

Experimental Results for $PD \parallel \Sigma w_j C_j$

Tables 1 to 5 show the frequencies of each algorithm producing the best solution. The data in the columns are the number of instances (out of 20 for each combination of n and m) for which the corresponding algorithms produce the best solutions. Note that for each instance, there might be more than one algorithms that produce the best solution. Thus, the sum of the entries in each row could be larger than 20.

Tables 6 to 10 show the positive ratios of the algorithms that are higher than the average costs of the best algorithms. In addition to the same field labels defined for Tables 1 to 5, we introduce five more fields:

- T_1 : The average running time (seconds) per instance for all five priority rules together.
- T_2 : The average running time (seconds) per instance for LP_1 .
- T_3 : The average running time (seconds) per instance for LP'_2 .
- T_4 : The average running time (seconds) per instance for LP''_2 .
- T_5 : The average running time (seconds) per instance for LP'''_2 .

The entries in the columns of the above four fields are simply the average running time in seconds per instance, as defined above. Now focus on the columns corresponding to the algorithms. For these columns, a “–” indicates that an algorithm produces the minimum average objective cost. Note that each row has one and only one “–” for each combination of n and m . In each row, the entries other than “–” are computed as:

$$\frac{\text{The average cost of corresponding algorithm} - \text{The minimum average cost}}{\text{The minimum average cost}} \times 100.$$

Table 11 shows the percentages that the average costs of LP''_2 and LP'''_2 are larger than that of LP'_2 .

Table 1: The frequency that each algorithm performs the best for instances of group G_1

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2
20	2	0	0	0	0	8	9	5	0	0
	5	0	0	0	0	6	12	7	0	0
	10	0	0	0	0	5	10	5	1	0
	20	0	0	0	0	5	12	6	0	0
50	2	0	0	0	0	4	8	11	1	0
	5	0	0	0	0	2	10	9	2	0
	10	0	0	0	0	3	11	10	0	0
	20	0	0	0	0	2	9	9	0	0
100	2	0	0	0	0	1	2	18	0	0
	5	0	0	0	0	0	4	16	0	0
	10	0	0	0	0	1	2	17	0	0
	20	0	0	0	0	0	4	16	0	0
200	2	0	0	0	0	0	1	19	0	0
	5	0	0	0	0	0	2	18	0	0
	10	0	0	0	0	0	0	20	0	0
	20	0	0	0	0	0	0	20	0	0

Table 2: The frequency that each algorithm performs the best for instances of group G_2

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2
20	2	0	0	0	0	8	9	6	0	0
	5	0	0	0	0	5	8	7	0	0
	10	0	0	0	0	6	10	6	1	0
	20	0	0	0	0	4	8	10	0	0
50	2	0	0	0	0	5	11	9	0	0
	5	0	0	0	0	6	8	10	1	0
	10	0	0	0	0	4	6	12	0	0
	20	0	0	0	0	3	4	13	0	0
100	2	0	0	0	0	2	8	12	0	0
	5	0	0	0	0	3	5	15	0	0
	10	0	0	0	0	0	6	14	0	0
	20	0	0	0	0	3	5	12	0	0
200	2	0	0	0	0	1	4	15	0	0
	5	0	0	0	0	0	1	19	0	0
	10	0	0	0	0	0	0	20	1	0
	20	0	0	0	0	0	0	20	0	0

Table 3: The frequency that each algorithm performs best for instances of group G_3

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2
20	2	2	1	2	0	3	7	5	0	0
	5	1	2	2	1	6	4	5	0	0
	10	2	2	1	0	9	3	6	1	0
	20	2	1	1	0	7	4	5	0	0
50	2	2	1	0	2	9	6	4	0	0
	5	0	2	0	0	6	7	8	0	0
	10	2	1	0	0	8	5	7	0	0
	20	1	2	0	0	6	7	7	0	0
100	2	1	3	0	0	16	2	2	0	0
	5	2	0	0	0	8	5	8	0	0
	10	0	0	0	0	12	3	5	0	0
	20	0	0	0	0	11	1	8	0	0
200	2	1	0	0	0	19	1	1	0	0
	5	0	1	0	0	16	1	3	0	0
	10	0	0	0	0	13	3	4	0	0
	20	0	0	0	0	17	2	2	0	0

