SCIENCE CHINA
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• **Supplementary File** •

Design of UCA Structure with Maximum Capacity for mmWave LOS MIMO Systems

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Appendix A Derivation of $d'_{m,n}$

From Figure A1, $d'_{m,n}$ can be formulated as

$$d'_{m,n} = \sqrt{\left(\frac{d_r}{2}\right)^2 + \left(\frac{d_t}{2}\right)^2 + \frac{d_r d_t}{2}\cos\Delta\theta_{m,n}}, \tag{A1}$$

$$\Delta\theta_{m,n} = \theta_{t,n} - \theta_{r,m}, \tag{A2}$$

$$\theta_{t,n} = \theta_{t,0} + \frac{2\pi}{N}n, \tag{A3}$$

$$\theta_{r,m} = \theta_{r,0} + \frac{2\pi}{M}m, \tag{A4}$$

$$\theta_{t,0} = \delta_t, \tag{A5}$$

$$\theta_{r,0} = \delta_r, \tag{A6}$$

where $\theta_{t,n}$ and $\theta_{r,m}$ respectively represent the angle between the nth antenna at the *transmitter* (Tx) and the mth antenna at the *receiver* (Rx) and the positive x-axis direction, and $\Delta\theta_{m,n}$ is the angle between $\theta_{r,m}$ and $\theta_{t,n}$.

If we let $\delta_t = \delta_r = 0$, $d'_{m,n}$ can be expressed as

$$d'_{m,n} = \sqrt{\left(\frac{d_r}{2}\right)^2 + \left(\frac{d_t}{2}\right)^2 + \frac{d_r d_t}{2}\cos\left(\frac{2\pi n}{N} - \frac{2\pi m}{M}\right)}, \tag{A7}$$

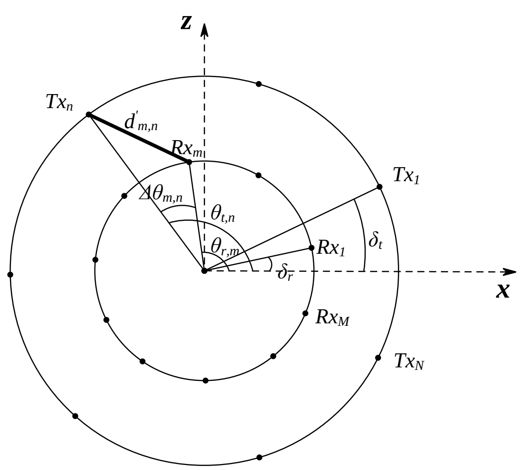

Figure A1 Antenna spacing of UCA after panning

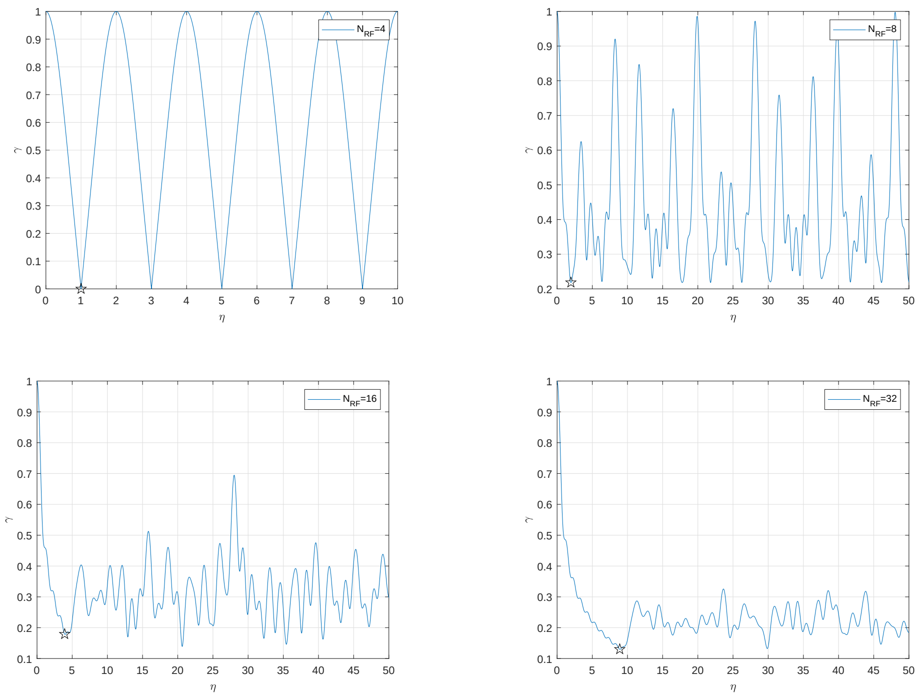

Figure B1 The curve of γ versus η

Appendix B Simulation proof of the relationship between γ and η

In order to test this hypothesis, we conduct a simulation, and the results are shown in Figure B1. We set the carrier frequency to 75GHz, the distance L between the Tx and the Rx to be 100m. The four graphs in Figure B1 are simulation results under the different conditions, i.e., the number of *radio frequency* (RF) chains is 4, 8, 16 and 32. The simulation results show that γ has a trend of decreasing first and then fluctuating, and we choose the pentagon in the Figure B1, as the approximate optimal solution of η, denoted by η^{opt}.

The approximate optimal solution has the following two advantages.

