

# Stable Prediction in High-Dimensional Linear Models-Supplementary Materials

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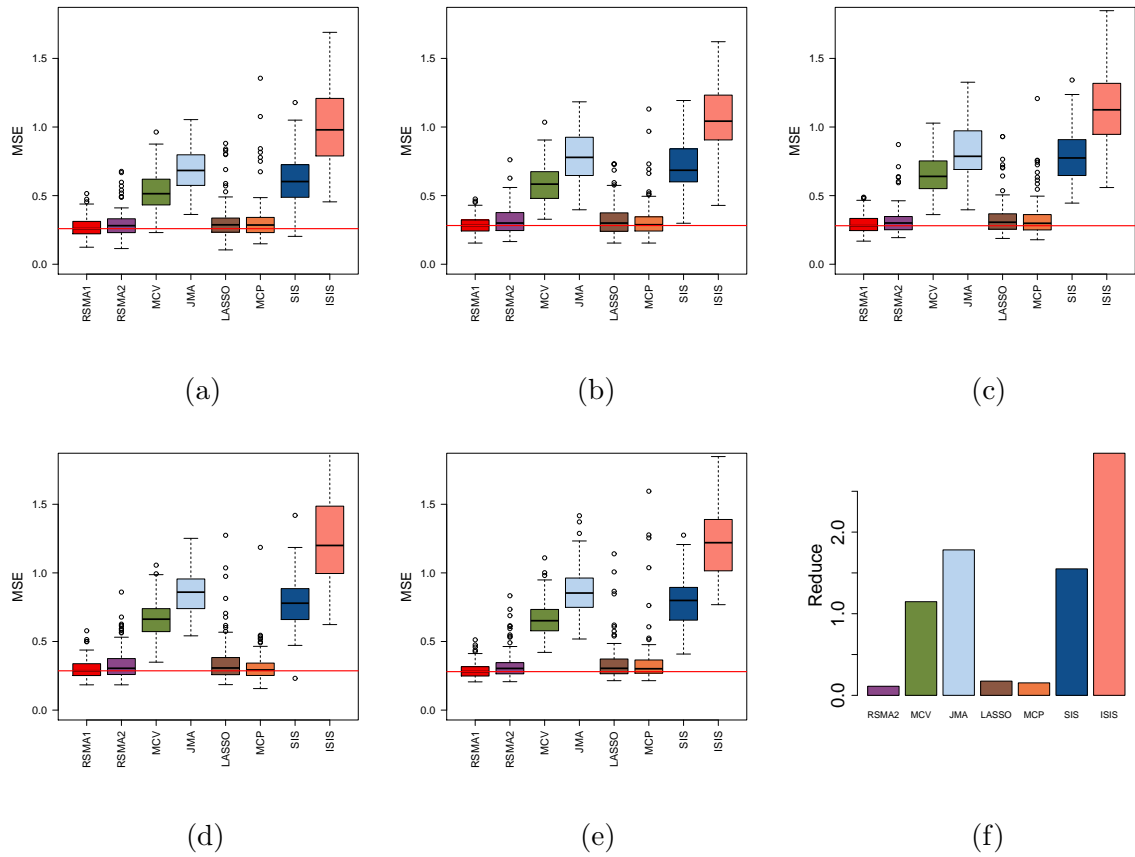


Figure 1: MSE for Example 2 with  $\rho = -0.7$  and  $p = 500$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