Table 4: The frequency that each algorithm performs the best for instances of group G_4

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2
20	2	1	3	2	0	9	2	3	1	1
	5	2	2	1	1	7	1	1	2	0
	10	3	2	1	0	6	2	4	0	0
	20	2	2	1	0	9	3	3	0	1
50	2	1	1	1	0	12	2	3	0	0
	5	2	1	1	0	10	3	3	1	0
	10	0	0	0	0	12	3	5	0	0
	20	1	0	0	0	13	4	2	0	0
100	2	0	0	0	0	14	3	4	0	0
	5	0	1	0	0	12	2	5	0	0
	10	0	0	0	0	15	1	4	0	0
	20	1	0	0	0	18	0	1	0	0
200	2	0	0	0	0	16	1	3	0	0
	5	1	0	0	0	18	0	1	0	0
	10	0	0	0	0	17	1	2	0	0
	20	0	0	0	0	18	1	1	0	0

 Table 5: The frequency that each algorithm performs best for instances of group G_5

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2
20	2	1	0	0	0	2	9	7	0	0
	5	0	0	0	0	0	9	11	0	0
	10	0	0	0	0	0	11	9	0	0
	20	0	0	0	0	1	9	10	1	0
50	2	0	0	0	0	0	8	12	0	0
	5	0	0	0	0	1	10	9	0	0
	10	0	0	0	0	0	8	12	0	0
	20	0	0	0	0	0	6	14	0	0
100	2	0	0	0	0	0	6	14	0	0
	5	0	0	0	0	0	7	13	0	0
	10	0	0	0	0	0	3	17	1	0
	20	0	0	0	0	0	1	19	0	0
200	2	0	0	0	0	0	2	18	0	0
	5	0	0	0	0	0	1	19	0	0
	10	0	0	0	0	0	2	18	0	0
	20	0	0	0	0	0	1	19	0	0

Table 6: Comparison of average costs in percentage and average running times for group G_1

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2	T_1	T_2	T_3	T_4	T_5
20	2	5.27	5.96	3.05	5.28	0.06	-	0.12	0.21	2.39	0.00	0.18	0.43	0.07	0.01
	5	8.53	8.41	4.82	17.10	2.42	-	0.47	1.26	2.85	0.00	0.38	1.35	0.29	0.06
	10	11.27	10.73	8.72	20.10	2.66	-	1.22	2.52	3.39	0.02	1.00	2.46	0.43	0.08
	20	15.93	14.33	9.22	22.91	3.20	-	0.03	2.11	2.26	0.02	2.78	6.92	0.87	0.15
50	2	4.49	5.77	3.56	7.13	0.92	0.62	-	0.98	2.11	0.02	1.10	3.07	0.63	0.20
	5	6.28	8.79	4.52	13.47	1.10	-	1.53	4.83	7.44	0.02	2.89	9.58	1.78	0.42
	10	9.18	11.11	6.83	17.97	1.79	-	0.17	0.53	1.43	0.03	6.59	19.33	3.34	0.68
	20	11.59	14.00	8.98	23.14	3.27	-	2.36	3.25	6.20	0.02	12.35	41.51	6.90	1.21
100	2	3.01	4.75	2.08	8.86	1.51	1.23	-	1.02	4.72	0.04	6.10	16.24	3.73	1.26
	5	6.80	8.02	3.48	17.01	3.97	1.91	-	0.90	4.54	0.07	10.25	40.35	8.18	2.24
	10	7.37	9.91	4.16	20.47	4.17	2.47	-	0.74	3.35	0.08	27.84	83.68	15.09	3.46
	20	8.84	11.76	5.21	24.48	4.66	2.74	-	0.29	1.25	0.09	72.30	192	32.81	5.95
200	2	2.86	4.10	3.33	8.48	0.94	0.70	-	1.12	1.75	0.08	45.87	117	28.94	10.92
	5	6.63	7.36	4.09	16.60	3.83	2.79	-	1.21	2.78	0.09	72.33	229	48.48	14.11
	10	6.72	8.67	6.88	22.71	4.33	2.46	-	1.08	2.03	0.10	144	481	83.95	21.65
	20	9.23	9.61	7.64	27.39	6.56	3.02	-	1.52	2.88	0.09	396	1454	160	31.54

