Exhibit

Illustrative example of the instance generator for the VRPSLDW

This exhibit exemplifies not only how the instance generator works, but also provides a valid argument regarding the quality of the BK solutions. For designing instances with BK high quality feasible solutions the following principles for designing routes are considered:

1. Optimal solutions are those in which the customer's demand is split as few as possible. This is consistent with the results of Archetti *et al*. (2006)[1] for the split delivery routing problem.
2. Customers assigned to the same route are close to each other.
3. Routes with full truck load are preferable in optimal solutions.

We design a trial example with 15 customers, which allow us to illustrate its operation. The following assumptions are hold:

- There are 4 vehicles (two of 15 units of capacity and two of 30 units).
- Each vehicle can complete at most 3 tours.
- The total collection capacity of the system is 1,620 units per week.
- There are 15 customers (F1).
- We want to create customers with balanced demand (F2).
- We desire tight date windows (F3).
- We desire a collection capacity above 15% over the total customer demand (F4).
- We desire customers with low dispersion (F5).

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[1] Archetti C., M. W. P. Savelsbergh and M. G. Speranza (2006). Worst-Case Analysis for Split Delivery Vehicle Routing Problems. *Transportation Science* 40(2): 226–234.

Step 1 – Compute customer demand considering factors F1, F2 and F4.

The total demand generated is of 1,381, which represents the 85.25% of the collection capacity of the systems. The demand for each customer is given in Table 1.

Table 1 Demand for each customer.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
85	87	85	110	105	97	90	74	100	86	92	92	99	90	89	**1381**

Step 2 – Form the LOAD matrix (i.e. the solution).

The load matrix is initialized by taking the first customer and inserting a portion of its demand into the first route where both the largest load may be allocated and the remaining capacity of the vehicle is minimized. The procedure is repeated until the entire total customer's demand has been allocated. Then, the data structure is updated and the next customer in a lexicographical order is selected.

The solution is given in Table 2. To define the date windows, for each customer the procedure identifies the first and last day in which it is visited; then under the factor F3 consideration it determines the width of the date windows. Note that to ensure full truckload routes some customer demands were adjusted. Table 3 shows the new demand for each customer.

Table 3 New demand for each customer.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
90	90	90	120	105	97	90	75	105	90	92	96	105	90	90	**1425**

It should be noted that the difference between the customer with the higher demand (customer 4) and the customer with the lower demand (customer 8) is 45 units, and in average each customer holds a demand of 95 units, thus the customer demand is considered to be balanced.

Table 2 LOAD matrix (solution) of the trial example.

	Route	Customers 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\bar{Q}_k$	Q_k
Day 1	π^1_{11}								15								0	15
	π^1_{12}						7					2	6				0	15
	π^1_{13}									15							0	15
	π^1_{21}					15											0	15
	π^1_{22}					15											0	15
	π^1_{23}					15											0	15
	π^1_{31}	30															0	30
	π^1_{32}	30															0	30
	π^1_{33}	30															0	30
	π^1_{41}								30								0	30
	π^1_{42}								30								0	30
	π^1_{43}														30		0	30
Day 2	π^2_{11}					15											0	15
	π^2_{12}					15											0	15
	π^2_{13}					15											0	15
	π^2_{21}					15											0	15
	π^2_{22}													15			0	15
	π^2_{23}													15			0	15
	π^2_{31}													30			0	30
	π^2_{32}													30			0	30
	π^2_{33}														30		0	30
	π^2_{41}														30		0	30
	π^2_{42}														30		0	30
	π^2_{43}												30				0	30
Day 3	π^3_{11}													15			0	15
	π^3_{12}													15			0	15
	π^3_{13}													15			0	15
	π^3_{21}													15			0	15
	π^3_{22}													15			0	15
	π^3_{23}							15									0	15
	π^3_{31}												30				0	30
	π^3_{32}												30				0	30
	π^3_{33}					30											0	30
	π^3_{41}						30										0	30
	π^3_{42}						30										0	30
	π^3_{43}		30														0	30
Day 4	π^4_{11}							15									0	15

