Appendix for "Further Improvements of Finite Sample Approximation of Central Limit Theorems for Envelopment Estimator"

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Appendix A Details and Additional Results from Simulations for VRS-DEA Estimator

We conduct extensive Monte-Carlo experiments to compare the performance of our proposed method with those in Kneip et al. (2015), Simar and Zelenyuk (2020) and Nguyen et al. (2022). Following Simar and Zelenyuk (2020), we choose the Cobb-Douglas production function (VRS) to simulate the data. Formally,

$$y_i^{\partial}(x_i) = \prod_{j=1}^{p} x_{ji}^{\beta_j}, \tag{A.1}$$

where $1 \times p$ vector $x_i = (x_{1i}, x_{2i}, \ldots, x_{pi})$ and $x_{ji} \overset{\text{iid}}{\sim} \text{Unif}(0,1)$, $\forall\, j \in 1, \ldots, p$. In addition, we generate the true inefficiency scores from $\lambda_i \sim |N(0,1)| + 1$ (i.e., we generate the true inefficiency from the half normal distribution and then shift it up by 1); it can be shown that the simple mean true efficiency equal to $1 + \sqrt{\frac{2}{\pi}}$. The observed output is $y_i = y^{\partial}(x_i)/\lambda_i$, i.e., by projecting $y^{\partial}(x_i)$ from the frontier into the production set. Thus a simulated sample $\mathcal{S}_n = \{x_i, y_i\}_{i=1}^n$ is created. The values of β_j's are set the same as Simar and Zelenyuk (2020) so that we can directly compare our method with the correction method in Simar and Zelenyuk (2020).

To be more clear, for each experiment (i.e., for each combination of p, q and n) we perform $2,000$ trials, drawing on each trial a sample of size n and checking whether the estimated confidence interval covers the simple mean true efficiency. We then take the average of each trial's coverage over $2,000$ trials. Finally, we consider 0.90, 0.95 and 0.99 confidence intervals.

Tables A.1–A.4 present additional results not shown in the paper. Table A.1 lists the values of β_j's used by Simar and Zelenyuk (2020) for each dimension p. Table A.2 presents the coverages of estimated confidence intervals for the simple mean efficiency using VRS-DEA estimator across different sample sizes and dimensions. Table A.3 presents simple mean efficiency estimates and its estimated bias using VRS-DEA estimator, while Table A.4 presents mean standard deviation of estimated efficiency using VRS-DEA estimator.

Table A.1: The Values of β_j

p	1	2	3	4	5	7
β_1	0.4	0.4	0.4	0.4	0.4	0.05
β_2		0.2	0.2	0.2	0.2	0.1
β_3			0.1	0.1	0.1	0.15
β_4				0.15	0.15	0.2
β_5					0.05	0.125
β_6						0.075
β_7						0.025

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
1	1	10	0.506	0.571	0.667	0.576	0.624	0.726	0.674	0.723	0.810
1	1	20	0.607	0.643	0.714	0.682	0.715	0.783	0.787	0.821	0.867
1	1	50	0.713	0.731	0.778	0.797	0.814	0.851	0.893	0.901	0.925
1	1	100	0.785	0.792	0.826	0.856	0.863	0.885	0.937	0.941	0.952
1	1	200	0.812	0.815	0.839	0.884	0.886	0.901	0.957	0.959	0.963
1	1	300	0.838	0.838	0.857	0.908	0.909	0.921	0.971	0.972	0.975
1	1	500	0.845	0.846	0.856	0.916	0.916	0.921	0.977	0.977	0.979
1	1	1000	0.858	0.858	0.866	0.909	0.909	0.916	0.976	0.976	0.978

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
1	1	10	0.550	0.640	0.752	0.621	0.715	0.803	0.739	0.809	0.878
1	1	20	0.646	0.692	0.757	0.718	0.753	0.823	0.817	0.852	0.892
1	1	50	0.734	0.750	0.796	0.808	0.828	0.863	0.902	0.916	0.943
1	1	100	0.803	0.807	0.826	0.867	0.871	0.897	0.943	0.947	0.958
1	1	200	0.817	0.820	0.841	0.890	0.892	0.907	0.958	0.959	0.967
1	1	300	0.847	0.849	0.864	0.910	0.912	0.922	0.973	0.974	0.978
1	1	500	0.844	0.847	0.856	0.916	0.916	0.924	0.978	0.979	0.980
1	1	1000	0.860	0.860	0.868	0.914	0.914	0.917	0.976	0.976	0.979

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
2	1	10	0.273	0.339	0.503	0.317	0.390	0.566	0.398	0.484	0.667
2	1	20	0.374	0.442	0.584	0.443	0.511	0.661	0.556	0.622	0.783
2	1	50	0.508	0.550	0.656	0.577	0.625	0.739	0.721	0.773	0.871
2	1	100	0.629	0.661	0.722	0.702	0.733	0.806	0.827	0.856	0.911
2	1	200	0.709	0.729	0.778	0.786	0.806	0.857	0.897	0.907	0.937
2	1	300	0.738	0.747	0.788	0.811	0.826	0.859	0.914	0.920	0.947
2	1	500	0.768	0.779	0.820	0.856	0.865	0.894	0.942	0.947	0.961
2	1	1000	0.828	0.833	0.851	0.897	0.900	0.911	0.956	0.960	0.969

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
2	1	10	0.399	0.531	0.703	0.463	0.592	0.759	0.564	0.687	0.845
2	1	20	0.503	0.606	0.746	0.574	0.688	0.813	0.702	0.790	0.896
2	1	50	0.607	0.671	0.775	0.696	0.759	0.851	0.826	0.868	0.931
2	1	100	0.684	0.723	0.803	0.772	0.811	0.871	0.888	0.913	0.953
2	1	200	0.754	0.773	0.828	0.833	0.852	0.886	0.926	0.940	0.962
2	1	300	0.770	0.786	0.829	0.852	0.864	0.895	0.935	0.943	0.962
2	1	500	0.792	0.803	0.829	0.871	0.877	0.902	0.953	0.958	0.973
2	1	1000	0.844	0.849	0.867	0.902	0.905	0.921	0.968	0.969	0.975

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
3	1	10	0.177	0.254	0.450	0.218	0.293	0.510	0.277	0.367	0.625
3	1	20	0.264	0.371	0.598	0.330	0.445	0.685	0.441	0.560	0.808
3	1	50	0.458	0.545	0.736	0.541	0.635	0.818	0.695	0.783	0.926
3	1	100	0.612	0.679	0.797	0.694	0.750	0.867	0.822	0.878	0.953
3	1	200	0.724	0.771	0.855	0.803	0.848	0.922	0.916	0.947	0.981
3	1	300	0.766	0.807	0.872	0.853	0.875	0.926	0.939	0.960	0.984
3	1	500	0.837	0.857	0.910	0.903	0.919	0.952	0.968	0.978	0.991
3	1	1000	0.867	0.879	0.919	0.929	0.938	0.963	0.985	0.989	0.995