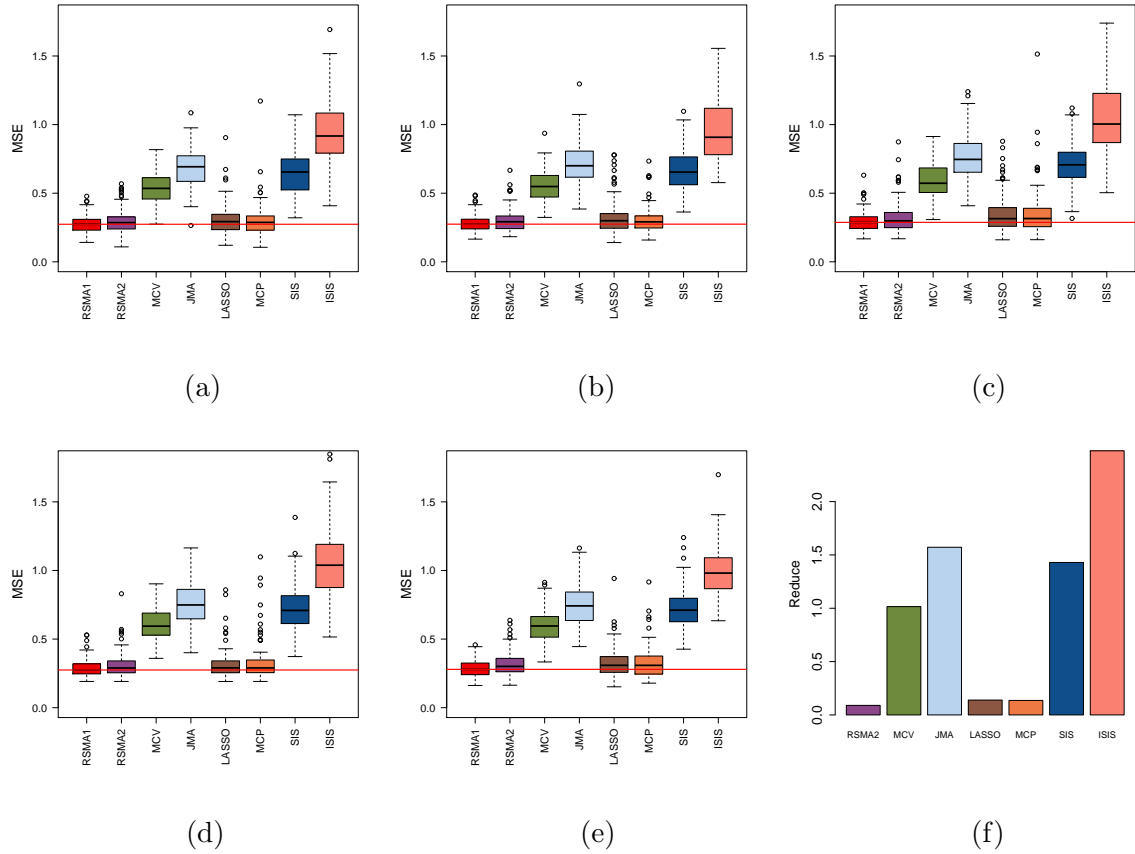


Figure 2: MSE for Example 2 with  $\rho = 0$  and  $p = 500$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