Day	π	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
	π^4_{12}							15									0	15
	π^4_{13}							15									0	15
	π^4_{21}							15									0	15
	π^4_{22}							15									0	15
	π^4_{23}																15	15
	π^4_{31}		30														0	30
	π^4_{32}		30														0	30
	π^4_{33}			30													0	30
	π^4_{41}			30													0	30
	π^4_{42}			30													0	30
	π^4_{43}				30												0	30
Day 5	π^5_{11}																15	15
	π^5_{12}																15	15
	π^5_{13}																15	15
	π^5_{21}																15	15
	π^5_{22}																15	15
	π^5_{23}																15	15
	π^5_{31}				30												0	30
	π^5_{32}				30												0	30
	π^5_{33}				30												0	30
	π^5_{41}											30					0	30
	π^5_{42}											30					0	30
	π^5_{43}											30					0	30
Day 6	π^6_{11}																15	15
	π^6_{12}																15	15
	π^6_{13}																15	15
	π^6_{21}																15	15
	π^6_{22}																15	15
	π^6_{23}																15	15
	π^6_{31}								30								0	30
	π^6_{32}								30								0	30
	π^6_{33}								30								0	30
	π^6_{41}									30							0	30
	π^6_{42}									30							0	30
	π^6_{43}									30							0	30
	ω_i	90	90	90	120	105	97	90	75	105	90	92	96	105	90	90		

In this step a solution has been designed and the remaining of the procedure derives the instance data for that solution.

Step 3 – Form clusters and locate customers.

The size of the area to locate customers ranges from (-50 to 50) for x and y, the depot is located at (0, 0). To ensure high quality feasible routes, all customers assigned to the same route are grouped into clusters. The coordinates of the clusters are calculated, and its amplitude is defined by factor F5. Table 4 shows the clusters that were identified and the center of the clusters. It should be noted that clusters are fairly distributed in the area. Therefore, two imaginary circles are drawn with radius of 50 and 25 for the outer and the inner respectively. Figure 1 shows the location of the clusters.

Table 4 Clusters.

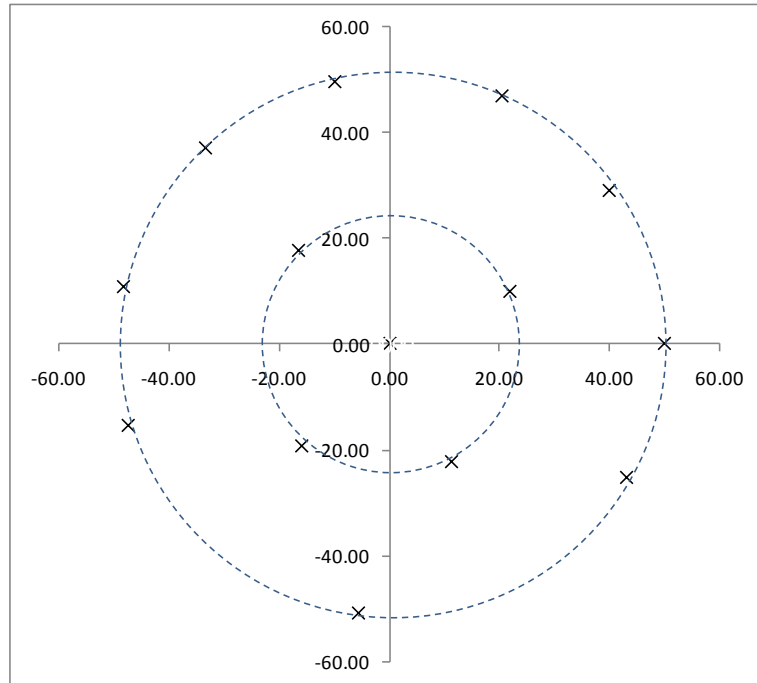| Cluster	Elements	Center of the cluster
1	{1}	(50.00, 0.00)
2	{2}	(40.00, 29.05)
3	{3}	(20.40, 47.00)
4	{4}	(-9.89, 49.68)
5	{5}	(-33.35, 37.26)
6	{6, 11, 12}	(-48.26, 10.97)
7	{7}	(-47.62, -15.24)
8	{8}	(-5.52, -50.97)
9	{9}	(43.12, -25.32)
10	{10}	(22.00, 10.00)
11	{13}	(-16.61, 17.87)
12	{14}	(-15.83, -19.35)
13	{15}	(11.20, -22.35)

Figure 1 Location of clusters

Now we allocate the customers around the center of their corresponding clusters according to factor F5. The coordinates of the customers are given in Table 5.

Table 5 Coordinates of customers.