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
3	1	10	0.361	0.498	0.717	0.420	0.560	0.767	0.531	0.655	0.833
3	1	20	0.498	0.648	0.834	0.577	0.719	0.885	0.688	0.812	0.947
3	1	50	0.665	0.776	0.896	0.742	0.841	0.943	0.852	0.920	0.980
3	1	100	0.744	0.824	0.914	0.828	0.890	0.954	0.922	0.956	0.989
3	1	200	0.805	0.864	0.932	0.886	0.925	0.966	0.959	0.982	0.994
3	1	300	0.823	0.854	0.915	0.892	0.917	0.964	0.967	0.978	0.994
3	1	500	0.859	0.878	0.926	0.915	0.935	0.963	0.979	0.985	0.997
3	1	1000	0.870	0.886	0.925	0.937	0.947	0.966	0.983	0.988	0.996

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
4	1	10	0.108	0.166	0.366	0.131	0.193	0.427	0.171	0.266	0.517
4	1	20	0.148	0.234	0.504	0.185	0.284	0.597	0.257	0.384	0.735
4	1	50	0.306	0.425	0.707	0.376	0.526	0.806	0.521	0.687	0.925
4	1	100	0.451	0.573	0.800	0.547	0.682	0.893	0.721	0.837	0.968
4	1	200	0.634	0.728	0.875	0.730	0.823	0.950	0.874	0.943	0.994
4	1	300	0.713	0.791	0.893	0.804	0.865	0.953	0.913	0.957	0.995
4	1	500	0.799	0.849	0.931	0.879	0.923	0.977	0.967	0.984	0.998
4	1	1000	0.851	0.883	0.941	0.918	0.943	0.975	0.980	0.987	0.998

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
4	1	10	0.319	0.439	0.639	0.388	0.485	0.675	0.482	0.578	0.731
4	1	20	0.407	0.554	0.805	0.464	0.625	0.869	0.586	0.722	0.933
4	1	50	0.608	0.771	0.928	0.697	0.837	0.964	0.815	0.914	0.995
4	1	100	0.717	0.850	0.954	0.805	0.909	0.979	0.910	0.969	0.997
4	1	200	0.822	0.911	0.977	0.904	0.955	0.994	0.970	0.995	1.000
4	1	300	0.835	0.900	0.968	0.901	0.953	0.992	0.972	0.993	1.000
4	1	500	0.868	0.928	0.971	0.935	0.966	0.989	0.982	0.992	0.999
4	1	1000	0.870	0.903	0.949	0.924	0.950	0.980	0.980	0.992	0.999

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
5	1	10	0.065	0.108	0.287	0.083	0.136	0.336	0.108	0.176	0.402
5	1	20	0.092	0.165	0.464	0.118	0.204	0.555	0.176	0.294	0.685
5	1	50	0.174	0.293	0.682	0.227	0.389	0.794	0.349	0.556	0.912
5	1	100	0.357	0.515	0.843	0.459	0.635	0.932	0.625	0.819	0.989
5	1	200	0.544	0.687	0.912	0.646	0.796	0.973	0.822	0.933	0.997
5	1	300	0.671	0.784	0.930	0.765	0.871	0.979	0.894	0.967	0.999
5	1	500	0.776	0.861	0.963	0.857	0.928	0.990	0.961	0.989	1.000
5	1	1000	0.841	0.904	0.975	0.917	0.964	0.994	0.984	0.996	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
5	1	10	0.275	0.360	0.509	0.322	0.401	0.543	0.409	0.472	0.593
5	1	20	0.350	0.517	0.802	0.427	0.576	0.853	0.548	0.685	0.911
5	1	50	0.536	0.732	0.945	0.617	0.799	0.974	0.764	0.895	0.994
5	1	100	0.724	0.891	0.987	0.814	0.940	0.996	0.922	0.984	1.000
5	1	200	0.823	0.931	0.993	0.890	0.975	0.999	0.970	0.997	1.000
5	1	300	0.838	0.938	0.995	0.905	0.978	1.000	0.977	0.999	1.000
5	1	500	0.870	0.939	0.990	0.927	0.978	0.998	0.982	0.997	1.000
5	1	1000	0.846	0.910	0.963	0.911	0.952	0.991	0.971	0.995	1.000

Table A.2: Empirical Coverages for the Simple Mean Efficiency using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
7	1	10	0.036	0.072	0.190	0.046	0.085	0.213	0.064	0.109	0.251
7	1	20	0.035	0.084	0.382	0.048	0.111	0.453	0.080	0.175	0.572
7	1	50	0.079	0.184	0.674	0.115	0.246	0.787	0.194	0.391	0.917
7	1	100	0.178	0.350	0.862	0.242	0.456	0.937	0.378	0.663	0.990
7	1	200	0.344	0.575	0.947	0.451	0.703	0.991	0.646	0.887	1.000
7	1	300	0.499	0.739	0.980	0.623	0.850	0.997	0.815	0.973	1.000
7	1	500	0.670	0.859	0.992	0.774	0.933	1.000	0.925	0.992	1.000
7	1	1000	0.813	0.920	0.995	0.887	0.970	1.000	0.971	0.998	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
7	1	10	0.240	0.288	0.359	0.296	0.336	0.401	0.441	0.466	0.506
7	1	20	0.323	0.435	0.719	0.393	0.492	0.764	0.518	0.604	0.820
7	1	50	0.492	0.689	0.957	0.568	0.770	0.977	0.724	0.878	0.994
7	1	100	0.668	0.874	0.999	0.751	0.927	1.000	0.883	0.982	1.000
7	1	200	0.810	0.962	1.000	0.886	0.987	1.000	0.963	0.999	1.000
7	1	300	0.863	0.976	1.000	0.919	0.992	1.000	0.977	1.000	1.000
7	1	500	0.873	0.972	0.999	0.922	0.989	1.000	0.979	1.000	1.000
7	1	1000	0.838	0.944	0.989	0.896	0.974	0.998	0.964	0.994	1.000

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
1	1	10	1.3842	-0.2451	1.6293	1.8065	1.7979
1	1	20	1.5007	-0.2025	1.7032	1.7965	1.7979
1	1	50	1.6200	-0.1443	1.7642	1.8014	1.7979
1	1	100	1.6745	-0.1018	1.7762	1.7956	1.7979
1	1	200	1.7183	-0.0709	1.7892	1.7982	1.7979
1	1	300	1.7345	-0.0563	1.7908	1.7969	1.7979
1	1	500	1.7525	-0.0421	1.7946	1.7981	1.7979
1	1	1000	1.7682	-0.0276	1.7958	1.7974	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
1	1	10	1.4806	-0.2508	1.7314	1.8065	1.7979
1	1	20	1.5475	-0.2045	1.7520	1.7965	1.7979
1	1	50	1.6368	-0.1447	1.7815	1.8014	1.7979
1	1	100	1.6824	-0.1019	1.7843	1.7956	1.7979
1	1	200	1.7220	-0.0710	1.7929	1.7982	1.7979
1	1	300	1.7368	-0.0564	1.7932	1.7969	1.7979
1	1	500	1.7539	-0.0421	1.7960	1.7981	1.7979
1	1	1000	1.7688	-0.0276	1.7964	1.7974	1.7979

