, which has been widely used as one of the multiple equilibria selection criterion. The $LMSE$ from the perspective of the types that have the most incentives to reveal their identities to refine the pure-strategy perfect Bayesian equilibria. In our paper, the l-type incumbent has more incentive to reveal her type than h-type incumbent, thus, we from the perspective of the l-type incumbent to find the unique optimal equilibrium. We can further understand the concept from the l-type incumbent's incentive. For the l-type incumbent, if the revealing type is always beneficial for her, she will always have an incentive to reveal her type no matter how the h-type incumbent mimics.

The definition of *Lexicographically Maximum Sequential Equilibrium* ($LMSE$) can be demonstrated as below. In a signalling game (G), we denote the set of pure-strategy perfect Bayesian equilibria by $PBE(G)$ and the set of types by $\{h, l\}$. Furthermore, we denote the i-type player's profit by $\pi_i(\sigma)$, where the strategy profile $\sigma \in PBE(G)$. If $\pi_l(\sigma) > \pi_l(\sigma')$, or $\pi_l(\sigma) = \pi_l(\sigma')$ and $\pi_h(\sigma) > \pi_h(\sigma')$, we can deem that the strategy profile σ *lexicographically dominates* (*l-dominates*) the strategy profile σ'. Otherwise, the strategy profile σ' is an $LMSE$ if there does not exist any profile $\sigma \in PBE(G)$ which *l-dominates* the strategy profile σ'.

Appendix C and Appendix D have characterized the most profitable separating equilibrium and pooling equilibrium respectively, according to the definition of $LMSE$, it is easy to certify that the others separating equilibria and pooling equilibria are *l-dominated* by the most profitable separating equilibrium and the most profitable pooling equilibrium. Such that, next, we just need to use the concept of $LMSE$ to find the unique outcome.

We use $\max \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{LS}}]$ to represent the incumbent's maximal expected profit under the case of separating when $s = l$, and $\max \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{LP}}]$ represents the pooling case. In order to figure out the most-efficient outcome, we assume that the most profitable pooling equilibrium *l-dominates* the most profitable separating equilibrium, which means $\max \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{LP}}] > \max \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{LS}}]$. Equivalently, $\frac{(\tau-\mathit{\Delta}-2\theta\mathit{\Delta})^2}{8}\bar{A} > \frac{(\tau-\mathit{\Delta})^2(1-\mathcal{D}^2)}{8}\bar{A}^2$ where $\frac{(1-\theta)\tau}{2-3\theta-\theta^2} < \mathit{\Delta} < \frac{\tau}{2+\theta}$. And the inequation holds if and only if the following inequation exists:

$$\left[(\theta^2+\theta+6)^2-32\right]\mathit{\Delta}^2 + \left[2(1-\theta)(\theta^2+\theta+6)-16\right]\tau\mathit{\Delta} + (1-\theta)^2\tau^2 > 0. \tag{10}$$

Equivalently, $\mathit{\Delta}$ must satisfy

$$\mathit{\Delta} < \mathit{\Delta}_1 = \frac{\left[8-(1-\theta)(\theta^2+\theta+6)-4\sqrt{2(1-\theta)^2-(1-\theta)(\theta^2+\theta+6)+4}\right]\tau}{(\theta^2+\theta+6)^2-32} \tag{11}$$

or

$$\mathit{\Delta} > \mathit{\Delta}_2 = \frac{\left[8-(1-\theta)(\theta^2+\theta+6)+4\sqrt{2(1-\theta)^2-(1-\theta)(\theta^2+\theta+6)+4}\right]\tau}{(\theta^2+\theta+6)^2-32} \tag{12}$$

Which illuminates that the pooling equilibrium *l-dominates* the separating equilibrium if ans only if there exists an intersection between the set of $\frac{(1-\theta)\tau}{2-3\theta-\theta^2} < \mathit{\Delta} < \frac{\tau}{2+\theta}$ and the set of $\mathit{\Delta} < \mathit{\Delta}_1$ or $\mathit{\Delta} > \mathit{\Delta}_2$. For concision, we mark the coefficients as $a_1 = \frac{1-\theta}{2-3\theta-\theta^2}$, $a_2 = \frac{1}{2+\theta}$, $a_3 = \frac{8-(1-\theta)(\theta^2+\theta+6)-4\sqrt{2(1-\theta)^2-(1-\theta)(\theta^2+\theta+6)+4}}{(\theta^2+\theta+6)^2-32}$ and $a_4 = \frac{8-(1-\theta)(\theta^2+\theta+6)+4\sqrt{2(1-\theta)^2-(1-\theta)(\theta^2+\theta+6)+4}}{(\theta^2+\theta+6)^2-32}$.

