

# Preprocessing Algorithm and Tightening Constraints for Multiperiod Blend Scheduling: Cost Minimization

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## Supporting material

**Table S1.**  $M^C$  for the motivating example

	Objective and constraints in $M^C$	Description
<i>min</i>	$10\tilde{F}_{S1,J1,0} + 10\tilde{F}_{S1,J1,1} + 5\tilde{F}_{S2,J1,0} + 5\tilde{F}_{S2,J1,1}$	Objective
<i>s.t</i>	$I_{J1,1} = \tilde{F}_{S1,J1,0} + \tilde{F}_{S2,J1,0} - \bar{F}_{J1,P1,0}$ $I_{J1,2} = I_{J1,1} + \tilde{F}_{S1,J1,1} + \tilde{F}_{S2,J1,1} - \bar{F}_{J1,P1,1}$	Eqn. (1)
	$\tilde{I}_{S1,1} = 1 - \tilde{F}_{S1,J1,0}$ $\tilde{I}_{S1,2} = \tilde{I}_{S1,1} - \tilde{F}_{S1,J1,1}$ $\tilde{I}_{S2,1} = 1 - \tilde{F}_{S2,J1,0}$ $\tilde{I}_{S2,2} = \tilde{I}_{S2,1} - \tilde{F}_{S2,J1,1}$	Eqn. (2)
	$\bar{I}_{P1,1} = \bar{F}_{J1,P1,0}$ $\bar{I}_{P1,2} = \bar{I}_{P1,1} + \bar{F}_{J1,P1,2} - 1$	Eqn. (3)
	$\tilde{F}_{S1,J1,0} \leq X_{S1,J1,0}$ $\tilde{F}_{S2,J1,0} \leq X_{S2,J1,0}$ $\tilde{F}_{S1,J1,1} \leq X_{S1,J1,1}$ $\tilde{F}_{S2,J1,1} \leq X_{S2,J1,1}$	Eqn. (4)
	$\bar{F}_{J1,P1,0} \leq \bar{X}_{J1,P1,0}$ $\bar{F}_{J1,P1,1} \leq \bar{X}_{J1,P1,1}$	
	$X_{S1,J1,0} \leq 1 - \bar{X}_{J1,P1,0}$ $X_{S2,J1,0} \leq 1 - \bar{X}_{J1,P1,0}$ $X_{S1,J1,1} \leq 1 - \bar{X}_{J1,P1,1}$ $X_{S2,J1,1} \leq 1 - \bar{X}_{J1,P1,1}$	Eqn. (5)
	$I_{J1,1}C_{Q1,J1,1} = \tilde{F}_{S1,J1,0} + 0.8\tilde{F}_{S2,J1,0}$ $I_{J1,2}C_{Q1,J1,2} = I_{J1,1}C_{Q1,J1,1} + \tilde{F}_{S1,J1,1} + 0.8\tilde{F}_{S2,J1,1} - \bar{F}_{J1,P1,1}C_{Q1,J1,1}$	Eqn. (6)
	$C_{Q1,J1,1} \geq 0.9 - M(1 - \bar{X}_{J1,P1,1})$	Eqn. (7)

Eqn. (27) – (29) for multiple due times:

$$\sum_{t' \leq t} \hat{F}_{s,p,t'} \geq \hat{\omega}_{s,p,t}, \quad s, p, t \text{ s.t. } \delta_{p,t} > 0 \quad (27')$$

$$\sum_{s \in \mathbf{S}_{p,q}^U} \sum_{t' \leq t} (\hat{\sigma}_{p,q}^U - \sigma_{s,q}) \hat{F}_{s,p,t'} \geq (\hat{\sigma}_{p,q}^U - \sigma_{p,q}^U) \omega_{p,t} + \sum_{s \notin \mathbf{S}_{p,q}^U} (\sigma_{s,q} - \hat{\sigma}_{p,q}^U) \hat{\omega}_{s,p,t}, \quad (28')$$

$$p, q \in \mathbf{Q}_p^M, t \text{ s.t. } \delta_{p,t} > 0$$

$$\sum_{s \in \mathbf{S}_{p,q}^L} \sum_{t' \leq t} (\sigma_{s,q} - \hat{\sigma}_{p,q}^L) \hat{F}_{s,p} \geq (\sigma_{p,q}^L - \hat{\sigma}_{p,q}^L) \omega_{p,t} + \sum_{s \notin \mathbf{S}_{p,q}^L} (\hat{\sigma}_{p,q}^L - \sigma_{s,q}) \hat{\omega}_{s,p,t}, \quad (29')$$