NOTE: $\widehat{\mu}_n$: Mean efficiency constructed using n observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
2	1	10	1.2178	-0.2600	1.4778	1.8008	1.7979
2	1	20	1.3300	-0.2686	1.5986	1.8005	1.7979
2	1	50	1.4645	-0.2365	1.7010	1.7980	1.7979
2	1	100	1.5489	-0.1985	1.7474	1.7992	1.7979
2	1	200	1.6127	-0.1578	1.7705	1.7985	1.7979
2	1	300	1.6424	-0.1358	1.7782	1.7969	1.7979
2	1	500	1.6773	-0.1116	1.7889	1.7991	1.7979
2	1	1000	1.7109	-0.0825	1.7935	1.7977	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
2	1	10	1.3961	-0.2752	1.6713	1.8008	1.7979
2	1	20	1.4393	-0.2765	1.7158	1.8005	1.7979
2	1	50	1.5172	-0.2395	1.7567	1.7980	1.7979
2	1	100	1.5783	-0.1997	1.7780	1.7992	1.7979
2	1	200	1.6291	-0.1584	1.7875	1.7985	1.7979
2	1	300	1.6540	-0.1362	1.7902	1.7969	1.7979
2	1	500	1.6849	-0.1118	1.7967	1.7991	1.7979
2	1	1000	1.7152	-0.0826	1.7977	1.7977	1.7979