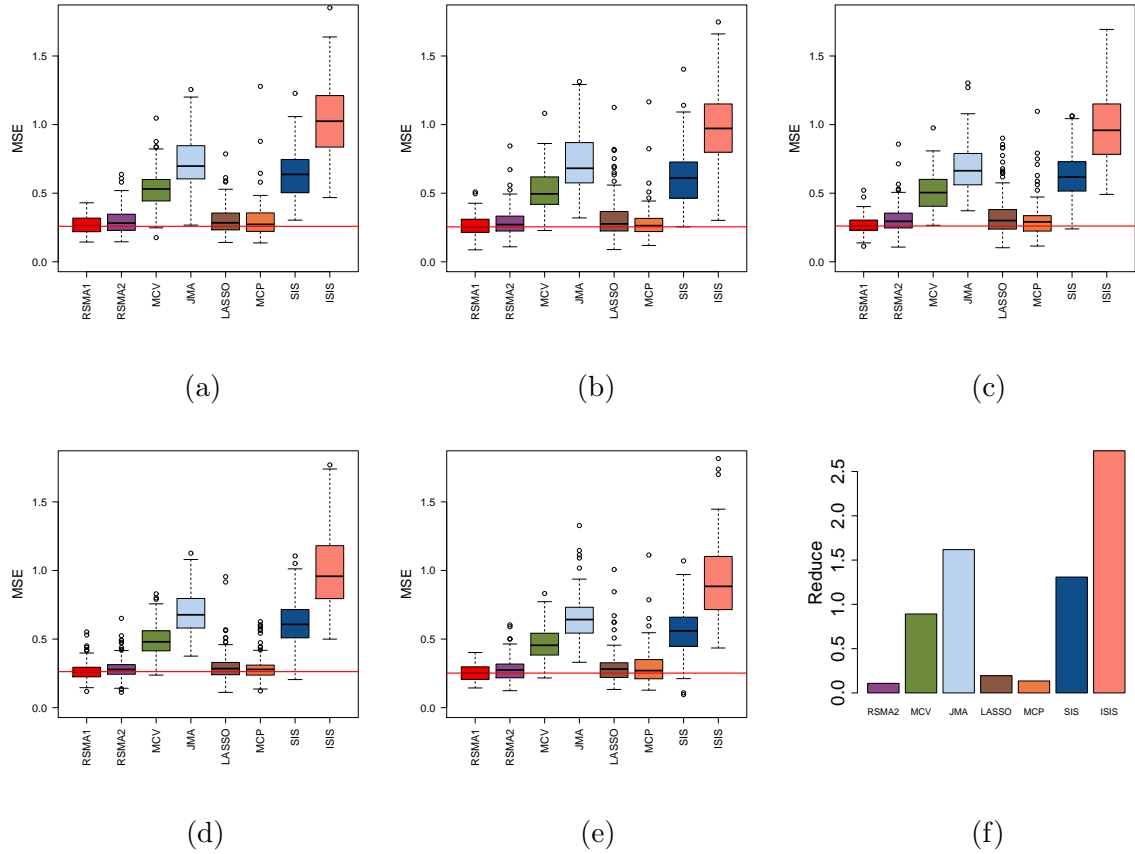


Figure 3: MSE for Example 2 with  $\rho = 0.7$  and  $p = 500$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

