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• **Supplementary File** •

Secure transmission for heterogeneous cellular network with limited feedback

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Appendix A Proof of Lemma 1:

According to the total probability theorem and the fact that$||\hat{h}_{io}||^2$ and $|\hat{h}_{iu}^H h_{\hat{l}}|^2$ are independent, the $||h_{io}^*||^2$ can be expressed as:

$$\begin{aligned}&\mathrm{P}(||h_{iu}^H||^2|\hat{h}_{iu}^H h_{\hat{l}}|^2 \leqslant x)\\ &\overset{(a)}{=} \int_0^\infty \mathrm{P}(|\hat{h}_{iu}^H h_{\hat{l}}|^2 \leqslant x/t|||h_{iu}^H||^2 = t)\mathrm{f}_{||h_{iu}^H||^2}(t)dt\\ &= \int_0^\infty \mathrm{F}_{|\hat{h}_{iu}^H h_{\hat{l}}|^2}(x/t)\mathrm{f}_{||h_{iu}^H||^2}(t)dt\end{aligned} \tag{A1}$$

Where (a) holds for the total probability theorem. Such we have the CDF of $|\hat{h}_{iu}^H h_{\hat{l}}|^2$ in (5), and $||\hat{h}_{io}||^2 \sim Gamma(N_i, 1)$. Due to piecewise function in(5), we forms the integral in (A1) into:

$$\begin{aligned}&\mathrm{P}(||h_{iu}^H||^2|\hat{h}_{iu}^H h_{\hat{l}}|^2 \leqslant x)\\ &= \int_x^{x/(1-\varepsilon)} (1 - 2^B(1-x/t)^{N_i-1})\frac{t^{N_i-1}e^{-t}}{\Gamma(N_{d,i})}dt + \int_0^x \mathrm{f}_{||h_{iu}^H||^2}(t)dt\\ &\overset{(b)}{=} F_\gamma(N_i, \tfrac{x}{1-\varepsilon}) - 2^B e^{-x} F_\gamma(N_i, \tfrac{\varepsilon x}{1-\varepsilon})\\ &= \sum_{m=0}^{N_i-1} \frac{(\frac{x}{1-\varepsilon})^m e^{-\frac{x}{1-\varepsilon}}(2^B\varepsilon^m - 1)}{\Gamma(m+1)} + 1 - 2^{B_i}e^{-x}\end{aligned} \tag{A2}$$

Where (b) comes from the definition of the CDF of gamma distribution. And $F_\gamma(N_i, x)$ represents the CDF of Gamma distribution with parameter N_i, which is given by $F_\gamma(N_i, x) = 1 - \sum_{m=0}^{N_i-1} \frac{x^m e^{-x}}{m!}$. We noted that the property of x and B_i can be analyzed easier under fixed N_i because of the characteristic of Gamma distribution.

Appendix B Proof of Lemma 2:

We bring the CDF of $||h_{io}^*||^2$ into (10), and the expression contains the power of three added random variables, which is hard to solve. First, we pull back the influence of estimation error into $I_{err,average} = P_i|X_{iu}|^{-\alpha_i}\sigma_m^2$ and get the lower bound of coverage probability[36]. We find that the formula contains power of two added random variables. Therefore, (12) transforms into:

$$\begin{aligned}&p_c^i \geqslant E_{I_{ui}}(\sum_{m=0}^{N-1} \frac{(\frac{\lambda}{1-\varepsilon})^m e^{-\frac{\lambda}{1-\varepsilon}}(2^B\varepsilon^m - 1)}{\Gamma(m+1)} + 1 - 2^B e^{-\lambda})\\ &= \sum_{m=0}^{N-1} \frac{E_{I_{ui}}((\frac{\lambda}{1-\varepsilon})^m e^{-\frac{\lambda}{1-\varepsilon}})(2^B\varepsilon^m - 1)}{\Gamma(m+1)} + 1 - 2^B E_{I_{ui}}(e^{-\lambda})\\ &\overset{(a)}{=} \sum_{m=0}^{N-1}\sum_{p=0}^{m} \begin{pmatrix} m \\ p \end{pmatrix} \frac{(-1)^m L_{I_{ui}}^{(m)}(\frac{K}{1-\varepsilon})(\frac{K\sigma_1^2}{1-\varepsilon})^{m-p} e^{-\frac{K\sigma_1^2}{1-\varepsilon}}(2^B\varepsilon^m - 1)}{\Gamma(m+1)} + 1 - 2^B e^{-K\sigma_1^2} L_{I_{ui}}(K)\end{aligned} \tag{B1}$$

Which $\sigma_1^2 = \sigma_u^2 + P_i|X_{iu}|^{-\alpha_i}\sigma_m^2$. The process (a) is obtained by[38] with p-order derivative of Laplace transform of $\mathcal{L}_{I_o}(s)$. The Laplace transform is given by:

$$\begin{aligned}&\mathcal{L}_{I_o}(s) \overset{(b)}{=} \prod_{j=1}^{K} E_{\theta_j^o}[\exp(-K\sum_{j\in\theta_b, j\neq i} P_j||h_{ju}||^2|x_j|^{-\alpha_j})]\\ &\overset{(c)}{=} \prod_{j=1}^{K} \exp(-2\pi\lambda_j \int_x^\infty (1 - \varpi(P_j))rdr)\end{aligned} \tag{B2}$$

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Which $\varpi(P_j)=\int_0^\infty e^{-Kpr^{-\alpha_j}} f_{P_j\|h_{uj}\|^2}(p)dp$ with $P_j\|h_{uj}\|^2 \sim Gamma(N_j, P_j)$. And (b) is obtained by the definition of Laplace transform, and (c) is using the Probability Generating functional (PGFL) of Stochastic Geometric theory, expressed as:

$$G[f(x)] = E\left(\prod_{x\in\Phi} f(x)\right) = \exp\left(-\lambda\int_{R^2}(1-f(x))dx\right) \tag{B3}$$