NOTE: $\widehat{\mu}_n$: Mean efficiency constructed using n observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
3	1	10	1.1287	-0.2303	1.3590	1.8022	1.7979
3	1	20	1.2165	-0.2841	1.5005	1.7961	1.7979
3	1	50	1.3423	-0.2985	1.6408	1.8005	1.7979
3	1	100	1.4341	-0.2723	1.7064	1.7964	1.7979
3	1	200	1.5097	-0.2393	1.7490	1.7970	1.7979
3	1	300	1.5493	-0.2162	1.7655	1.7972	1.7979
3	1	500	1.5929	-0.1885	1.7814	1.7990	1.7979
3	1	1000	1.6388	-0.1516	1.7904	1.7978	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
3	1	10	1.3608	-0.2529	1.6137	1.8022	1.7979
3	1	20	1.3858	-0.3004	1.6863	1.7961	1.7979
3	1	50	1.4414	-0.3069	1.7483	1.8005	1.7979
3	1	100	1.4976	-0.2769	1.7744	1.7964	1.7979
3	1	200	1.5494	-0.2417	1.7911	1.7970	1.7979
3	1	300	1.5795	-0.2178	1.7973	1.7972	1.7979
3	1	500	1.6140	-0.1895	1.8035	1.7990	1.7979
3	1	1000	1.6520	-0.1521	1.8040	1.7978	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations where $n_\kappa = \lfloor n^{2\kappa} \rfloor$; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
4	1	10	1.0681	-0.1741	1.2421	1.8001	1.7979
4	1	20	1.1335	-0.2490	1.3825	1.7939	1.7979
4	1	50	1.2414	-0.3119	1.5533	1.8033	1.7979
4	1	100	1.3202	-0.3191	1.6393	1.7972	1.7979
4	1	200	1.3987	-0.3046	1.7033	1.7990	1.7979
4	1	300	1.4424	-0.2894	1.7318	1.7978	1.7979
4	1	500	1.4908	-0.2657	1.7565	1.7984	1.7979
4	1	1000	1.5498	-0.2289	1.7787	1.7973	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
4	1	10	1.3464	-0.1989	1.5453	1.8001	1.7979
4	1	20	1.3551	-0.2724	1.6275	1.7939	1.7979
4	1	50	1.3900	-0.3277	1.7176	1.8033	1.7979
4	1	100	1.4265	-0.3296	1.7562	1.7972	1.7979
4	1	200	1.4719	-0.3111	1.7830	1.7990	1.7979
4	1	300	1.5004	-0.2942	1.7946	1.7978	1.7979
4	1	500	1.5345	-0.2689	1.8035	1.7984	1.7979
4	1	1000	1.5789	-0.2307	1.8096	1.7973	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations where $n_\kappa = \lfloor n^{2\kappa} \rfloor$; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
5	1	10	1.0382	-0.1248	1.1630	1.7995	1.7979
5	1	20	1.0892	-0.2140	1.3033	1.7972	1.7979
5	1	50	1.1690	-0.2988	1.4678	1.7982	1.7979
5	1	100	1.2462	-0.3383	1.5845	1.7972	1.7979
5	1	200	1.3193	-0.3417	1.6610	1.7960	1.7979
5	1	300	1.3640	-0.3385	1.7025	1.7981	1.7979
5	1	500	1.4126	-0.3229	1.7354	1.7974	1.7979
5	1	1000	1.4792	-0.2931	1.7724	1.7981	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
5	1	10	1.3450	-0.1475	1.4925	1.7995	1.7979
5	1	20	1.3419	-0.2411	1.5830	1.7972	1.7979
5	1	50	1.3601	-0.3219	1.6819	1.7982	1.7979
5	1	100	1.3928	-0.3560	1.7488	1.7972	1.7979
5	1	200	1.4269	-0.3540	1.7809	1.7960	1.7979
5	1	300	1.4539	-0.3482	1.8022	1.7981	1.7979
5	1	500	1.4829	-0.3299	1.8127	1.7974	1.7979
5	1	1000	1.5294	-0.2975	1.8269	1.7981	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations where $n_\kappa = \lfloor n^{2\kappa} \rfloor$; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.3: Simple Mean Efficiency and Bias using VRS-DEA Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
7	1	10	1.0159	-0.0717	1.0877	1.8017	1.7979
7	1	20	1.0397	-0.1448	1.1844	1.8001	1.7979
7	1	50	1.0964	-0.2558	1.3522	1.7971	1.7979
7	1	100	1.1496	-0.3201	1.4696	1.7950	1.7979
7	1	200	1.2056	-0.3641	1.5697	1.7974	1.7979
7	1	300	1.2482	-0.3809	1.6291	1.7980	1.7979
7	1	500	1.2909	-0.3877	1.6786	1.7976	1.7979
7	1	1000	1.3542	-0.3807	1.7349	1.7972	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
7	1	10	1.3541	-0.0884	1.4425	1.8017	1.7979
7	1	20	1.3448	-0.1718	1.5167	1.8001	1.7979
7	1	50	1.3494	-0.2884	1.6377	1.7971	1.7979
7	1	100	1.3619	-0.3510	1.7129	1.7950	1.7979
7	1	200	1.3799	-0.3902	1.7702	1.7974	1.7979
7	1	300	1.4003	-0.4037	1.8040	1.7980	1.7979
7	1	500	1.4212	-0.4065	1.8277	1.7976	1.7979
7	1	1000	1.4550	-0.3944	1.8494	1.7972	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations where $n_\kappa = \lfloor n^{2\kappa} \rfloor$; $\widehat{B}_n$: Bias constructed using n observation; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
1	1	10	0.4459	0.5154	0.6867	0.3899	0.4712	0.6268	0.5841	0.6028
1	1	20	0.5052	0.5485	0.6576	0.4671	0.5143	0.6165	0.5941	0.6028
1	1	50	0.5510	0.5710	0.6345	0.5340	0.5548	0.6164	0.6007	0.6028
1	1	100	0.5678	0.5774	0.6186	0.5591	0.5689	0.6095	0.6005	0.6028
1	1	200	0.5814	0.5859	0.6130	0.5771	0.5816	0.6085	0.6029	0.6028
1	1	300	0.5854	0.5883	0.6091	0.5826	0.5854	0.6062	0.6022	0.6028
1	1	500	0.5900	0.5916	0.6070	0.5884	0.5899	0.6053	0.6026	0.6028
1	1	1000	0.5942	0.5949	0.6047	0.5934	0.5941	0.6039	0.6024	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
2	1	10	0.3354	0.4310	0.7017	0.2994	0.4146	0.6638	0.5794	0.6028
2	1	20	0.4282	0.5101	0.7229	0.3731	0.4696	0.6639	0.5956	0.6028
2	1	50	0.4968	0.5525	0.6895	0.4572	0.5185	0.6465	0.5976	0.6028
2	1	100	0.5309	0.5678	0.6664	0.5048	0.5440	0.6381	0.6011	0.6028
2	1	200	0.5520	0.5746	0.6445	0.5357	0.5591	0.6270	0.6022	0.6028
2	1	300	0.5605	0.5770	0.6345	0.5483	0.5652	0.6216	0.6018	0.6028
2	1	500	0.5704	0.5813	0.6260	0.5621	0.5732	0.6172	0.6028	0.6028
2	1	1000	0.5794	0.5852	0.6168	0.5745	0.5804	0.6117	0.6026	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
3	1	10	0.2455	0.3423	0.6674	0.2584	0.3712	0.6836	0.5861	0.6028
3	1	20	0.3474	0.4538	0.7551	0.3057	0.4341	0.7129	0.5916	0.6028
3	1	50	0.4425	0.5366	0.7622	0.3898	0.4992	0.7067	0.6012	0.6028
3	1	100	0.4863	0.5587	0.7269	0.4426	0.5235	0.6800	0.5998	0.6028
3	1	200	0.5179	0.5712	0.6977	0.4855	0.5430	0.6627	0.6008	0.6028
3	1	300	0.5324	0.5750	0.6802	0.5058	0.5512	0.6515	0.6020	0.6028
3	1	500	0.5464	0.5782	0.6633	0.5262	0.5596	0.6416	0.6028	0.6028
3	1	1000	0.5599	0.5802	0.6436	0.5464	0.5673	0.6292	0.6024	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
4	1	10	0.1608	0.2407	0.5512	0.2320	0.3203	0.6327	0.5823	0.6028
4	1	20	0.2620	0.3658	0.7257	0.2582	0.3808	0.7273	0.5888	0.6028
4	1	50	0.3733	0.4889	0.7979	0.3247	0.4640	0.7502	0.6010	0.6028
4	1	100	0.4295	0.5367	0.7863	0.3772	0.5028	0.7325	0.6007	0.6028
4	1	200	0.4710	0.5617	0.7556	0.4246	0.5272	0.7064	0.6017	0.6028
4	1	300	0.4913	0.5708	0.7385	0.4502	0.5384	0.6945	0.6029	0.6028
4	1	500	0.5122	0.5773	0.7155	0.4779	0.5487	0.6785	0.6032	0.6028
4	1	1000	0.5320	0.5793	0.6854	0.5063	0.5565	0.6576	0.6023	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
5	1	10	0.1012	0.1626	0.4212	0.2171	0.2817	0.5528	0.5883	0.6028
5	1	20	0.2060	0.3014	0.6920	0.2383	0.3461	0.7335	0.5918	0.6028
5	1	50	0.3147	0.4368	0.8146	0.2846	0.4326	0.7918	0.5993	0.6028
5	1	100	0.3809	0.5111	0.8382	0.3314	0.4881	0.7923	0.5994	0.6028
5	1	200	0.4302	0.5503	0.8167	0.3774	0.5185	0.7643	0.6015	0.6028
5	1	300	0.4552	0.5679	0.8024	0.4047	0.5346	0.7509	0.6024	0.6028
5	1	500	0.4791	0.5781	0.7757	0.4332	0.5449	0.7278	0.6020	0.6028
5	1	1000	0.5068	0.5856	0.7412	0.4689	0.5556	0.7010	0.6028	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table A.4: Mean Standard Deviation of Estimated Efficiency using VRS-DEA Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
7	1	10	0.0475	0.0869	0.2562	0.2128	0.2524	0.4396	0.5841	0.6028
7	1	20	0.1246	0.1944	0.5560	0.2212	0.2925	0.6692	0.5900	0.6028
7	1	50	0.2328	0.3490	0.8167	0.2492	0.3847	0.8558	0.5990	0.6028
7	1	100	0.2985	0.4396	0.8915	0.2778	0.4495	0.8913	0.5987	0.6028
7	1	200	0.3568	0.5109	0.9111	0.3140	0.5019	0.8827	0.6006	0.6028
7	1	300	0.3870	0.5437	0.9157	0.3369	0.5265	0.8765	0.6032	0.6028
7	1	500	0.4175	0.5702	0.8972	0.3635	0.5457	0.8504	0.6017	0.6028
7	1	1000	0.4536	0.5924	0.8615	0.4001	0.5620	0.8111	0.6018	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Appendix B Details and Additional Results from Simulations for FDH Estimator

In this appendix, we conduct extensive Monte-Carlo experiments to illustrate the performance of our proposed method for FDH estimator. The technology and data generation process are the same as for VRS-DEA estimator presented in Appendix A. Instead of using VRS-DEA estimator, here we use FDH estimator. Note that in this situation, according to Kneip et al. (2015), FDH estimator is also consistent but with a slower convergence rate compared to VRS-DEA estimator.

Tables B.5–B.7 present additional results not shown in the paper. Table B.5 presents the coverages of estimated confidence intervals for the simple mean efficiency using FDH estimator across different sample sizes and dimensions. Table B.6 presents simple mean efficiency estimates and bias using FDH estimator, while Table B.7 presents mean standard deviation of estimated efficiency using FDH estimator.