- If this pentagon is moved to the left, the average correlation coefficient will increase, thereby reducing the channel capacity.
- If this pentagon moves to the right, there is not necessarily a value with a smaller average correlation coefficient. Even if there is a value with a smaller average correlation coefficient, the diameter will increase a lot, thereby increasing the size of the *uniform circular array* (UCA).

Appendix C Statistical results of η^{opt}

We draw the curve of γ versus η and obtain η^{opt} when N_{RF} is equal to 2, 4, ..., 64. The statistical results are shown in Table C1. Fit the data in Table C1, and the result of the linear fitting is shown in Figure C1.

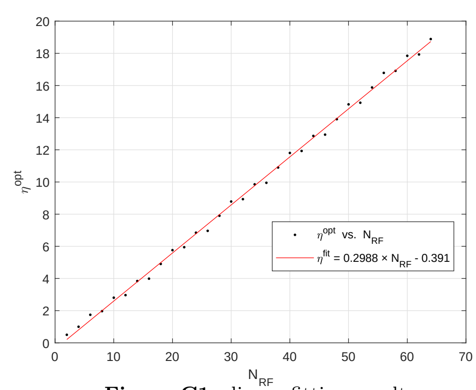

Figure C1 linear fitting result

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Table C1 Correspondence between η^{opt} and N_{RF}

N_{RF}	η^{opt}	N_{RF}	η^{opt}	N_{RF}	η^{opt}
2	0.5	24	6.8482	46	12.9448
4	1	26	6.9626	48	13.9044
6	1.7437	28	7.9004	50	14.8270
8	1.9696	30	8.7854	52	14.9282
10	2.8060	32	8.9322	54	15.8769
12	2.9655	34	9.8544	56	16.7834
14	3.8423	36	9.9506	58	16.9091
16	3.9889	38	10.9002	60	17.8461
18	4.9050	40	11.8055	62	17.9286
20	5.7607	42	11.9289	64	18.8873
22	5.9450	44	12.8654		

Appendix D Proof of the linear relationship between η^{opt} and N_{RF}

Because when $\eta = \eta^{opt}$, the sub-channel correlation is the approximately minimum. Therefore, exploring the simplified algorithm to solve η^{opt} should also be based on the expression of sub-channel correlation, which is (D1).

$$\mathbf{h}_{n_1}^H \mathbf{h}_{n_2} = \sum_{m=1}^{M} \cos\left(\pi\eta \sin\frac{(n_2-n_1)\pi}{N} \sin\left(\frac{2\pi m}{M} - \frac{(n_1+n_2)\pi}{N}\right)\right), \tag{D1}$$

Each cumulative term in (D1) has the similar form

$$\cos\left(\pi\eta \sin\frac{(n-1)\pi}{N_{RF}} \sin\frac{(2m-n-1)\pi}{N_{RF}}\right). \tag{D2}$$

And we can get the first zero point of each cumulative term

$$\eta\left(n, m, N_{RF}\right) = \frac{1}{2\sin\frac{(n-1)\pi}{N_{RF}} \sin\frac{(2m-n-1)\pi}{N_{RF}}}. \tag{D3}$$

Due to $m \in [1, N_{RF}]$, $\sin\frac{(2m-n-1)\pi}{N_{RF}}$ is a variable with a value from 0 to 1. We can replace it with a random variable β_m. The probability distribution function of β_m is

$$f_r\left(\beta_m\right) = \begin{cases} \frac{2}{\pi\sqrt{1-\beta_m^2}} & 0 < \beta_m < 1 \\ 0 & \text{others} \end{cases}. \tag{D4}$$

Therefore, η^{opt} can be expressed as

$$\begin{aligned} \eta^{opt} &= \frac{1}{2\sin\frac{\pi}{N_{RF}} \sin\frac{(2m-3)\pi}{N_{RF}}} + \Delta \\ &= \frac{1}{2\sin\frac{\pi}{N_{RF}}\beta_m} + \Delta = \frac{N_{RF}}{2\pi\beta_m} + \Delta. \end{aligned} \tag{D5}$$

From (D5), η^{opt} will increase as N_{RF} increases. Especially when N_{RF} is large, η^{opt} and N_{RF} are roughly linear.

Appendix E Simulation details

In this part, we provide some simulation details for Figure 1(a)(b)(c) in the letter. For the *spectral efficiency* (SE) of the Rayleigh fading channel, we generate the channel matrix 1000 times to calculate the average of their SE by (E1). Figure 1(a) shows the curve of SE versus *Signal to noise ratio* (SNR).

$$\mathcal{R}\left(\mathbf{H}\right) = \mathbb{E}\left(\log_2\left(\left|\mathbf{I}_{N_{RF}} + \frac{\rho}{\sigma^2 N_{RF}}\mathbf{H}^H\mathbf{H}\right|\right)\right). \tag{E1}$$

In the *line-of-sight* (LOS) environment, we can ignore the multipath effect, so the simulation conditions will be simple. We adopt a 8×8 baseband system, apply *quadrature phase shift keying* (QPSK) modulation, and utilize the *minimum mean square error* (MMSE) criterion for decoding at the Rx. We use the *multiple input multiple output* (MIMO) transmission scheme of space division multiplexing. Each RF chain transmits 10^6 bits of data and counts the *bit error rate* (BER) under different SNR. The simulation result is shown in Figure 1(b).

We first calculate the singular values of $\mathbf{H}^H\mathbf{H}$ and sort them in ascending order, then normalize their accumulated value, and finally draw the *cumulative distribution function* (CDF) as shown in Figure 1(c).