Such that, $\frac{(1-\theta)\tau}{2-3\theta-\theta^2} = a_1\tau < \varDelta < \frac{\tau}{2+\theta} = a_2\tau$, $\varDelta_1 = a_3\tau$ and $\varDelta_2 = a_4\tau$ We use a figure to intuitively describe the relationship among these coefficients.

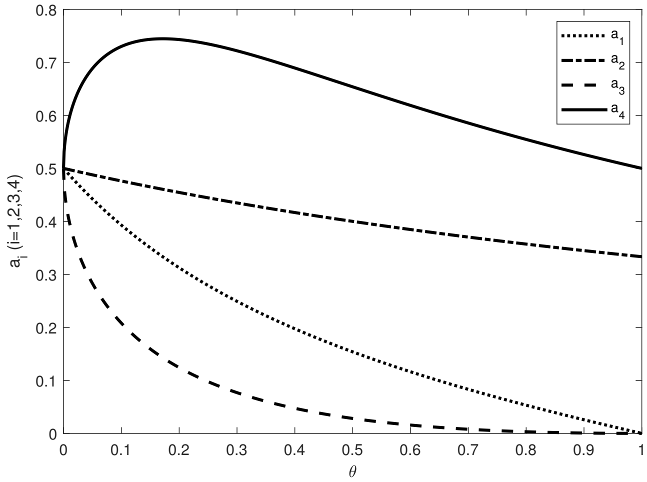


Fig. E1: The comparison of these coefficients.

From the Fig. E1, we can find that there is no intersection between $\frac{(1-\theta)\tau}{2-3\theta-\theta^2} < \varDelta < \frac{\tau}{2+\theta}$ and $\varDelta < \varDelta_1$ or $\varDelta > \varDelta_2$, which indicates that $\max \mathrm{E}[\varPi_{il}^{\mathrm{LP}}] > \max \mathrm{E}[\varPi_{il}^{\mathrm{LS}}]$ never holds, which means that the separating equilibrium is an *LMSE* and the most profitable separating equilibrium *l-dominates* the most profitable pooling equilibrium.

F Proof of Proposition 2

The two retailers' total sourcing quantities under no information leakage are: $Q_h^{\mathrm{NL}} = q_{ih}^{\mathrm{NL}} + q_e^{\mathrm{NL}} = \frac{2(\tau+\varDelta)-(1-\theta)\varDelta}{3}\bar{A}$ when $s = h$ and $Q_l^{\mathrm{NL}} = q_{il}^{\mathrm{NL}} + q_e^{\mathrm{NL}} = \frac{2(\tau-\varDelta)+\theta\varDelta}{3}\bar{A}$ when $s = l$. And the two retailers' total sourcing quantities under information leakage are: $Q_h^{\mathrm{L}} = q_{ih}^{\mathrm{L}} + q_{eh}^{\mathrm{L}} = \frac{3(\tau+\varDelta)}{4}\bar{A}$, $Q_h^{\mathrm{L}} = q_{ih}^{\mathrm{L}} + q_{eh}^{\mathrm{L}} = \frac{3(\tau+\varDelta)}{4}\bar{A}$ when $s = h$, $Q_l^{\mathrm{L}} = q_{il}^{\mathrm{L}} + q_{el}^{\mathrm{L}} = \frac{3\tau+\varDelta-2\sqrt{(\tau+2\varDelta)\varDelta}}{4}\bar{A}$ when $s = l$ and $0 < \varDelta < \frac{\tau}{2}$, and $Q_l^{\mathrm{L}} = q_{il}^{\mathrm{L}} + q_{el}^{\mathrm{L}} = \frac{3(\tau-\varDelta)}{4}\bar{A}$ when $s = l$ and $\frac{\tau}{2} \le \varDelta < 1$.

