SUPPLEMENTARY MATERIALS

Modified Bogoliubov-de Gennes Treatment for Majorana Conductances in Three-Terminal Transports

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This supplementary material provides more detailed considerations on the Bogoliubov-de Gennes (BdG) treatment for the nonlocal transport through Majorana zero modes (MZMs). Specifically, we start our analysis in Sec. I with the toy model of a pair of MZMs coupled to two quantum dots (QDs), showing the coexistence and interplay of the both channels of teleportation and Andreev process (together with insight from a POVM measurement perspective). The existence of the teleportation channel differs from its vanishing picture in the BdG treatment. In Sec. II we present an analysis based on the single-electron wavefunction approach of quantum transport, which is actually a time-dependent version of the S matrix scattering theory. In the presence of Andreev process, we show that inserting the conventional BdG treatment in this wavefunction approach (for the *central device* state evolution) can recover, as a stationary limit, the usual steady-state result from either the S matrix scattering theory or the nonequilibrium Green's function technique. However, the wavefunction approach can show very clearly how the *redundancy* of the "negative" eigenenergy states is involved in the treatment and that, without loss of any physics, e.g., the essential Andreev process in this context, eliminating this redundancy (as done in our modified BdG treatment) would result in different predictions. In Sec. III, more conceptually using two models (say, the Kitaev and the BCS models), we further analyze the dynamical picture of the BdG treatment for the electron-hole excitation in a superconductor by assuming an electron injection from outside (reservoir or transport lead). We show that the condition required to support the dynamical picture is not reasonable. Finally, in Sec. IV of this supplementary material, we add a few technical particulars for solving the Majorana master equation (MME) and deriving the Majorana energy (ϵ_M) dependence of the teleportation-channel conductance.

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I. ANALYSIS OF A TOY MODEL

Let us consider first the simple toy model analyzed in Ref. [1], where the dynamics of charge transfer between two QDS mediated by a pair of MZMz (γ_1 and γ_2) was analytically solved, and the issue of Majorana teleportation was highlighted in particular.

A. Number-State Treatment and Teleportation

The model analyzed in Ref. [1] is described by the following low-energy effective Hamiltonian

$$H = i\frac{\epsilon_M}{2}\gamma_1\gamma_2 + \sum_{\alpha=1,2}\left[\epsilon_\alpha d_\alpha^\dagger d_\alpha + \lambda_\alpha(d_\alpha^\dagger - d_\alpha)\gamma_\alpha\right]. \quad (1)$$

Here γ_1 and γ_2 are the Majorana operators associated with the MZMs which are assumed in our work emerging at the ends of a quantum wire and have an interaction energy ϵ_M. $d_1(d_1^\dagger)$ and $d_2(d_2^\dagger)$ are the annihilation (creation) operators of the two single-level quantum dots, while λ_1 and λ_2 are their coupling amplitudes to the MZMs. The Majorana operators are related to the regular complex fermion operators through the transformation of $\gamma_1 = f + f^\dagger$ and $\gamma_2 = i(f - f^\dagger)$. We then reexpress Eq. (1) as

$$\begin{aligned} H \;=\; & \epsilon_M(f^\dagger f - \frac{1}{2}) + \sum_{\alpha=1,2}[\epsilon_\alpha d_\alpha^\dagger d_\alpha + \lambda_\alpha(d_\alpha^\dagger f + f^\dagger d_\alpha)] \\ & -\lambda_1(d_1^\dagger f^\dagger + f d_1) + \lambda_2(d_2^\dagger f^\dagger + f d_2)\,. \end{aligned} \quad (2)$$

In terms of the occupation-number-state representation $|n_1, n_f, n_2\rangle$, totally we have eight basis states, i.e., with the electron numbers of the left dot, central quasiparticle, and the right dot (n_1, n_f and n_2) being "0" or "1", respectively. To highlight the most challenging issue of nonlocality, let us consider the case of $\epsilon_M = 0$. Starting with the state $|100\rangle$, the quantum evolution will be restricted within the odd-parity subspace expanded by $\{|100\rangle, |010\rangle, |001\rangle, |111\rangle\}$. It is straightforward to obtain the occupation probabilities of the left and right dots, respectively, as

$$
\begin{aligned}
P_1(t) &= P_{100}(t) + P_{111}(t) \\
&= 1 - \frac{4\lambda_1^2}{\epsilon_1^2 + 4\lambda_1^2} \sin^2\left(\frac{\sqrt{\epsilon_1^2 + 4\lambda_1^2}}{2} t\right), \\
P_2(t) &= P_{001}(t) + P_{111}(t) \\
&= \frac{4\lambda_2^2}{\epsilon_2^2 + 4\lambda_2^2} \sin^2\left(\frac{\sqrt{\epsilon_2^2 + 4\lambda_2^2}}{2} t\right). \qquad (3)
\end{aligned}
$$