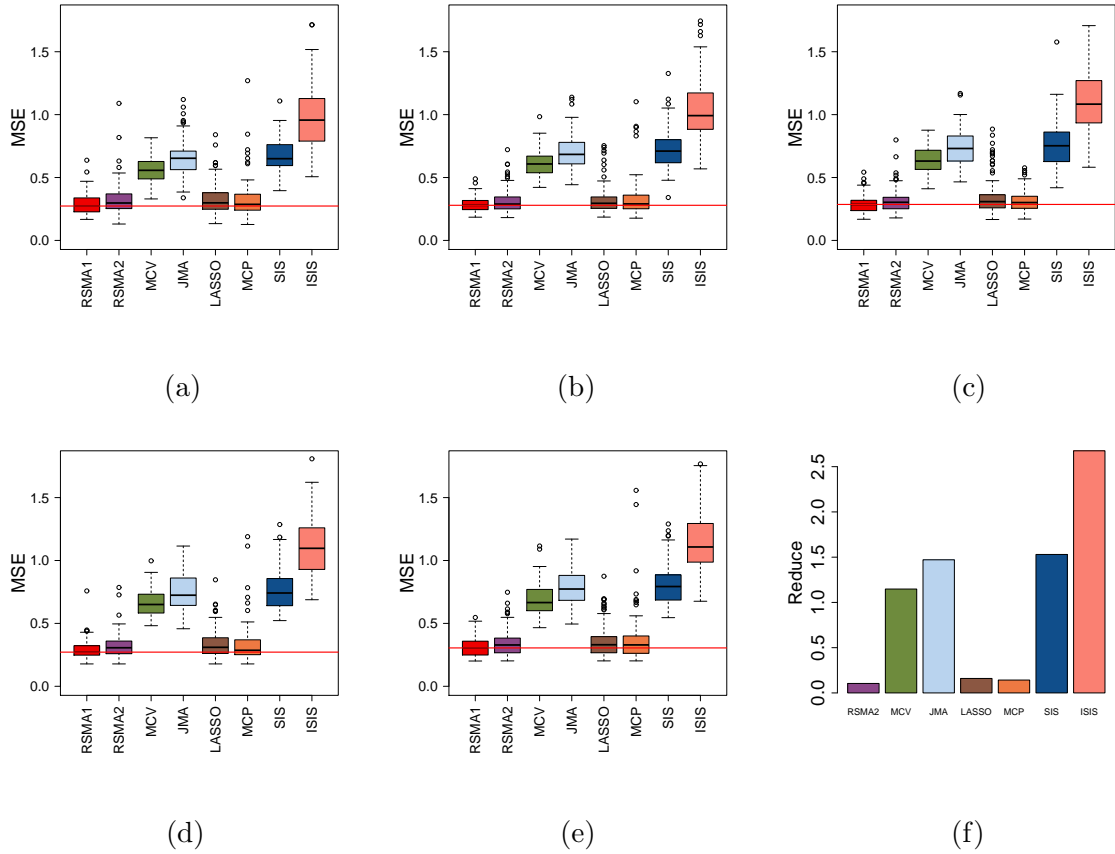


Figure 4: MSE for Example 2 with  $\rho = -0.7$  and  $p = 2000$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

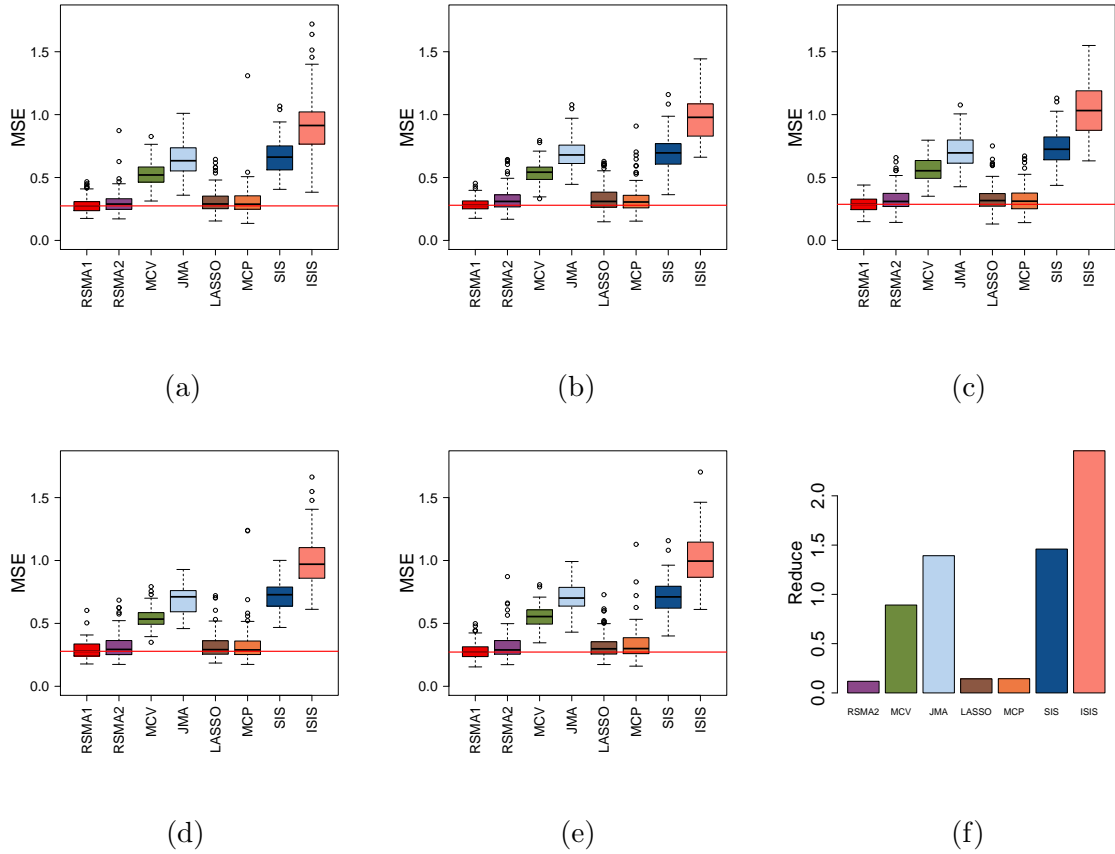


Figure 5: MSE for Example 2 with  $\rho = 0$  and  $p = 2000$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