Table 7: Comparison of average costs in percentage and average running times for group G_2

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2	T_1	T_2	T_3	T_4	T_5
20	2	2.80	3.03	2.42	6.92	1.51	1.61	-	0.31	2.32	0.00	0.11	0.38	0.07	0.01
	5	3.62	4.62	3.26	7.23	-	0.23	0.15	0.37	1.77	0.00	0.53	1.65	0.23	0.04
	10	4.25	4.72	3.45	8.92	0.13	-	0.15	0.41	1.88	0.02	2.16	6.05	0.51	0.10
	20	4.87	6.06	4.53	12.06	-	0.16	0.23	0.26	1.60	0.02	4.71	14.51	1.77	0.27
	2	3.06	4.30	3.02	8.68	1.03	0.68	-	0.74	3.30	0.02	0.88	3.11	0.69	0.22
	5	4.95	4.97	4.02	9.59	-	0.03	0.07	0.80	2.43	0.02	3.21	10.04	1.75	0.39
50	10	5.08	6.90	5.30	11.90	0.41	0.12	-	0.74	2.30	0.03	10.77	34.95	4.20	0.81
	20	6.44	7.43	6.67	13.65	0.39	0.04	-	0.88	2.14	0.05	52.51	172	12.28	2.14
	2	2.99	4.64	3.87	6.91	0.36	0.99	-	1.16	3.28	0.02	6.04	16.23	3.89	1.27
	5	4.14	4.95	4.34	12.57	0.83	0.74	-	1.20	3.32	0.05	14.55	46.55	8.34	2.07
100	10	5.96	6.68	4.97	14.56	0.51	0.42	-	1.41	2.90	0.10	45.22	167	18.13	4.01
	20	4.82	7.61	5.05	18.37	1.18	-	0.08	0.88	2.84	0.09	204	663	55.74	8.99
	2	1.67	3.24	2.87	7.72	1.06	0.93	-	0.73	3.73	0.09	40.76	104	28.29	10.20
	5	4.17	5.63	5.52	13.77	0.81	1.05	-	1.47	3.76	0.09	77.55	285	53.63	15.91
200	10	5.54	7.09	5.60	14.29	1.12	0.73	-	1.21	3.32	0.08	311	1072	119	26.74
	20	4.28	6.36	4.18	15.33	0.57	0.72	-	1.29	2.91	0.07	1393	5278	400	65.02

Table 8: Comparison of average costs in percentage and average running times for group G_3