Applying [28, 3.191], $\varpi(P_j)$ can simplified into:

$$\begin{aligned}&\varpi(P_j)=\int_0^\infty \frac{1}{\Gamma(N_j){P_j}^{N_j}} e^{-Kpr^{-\alpha_j}} p^{N_j-1}e^{-p/P_j}dp\\&={P_j}^{-N_j}(\tfrac{1}{P_j}+Kr^{-\alpha_j})^{-N_j}=(1+KP_jr^{-\alpha_j})^{-N_j}\end{aligned} \tag{B4}$$

And using [28, 3.194], the Laplace transform is calculated as:

$$\mathcal{L}_{I_o}(s)=\mathrm{E}_{I_o}\left[e^{-sI_o}\right]=\prod_{j\in\mathcal{K}}\mathcal{L}_{I_{jo}}(s) \tag{B5}$$

So, the Laplace transform is obtained by (B2) and (B4), usingsimplified as (15). with the help of the[21], we can further obtain the expression of p-order Laplace transform as:

$$\begin{aligned}&L_{I_{ui}}^{(p)}(s)=\sum_{j=1}^{k}\pi\lambda_j\sum_{z=0}^{p}L_{I_{ui}}^{(z)}(s)(p-z)!\int_x^\infty\frac{(P_jr^{-\alpha_j})^{p-z}}{(1+sP_jr^{-\alpha_j})^{N_j+p-z}}dr\\&=\sum_{j=1}^{k}\pi\lambda_j\sum_{z=0}^{p}L_{I_{ui}}^{(z)}(s)\int_{P_jx^{-\alpha_j}}^\infty\frac{(p-z)!(y)^{p-z+1/\alpha_j+1}}{(P_j)^{-1/\alpha_j}(1+sy)^{N_j+p-z}}dr\\&=\sum_{z=0}^{p}\sum_{j=1}^{k}\pi\lambda_jL_{I_{ui}}^{(z)}(s)\frac{(p-z)!(P_jx^{-\alpha_j})^{-N_j-1/\alpha_j}}{(P_j)^{-1/\alpha_j}s^{N_j+p-z}(N_j+1/\alpha_j)}{}_2F_1(N_j+p-z,N_j+1/\alpha_j;N_j+1/\alpha_j+1;-\frac{1}{sP_jx^{-\alpha_j}})\end{aligned} \tag{B6}$$

Appendix C proof of lemma3

As (18) is obtained, using the PGFL over PPP in (a) and the Jensen inequality in (b), the upper bound is satisfied:

$$\begin{aligned}&E_{r,I_{ei}}(1-\prod_{e\in\theta_e}P(|\hat{h}_{ie}^Hh_{\hat{l}}|^2<\frac{(2^{R_e}-1)(I_{ei}+\sigma_e^2)}{P_i|Y_{ie}|^{-\alpha_i}}))\\&\overset{(a)}{=}1-\exp(-\pi\lambda_e\int_0^\infty 1-P(|\hat{h}_{ie}^Hh_{\hat{l}}|^2<\frac{(2^{R_e}-1)(I_{ei}+\sigma_e^2)}{P_i|Y_{ie}|^{-\alpha_i}})dy)\\&\overset{(b)}{\geqslant}1-\exp(-\pi\lambda_e\int_0^\infty E_{I_{ei}}(\exp(-\frac{(2^{R_e}-1)(I_{ei}+\sigma_e^2)}{P_i|Y_{ie}|^{-\alpha_i}}))dy)\\&=1-\exp(-\pi\lambda_e\int_0^\infty(\exp(-\frac{(2^{R_e}-1)\sigma_e^2}{P_i|Y_{ie}|^{-\alpha_i}})L_{I_{ei}}(\frac{(2^{R_e}-1)}{P_i|Y_{ie}|^{-\alpha_i}}))dy)\end{aligned} \tag{C1}$$

Similarly, we can further get the expression of $\mathcal{L}_{I_{ei}}$ as:

$$\begin{aligned}&L_{I_{ei}}(s)=\exp(-\pi\sum_{j=1}^{K}\lambda_j\int_0^\infty r(1-(1+sP_jr^{-\alpha_j})^{-N_j})dr)\\&=\exp(-\pi\sum_{j=1}^{K}\lambda_j1/\alpha_j(sP_j)^{2/\alpha_j}\sum_{k=1}^{N_j}B(1-2/\alpha_j,k+2/\alpha_j-1)\end{aligned} \tag{C2}$$

As for the lower bound described above, we can show the expression with the help of $P(|\hat{h}_{ie}^Hh_{\hat{l}}|^2<\frac{(2^{R_e}-1)(I_{ei}+\sigma_e^2)}{P_i|Y_{ie}|^{-\alpha_i}})$ calculated above:

$$\begin{aligned}&p_s^i\leqslant\int_0^\infty P(\log_2(1+SNR_e)>R_e)f_e(r)dr\\&=\int_0^\infty 2\pi\lambda\exp(-\frac{(2^{R_e}-1)\sigma_e^2}{P_i|r|^{-\alpha_i}}-\pi\sum_{j=1}^{K}\lambda_j1/\alpha_j(k_2P_j)^{2/\alpha_j}\sum_{k=1}^{N_j}B(1-2/\alpha_j,k+2/\alpha_j-1))-\pi\lambda_er^2)dr\end{aligned} \tag{C3}$$