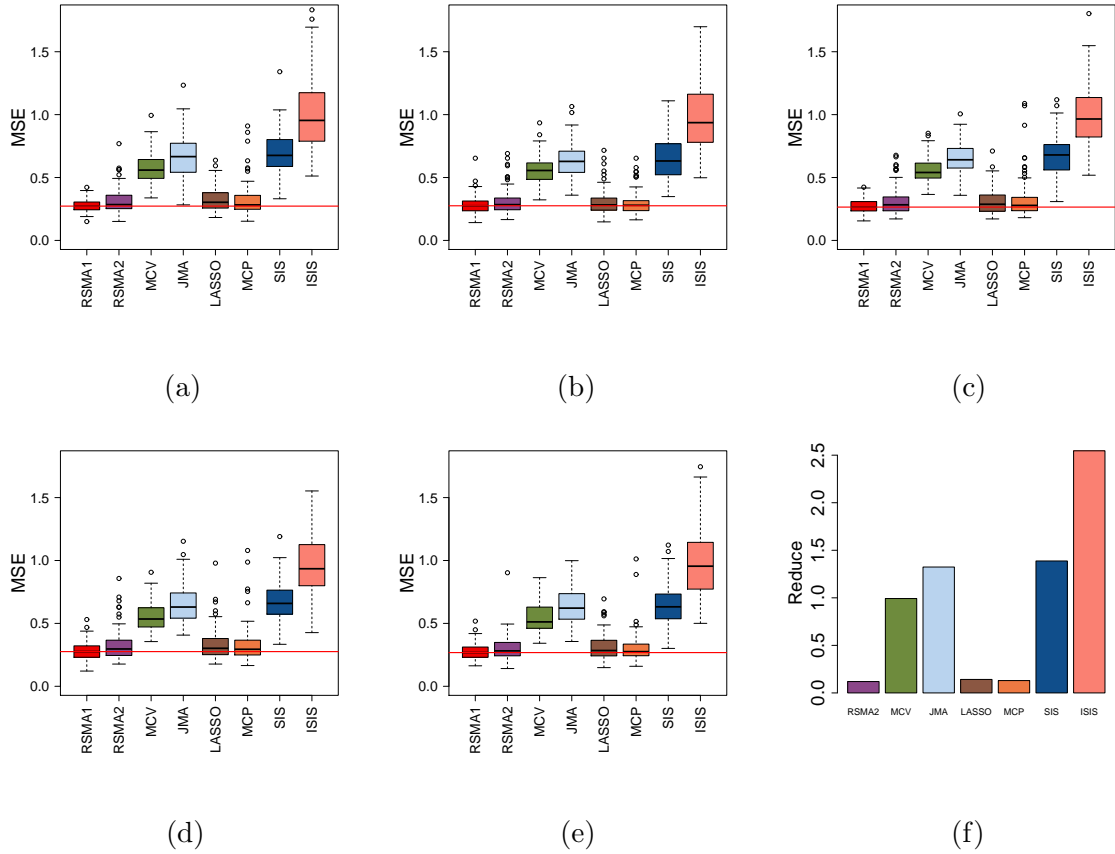


Figure 6: MSE for Example 2 with  $\rho = 0.7$  and  $p = 2000$ . Subfigures (a)-(e) are results at  $h$  from 1 to 5. The red horizontal lines represent the median values of MSE for RSMA1. Subfigure (f) shows the average of relative MSE reduction of RSMA1 compared with other methods considered.

Table 1: Simulation results for Example 4,  $p = 500, 1000$ . Scenario A: the error term has a  $t$  distribution with 3 degrees of freedom; Scenario B: the error term has a normal distribution  $N(0, \sigma_i^2)$  with  $\sigma_i = x_{1i}^2$ .  $10\times$  standard errors are reported in parentheses. The smallest MSEs and standard errors are marked in bold.