$$p, q \in \mathbf{Q}_p^M, t \text{ s.t. } \delta_{p,t} > 0$$

**Table S2.** Property data for Instance 1 - 5

Property	S1	S2	S3	S4	S5	S6	S7
Q1	0	0	1	0	0.92	0.86	0
Q2	0	0	0.96	0	0.91	0.71	0
Q3	0	0	0.85	0	0.8	0.85	0
Q4	0.33	0.07	0.81	0.52	0.84	0.97	1
Q5	0.24	0.13	0.87	1	0.9	0.91	0.16
Q6	0.04	0.08	0.7	0	0.66	0.83	0.03
Q7	1	1	0.97	0	1	1	0

**Note:** Values in the original literature are scaled to [0,1]

**Table S3.** Property data for Instance 6 - 12

Property	S1	S2	S3	S4	S5	S6	S7	S8	S9
Q1	0.57	0.69	0.74	0.76	0.63	0.78	0.96	1	0.61
Q2	0.32	0.16	0.2	0.12	0.4	0.04	0.03	0.25	1
Q3	1	0.5	0	0.06	0	0	0.09	0.02	0.38
Q4	0.65	0.82	1	0	0.08	0	0	0.21	0.08
Q5	0.52	0.66	1	0	0	0	0	0.4	0.5
Q6	0.04	1	0.04	0	0.017	0.03	0	0	0
Q7	0.92	0.83	0.76	0.98	0.93	0.89	0.71	0.84	1
Q8	0.43	0.79	0.3	0.45	0.25	0.46	0.37	0.69	1
Q9	0.01	0.04	0.11	0.07	0	0	0	1	0.05

**Note:** Values in the original literature are scaled to [0,1]

**Table S4.** Property data for Instance 13 - 15

Property	S1	S2	S3	S4	S5	S6	S7	S8
Q1	0.4	0.67	0.87	0.41	1	0.42	0.64	0.84

**Note:** Values in the original literature are scaled to [0,1]

**Table S5.** Property data for Instance 13 - 15

Property	S1	S2
Q1	0.06	0.26

**Table S6.** Product specification for Instances 1 - 5

Property	P1	P2	P3
Q1	[0, 0.8]	[0, 0.8]	[0, 0.8]
Q2	[0, 0.79]	[0, 0.79]	[0, 0.79]
Q3	[0.87,1]	[0.91,1]	[0.93,1]
Q4	[0, 0.9]	[0, 0.9]	[0, 0.9]
Q5	[0.89,1]	[0.92,1]	[0.95,1]
Q6	[0.89,1]	[0.89,1]	[0.89,1]
Q7	[0, 0.21]	[0, 0.21]	[0, 0.21]

**Note:** Values in the original literature are scaled to [0,1]

**Table S7.** Product specification for Instances 6 - 12

Property	P1	P2	P3	P4
Q1	[0, 0.73]	[0, 0.74]	[0, 0.72]	[0, 0.68]
Q2	[0, 0.39]	[0, 0.39]	[0, 0.39]	[0, 0.39]
Q3	[0, 0.56]	[0, 0.62]	[0, 0.55]	[0, 0.62]
Q4	[0, 0.72]	[0, 0.77]	[0, 0.78]	[0, 0.75]
Q5	[0, 0.73]	[0, 0.75]	[0, 0.88]	[0, 0.83]
Q6	[0, 0.84]	[0, 0.76]	[0, 0.84]	[0, 0.76]
Q7	[0.74, 1]	[0.74, 1]	[0.73, 1]	[0.74, 1]
Q8	[0, 0.95]	[0, 0.91]	[0, 0.90]	[0, 0.94]
Q9	[0, 0.10]	[0, 0.10]	[0, 0.12]	[0, 0.11]

**Note:** Values in the original literature are scaled to [0,1]

**Table S8.** Product specification for Instances 13 - 15

Property	P1	P2	P3
Q1	[0.40, 0.47]	[0.57, 0.67]	[0.80, 0.87]