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2	T_1	T_2	T_3	T_4	T_5
20	2	1.46	1.71	1.98	2.37	0.05	0.16	-	0.13	1.33	0.00	0.19	0.46	0.04	0.01
	5	1.73	2.43	1.29	3.55	-	0.00	0.02	0.13	0.79	0.00	0.31	1.23	0.10	0.02
	10	2.07	2.34	1.34	3.10	-	0.04	0.27	0.38	1.32	0.00	0.96	3.45	0.23	0.05
	20	1.92	1.89	1.64	4.25	0.23	-	0.57	0.64	1.46	0.00	2.93	8.14	0.48	0.10
50	2	1.42	2.02	1.86	3.29	0.40	0.37	-	0.57	3.84	0.00	1.05	2.84	0.37	0.13
	5	2.47	3.12	2.34	3.55	0.11	0.04	-	0.13	3.34	0.00	3.35	8.61	0.89	0.23
	10	2.69	2.76	2.34	3.56	0.28	-	0.29	0.58	2.45	0.01	5.89	20.14	1.27	0.32
	20	2.17	2.11	1.67	4.48	0.47	0.16	-	0.57	3.95	0.01	24.13	61.29	2.41	0.56
100	2	1.15	2.16	1.37	2.33	0.45	0.41	-	1.00	3.99	0.00	3.92	15.48	2.63	0.81
	5	2.05	2.66	2.18	3.98	-	0.10	0.11	0.97	4.11	0.01	12.45	33.78	4.90	1.23
	10	2.65	3.17	2.49	4.37	0.06	-	0.22	0.78	4.16	0.01	24.37	65.53	7.09	1.89
	20	2.79	2.59	1.85	4.37	0.34	0.21	-	1.02	4.11	0.01	68.39	260	17.71	2.66
200	2	0.75	1.54	1.46	2.52	0.27	0.02	-	0.85	3.64	0.02	32.70	95.11	28.09	8.67
	5	1.70	2.46	2.01	3.83	0.01	0.25	-	0.96	3.95	0.02	79.11	221	44.38	13.55
	10	2.15	3.11	2.36	4.58	0.23	0.45	-	0.92	3.53	0.03	292	787	68.20	17.94
	20	1.84	2.22	1.76	4.18	-	0.02	0.01	0.82	3.22	0.07	1248	3640	224	33.75

Table 9: Comparison of average costs in percentage and average running times for group G_4

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2	T_1	T_2	T_3	T_4	T_5
20	2	2.24	1.65	1.69	2.38	-	0.09	0.27	0.56	0.67	0.00	0.09	0.33	0.05	0.01
	5	1.86	1.83	1.80	1.74	-	0.12	0.16	0.29	1.83	0.00	0.43	1.21	0.13	0.02
	10	1.70	1.67	1.47	4.01	-	0.11	0.05	0.27	2.78	0.00	1.52	5.14	0.38	0.07
	20	1.89	1.68	1.36	6.76	-	0.09	0.02	0.68	1.06	0.00	2.52	9.30	0.68	0.12
50	2	1.11	1.57	1.63	1.83	-	0.36	0.17	0.63	3.79	0.00	1.06	3.08	0.54	0.17
	5	1.90	2.45	2.28	3.24	-	0.25	0.14	0.80	2.84	0.00	2.11	7.87	1.05	0.27
	10	2.12	2.13	1.92	4.45	-	0.17	0.01	0.46	3.45	0.00	9.37	24.36	2.54	0.54
	20	1.78	2.21	2.20	6.52	-	0.26	0.02	0.89	3.06	0.01	26.19	77.47	5.13	1.03
100	2	1.03	1.43	1.09	1.39	-	0.08	0.02	1.24	4.62	0.00	3.76	14.66	2.92	1.22
	5	1.98	1.92	1.69	3.39	-	0.25	0.18	0.89	3.46	0.01	14.30	42.03	6.34	1.77
	10	1.98	2.28	2.11	3.54	-	0.55	0.04	0.91	3.76	0.01	44.75	113	10.38	2.64
	20	1.21	2.43	2.17	4.21	-	0.20	0.13	0.59	3.41	0.01	152	375	32.72	5.38
200	2	0.89	1.34	1.12	1.34	-	0.07	0.06	0.93	4.77	0.02	36.79	95.98	25.53	8.71
	5	1.22	2.24	2.17	3.19	-	0.15	0.14	1.33	4.42	0.02	80.89	215	49.16	15.88
	10	1.55	2.51	2.22	4.57	-	0.04	0.01	0.66	4.05	0.03	369	917	105	25.79
	20	1.30	1.99	1.92	4.65	-	0.08	0.06	0.88	3.78	0.07	1699	4471	287	39.59

