

1 Supplemental material

The supplemental material contains more details regarding the test set used in the numerical comparison as well as detailed results showing how each method performed for each individual test problem.

2 Test set

The main properties of the test problems are shown in Table 1, showing the number of integer variables m^{int} , binary variables m^{bin} , continuous variables n , and total variables $n + m$. The table also shows the number of constraints, the number of nonlinear functions, *i.e.*, nonlinear objective function and constraint functions, and the ratio between number of variables present in a nonlinear term and the total of variables $\frac{n_{nonlin}}{n+m}$.

Table 1: List of instances in convex MINLP test set.

Instance	m^{int}	m^{bin}	n	$n + m$	ncons	nnlfunc	$\frac{n_{nonlin}}{n+m}$
cvxnonsep_normcon40	20	0	20	40	1	1	1
cvxnonsep_nsig40	20	0	20	40	1	1	1
cvxnonsep_pcon40	20	0	20	40	1	1	1
cvxnonsep_psig40	20	0	20	40	0	1	1
cvxnonsep_normcon30	15	0	15	30	1	1	1
cvxnonsep_nsig30	15	0	15	30	1	1	1
cvxnonsep_pcon30	15	0	15	30	1	1	1
cvxnonsep_psig30	15	0	15	30	0	1	1
cvxnonsep_normcon20	10	0	10	20	1	1	1
cvxnonsep_nsig20	10	0	10	20	1	1	1
cvxnonsep_pcon20	10	0	10	20	1	1	1
cvxnonsep_psig20	10	0	10	20	0	1	1
du-opt	13	0	7	20	9	1	1
du-opt5	13	0	7	20	9	1	1
ex1223b	0	4	3	7	9	5	1
ibs2	0	1500	1510	3010	1821	10	0.996678
squff030-150	0	30	4500	4530	4650	1	0.993377
squff020-150	0	20	3000	3020	3150	1	0.993377
smallinvSNPr1b010-011	100	0	1	101	4	1	0.990099
smallinvSNPr1b020-022	100	0	1	101	4	1	0.990099
smallinvSNPr1b050-055	100	0	1	101	4	1	0.990099
smallinvSNPr1b100-110	100	0	1	101	4	1	0.990099
smallinvSNPr1b150-165	100	0	1	101	4	1	0.990099
smallinvSNPr1b200-220	100	0	1	101	4	1	0.990099
smallinvSNPr2b010-011	100	0	1	101	4	1	0.990099
smallinvSNPr2b020-022	100	0	1	101	4	1	0.990099
smallinvSNPr2b050-055	100	0	1	101	4	1	0.990099