**Note:** Values in the original literature are scaled to [0,1]

**Table S9.** Product specification for Instances 16 - 20

Property	P1	P2
Q1	[0, 0.16]	[0, 1]

**Table S10.** Variable cost coefficient  $\tilde{\beta}_{s,j}$ 

Stream	Instance 1 - 5	Instance 6 - 12	Instance 13 - 15	Instance 16 - 20
S1	29.2	20	3	2
S2	11.5	24	5	1
S3	20	30	6.5	–
S4	22	25	3.1	–
S5	25	22	7.5	–
S6	19.7	27	3.17	–
S7	24.5	50	4.83	–
S8	–	50	6.33	–
S9	–	22.5	–	–

**Note:** In tested instances,  $\tilde{\beta}_{s,j}$  are the same for every  $j$ .

**Table S11.** Fixed cost coefficient  $\tilde{\alpha}_{s,j}$ 

	Instance 5	Instance 15	Instance 18
$\tilde{\alpha}_{s,j}$	10	1	0.5

**Note:** In tested instances,  $\tilde{\alpha}_{s,j}$  are the same for every  $(s, j)$ .

**Table S12.** Stream availability and product demand for Instance 1 and 5

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S7	50	30	20	-	-	-	-
P1	-	-	-	70	70	70	70
P2	-	-	-	50	50	50	50
P3	-	-	-	30	30	30	30

**Table S13.** Stream availability and product demand for Instance 2

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S7	50	30	20	-	-	-	-
P1	-	-	-	75	75	75	75
P2	-	-	-	55	55	55	55
P3	-	-	-	35	35	35	35

**Table S14.** Stream availability and product demand for Instance 3

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S7	50	30	20	-	-	-	-
P1	-	-	-	80	80	80	80
P2	-	-	-	60	60	60	60
P3	-	-	-	40	40	40	40

**Table S15.** Stream availability and product demand for Instance 4

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S7	50	30	20	-	-	-	-
P1	-	-	-	85	85	85	85
P2	-	-	-	65	65	65	65
P3	-	-	-	45	45	45	45

**Table S16.** Stream availability and product demand for Instance 6

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	70	70	70	70
P2	-	-	-	50	50	50	50
P3	-	-	-	30	30	30	30
P4	-	-	-	40	40	40	40

**Table S17.** Stream availability and product demand for Instance 7

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	75	75	75	75
P2	-	-	-	55	55	55	55
P3	-	-	-	35	35	35	35
P4	-	-	-	45	45	45	45

**Table S18.** Stream availability and product demand for Instance 8

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	80	80	80	80
P2	-	-	-	60	60	60	60
P3	-	-	-	40	40	40	40
P4	-	-	-	50	50	50	50

**Table S19.** Stream availability and product demand for Instance 9

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	85	85	85	85
P2	-	-	-	65	65	65	65
P3	-	-	-	45	45	45	45
P4	-	-	-	55	55	55	55

**Table S20.** Stream availability and product demand for Instance 10

Stream/Prodoct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	90	90	90	90
P2	-	-	-	70	70	70	70
P3	-	-	-	50	50	50	50
P4	-	-	-	60	60	60	60

**Table S21.** Stream availability and product demand for Instance 11

Stream/Prodoct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	70	70	70	70
P2	-	-	-	50	50	50	50
P3	-	-	-	35	35	35	35
P4	-	-	-	40	40	40	40

**Table S22.** Stream availability and product demand for Instance 12

Stream/Prodoct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S9	50	30	20	-	-	-	-
P1	-	-	-	70	70	70	70
P2	-	-	-	50	50	50	50
P3	-	-	-	30	30	30	30
P4	-	-	-	45	45	45	45

**Table S23.** Stream availability and product demand for Instance 13

Stream/Prodoct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S3	50	30	20	-	-	-	-
P1	-	-	-	70	70	70	70
P2	-	-	-	50	50	50	50
P3	-	-	-	30	30	30	30
P4	-	-	-	30	30	30	30

**Table S24.** Stream availability and product demand for Instance 14

Stream/Prodoct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1 to S3	50	30	20	-	-	-	-
P1	-	-	-	65	70	70	70
P2	-	-	-	45	50	50	50
P3	-	-	-	25	30	30	30
P4	-	-	-	25	30	30	30