If $s = h$, it is obvious that $Q_h^{\mathrm{NL}} = \frac{8(\tau+\varDelta)-4(1-\theta)\varDelta}{12}\bar{A} < Q_h^{\mathrm{L}} = \frac{9(\tau+\varDelta)}{12}\bar{A}$.

If $s = l$, $\frac{1-\delta+2\theta\delta}{\delta(2\theta+1)} \ge 1$ when $\delta \le 0.5$, which means that $0 < \rho < 1 < \frac{1-\delta+2\theta\delta}{\delta(2\theta+1)}$ (i.e., $0 < \varDelta < \frac{\tau}{2}$) always holds. Under such a condition, $Q_l^{\mathrm{NL}} < Q_l^{\mathrm{L}}$ if and only if $(49-88\theta+16\theta^2)\varDelta^2 - (14+8\theta)\tau\varDelta + \tau^2 > 0$. There is $(49-88\theta+16\theta^2)\varDelta^2-(14+8\theta)\tau\varDelta+\tau^2 = 0$ when $\varDelta_{1,2} = \frac{7+4\theta\pm 24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau$. (1) $49-88\theta+16\theta^2 < 0$ when $\theta \in (\frac{11-6\sqrt{2}}{4}, 1)$, $\frac{7+4\theta+24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau < 0$ and $\frac{7+4\theta-24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau > \frac{\tau}{2}$, $Q_l^{\mathrm{NL}} < Q_l^{\mathrm{L}}$ always holds when $0 < \varDelta < \frac{\tau}{2}$. (2) $49-88\theta+16\theta^2 = 0$ when $\theta = \frac{11-6\sqrt{2}}{4}$, $-(14+8\theta)\tau\varDelta + \tau^2 > 0$ i.e., $Q_l^{\mathrm{NL}} < Q_l^{\mathrm{L}}$ always holds. (3) $49-88\theta+16\theta^2 > 0$ when $\theta \in (0, \frac{11-6\sqrt{2}}{4})$, there are $\frac{7+4\theta-24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau > 0$ and $\frac{7+4\theta+24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau < \frac{\tau}{2}$ if $\theta \in (0, (\frac{\sqrt{29}}{2}-3)^2)$, $\frac{7+4\theta-24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau < 0$ and $\frac{7+4\theta+24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau < \frac{\tau}{2}$ if $\theta \in ((\frac{\sqrt{29}}{2}-3)^2, 0.16565)$, $\frac{7+4\theta-24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau < 0$ and $\frac{7+4\theta+24\sqrt{\theta}}{16\theta^2-88\theta+49}\tau > \frac{\tau}{2}$ if $\theta \in (0.16565, \frac{11-6\sqrt{2}}{4})$.

If $s = l$, $\frac{1-\delta+2\theta\delta}{\delta(2\theta+1)} < 1$ when $\delta > 0.5$, both $0 < \Delta < \frac{\tau}{2}$ ($0 < \rho <$), which means that $Q_l^{\mathrm{L}} = \frac{3\tau+\Delta-2\sqrt{(\tau+2\Delta)\Delta}}{4}\bar{A}$ $\frac{3\tau+\Delta-2\sqrt{(\tau+2\Delta)\Delta}}{4}\bar{A}$ when $0 < \Delta < \frac{\tau}{2}$ (i.e., $0 < \rho < \frac{1-\delta+2\theta\delta}{\delta(2\theta+1)}$), and $Q_l^{\mathrm{L}} = \frac{3(\tau-\Delta)}{4}\bar{A}$ $\frac{\tau}{2} \leq \Delta < 1$ (i.e., $\frac{1-\delta+2\theta\delta}{\delta(2\theta+1)} \leq \rho < 1$). Under the condition of $0 < \Delta < \frac{\tau}{2}$, the results are similar to the situation of $0 < \delta \leq 0.5$, so we omit it. Under the condition of $\frac{\tau}{2} \leq \Delta < 1$ ($\rho \leq \frac{1-\delta+2\theta\delta}{\delta(2\theta+1)}$), $Q_l^{\mathrm{L}} > Q_l^{\mathrm{NL}}$ when $0 < \theta < 0.25$ and $\frac{1-\delta+2\theta\delta}{\delta(2\theta+1)} < \rho < \dfrac{1-\delta+2\theta\delta}{6\theta\delta}$, otherwise, $Q_l^{\mathrm{L}} < Q_l^{\mathrm{NL}}$.