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
1	1	10	0.286	0.338	0.482	0.331	0.387	0.540	0.414	0.481	0.651
1	1	20	0.369	0.416	0.551	0.430	0.488	0.642	0.553	0.619	0.773
1	1	50	0.460	0.502	0.614	0.546	0.594	0.710	0.690	0.736	0.839
1	1	100	0.546	0.583	0.665	0.644	0.669	0.750	0.783	0.808	0.877
1	1	200	0.649	0.664	0.726	0.736	0.753	0.801	0.855	0.872	0.916
1	1	300	0.680	0.694	0.742	0.767	0.778	0.821	0.887	0.895	0.923
1	1	500	0.724	0.734	0.767	0.809	0.819	0.847	0.920	0.926	0.947
1	1	1000	0.782	0.789	0.810	0.863	0.865	0.888	0.948	0.948	0.961

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
1	1	10	0.430	0.535	0.683	0.494	0.600	0.749	0.600	0.695	0.835
1	1	20	0.519	0.599	0.727	0.588	0.670	0.792	0.702	0.771	0.877
1	1	50	0.599	0.652	0.750	0.683	0.727	0.820	0.796	0.838	0.905
1	1	100	0.645	0.679	0.766	0.743	0.773	0.842	0.855	0.881	0.935
1	1	200	0.724	0.742	0.795	0.797	0.816	0.866	0.910	0.922	0.954
1	1	300	0.737	0.754	0.798	0.820	0.834	0.872	0.917	0.927	0.953
1	1	500	0.773	0.781	0.819	0.855	0.863	0.891	0.946	0.948	0.962
1	1	1000	0.811	0.817	0.844	0.883	0.887	0.903	0.960	0.962	0.972

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
2	1	10	0.117	0.169	0.368	0.140	0.205	0.427	0.204	0.284	0.517
2	1	20	0.139	0.196	0.432	0.175	0.249	0.525	0.252	0.345	0.667
2	1	50	0.216	0.300	0.562	0.276	0.380	0.683	0.401	0.535	0.849
2	1	100	0.321	0.408	0.683	0.399	0.512	0.797	0.575	0.716	0.933
2	1	200	0.447	0.543	0.782	0.557	0.671	0.872	0.758	0.846	0.975
2	1	300	0.538	0.622	0.780	0.648	0.729	0.885	0.801	0.883	0.977
2	1	500	0.629	0.695	0.834	0.731	0.794	0.916	0.878	0.930	0.985
2	1	1000	0.724	0.765	0.874	0.817	0.866	0.941	0.945	0.964	0.991

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
2	1	10	0.355	0.448	0.650	0.418	0.507	0.705	0.523	0.604	0.771
2	1	20	0.376	0.493	0.740	0.445	0.559	0.808	0.556	0.668	0.897
2	1	50	0.482	0.620	0.855	0.567	0.706	0.914	0.701	0.830	0.974
2	1	100	0.618	0.731	0.905	0.702	0.812	0.951	0.830	0.919	0.993
2	1	200	0.721	0.810	0.939	0.804	0.889	0.975	0.922	0.967	0.997
2	1	300	0.762	0.825	0.943	0.829	0.904	0.975	0.943	0.973	0.995
2	1	500	0.818	0.870	0.950	0.894	0.934	0.981	0.968	0.987	0.998
2	1	1000	0.853	0.891	0.954	0.923	0.951	0.985	0.985	0.991	0.998

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
3	1	10	0.047	0.077	0.232	0.059	0.095	0.282	0.084	0.133	0.343
3	1	20	0.052	0.095	0.346	0.068	0.121	0.420	0.111	0.189	0.553
3	1	50	0.066	0.123	0.466	0.089	0.168	0.598	0.156	0.279	0.787
3	1	100	0.081	0.162	0.585	0.119	0.246	0.753	0.227	0.425	0.929
3	1	200	0.191	0.321	0.766	0.265	0.434	0.889	0.434	0.669	0.991
3	1	300	0.259	0.420	0.817	0.351	0.550	0.921	0.567	0.786	0.994
3	1	500	0.386	0.542	0.872	0.501	0.683	0.962	0.717	0.887	0.999
3	1	1000	0.557	0.702	0.921	0.683	0.822	0.979	0.866	0.958	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
3	1	10	0.269	0.321	0.467	0.319	0.368	0.504	0.416	0.460	0.569
3	1	20	0.288	0.390	0.688	0.357	0.457	0.746	0.479	0.562	0.826
3	1	50	0.346	0.505	0.874	0.424	0.591	0.931	0.566	0.740	0.981
3	1	100	0.433	0.645	0.948	0.538	0.750	0.981	0.701	0.884	0.997
3	1	200	0.601	0.810	0.989	0.704	0.894	0.996	0.873	0.976	1.000
3	1	300	0.695	0.859	0.992	0.794	0.930	0.999	0.911	0.987	1.000
3	1	500	0.784	0.921	0.995	0.870	0.972	1.000	0.969	0.997	1.000
3	1	1000	0.868	0.950	0.995	0.933	0.979	1.000	0.982	1.000	1.000

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
4	1	10	0.022	0.036	0.113	0.026	0.046	0.130	0.036	0.065	0.159
4	1	20	0.013	0.024	0.153	0.017	0.029	0.200	0.025	0.049	0.275
4	1	50	0.013	0.021	0.276	0.015	0.035	0.381	0.027	0.069	0.575
4	1	100	0.009	0.023	0.333	0.013	0.037	0.483	0.029	0.088	0.764
4	1	200	0.011	0.035	0.454	0.018	0.054	0.673	0.044	0.142	0.929
4	1	300	0.024	0.062	0.598	0.036	0.113	0.824	0.090	0.267	0.985
4	1	500	0.042	0.108	0.684	0.070	0.172	0.891	0.147	0.396	0.998
4	1	1000	0.123	0.250	0.824	0.185	0.387	0.965	0.347	0.686	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
4	1	10	0.216	0.240	0.298	0.289	0.312	0.358	0.447	0.462	0.497
4	1	20	0.163	0.207	0.457	0.210	0.259	0.499	0.324	0.379	0.577
4	1	50	0.177	0.269	0.741	0.230	0.339	0.821	0.368	0.486	0.911
4	1	100	0.171	0.306	0.857	0.226	0.405	0.939	0.371	0.590	0.984
4	1	200	0.229	0.436	0.957	0.301	0.566	0.989	0.478	0.788	1.000
4	1	300	0.319	0.572	0.986	0.418	0.713	0.999	0.614	0.907	1.000
4	1	500	0.399	0.687	0.997	0.505	0.829	1.000	0.714	0.963	1.000
4	1	1000	0.600	0.859	1.000	0.722	0.947	1.000	0.890	0.997	1.000