	$p$	RSMA1	RSMA2	MCV	JMA	LASSO	MCP	SIS	ISIS
Scenario A	500	0.84	<b>0.78</b>	1.03	2.23	1.10	1.37	1.00	1.94
		(0.57)	<b>(0.53)</b>	(0.69)	(1.37)	(0.67)	(0.78)	(1.00)	(1.20)
	1000	0.92	<b>0.81</b>	1.59	2.29	1.24	1.40	1.13	1.88
		(0.59)	<b>(0.57)</b>	(0.97)	(1.54)	(0.74)	(0.91)	(1.02)	(1.20)
Scenario B	500	0.77	<b>0.71</b>	0.98	2.09	1.04	1.21	0.88	1.67
		(0.42)	<b>(0.42)</b>	(0.51)	(0.69)	(0.61)	(0.68)	(0.69)	(0.93)
	1000	0.91	<b>0.85</b>	1.52	2.25	1.27	1.56	1.12	1.90
		<b>(0.42)</b>	(0.44)	(0.51)	(1.06)	(0.61)	(0.68)	(0.69)	(0.93)

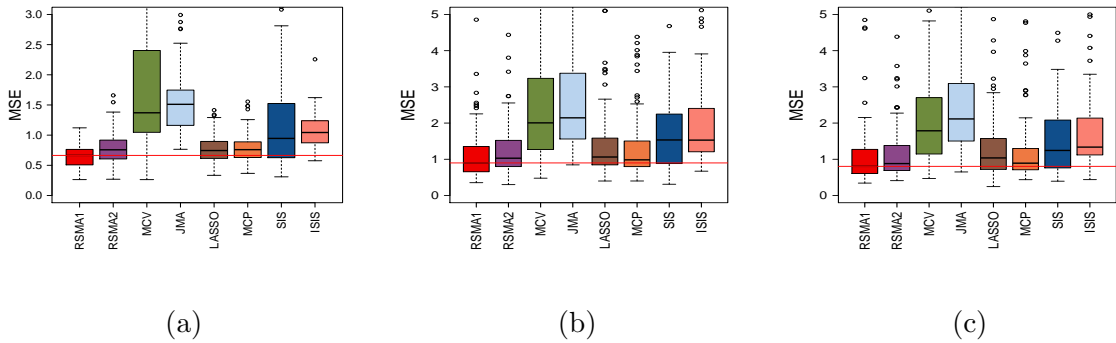


Figure 7: Simulation results for Example 5 with three distributions: (a) normal distribution  $N(0, 1.5)$ ; (b)  $t$  distribution with 3 degrees of freedom; (c) mixture distribution of normal and  $t$  distribution with degrees of freedom 2.