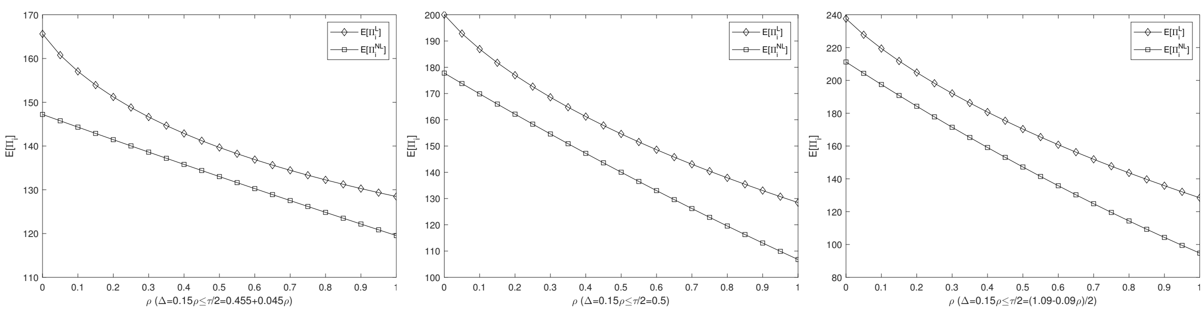
G Proof of Proposition 3

For concision, we assume that the manufacturer's wholesale price is equal to zero when we are investigating the incumbent retailer's information management preference. From the above proof, we can directly know the incumbent's expected profits.

When $s = h$, we assume that the incumbent's expected profit under the condition of information leakage is larger than no information leakage, i.e., $\mathrm{E}[\Pi_{ih}^{\mathrm{L}}] > \mathrm{E}[\Pi_{ih}^{\mathrm{NL}}]$, equivalently, $(3-2\sqrt{2})\tau > \left(4\sqrt{2}-2\sqrt{2}\theta-3\right)\Delta$. Thus, if $4\sqrt{2}-2\sqrt{2}\theta-3 \leq 0$ (i.e., $\theta \geq \frac{4\sqrt{2}-3}{2\sqrt{2}}$), it is easy to find that the inequation always holds, which means that the incumbent's expected profit is higher under the condition of information leakage than no information leakage. While when $4\sqrt{2}-2\sqrt{2}\theta-3 > 0$ (i.e., $\theta < \frac{4\sqrt{2}-3}{2\sqrt{2}}$), the inequation holds if and only if $\Delta < \frac{(3-2\sqrt{2})\tau}{4\sqrt{2}-2\sqrt{2}\theta-3}$, i.e., $\rho < \frac{(3-2\sqrt{2})(1-\delta+2\theta\delta)}{6(\sqrt{2}-1)(1-\theta)\delta}$, which means that the incumbent will earn more under the condition of information leakage when $\rho < \frac{(3-2\sqrt{2})(1-\delta+2\theta\delta)}{6(\sqrt{2}-1)(1-\theta)\delta}$ where $\theta < \frac{4\sqrt{2}-3}{2\sqrt{2}}$. Otherwise, the incumbent's profit is worse under the condition of information leakage than no information leakage.

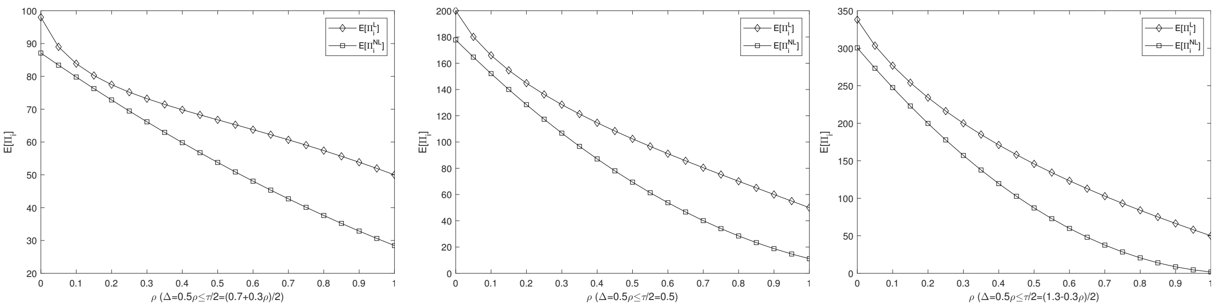
When $s = l$, the incumbent's expected profits under the two scenarios are