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
5	1	50	0.004	0.009	0.187	0.005	0.011	0.251	0.009	0.025	0.381
5	1	100	0.002	0.003	0.212	0.002	0.005	0.364	0.004	0.017	0.607
5	1	200	0.003	0.008	0.347	0.004	0.017	0.576	0.011	0.037	0.865
5	1	300	0.004	0.013	0.418	0.006	0.020	0.686	0.014	0.055	0.951
5	1	500	0.009	0.025	0.603	0.016	0.050	0.874	0.034	0.129	0.998
5	1	1000	0.025	0.072	0.839	0.038	0.136	0.982	0.090	0.364	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
5	1	50	0.120	0.165	0.606	0.164	0.229	0.669	0.303	0.385	0.772
5	1	100	0.105	0.173	0.807	0.148	0.248	0.888	0.270	0.428	0.962
5	1	200	0.124	0.264	0.938	0.180	0.380	0.981	0.341	0.608	0.998
5	1	300	0.150	0.325	0.980	0.217	0.460	0.998	0.386	0.720	1.000
5	1	500	0.228	0.493	0.997	0.307	0.663	1.000	0.521	0.890	1.000
5	1	1000	0.397	0.767	1.000	0.522	0.887	1.000	0.751	0.985	1.000

Table B.5: Empirical Coverages for the Simple Mean Efficiency using FDH Estimator (continued)

Panel A: Without Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3	Sol1	Sol2	Sol3
7	1	50	0.003	0.004	0.139	0.003	0.004	0.177	0.003	0.011	0.251
7	1	100	0.001	0.002	0.149	0.001	0.003	0.234	0.001	0.005	0.370
7	1	200	0.001	0.004	0.272	0.002	0.005	0.447	0.004	0.011	0.700
7	1	300	0.001	0.002	0.264	0.001	0.003	0.475	0.002	0.006	0.800
7	1	500	0.002	0.003	0.498	0.003	0.004	0.797	0.003	0.017	0.984
7	1	1000	0.007	0.017	0.785	0.009	0.031	0.981	0.018	0.062	1.000

Panel B: With Data Sharpening

			0.90			0.95			0.99		
p	q	n	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6	Sol4	Sol5	Sol6
7	1	50	0.174	0.200	0.472	0.256	0.287	0.539	0.445	0.476	0.661
7	1	100	0.088	0.124	0.647	0.139	0.184	0.729	0.281	0.348	0.823
7	1	200	0.133	0.206	0.898	0.201	0.298	0.953	0.369	0.517	0.987
7	1	300	0.104	0.186	0.947	0.161	0.287	0.982	0.313	0.497	0.998
7	1	500	0.163	0.308	0.997	0.233	0.451	1.000	0.423	0.713	1.000
7	1	1000	0.239	0.525	1.000	0.333	0.700	1.000	0.565	0.915	1.000

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
1	1	10	1.2474	-0.2312	1.4787	1.8009	1.7979
1	1	20	1.3494	-0.2402	1.5896	1.7994	1.7979
1	1	50	1.4675	-0.2152	1.6828	1.7966	1.7979
1	1	100	1.5447	-0.1856	1.7302	1.7956	1.7979
1	1	200	1.6084	-0.1518	1.7602	1.7967	1.7979
1	1	300	1.6395	-0.1320	1.7716	1.7982	1.7979
1	1	500	1.6708	-0.1085	1.7793	1.7972	1.7979
1	1	1000	1.7058	-0.0823	1.7881	1.7979	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
1	1	10	1.4215	-0.2439	1.6654	1.8009	1.7979
1	1	20	1.4556	-0.2467	1.7023	1.7994	1.7979
1	1	50	1.5198	-0.2176	1.7375	1.7966	1.7979
1	1	100	1.5740	-0.1866	1.7607	1.7956	1.7979
1	1	200	1.6247	-0.1523	1.7770	1.7967	1.7979
1	1	300	1.6511	-0.1323	1.7834	1.7982	1.7979
1	1	500	1.6783	-0.1087	1.7870	1.7972	1.7979
1	1	1000	1.7100	-0.0824	1.7924	1.7979	1.7979

NOTE: $\widehat{\mu}_n$: Mean efficiency constructed using n observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
2	1	10	1.0936	-0.1649	1.2584	1.7933	1.7979
2	1	20	1.1459	-0.2131	1.3590	1.7986	1.7979
2	1	50	1.2380	-0.2662	1.5042	1.7975	1.7979
2	1	100	1.3107	-0.2826	1.5933	1.7992	1.7979
2	1	200	1.3867	-0.2773	1.6640	1.7986	1.7979
2	1	300	1.4225	-0.2657	1.6881	1.7973	1.7979
2	1	500	1.4703	-0.2487	1.7190	1.7980	1.7979
2	1	1000	1.5291	-0.2205	1.7496	1.7988	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
2	1	10	1.3719	-0.1859	1.5578	1.7933	1.7979
2	1	20	1.3638	-0.2319	1.5957	1.7986	1.7979
2	1	50	1.3886	-0.2789	1.6675	1.7975	1.7979
2	1	100	1.4185	-0.2910	1.7095	1.7992	1.7979
2	1	200	1.4609	-0.2824	1.7433	1.7986	1.7979
2	1	300	1.4826	-0.2695	1.7521	1.7973	1.7979
2	1	500	1.5152	-0.2513	1.7665	1.7980	1.7979
2	1	1000	1.5592	-0.2221	1.7813	1.7988	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
3	1	10	1.0357	-0.0987	1.1344	1.8014	1.7979
3	1	20	1.0644	-0.1497	1.2141	1.7985	1.7979
3	1	50	1.1198	-0.2268	1.3466	1.7956	1.7979
3	1	100	1.1677	-0.2707	1.4384	1.7967	1.7979
3	1	200	1.2314	-0.3057	1.5370	1.7994	1.7979
3	1	300	1.2633	-0.3153	1.5786	1.7978	1.7979
3	1	500	1.3092	-0.3206	1.6298	1.7977	1.7979
3	1	1000	1.3685	-0.3148	1.6833	1.7980	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
3	1	10	1.3622	-0.1184	1.4806	1.8014	1.7979
3	1	20	1.3485	-0.1711	1.5196	1.7985	1.7979
3	1	50	1.3460	-0.2488	1.5948	1.7956	1.7979
3	1	100	1.3545	-0.2899	1.6444	1.7967	1.7979
3	1	200	1.3784	-0.3209	1.6992	1.7994	1.7979
3	1	300	1.3916	-0.3282	1.7198	1.7978	1.7979
3	1	500	1.4146	-0.3308	1.7454	1.7977	1.7979
3	1	1000	1.4483	-0.3219	1.7702	1.7980	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
4	1	10	1.0109	-0.0435	1.0544	1.8012	1.7979
4	1	20	1.0220	-0.0697	1.0917	1.7958	1.7979
4	1	50	1.0484	-0.1317	1.1801	1.7974	1.7979
4	1	100	1.0755	-0.1837	1.2592	1.7967	1.7979
4	1	200	1.1095	-0.2342	1.3437	1.7973	1.7979
4	1	300	1.1366	-0.2633	1.3999	1.7982	1.7979
4	1	500	1.1677	-0.2943	1.4619	1.7979	1.7979
4	1	1000	1.2135	-0.3226	1.5360	1.7979	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_n$	$\widehat{B}_n$	$\widehat{\mu}_n - \widehat{B}_n$	μ_s	μ
4	1	10	1.3675	-0.0540	1.4215	1.8012	1.7979
4	1	20	1.3432	-0.0843	1.4275	1.7958	1.7979
4	1	50	1.3234	-0.1517	1.4752	1.7974	1.7979
4	1	100	1.3195	-0.2058	1.5253	1.7967	1.7979
4	1	200	1.3235	-0.2561	1.5796	1.7973	1.7979
4	1	300	1.3308	-0.2844	1.6152	1.7982	1.7979
4	1	500	1.3386	-0.3136	1.6522	1.7979	1.7979
4	1	1000	1.3572	-0.3387	1.6960	1.7979	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
5	1	50	1.0188	-0.0782	1.0971	1.7965	1.7979
5	1	100	1.0365	-0.1208	1.1573	1.7959	1.7979
5	1	200	1.0535	-0.1737	1.2272	1.7966	1.7979
5	1	300	1.0741	-0.2034	1.2775	1.7980	1.7979
5	1	500	1.0959	-0.2430	1.3389	1.7981	1.7979
5	1	1000	1.1360	-0.2913	1.4273	1.7981	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
5	1	50	1.3286	-0.0934	1.4220	1.7965	1.7979
5	1	100	1.3171	-0.1407	1.4578	1.7959	1.7979
5	1	200	1.3094	-0.1971	1.5065	1.7966	1.7979
5	1	300	1.3116	-0.2277	1.5393	1.7980	1.7979
5	1	500	1.3154	-0.2678	1.5832	1.7981	1.7979
5	1	1000	1.3277	-0.3153	1.6430	1.7981	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations; $\widehat{B}_n$: Bias constructed using n observations; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.6: Simple Mean Efficiency and Bias using FDH Estimator (continued)