smallinvSNPr2b100-110	100	0	1	101	4	1	0.990099
smallinvSNPr2b150-165	100	0	1	101	4	1	0.990099
smallinvSNPr2b200-220	100	0	1	101	4	1	0.990099
smallinvSNPr3b010-011	100	0	1	101	4	1	0.990099
smallinvSNPr3b020-022	100	0	1	101	4	1	0.990099
smallinvSNPr3b050-055	100	0	1	101	4	1	0.990099
smallinvSNPr3b100-110	100	0	1	101	4	1	0.990099
smallinvSNPr3b150-165	100	0	1	101	4	1	0.990099
smallinvSNPr3b200-220	100	0	1	101	4	1	0.990099
smallinvSNPr4b010-011	100	0	1	101	4	1	0.990099
smallinvSNPr4b020-022	100	0	1	101	4	1	0.990099
smallinvSNPr4b050-055	100	0	1	101	4	1	0.990099
smallinvSNPr4b100-110	100	0	1	101	4	1	0.990099
smallinvSNPr4b150-165	100	0	1	101	4	1	0.990099
smallinvSNPr4b200-220	100	0	1	101	4	1	0.990099
smallinvSNPr5b010-011	100	0	1	101	4	1	0.990099
smallinvSNPr5b020-022	100	0	1	101	4	1	0.990099
smallinvSNPr5b050-055	100	0	1	101	4	1	0.990099
smallinvSNPr5b100-110	100	0	1	101	4	1	0.990099
smallinvSNPr5b150-165	100	0	1	101	4	1	0.990099
smallinvSNPr5b200-220	100	0	1	101	4	1	0.990099
squff030-100	0	30	3000	3030	3100	1	0.990099
squff040-080	0	40	3200	3240	3280	1	0.987654
squff015-080	0	15	1200	1215	1280	1	0.987654
squff010-080	0	10	800	810	880	1	0.987654
squff015-060	0	15	900	915	960	1	0.983607
squff020-050	0	20	1000	1020	1050	1	0.980392
squff025-040	0	25	1000	1025	1040	1	0.97561
squff020-040	0	20	800	820	840	1	0.97561
squff010-040	0	10	400	410	440	1	0.97561
smallinvDAXr1b010-011	30	0	1	31	4	1	0.967742
smallinvDAXr1b020-022	30	0	1	31	4	1	0.967742
smallinvDAXr1b050-055	30	0	1	31	4	1	0.967742
smallinvDAXr1b100-110	30	0	1	31	4	1	0.967742
smallinvDAXr1b150-165	30	0	1	31	4	1	0.967742
smallinvDAXr1b200-220	30	0	1	31	4	1	0.967742
smallinvDAXr2b010-011	30	0	1	31	4	1	0.967742
smallinvDAXr2b020-022	30	0	1	31	4	1	0.967742
smallinvDAXr2b050-055	30	0	1	31	4	1	0.967742
smallinvDAXr2b100-110	30	0	1	31	4	1	0.967742
smallinvDAXr2b150-165	30	0	1	31	4	1	0.967742
smallinvDAXr2b200-220	30	0	1	31	4	1	0.967742
smallinvDAXr3b010-011	30	0	1	31	4	1	0.967742
smallinvDAXr3b020-022	30	0	1	31	4	1	0.967742
smallinvDAXr3b050-055	30	0	1	31	4	1	0.967742
smallinvDAXr3b100-110	30	0	1	31	4	1	0.967742

smallinvDAXr3b150-165	30	0	1	31	4	1	0.967742
smallinvDAXr3b200-220	30	0	1	31	4	1	0.967742
smallinvDAXr4b010-011	30	0	1	31	4	1	0.967742
smallinvDAXr4b020-022	30	0	1	31	4	1	0.967742
smallinvDAXr4b050-055	30	0	1	31	4	1	0.967742
smallinvDAXr4b100-110	30	0	1	31	4	1	0.967742
smallinvDAXr4b150-165	30	0	1	31	4	1	0.967742
smallinvDAXr4b200-220	30	0	1	31	4	1	0.967742
smallinvDAXr5b010-011	30	0	1	31	4	1	0.967742
smallinvDAXr5b020-022	30	0	1	31	4	1	0.967742
smallinvDAXr5b050-055	30	0	1	31	4	1	0.967742
smallinvDAXr5b100-110	30	0	1	31	4	1	0.967742
smallinvDAXr5b150-165	30	0	1	31	4	1	0.967742
smallinvDAXr5b200-220	30	0	1	31	4	1	0.967742
squff025-030	0	25	750	775	780	1	0.967742
squff025-025	0	25	625	650	650	1	0.961538
squff010-025	0	10	250	260	275	1	0.961538
fac2	0	12	54	66	33	1	0.818182
fac3	0	12	54	66	33	1	0.818182
fac1	0	6	16	22	18	1	0.727273
ex1223	0	4	7	11	13	5	0.636364
st_e14	0	4	7	11	13	5	0.636364
batchdes	0	9	10	19	19	2	0.526316
cvxnonsep_pcon20r	10	0	29	39	20	19	0.512821
cvxnonsep_pcon30r	15	0	44	59	30	29	0.508475
cvxnonsep_pcon40r	20	0	59	79	40	39	0.506329
cvxnonsep_psig40r	20	0	62	82	42	41	0.5
cvxnonsep_normcon40r	20	0	60	80	41	40	0.5
cvxnonsep_nsig40r	20	0	60	80	41	40	0.5
cvxnonsep_psig30r	15	0	47	62	32	31	0.5
cvxnonsep_normcon30r	15	0	45	60	31	30	0.5
cvxnonsep_nsig30r	15	0	45	60	31	30	0.5
cvxnonsep_psig20r	10	0	32	42	22	21	0.5
cvxnonsep_normcon20r	10	0	30	40	21	20	0.5
cvxnonsep_nsig20r	10	0	30	40	21	20	0.5
st_miqp4	3	0	3	6	4	1	0.5

3 Detailed results

Details regarding the computational results are given in Table 2, showing the number of iterations, time and final gap for each test problem and method. The sign * indicates that the time limit was exceeded. The gap reported is calculated as $|LB - UB|/|LB|$ where LB and UB are the lower and upper bounds, respectively.