$$\mathrm{E}[\Pi_{il}^{\mathrm{L}}] = \begin{cases} \frac{(\tau-\Delta)^2(1-\mathcal{D}^2)\bar{A}^2}{8} & \text{if } \Delta \leq \frac{\tau}{2}, \\ \frac{(\tau-\Delta)^2\bar{A}^2}{8} & \text{if } \Delta > \frac{\tau}{2} \end{cases}$$



(a) An example of the incumbent's profit comparison when $\theta = 0.2$.
(b) An example of the incumbent's profit comparison when $\theta = 0.5$.
(c) An example of the incumbent's profit comparison when $\theta = 0.8$.

Fig. G1: Example of a three-part figure with individual sub-captions showing that the incumbent will earn more under the condition of information leakage than no information leakage when $\delta = 0.15$.

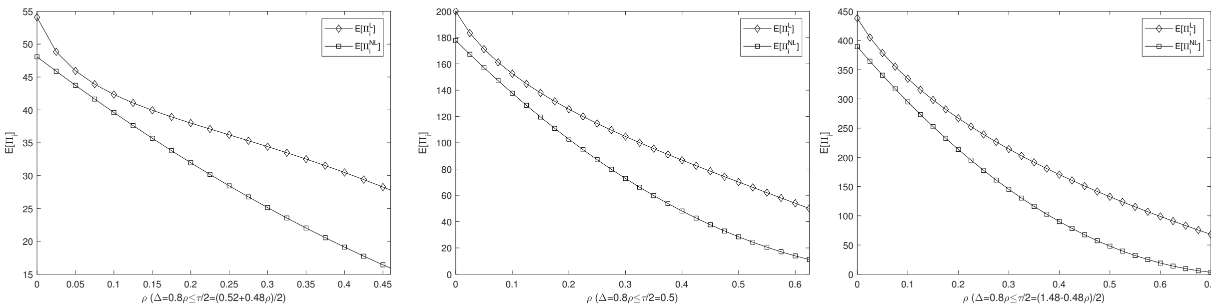


(a) An example of the incumbent's profit comparison when $\theta = 0.2$.

(b) An example of the incumbent's profit comparison when $\theta = 0.5$.

(c) An example of the incumbent's profit comparison when $\theta = 0.8$.

Fig. G2: Example of a three-part figure with individual sub-captions showing that the incumbent will earn more under the condition of information leakage than no information leakage when $\delta = 0.5$.



(a) An example of the incumbent's profit comparison when $\theta = 0.2$.

(b) An example of the incumbent's profit comparison when $\theta = 0.5$.

(c) An example of the incumbent's profit comparison when $\theta = 0.8$.

Fig. G3: Example of a three-part figure with individual sub-captions showing that the incumbent will earn more under the condition of information leakage than no information leakage when $\delta = 0.8$.

and

$$\mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{NL}}] = \frac{(\tau - \mathit{\Delta} - \theta\mathit{\Delta})^2 \bar{A}^2}{9}$$