Panel A: Without Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
7	1	50	1.0082	-0.0393	1.0475	1.8019	1.7979
7	1	100	1.0126	-0.0648	1.0774	1.7979	1.7979
7	1	200	1.0230	-0.1003	1.1233	1.7985	1.7979
7	1	300	1.0313	-0.1261	1.1574	1.7978	1.7979
7	1	500	1.0425	-0.1623	1.2048	1.7982	1.7979
7	1	1000	1.0645	-0.2169	1.2814	1.7985	1.7979

Panel B: With Data Sharpening

p	q	n	$\widehat{\mu}_{n_\kappa}$	$\widehat{B}_n$	$\widehat{\mu}_{n_\kappa} - \widehat{B}_n$	μ_s	μ
7	1	50	1.3523	-0.0490	1.4013	1.8019	1.7979
7	1	100	1.3384	-0.0791	1.4175	1.7979	1.7979
7	1	200	1.3342	-0.1202	1.4544	1.7985	1.7979
7	1	300	1.3269	-0.1492	1.4761	1.7978	1.7979
7	1	500	1.3227	-0.1892	1.5119	1.7982	1.7979
7	1	1000	1.3214	-0.2477	1.5691	1.7985	1.7979

NOTE: $\widehat{\mu}_{n_\kappa}$: Mean efficiency constructed using n_κ observations; $\widehat{B}_n$: Bias constructed using n observation; μ_s: Sample mean of true efficiency; μ: Population mean of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
1	1	10	0.3574	0.4318	0.6712	0.3165	0.4068	0.6269	0.5813	0.6028
1	1	20	0.4383	0.5039	0.7025	0.3836	0.4608	0.6440	0.5976	0.6028
1	1	50	0.4936	0.5404	0.6778	0.4541	0.5056	0.6355	0.5970	0.6028
1	1	100	0.5248	0.5575	0.6574	0.4988	0.5335	0.6295	0.5993	0.6028
1	1	200	0.5473	0.5683	0.6405	0.5311	0.5529	0.6232	0.6014	0.6028
1	1	300	0.5572	0.5729	0.6322	0.5451	0.5612	0.6194	0.6026	0.6028
1	1	500	0.5655	0.5760	0.6222	0.5573	0.5679	0.6134	0.6022	0.6028
1	1	1000	0.5757	0.5816	0.6144	0.5708	0.5768	0.6093	0.6025	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
2	1	10	0.1925	0.2574	0.5335	0.2461	0.3202	0.5895	0.5806	0.6028
2	1	20	0.2777	0.3539	0.6760	0.2693	0.3603	0.6662	0.5928	0.6028
2	1	50	0.3659	0.4547	0.7480	0.3191	0.4263	0.6990	0.5971	0.6028
2	1	100	0.4189	0.5066	0.7587	0.3675	0.4702	0.7047	0.6007	0.6028
2	1	200	0.4582	0.5363	0.7415	0.4120	0.5003	0.6925	0.6022	0.6028
2	1	300	0.4760	0.5456	0.7240	0.4348	0.5120	0.6800	0.6017	0.6028
2	1	500	0.4964	0.5555	0.7057	0.4618	0.5261	0.6686	0.6026	0.6028
2	1	1000	0.5180	0.5631	0.6809	0.4918	0.5397	0.6527	0.6029	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
3	1	10	0.0969	0.1401	0.3518	0.2275	0.2730	0.4950	0.5802	0.6028
3	1	20	0.1709	0.2302	0.5598	0.2329	0.2973	0.6216	0.5928	0.6028
3	1	50	0.2596	0.3471	0.7336	0.2571	0.3607	0.7330	0.5965	0.6028
3	1	100	0.3114	0.4140	0.7879	0.2811	0.4052	0.7592	0.5985	0.6028
3	1	200	0.3629	0.4753	0.8131	0.3168	0.4518	0.7678	0.6016	0.6028
3	1	300	0.3873	0.4999	0.8121	0.3374	0.4712	0.7624	0.6019	0.6028
3	1	500	0.4159	0.5254	0.8037	0.3653	0.4932	0.7525	0.6030	0.6028
3	1	1000	0.4479	0.5476	0.7806	0.4007	0.5142	0.7319	0.6030	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
4	1	10	0.0356	0.0568	0.1606	0.2162	0.2372	0.3473	0.5814	0.6028
4	1	20	0.0759	0.1044	0.3175	0.2106	0.2369	0.4529	0.5865	0.6028
4	1	50	0.1530	0.2039	0.5704	0.2159	0.2683	0.6312	0.5972	0.6028
4	1	100	0.2031	0.2752	0.6920	0.2276	0.3087	0.7184	0.5996	0.6028
4	1	200	0.2483	0.3421	0.7656	0.2429	0.3539	0.7645	0.6002	0.6028
4	1	300	0.2767	0.3825	0.8004	0.2569	0.3838	0.7854	0.6021	0.6028
4	1	500	0.3082	0.4265	0.8274	0.2753	0.4176	0.7990	0.6016	0.6028
4	1	1000	0.3483	0.4748	0.8370	0.3044	0.4556	0.7967	0.6031	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
5	1	50	0.0922	0.1224	0.4230	0.2053	0.2316	0.5274	0.5993	0.6028
5	1	100	0.1372	0.1841	0.5817	0.2088	0.2544	0.6511	0.6005	0.6028
5	1	200	0.1792	0.2503	0.7029	0.2167	0.2941	0.7454	0.6009	0.6028
5	1	300	0.2029	0.2879	0.7539	0.2234	0.3198	0.7815	0.6013	0.6028
5	1	500	0.2335	0.3374	0.8082	0.2347	0.3565	0.8181	0.6025	0.6028
5	1	1000	0.2745	0.4005	0.8581	0.2546	0.4054	0.8478	0.6029	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Table B.7: Mean Standard Deviation of Estimated Efficiency using FDH Estimator (continued)

