Supplementary Information

Derivation and implementation of the multiasperity adhesive contact model

1 Derivation of the model

We consider the indentation of a nominally flat elastic plane decorated with a distribution of spherical caps by a rigid, smooth and spherical probe. The rough surface is described by a collection of N spherical asperities having all the same radius of curvature R and prescribed positions (x_i, y_i, z_i). Locally, the indentation of each asperity is assumed to obey JKR's theory. Elastic interactions between micro-contacts are accounted for by introducing a shift of the position of the deformable surface seen by each asperity due to the deflection caused by the neighboring ones. Accordingly, the actual indentation δ_i of the i^{th} asperity is thus given by

$$\delta_i = \delta_i^0 + \sum_{j \neq i}^{N} \alpha_{ij}(\delta_j) \tag{1}$$

where $\delta_i^0 > 0$ is the indentation depth in the absence of any elastic coupling between micro-contacts. As shown in Fig. 1, δ_i^0 is a purely geometrical term simply given by the difference between the position of the two undeformed surfaces for the prescribed indentation depth Δ. The sum in the rhs of Eq. 1 represents the interaction term derived from JKR's theory. Instead of providing its exact expression, we derive α_{ij} from an asymptotic expansion that is based on the Green's tensor [1] for a point loading on an elastic incompressible half-plane. It writes

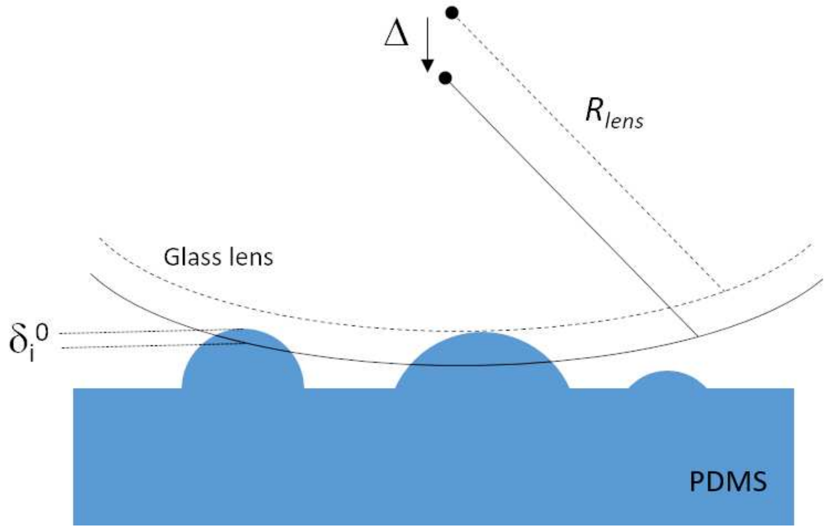

Figure 1: Sketch of the geometric configuration for the indentation of the patterned surface by a smooth rigid lens. Δ is the prescribed indentation depth taking as a reference for the vertical position of the indenting sphere the altitude at which the smooth surface is touching the uppermost asperity.

$$(\alpha_{ij}(\delta_j)) = -\frac{3}{4\pi E}\frac{1}{r_{ij}}F(\delta_j) \;\; ,i \neq j \tag{2}$$

where r_{ij} is the distance separating asperities i and j and where $F(\delta_j)$ is the normal indentation load of the j^{th} asperity as obtained from JKR's theory

$$F(\delta_j) = \frac{3RKa_j(\delta_j)\delta_j - Ka_j^3(\delta_j)}{2R} \tag{3}$$

where $K = 16/9E$ is an elastic constant and $a_j(\delta_j)$ is the contact radius of the j^{th} asperity. The latter can be determined by numerically inverting the explicit $\delta(a)$ JKR's relationship given by

$$\delta = \frac{a^2}{R} - \sqrt{\frac{8\pi a w}{3K}} \tag{4}$$

where w is the adhesion energy. From Eq 3, the interaction term $\alpha_{ij}(\delta_j)$ can be rewritten as

$$(\alpha_{ij}(\delta_j)) = -\frac{4}{3\pi}\frac{1}{r_{ij}}f(\delta_j) \ \ , i \neq j \tag{5}$$

with

$$f(\delta_j) = \frac{3Ra_j(\delta_j)\delta_j - a_j^3(\delta_j)}{2R} \tag{6}$$

This asymptotic expression of the interaction term is expected to be valid as long as the distance r_{ij} between two neighboring microcontacts is large with respect to their respective contact radii. From a comparison with the exact JKR expression [2] for vertical displacement of the free surface (Fig. 2), we found that the asymptotic expression provides a very accurate value of the surface displacement since $r_{ij} > 3a_{i,j}$, which is always verified experimentally.