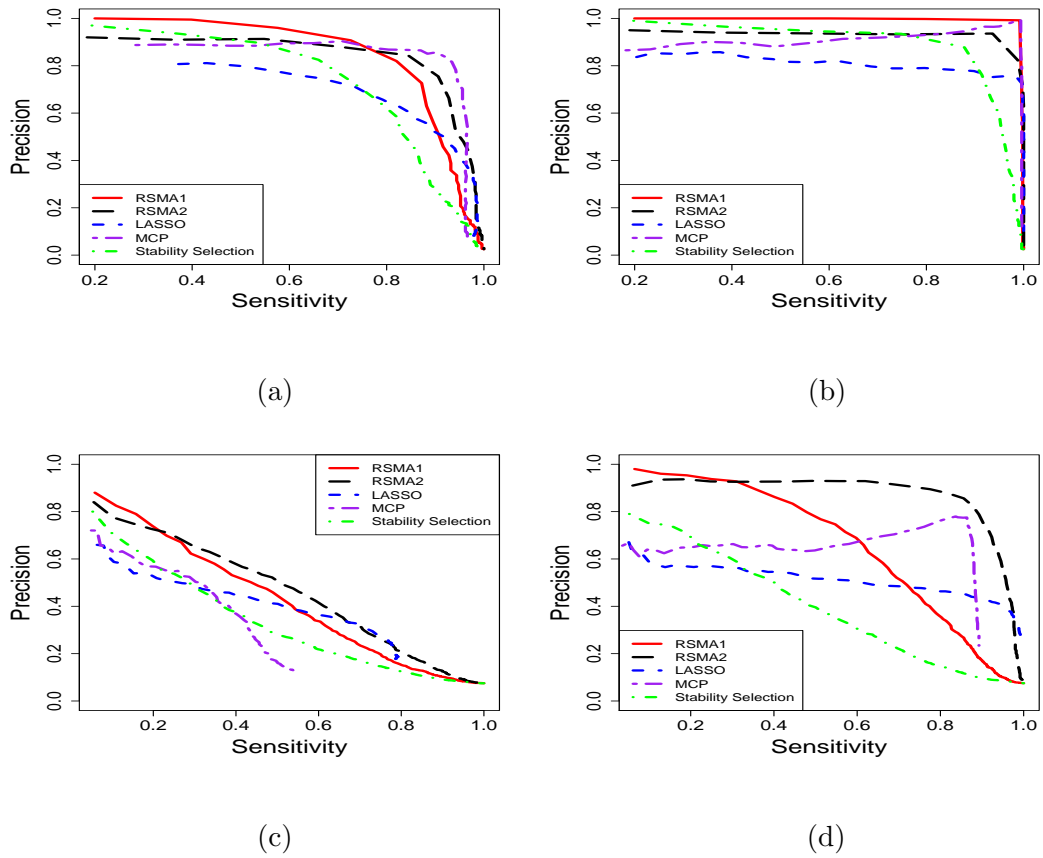
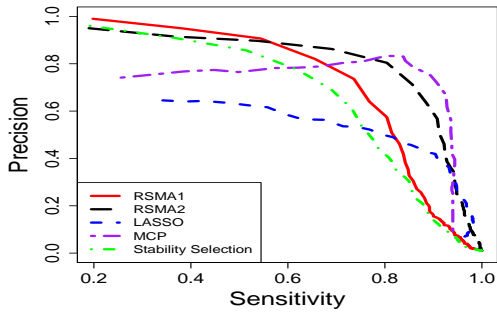
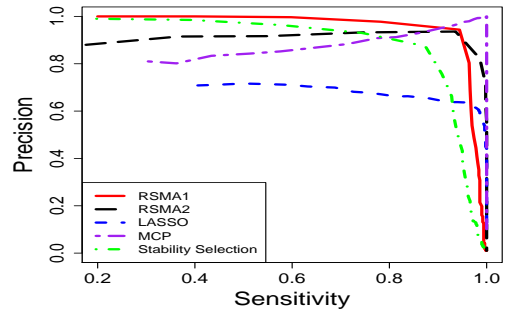


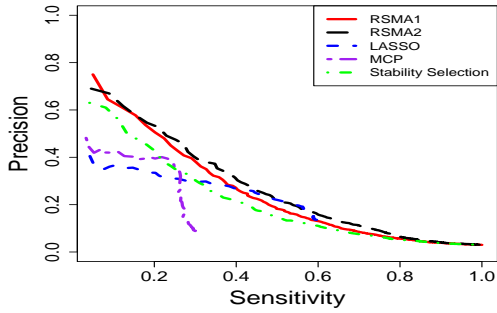
Figure 8: The precision-sensitivity curves comparing the ranking ability of all methods on the semi-real dataset for Leukaemia data with  $p = 200$  and error term following normal distribution. (a) SNR=2 and  $s=5$ ; (b) SNR=8 and  $s=5$ ; (c) SNR=2 and  $s=15$ ; (d) SNR=8 and  $s=15$ .