p	q	n	Sol1	Sol2	Sol3	Sol4	Sol5	Sol6	σ_s	σ
7	1	50	0.0425	0.0586	0.2621	0.2107	0.2220	0.4221	0.6013	0.6028
7	1	100	0.0746	0.1000	0.4305	0.2063	0.2253	0.5493	0.6008	0.6028
7	1	200	0.1064	0.1472	0.5807	0.2062	0.2411	0.6806	0.6016	0.6028
7	1	300	0.1254	0.1785	0.6563	0.2081	0.2576	0.7457	0.6016	0.6028
7	1	500	0.1506	0.2218	0.7411	0.2124	0.2852	0.8175	0.6023	0.6028
7	1	1000	0.1862	0.2861	0.8404	0.2218	0.3328	0.8968	0.6034	0.6028

NOTE: σ_s: Sample standard deviation of true efficiency; σ: Population standard deviation of true efficiency.

Appendix C Details and Additional Results from Simulations for CRS-DEA Estimator

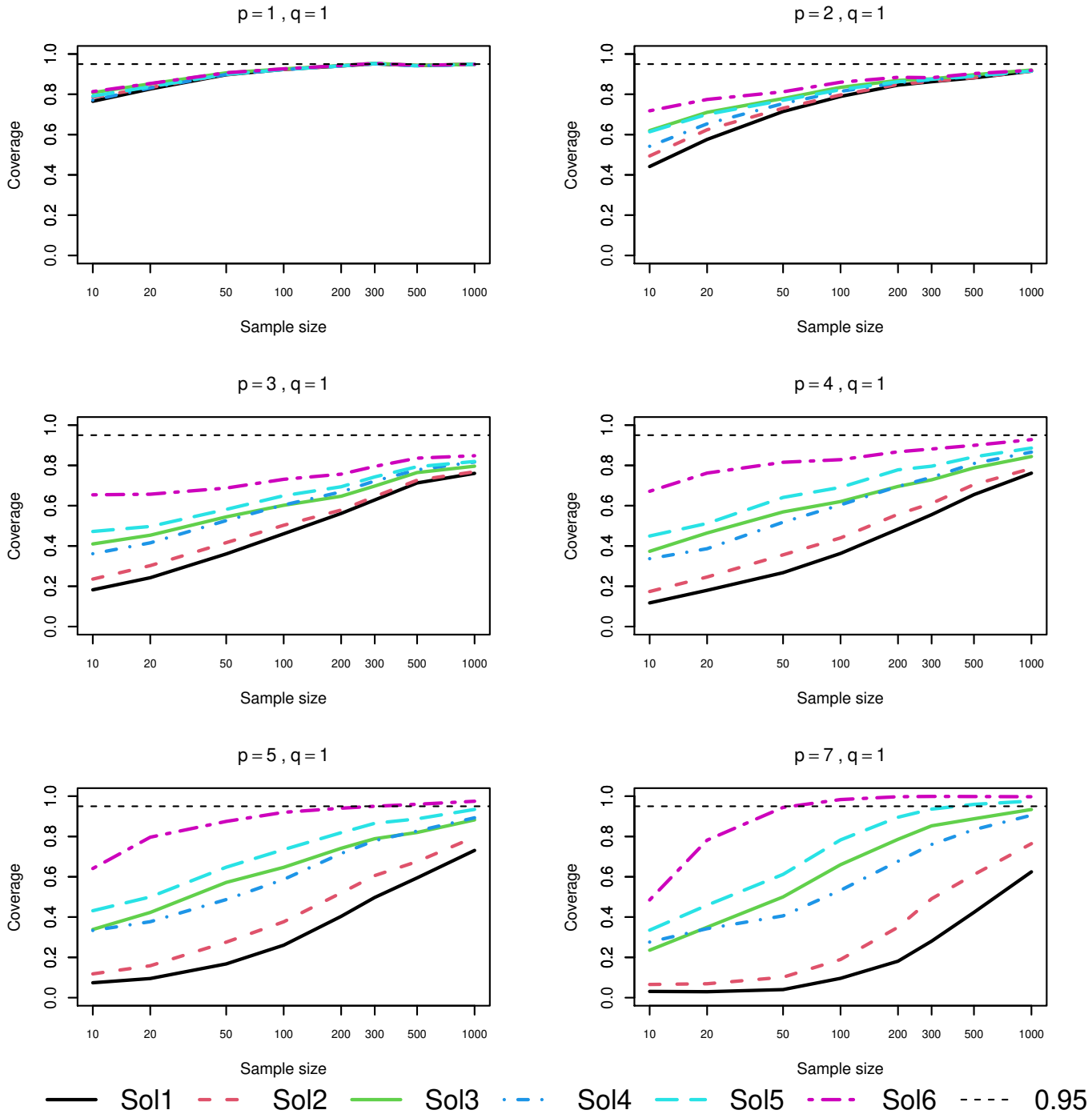
In this appendix, we conduct extensive Monte-Carlo experiments to illustrate the performance of our proposed method for CRS-DEA estimator. For CRS estimator, the technology needs to be globally constant returns to scale. Following Kneip et al. (2015), we use the following technology,

$$y_i^{\partial}(x_i) = \prod_{j=1}^{p} x_{ji}^{1/p}, \tag{C.1}$$

The inefficiency is generated similarly as for VRS-DEA estimator presented in Appendix A. Instead of using VRS-DEA estimator, here we use CRS-DEA estimator.

The results are summarized in Figure C.1, confirming the improved performance of our proposed method for CRS-DEA estimator.

Figure C.1: Empirical Coverages for the Simple Mean Efficiency using CRS-DEA Estimator



NOTE: A (base-10) logarithmic scale is used on the horizontal axis.

References

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