Table 10: Comparison of average costs in percentage and average running times for group G_5

n	m	$WSTP$	$WSMP$	$WSMC$	$WSPL$	$WECT$	LP_1	LP'_2	LP''_2	LP'''_2	T_1	T_2	T_3	T_4	T_5	
∞	2	1.78	1.49	1.30	6.84	0.75	0.59	-	0.12	0.50	0.00	0.11	0.37	0.05	0.01	
	5	5.91	9.37	4.44	9.67	2.82	1.48	-	0.33	1.96	0.00	0.49	1.32	0.21	0.04	
	20	10	5.30	7.32	5.18	16.22	0.21	0.43	-	0.33	1.50	0.02	0.94	3.31	0.51	0.10
		20	5.30	9.48	5.86	12.77	-	0.20	0.60	1.08	1.99	0.02	5.08	13.52	1.82	0.28
	2	3.86	5.35	3.52	12.06	2.26	1.83	-	0.56	4.02	0.02	0.76	2.86	0.59	0.19	
	5	7.00	9.57	7.30	15.70	2.21	2.48	-	0.45	2.48	0.02	3.08	9.04	1.53	0.36	
50	10	7.65	11.83	8.69	13.53	2.96	2.73	-	0.61	2.99	0.02	9.33	32.26	5.97	0.97	
		20	9.38	15.78	9.83	18.29	2.17	1.99	-	1.09	2.98	0.09	32.64	108	15.68	2.35
	100	2	3.01	4.99	3.22	14.94	1.77	1.28	-	0.94	4.98	0.03	4.82	14.72	3.53	1.19
		5	7.35	10.26	7.39	17.46	2.42	2.54	-	1.06	4.80	0.02	13.83	42.01	7.67	2.20
	200	10	7.90	12.93	10.49	15.52	2.19	2.20	-	0.73	3.11	0.09	39.37	124	21.76	4.09
		20	5.87	16.84	14.02	15.98	1.67	1.66	-	0.35	2.72	0.06	244	863	83.76	14.16
200	2	3.01	5.26	3.58	17.26	2.43	2.08	-	1.16	5.29	0.08	30.78	96.56	26.31	9.06	
	5	6.04	11.28	8.85	17.14	2.49	2.73	-	1.22	4.34	0.08	89.83	227	48.75	14.14	
	10	6.53	14.49	11.63	16.66	2.44	2.71	-	0.82	3.53	0.10	278	935	112	25.94	
		20	6.44	16.50	13.95	18.69	1.57	1.54	-	1.01	3.49	0.09	1755	5437	409	68.94

Table 11: The percentage that the average costs of LP_2'' and LP_2''' are larger than that of LP_2'

n	m	G_1		G_2		G_3		G_4		G_5	
		LP_2''	LP_2'''								
20	2	0.09	2.27	0.31	2.32	0.13	1.33	0.18	0.47	0.12	0.50
	5	0.79	2.38	0.22	1.62	0.11	0.77	0.13	1.67	0.33	1.96
	10	1.30	2.17	0.26	1.73	0.11	0.95	0.22	2.73	0.33	1.50
	20	2.08	2.23	0.03	1.37	0.07	0.99	0.66	1.04	0.42	1.39
50	2	0.98	2.11	0.74	3.30	0.57	3.84	0.46	3.62	0.56	4.02
	5	3.30	5.90	0.73	2.36	0.13	3.34	0.66	2.70	0.45	2.48
	10	0.36	1.26	0.74	2.30	0.29	2.16	0.45	3.44	0.61	2.99
	20	0.09	3.84	0.88	2.14	0.57	3.95	0.87	2.17	1.09	2.98
100	2	1.02	4.72	1.16	3.28	1.00	3.99	1.22	4.60	0.94	4.98
	5	0.9	4.54	1.20	3.32	0.86	4.00	0.71	3.28	1.06	4.80
	10	0.74	3.35	1.41	2.90	0.56	3.94	0.87	3.72	0.73	3.11
	20	0.29	1.25	0.80	2.72	1.02	4.11	0.46	3.28	0.35	2.72
200	2	1.12	1.75	0.73	3.73	0.85	3.64	0.87	4.71	1.16	5.29
	5	1.21	2.78	1.47	3.76	0.96	3.95	1.19	4.28	1.22	4.34
	10	1.08	2.03	1.21	3.32	0.92	3.53	0.65	4.04	0.82	3.53
	20	0.52	2.88	1.29	2.91	0.81	3.21	0.82	3.72	1.01	3.49