Table 2: Detailed results

Instance	OA			L-OA			Q-OA		
	Time [s]	Iterations	Gap	Time [s]	Iterations	Gap	Time [s]	Iterations	Gap
cvxnonsep_normcon40	900*	1856*	0.0123*	900*	549*	0.0076*	900*	535*	0.0075*
cvxnonsep_nsig40	360.863	663	0.0010	147.874	201	0.0010	121.554	144	0.0009
cvxnonsep_pcon40	119.204	290	0.0010	112.095	158	0.0009	108.194	148	0.0006
cvxnonsep_psig40	322.982	629	0.0010	168.723	210	0.0009	165.35	205	0.0009
cvxnonsep_normcon30	900*	1134*	0.0016*	538.453	455	0.0009	574.245	456	0.0006
cvxnonsep_nsig30	381.401	915	0.0010	42.286	78	0.0009	48.464	83	0.0009
cvxnonsep_pcon30	52.22	156	0.0010	41.269	72	0.0009	37.914	60	0.0008
cvxnonsep_psig30	266.827	694	0.0009	98.742	160	0.0007	89.545	141	0.0009
cvxnonsep_normcon20	104.701	311	0.0007	55.874	114	0.0010	49.974	102	0.0009
cvxnonsep_nsig20	35.909	119	0.0009	26.007	50	0.0010	16.489	32	0.0009
cvxnonsep_pcon20	19.183	63	0.0010	17.829	35	0.0009	13.166	25	0.0008
cvxnonsep_psig20	113.183	379	0.0008	9.25	19	0.0010	2.967	7	0.0008
du-opt	1.223	5	0.0005	4.066	9	0.0009	0.919	3	0.0000
du-opt5	1.203	5	0.0002	2.541	6	0.0008	0.89	3	0.0001
ex1223b	1.53	5	0.0000	2.225	5	0.0000	2.116	5	0.0000
squff030-150	900*	88*	3.8776*	900*	9*	1.7116*	900*	13*	2.0196*
squff020-150	900*	98*	2.7303*	900*	19*	1.4643*	900*	42*	1.4822*
smallinvSNPr1b010-011	1.894	6	0.0000	4.122	8	0.0000	2.35	5	0.0000
smallinvSNPr1b020-022	2.208	7	0.0000	4.63	9	0.0000	3.063	6	0.0000
smallinvSNPr1b050-055	3.18	10	0.0000	6.182	11	0.0000	4.249	8	0.0000
smallinvSNPr1b100-110	3.534	10	0.0000	7.272	13	0.0000	3.627	7	0.0008
smallinvSNPr1b150-165	3.412	11	0.0007	5.254	10	0.0003	2.535	5	0.0003
smallinvSNPr1b200-220	3.879	12	0.0006	5.994	12	0.0008	2.483	5	0.0008
smallinvSNPr2b010-011	1.194	4	0.0000	2.965	6	0.0000	1.819	4	0.0000
smallinvSNPr2b020-022	2.09	7	0.0000	4.63	9	0.0000	3.026	6	0.0000
smallinvSNPr2b050-055	3.051	10	0.0000	5.514	10	0.0000	2.483	5	0.0000
smallinvSNPr2b100-110	3.858	12	0.0003	5.91	11	0.0009	4.864	9	0.0007
smallinvSNPr2b150-165	3.212	10	0.0004	5.389	10	0.0000	2.532	5	0.0004
smallinvSNPr2b200-220	4.059	13	0.0004	6.539	12	0.0000	4.364	8	0.0000
smallinvSNPr3b010-011	2.301	7	0.0000	3.181	6	0.0000	2.73	5	0.0000
smallinvSNPr3b020-022	2.026	6	0.0000	3.673	7	0.0000	3.095	6	0.0000
smallinvSNPr3b050-055	3.084	10	0.0000	5.639	10	0.0000	4.318	8	0.0000
smallinvSNPr3b100-110	3.213	10	0.0006	4.998	9	0.0006	2.375	5	0.0006
smallinvSNPr3b150-165	3.412	11	0.0000	5.286	10	0.0009	3.156	6	0.0007
smallinvSNPr3b200-220	4.238	13	0.0000	6.489	12	0.0008	3.073	6	0.0001
smallinvSNPr4b010-011	7.176	20	0.0000	9.898	16	0.0000	9.05	14	0.0000
smallinvSNPr4b020-022	2.192	7	0.0000	3.523	7	0.0000	3.117	6	0.0000
smallinvSNPr4b050-055	3.477	11	0.0000	5.572	10	0.0003	3.733	7	0.0001