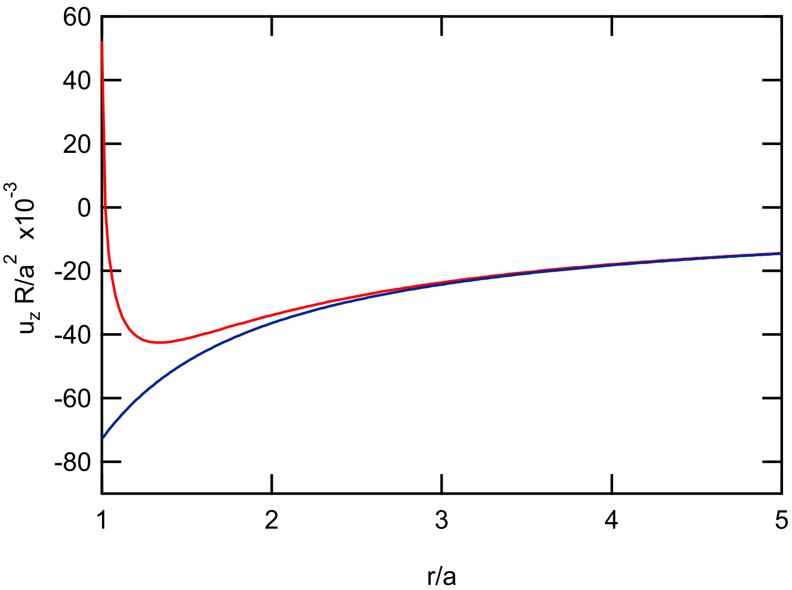

Figure 2: Normalized vertical displacement of the surface $u_z R/a^2$ calculated using the exact JKR relationship (red curve) and the asymptotic expression given in Eq. 1 (blue curve) for $P/(3\pi wR) = 0.5$ (R is the radius of the sphere, r is the radial coordinate and a is the contact radius).

2 Numerical implementation

Writing $V_i = z_i - d$, with d being the approach distance of the two surfaces at the location of the i^{th} asperity, the contact problem is the solution of the following set of non-linear equations

$$\mathbf{F}(\delta) = \delta - V - \mathbf{A}.\varphi(\delta) = 0 \tag{7}$$

with $\mathbf{A} = (\alpha_{ij}(\delta_j))$ and

$$\varphi(u) = f(u) \;\; u \geqslant 0 \;\; ; \;\; \varphi(u) = 0 \;\; u < 0 \tag{8}$$

This set of equations is solved using a Newton-Raphson algorithm as follows

$$\delta_{n+1} = \delta_n - \mathbf{J}_F^{-1}(\delta_n).\mathbf{F}(\delta_n) \tag{9}$$

where $\mathbf{J}_F$ is the Jacobian defined by

$$\mathbf{J}_F = \left(\frac{\partial F_i}{\partial \delta_k}\right) = \delta_{ik} - \mathbf{A}.\varphi'(\delta_k) \tag{10}$$

According to Eq. (10)

$$\mathbf{J}_F(\delta) = \mathbf{I} - \mathbf{Q} \tag{11}$$

with

$$\mathbf{Q} = \alpha_{ij}.\varphi'(\delta_k) \tag{12}$$

At each computation step, $\mathbf{J}_F^{-1}$ is calculated and numerically inverted in order to compute δ_{n+1} from δ_n. The calculation is repeated until $\sum |\delta_{n+1}$-$\delta_n|$ is lower than a prescribed convergency criterion.

References

[1] Landau, L.D. and Lifshitz, E.M., *Theory of elasticity* (Pergamon, 1986)

[2] Maugis, D., *Contact, adhesion and rupture of elastic solids* (Springer, Berlin, 1999)