respectively, where $\mathit{\Delta} \leq \frac{\tau}{1+\theta}$ (i.e., ρ) to ensure that the incumbent retailer's sourcing quantity is non negative. In the case of $\frac{\tau}{2} < \mathit{\Delta} \leq \frac{\tau}{1+\theta}$, i.e., $\frac{1-\delta+2\theta\delta}{(2\theta+1)\delta} < \rho \leq \frac{1-\delta+2\theta\delta}{3\theta\delta}$, we can easily prove that $\frac{(\tau-\mathit{\Delta})^2\bar{A}^2}{8} > \frac{(\tau-\mathit{\Delta}-\theta\mathit{\Delta})^2\bar{A}^2}{9}$ (i.e., $\mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{L}}] > \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{NL}}]$), which means that the incumbent retailer has the incentive to voluntarily share her information. In the case of $\mathit{\Delta} \leq \frac{\tau}{2}$ (i.e., $\rho \leq \frac{1-\delta+2\theta\delta}{(2\theta+1)\delta}$), for any δ, θ and ρ, there exists $\mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{L}}] > \mathrm{E}[\mathit{\Pi}_{il}^{\mathrm{NL}}]$ too. We randomly select several groups of data that meet the conditions to examine this conclusion, The results are shown in the following figures. For concision, we assume that $\omega = 0$ and $\bar{A} = 40$, we use the solid line with diamond in the following figures to represent the incumbent's expected profit under the scenario of information leakage and the solid line with square to represent the scenario of no information leakage. From those figures, we can find that the incumbent will realize a higher profit under the condition of information leakage when $\mathit{\Delta} \leq \frac{\tau}{2}$.

H Proof of Proposition 4

From the above proof, we can directly know the entrant's expected profits. When $s = h$, we assume that information leakage is higher than no information leakage for the entrant, i.e., $\mathrm{E}[\Pi_{mh}^{\mathrm{L}}] > \mathrm{E}[\Pi_{mh}^{\mathrm{NL}}]$ ($\frac{(\tau+\Delta)^2\bar{A}^2}{16} > \frac{(\tau-\Delta+2\theta\Delta)^2\bar{A}^2}{9}$), equivalently,

$$(7-8\theta)\Delta > \tau.$$

Thus, if $7-8\theta \leq 0$ (i.e., $\theta \geq \frac{7}{8}$), $\mathrm{E}[\Pi_{mh}^{\mathrm{L}}] > \mathrm{E}[\Pi_{mh}^{\mathrm{NL}}]$ never holds. But when $7-8\theta > 0$ (i.e., $\theta < \frac{7}{8}$), the above inequation holds if and only if $\Delta > \frac{\tau}{7-8\theta}$ (i.e., $\rho > \frac{1-\delta+2\theta\delta}{6(1-\theta)\delta}$) where $\theta < \frac{7}{8}$. This means that the entrant's profit is higher under the condition of information leakage than no information leakage if and only if $\rho > \frac{1-\delta+2\theta\delta}{6(1-\theta)\delta}$ where $\theta < \frac{7}{8}$, otherwise, the entrant's profit will be lower under the condition of information leakage than no information leakage.

When $s = l$, the entrant's expected profits are

$$\mathrm{E}[\Pi_{el}^{\mathrm{L}}] = \begin{cases} \frac{(\tau-\Delta)^2(1+\mathcal{D})^2\bar{A}^2}{16} & \text{if } \Delta \leq \frac{\tau}{2}, \\ \frac{(\tau-\Delta)^2\bar{A}^2}{16} & \text{if } \Delta > \frac{\tau}{2} \end{cases}$$

and

$$\mathrm{E}[\Pi_{il}^{\mathrm{NL}}] = \frac{(\tau-\Delta+2\theta\Delta)^2\bar{A}^2}{9}$$

separately. When $\Delta > \frac{\tau}{2}$, one can easily verify that $\frac{(\tau-\Delta+2\theta\Delta)^2\bar{A}^2}{9} > \frac{(\tau-\Delta)^2\bar{A}^2}{16}$, i.e., $\mathrm{E}[\Pi_{il}^{\mathrm{NL}}] > \mathrm{E}[\Pi_{el}^{\mathrm{L}}]$. When $\Delta \leq \frac{\tau}{2}$, we assume that no information leakage is better off than information leakage for the entrant retailer, that is, $\mathrm{E}[\Pi_{il}^{\mathrm{NL}}] > \mathrm{E}[\Pi_{el}^{\mathrm{L}}]$, equivalently,

$$\left(49+176\theta+64\theta^2\right)\Delta^2 - (14-16\theta)\tau\Delta + \tau^2 > 0.$$

It is plain to find that $(14-16\theta)^2\tau^2 - 4(49+176\theta+64\theta^2)\tau^2 = -752\theta\tau^2 < 0$, which indicates that the above inequation always holds. Such that, the entrant's profit will be lower if the manufacturer leaks information to him than doesn't when $s = l$.