smallinvSNPr4b100-110	3.686	12	0.0000	6.667	12	0.0000	4.724	9	0.0000
smallinvSNPr4b150-165	4.373	14	0.0000	5.88	11	0.0002	3.64	7	0.0000
smallinvSNPr4b200-220	3.351	11	0.0000	5.726	11	0.0000	3.077	6	0.0000
smallinvSNPr5b010-011	4.618	14	0.0000	9.93	16	0.0000	7.46	12	0.0000
smallinvSNPr5b020-022	2.841	9	0.0000	3.579	7	0.0000	3.15	6	0.0000
smallinvSNPr5b050-055	3.479	11	0.0000	5.51	10	0.0000	2.443	5	0.0000
smallinvSNPr5b100-110	2.751	8	0.0000	6.007	11	0.0000	3.657	7	0.0000
smallinvSNPr5b150-165	4.421	14	0.0005	5.848	11	0.0008	3.119	6	0.0005
smallinvSNPr5b200-220	3.785	12	0.0000	5.276	10	0.0000	3.023	6	0.0000
squff030-100	900*	113*	3.2451*	900*	11*	1.9290*	900*	48*	1.9899*
squff040-080	900*	123*	3.4900*	900*	10*	1.8806*	900*	46*	1.8806*
squff015-080	900*	152*	1.1302*	900*	71*	1.2469*	900*	75*	1.2660*
squff010-080	900*	181*	0.1943*	900*	121*	0.2657*	900*	132*	0.2509*
squff015-060	900*	200*	0.4635*	900*	130*	0.4371*	900*	110*	0.8024*
squff020-050	900*	190*	1.1339*	900*	88*	1.2583*	900*	99*	1.3194*
squff025-040	900*	199*	1.1967*	900*	60*	1.5676*	900*	98*	1.5676*
squff020-040	900*	231*	0.3661*	900*	112*	0.5885*	900*	114*	0.6238*
squff010-040	238.235	159	0.0000	205.645	104	0.0001	206.827	101	0.0001
smallinvDAXr1b010-011	11.335	35	0.0008	14.291	25	0.0001	12.691	22	0.0008
smallinvDAXr1b020-022	12.043	38	0.0005	14.295	25	0.0005	10.002	17	0.0005
smallinvDAXr1b050-055	14.076	43	0.0006	14.803	25	0.0005	10.445	18	0.0005
smallinvDAXr1b100-110	16.173	51	0.0008	12.489	22	0.0009	6.212	11	0.0009
smallinvDAXr1b150-165	14.641	46	0.0009	14.059	24	0.0009	3.493	7	0.0009
smallinvDAXr1b200-220	14.787	47	0.0009	11.347	20	0.0008	4.671	9	0.0007
smallinvDAXr2b010-011	12.981	40	0.0010	14.629	25	0.0001	12.975	22	0.0008
smallinvDAXr2b020-022	12.396	39	0.0005	12.641	22	0.0004	9.675	17	0.0005
smallinvDAXr2b050-055	13.484	42	0.0008	12.23	22	0.0009	10.662	18	0.0005
smallinvDAXr2b100-110	13.918	44	0.0010	12.966	22	0.0008	5.869	11	0.0010
smallinvDAXr2b150-165	16.614	51	0.0010	10.966	19	0.0010	3.617	7	0.0009
smallinvDAXr2b200-220	16.22	49	0.0009	11.232	20	0.0006	4.741	9	0.0007
smallinvDAXr3b010-011	12.487	38	0.0010	18.347	31	0.0007	12.795	22	0.0008
smallinvDAXr3b020-022	12.574	38	0.0005	13.228	23	0.0009	9.789	17	0.0005
smallinvDAXr3b050-055	14.313	44	0.0006	14.523	25	0.0008	10.837	18	0.0005
smallinvDAXr3b100-110	15.528	47	0.0009	13.817	24	0.0006	7.497	13	0.0010
smallinvDAXr3b150-165	15.892	48	0.0010	11.882	21	0.0009	4.153	8	0.0009
smallinvDAXr3b200-220	16.602	52	0.0009	11.495	20	0.0006	5.817	10	0.0009
smallinvDAXr4b010-011	11.106	35	0.0007	12.699	22	0.0007	13.449	22	0.0008
smallinvDAXr4b020-022	13.016	40	0.0005	14.626	25	0.0006	10.425	17	0.0005
smallinvDAXr4b050-055	15.155	45	0.0009	13.019	23	0.0006	10.559	18	0.0005
smallinvDAXr4b100-110	14.402	44	0.0009	11.205	20	0.0009	7.405	13	0.0010
smallinvDAXr4b150-165	15.547	48	0.0010	11.21	20	0.0009	4.091	8	0.0009
smallinvDAXr4b200-220	14.571	44	0.0008	11.565	21	0.0009	6.78	11	0.0009