Customer	X	y
DEP	0.00	0.00
1	49.00	-1.00
2	40.00	29.03
3	20.40	47.00
4	-9.89	49.01
5	-33.35	37.26
6	-48.78	10.97
7	-47.62	-15.24
8	-5.52	-49.69
9	43.12	-25.32
10	22.00	10.00
11	-47.00	11.06
12	-50.00	9.00
13	-17.48	17.87
14	-15.83	-19.35
15	11.20	-22.35

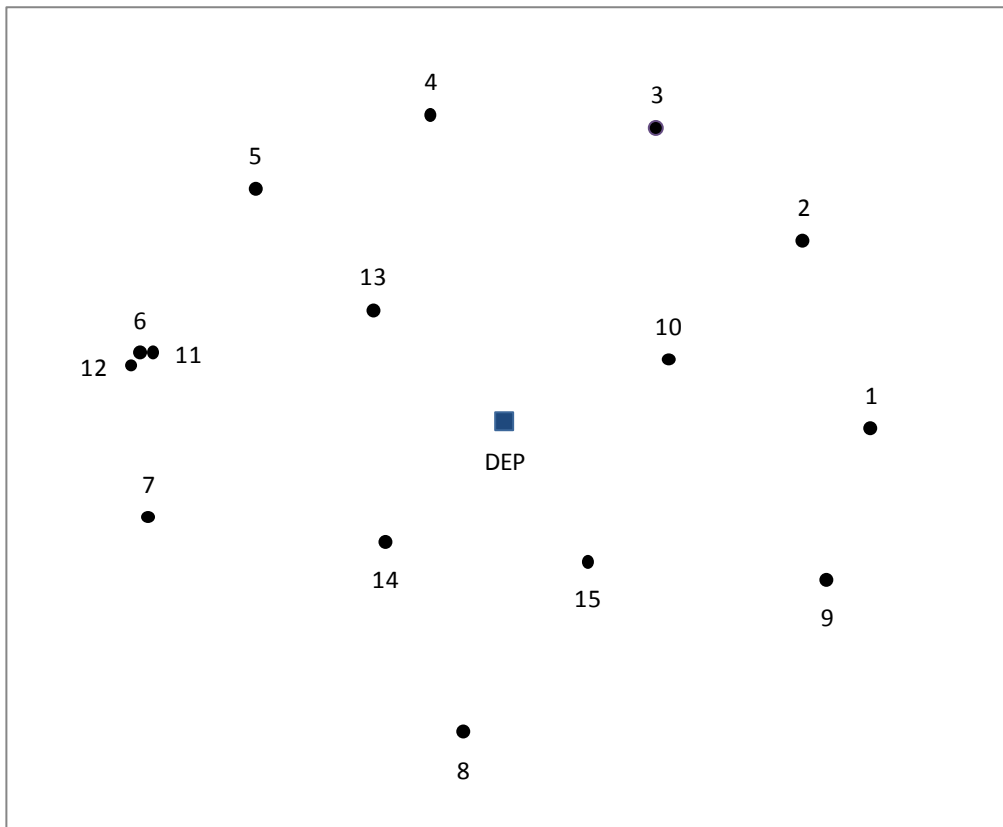In figure 2 we show the location of the customers.

Figure 2 Location of clusters

The distances are given in Table 6. With this information, the data instance is completed, which includes:

1. Number of customers.
2. Number of periods.
3. Number of vehicles.
4. Max number of tours per vehicle.
5. Capacity of each vehicle.
6. Unit cost per travel distance.
7. Max operation time (hrs) for each vehicle per day.
8. Demand for each customer.
9. Service time for each customer.
10. Date Windows.
11. Travel distance and time matrices.

Table 6 Distance matrix. To compute the travel time, we consider an average speed of 50 km/hr.