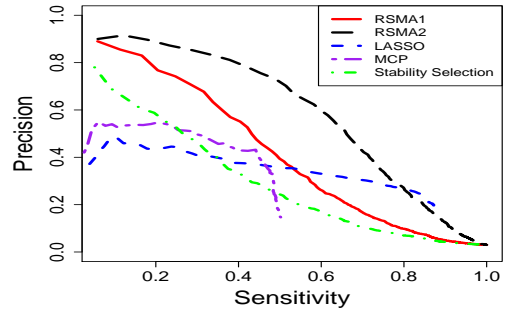
(a)



(b)



(c)



(d)

Figure 9: The precision-sensitivity curves comparing the ranking ability of all methods on the semi-real dataset for Leukaemia data with  $p = 500$  and error term following normal distribution. (a) SNR=2 and  $s=5$ ; (b) SNR=8 and  $s=5$ ; (c) SNR=2 and  $s=15$ ; (d) SNR=8 and  $s=15$ .

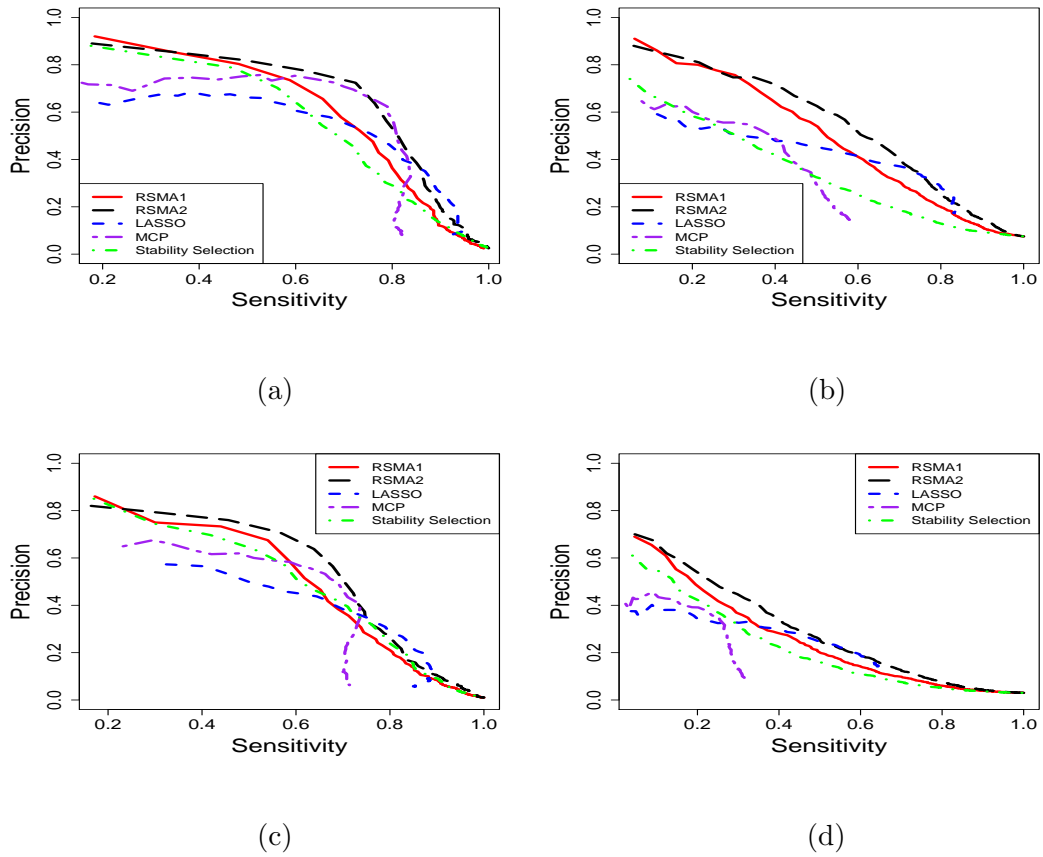


Figure 10: The precision-sensitivity curves comparing the ranking ability of all methods on the semi-real dataset for Leukaemia data with  $p = 200, 500$  and error term following  $t$  distribution with degrees of freedom 3. (a)  $p=200$  and  $s=5$ ; (b)  $p=200$  and  $s=15$ ; (c)  $p=500$  and  $s=5$ ; (d)  $p=500$  and  $s=15$ .