**Table S25.** Stream availability and product demand for Instance 15

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1	10	10	10	-	-	-	-
S2	30	30	30	-	-	-	-
P1	-	-	-	15	15	15	15
P2	-	-	-	15	15	15	15

**Table S26.** Stream availability and product demand for Instance 16

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1	10	10	10	-	-	-	-
S2	30	30	30	-	-	-	-
P1	-	-	-	15	15	15	15
P2	-	-	-	15	15	15	15

**Table S27.** Stream availability and product demand for Instance 17 and 18

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1	10	10	10	-	-	-	-
S2	30	30	30	-	-	-	-
P1	-	-	-	10	10	10	10
P2	-	-	-	20	20	20	20

**Table S28.** Stream availability and product demand for Instance 19

Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1	10	10	10	-	-	-	-
S2	30	30	30	-	-	-	-
P1	-	-	-	10	10	10	10
P2	-	-	-	10	10	10	10

**Table S29.** Stream availability and product demand for Instance 20

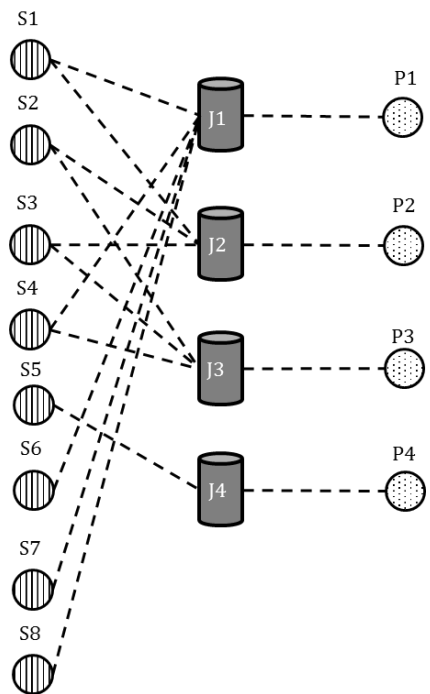
Stream/Prodouct	$\xi_{s,T0}$	$\xi_{s,T1}$	$\xi_{s,T2}$	$\delta_{p,T2}$	$\delta_{p,T3}$	$\delta_{p,T4}$	$\delta_{p,T5}$
S1	10	10	10	-	-	-	-
S2	30	30	30	-	-	-	-
P1	-	-	-	12	12	12	12
P2	-	-	-	15	15	15	15

## Network configurations

For Instance 1 to 12, we assign full connectivity between streams and blenders, and full connectivity between blenders and products. Blenders are not connected with other blenders, and direct flow from stream to product is not allowed.

Network configuration for Instance 13 to 15 is given below in Figure S1.

Instance 16 to 20 share the same network configuration, which is given in Figure 10. Capacity for stream storage, blenders and product storage are 150, 200 and 100, respectively, for all instances.



**Figure S1.** Network configuration for Instance 13-15.



**Table S30.** CPU time for  $M_x^C$  in seconds with solver BARON

	$M^C$	$M_P^C$	$M_{II P}^C$	$M_{I II P}^C$
Instance 1	7200.76	84.61	137.32	400.55
Instance 2	7200	441.87	257.26	370.12
Instance 3	7200	1014.35	1466.3	1454.1
Instance 4	7200	3273.49	3387	2914.81
Instance 5	7200	7200	7200.97	7201
Instance 6	7200.02	322.83	537.38	724.26
Instance 7	7200.01	1366.58	1908.35	7200.02
Instance 8	7200	1197.75	552.08	7200.2
Instance 9	671.3	5259.31	1533.67	4416.28
Instance 10	459.83	609.43	2307.38	1704.5
Instance 11	7200	7200	7200.01	7200
Instance 12	6242.21	7200.01	7200	7200.36
Instance 13	7200	7200	7200.54	7201.01
Instance 14	1106.41	5388.72	2454.77	4987.94
Instance 15	125.65	133.96	449.63	293.42
Instance 16	7200.01	4404.33	7201.13	7201.2
Instance 17	7200.55	108.44	86.35	137.15
Instance 18	7200.07	7200.84	1888.39	3514.83
Instance 19	7200.58	297.21	3433.6	1221.66
Instance 20	7200.68	751.15	427.56	1453.54