	DEP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DEP	0.00	49.01	49.42	51.24	50.00	50.00	50.00	50.00	50.00	50.00	24.17	48.28	50.80	25.00	25.00	25.00
1	49.01	0.00	31.35	55.87	77.26	90.80	98.51	97.66	73.10	25.02	29.15	96.75	99.50	69.11	67.38	43.41
2	49.42	31.35	0.00	26.59	53.74	73.81	90.60	98.17	90.94	54.44	26.19	88.84	92.20	58.55	73.88	58.90
3	51.24	55.87	26.59	0.00	30.36	54.63	78.00	92.20	100.11	75.80	37.03	76.39	80.00	47.79	75.60	69.96
4	50.00	77.26	53.74	30.36	0.00	26.24	54.40	74.51	98.80	91.29	50.39	53.08	56.65	32.05	68.62	74.41
5	50.00	90.80	73.81	54.63	26.24	0.00	30.48	54.40	91.29	98.80	61.69	29.54	32.80	25.05	59.25	74.41
6	50.00	98.51	90.60	78.00	54.40	30.48	0.00	26.24	74.51	98.80	70.79	1.78	2.32	32.05	44.78	68.62
7	50.00	97.66	98.17	92.20	74.51	54.40	26.24	0.00	54.40	91.29	74.05	26.31	24.36	44.78	32.05	59.25
8	50.00	73.10	90.94	100.11	98.80	91.29	74.51	54.40	0.00	54.40	65.73	73.56	73.64	68.62	32.05	32.05
9	50.00	25.02	54.44	75.80	91.29	98.80	98.80	91.29	54.40	0.00	41.15	97.18	99.24	74.41	59.25	32.05
10	24.17	29.15	26.19	37.03	50.39	61.69	70.79	74.05	65.73	41.15	0.00	69.01	72.01	40.26	47.88	34.10
11	48.28	96.75	88.84	76.39	53.08	29.54	1.78	26.31	73.56	97.18	69.01	0.00	3.64	30.29	43.54	67.11
12	50.80	99.50	92.20	80.00	56.65	32.80	2.32	24.36	73.64	99.24	72.01	3.64	0.00	33.71	44.40	68.76
13	25.00	69.11	58.55	47.79	32.05	25.05	32.05	44.78	68.62	74.41	40.26	30.29	33.71	0.00	37.26	49.40
14	25.00	67.38	73.88	75.60	68.62	59.25	44.78	32.05	32.05	59.25	47.88	43.54	44.40	37.26	0.00	27.20
15	25.00	43.41	58.90	69.96	74.41	74.41	68.62	59.25	32.05	32.05	34.10	67.11	68.76	49.40	27.20	0.00

Step 5 – Form ROUTE matrix and compute total cost.

Now we can compute the ROUTE matrix, see Table 7, we only show those routes used in the solution.

Table 7 Route matrix.

	Route	Customers														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Day 1	π^1_{11}								1							
	π^1_{12}					2						3	1			
	π^1_{13}									1						
	π^1_{21}					1										
	π^1_{22}					1										
	π^1_{23}					1										
	π^1_{31}	1														
	π^1_{32}	1														
	π^1_{33}	1														
	π^1_{41}								1							
	π^1_{42}								1							
	π^1_{43}														1	
Day 2	π^2_{11}					1										
	π^2_{12}					1										
	π^2_{13}					1										
	π^2_{21}					1										
	π^2_{22}													1		
	π^2_{23}													1		
	π^2_{31}														1	
	π^2_{32}														1	
	π^2_{33}															1
	π^2_{41}															1
	π^2_{42}															1
	π^2_{43}												1			
Day 3	π^3_{11}													1		
	π^3_{12}													1		
	π^3_{13}													1		
	π^3_{21}													1		
	π^3_{22}													1		
	π^3_{23}							1								
	π^3_{31}												1			
	π^3_{32}												1			

Day	Var															
	π^3_{33}						1									
	π^3_{41}						1									
	π^3_{42}						1									
	π^3_{43}		1													
Day 4	π^4_{11}							1								
	π^4_{12}							1								
	π^4_{13}							1								
	π^4_{21}							1								
	π^4_{22}							1								
	π^4_{31}		1													
	π^4_{32}		1													
	π^4_{33}			1												
	π^4_{41}			1												
	π^4_{42}			1												
	π^4_{43}				1											
Day 5	π^5_{31}				1											
	π^5_{32}				1											
	π^5_{33}				1											
	π^5_{41}											1				
	π^5_{42}											1				
	π^5_{43}											1				
Day 6	π^6_{31}									1						
	π^6_{32}									1						
	π^6_{33}									1						
	π^6_{41}										1					
	π^6_{42}										1					
	π^6_{43}										1					

The total cost of our solution is: 1,077.36.

Using AMPL – CPLEX to evaluate the solution

We have modeled this problem in AMPL and solve it in CPLEX. The adjusted problem after the *presolve* phase consists of 5,775 variables and 10,919 constraints. After 20 hr of computing time CPLEX explores 2,048,832 nodes however it was unable to find an integer solution. When we provide the BK solution to CPLEX the solution obtained was 1,077.36, which corresponds to the BK solution.

	B&B Nodes	Variables	Constraints	Time (s)	Solution	Optimality Gap (%)	Deviation from BK (%)
CPLEX	2048832	5775	10919	72000	Not found	Not info	Not info
BK+CPLEX	1396			36000	1077.36*	-----	0.00

Given the results obtained we don't have enough information to reject the hypothesis that our instance generator builds instances with a high quality upper bound.