smallinvDAXr5b010-011	12.773	40	0.0006	12.884	22	0.0007	14.052	22	0.0008
smallinvDAXr5b020-022	11.054	34	0.0005	10.71	19	0.0005	10.791	18	0.0005
smallinvDAXr5b050-055	13.882	42	0.0008	13.168	23	0.0010	9.071	15	0.0008
smallinvDAXr5b100-110	14.225	44	0.0009	11.124	20	0.0007	7.497	13	0.0009
smallinvDAXr5b150-165	17.016	51	0.0010	10.465	19	0.0010	6.072	11	0.0009
smallinvDAXr5b200-220	15.207	47	0.0010	10.104	18	0.0009	7.396	11	0.0009
squff025-030	900*	266*	0.4438*	900*	136*	0.6127*	900*	150*	0.6333*
squff025-025	900*	267*	0.4102*	900*	118*	0.6177*	900*	133*	0.7265*
squff010-025	132.307	145	0.0000	116.474	94	0.0007	123.704	94	0.0000
fac2	2.544	7	0.0000	3.546	7	0.0000	3.108	6	0.0000
fac3	2.347	7	0.0000	3.331	7	0.0000	2.965	6	0.0000
fac1	1.469	3	0.0000	1.171	3	0.0000	1.199	3	0.0000
ex1223	1.483	5	0.0000	2.181	5	0.0000	2.147	5	0.0000
st_e14	1.456	5	0.0000	2.174	5	0.0000	2.154	5	0.0000
batchdes	0.605	2	0.0000	0.612	2	0.0000	0.599	2	0.0000
cvxnonsep_pcon20r	1.816	7	0.0002	4.207	9	0.0003	1.942	5	0.0008
cvxnonsep_pcon30r	1.899	7	0.0002	4.474	9	0.0006	3.191	7	0.0007
cvxnonsep_pcon40r	1.607	6	0.0008	4.99	10	0.0006	4.484	9	0.0010
cvxnonsep_psig40r	1.273	5	0.0007	4.853	9	0.0003	2.215	5	0.0003
cvxnonsep_normcon40r	3.112	7	0.0000	5.694	9	0.0007	2.974	5	0.0007
cvxnonsep_nsig40r	2.341	8	0.0005	7.702	13	0.0007	2.589	6	0.0007
cvxnonsep_psig30r	1.855	7	0.0003	5.055	10	0.0002	2.528	6	0.0008
cvxnonsep_normcon30r	3.203	9	0.0000	6.422	12	0.0005	3.577	7	0.0009
cvxnonsep_nsig30r	1.645	5	0.0007	4.95	10	0.0006	2.39	6	0.0007
cvxnonsep_psig20r	1.831	6	0.0006	7.545	15	0.0008	2.486	6	0.0008
cvxnonsep_normcon20r	1.913	6	0.0000	5.456	11	0.0000	4.562	10	0.0009
cvxnonsep_nsig20r	1.449	5	0.0009	4.906	10	0.0006	2.211	5	0.0006
st_miqp4	0.386	2	0.0000	0.369	2	0.0000	0.375	2	0.0000
ibs2	900*	431*	0.0054*	900*	41*	0.1093*	103.888	16	0.0010