**Table S31.** CPU time for  $M_x^{SB}$  in seconds with solver BARON

	$M^{SB}$	$M_P^{SB}$	$M_{II P}^{SB}$	$M_{I II P}^{SB}$
Instance 1	1369.82	166.57	61.83	155.69
Instance 2	7200.01	7200.01	7200.03	7200.05
Instance 3	7200.01	7200.01	7200.03	7200.03
Instance 4	7200.01	7200.01	7200.03	7200.04
Instance 5	7200.01	7203.01	76.48	7201.2
Instance 6	408.17	794.68	858.14	1121.59
Instance 7	7200.08	4977.96	2036.91	7200.3
Instance 8	7200.09	7200.09	2333.08	7200.45
Instance 9	7200.07	7200.09	7200.27	7200.32
Instance 10	887.74	3111.01	1030.47	6499.18
Instance 11	5.9	7.75	364.55	12.78
Instance 12	7.74	7.86	13.2	8.42
Instance 13	5.91	11.45	363.83	11.98
Instance 14	1782.56	5297.24	554.71	7201.33
Instance 15	675.05	261.8	427.91	719.82
Instance 16	2514.71	4698.62	5207.1	4880.2
Instance 17	572.23	534.78	627.02	382.45
Instance 18	426.95	398.58	465.57	4730.3
Instance 19	7200	1729.1	2439.49	5736.67
Instance 20	1055.07	7201.14	1530.83	951.01

**Table S32.** CPU time for  $L_2M_x^C$  in seconds with solver CPLEX

	$L_2M^C$	$L_2M_P^C$	$L_2M_{IP}^C$	$L_2M_{IIP}^C$
Instance 1	7200.136	902.36	905.594	804.264
Instance 2	7200.092	2754.485	2562.397	987.47
Instance 3	7200.239	784.28	793.5	3186.278
Instance 4	7200.08	2532.058	2697.156	1120.176
Instance 5	7200.236	2438.894	2470.138	3392.823
Instance 6	779.347	657.147	654.382	742.831
Instance 7	58.916	66.903	67.498	7200.038
Instance 8	1.93	1.649	1.654	3.101
Instance 9	71.851	38.513	38.386	97.984
Instance 10	49.541	21.832	21.905	108.197
Instance 11	252.639	244.24	169.458	34.063
Instance 12	246.051	37.796	38.051	10.76
Instance 13	7.561	133.659	133.879	23.492
Instance 14	4.579	4.136	4.356	208.576
Instance 15	4.944	5.661	5.431	19.183
Instance 16	31.07	85.999	90.463	48.341
Instance 17	7200.052	1424.856	1425.785	1791.575
Instance 18	21.646	6.86	7.248	196.092
Instance 19	3.74	26.658	26.357	1.71
Instance 20	36.599	12.806	12.667	3671.4

**Table S33.** CPU time for  $L_2M_x^{SB}$  in seconds with solver CPLEX

	$L_2M^{SB}$	$L_2M_P^{SB}$	$L_2M_{IP}^{SB}$	$L_2M_{IIP}^{SB}$
Instance 1	145.874	588.465	595.98	385.42
Instance 2	1344.869	1835.007	1813.482	705.045
Instance 3	930.568	4457.964	4248.783	3149.107
Instance 4	7200.02	3803.238	3998.611	7200.03
Instance 5	221.501	174.874	113.308	967.713
Instance 6	1725.253	1312.426	1276.238	1327.331
Instance 7	7200.073	7200.076	7200.08	7200.079
Instance 8	7200.113	7200.069	7200.079	7200.084
Instance 9	7200.079	7200.069	7200.114	7200.068
Instance 10	90.507	40.269	39.812	63.69
Instance 11	56.095	17.065	16.807	51.829
Instance 12	21.831	50.162	49.13	41.433
Instance 13	204.868	539.631	384.225	2636.115
Instance 14	29.514	188.163	172.55	59.262
Instance 15	7200.04	7200.042	7200.044	7200.051
Instance 16	427.9	355.066	328.818	364.704
Instance 17	439.443	588.6	587.319	444.588
Instance 18	1171.911	455.845	447.919	432.812
Instance 19	7200.046	4548.141	4543.27	192.847
Instance 20	54.134	29.334	29.932	303.463