Remarkably, we find that the occupation probability of the right (left) dot does not depend on the occupation of the left (right) dot and the coupling strength λ_1 (λ_2). The charge transfer dynamics with the initial state $|100\rangle$ involves a transition from the intermediate state $|010\rangle$ to $|001\rangle$, which is nothing but the teleportation channel. It also involves a transition from the initial state $|100\rangle$ to $|111\rangle$, via the local Andreev process on the right side. Both processes lead to occupation of the right dot with identical effect, which is also identical to the result of the state transition of $|n_f n_2\rangle$ between $|10\rangle$ and $|01\rangle$, by setting $\lambda_1 = 0$ which cuts the coupling to the left dot completely. This observation implies an important result that, by coupling the MZMs to transport leads via the QDs, the currents in the left and right leads should have no cross correlation at the limit $\epsilon_M \to 0$, since the disturbance of current measurement on one side will not affect the current on the other side, because of the reason explained above.

Indeed, the "QD–MZMs–QD" segment has been considered in Ref. [5] to be embedded into a transport setup, by attaching the two QDs to transport leads. Based on the BdG treatment incorporated in the S matrix scattering theory, it was found that the cross correlation of currents (in the two leads) is $\propto (e^2/h)\epsilon_M^2/\Gamma$ (where Γ is the coupling rate to the leads), which implies a vanished cross correlation when $\epsilon_M \to 0$. The result of the vanished cross correlation at the limit $\epsilon_M \to 0$ was also concluded in the transport setup without introducing the quantum dots, i.e., by attaching the transport leads directly to the MZMs [6–8]. From an analysis to the S matrix elements, we notice that the result of the vanished cross correlation of currents in these works is commonly owing to the zero matrix elements of the teleportation and CAR channles at the limit $\epsilon_M \to 0$. Below, we present a heuristic discussion in terms of the picture of disconnected MZMs or, equivalently, destructive interference between the 'positive' and 'negative' energy states. We may stress that this 'BdG picture' is quite different from the 'channel degeneracy' revealed above in the number-state treatment.

B. BdG-Type Consideration

The feature of vanishing cross correlation at the limit $\epsilon_M \to 0$, in the transport setup whether or not embedding the quantum dots, is commonly rooted in the same type of BdG treatment for the effective coupling mediated by the MZMs. We therefore illustrate the reason by considering the setup with the "QD–MZMs–QD" segment. Following Ref. [5], the BdG Hamiltonian matrix of the central segment is given by

$$
H = \begin{pmatrix}
0 & i\epsilon_M & \lambda_1 & 0 & -\lambda_1^* & 0 \\
-i\epsilon_M & 0 & 0 & \lambda_2 & 0 & -\lambda_2^* \\
\lambda_1^* & 0 & \epsilon_1 & 0 & 0 & 0 \\
0 & \lambda_2^* & 0 & \epsilon_2 & 0 & 0 \\
-\lambda_1 & 0 & 0 & 0 & -\epsilon_1 & 0 \\
0 & -\lambda_2 & 0 & 0 & 0 & -\epsilon_2
\end{pmatrix}. \qquad (4)
$$

This corresponds to a use of the state basis $\{|\Phi_1\rangle, |\Phi_2\rangle, |e_1\rangle, |e_2\rangle, |h_1\rangle, |h_2\rangle\}$ for the MZMs and the two QDs (QD1 and QD2). In terms of the projection operators, the Hamiltonians of the MZMs and their coupling to the QDs are reexpressed, respectively, as

$$
\begin{aligned}
H_M &= i\epsilon_M|\Phi_1\rangle\langle\Phi_2| - i\epsilon_M|\Phi_2\rangle\langle\Phi_1|, \\
H_D &= \big(\epsilon_1|e_1\rangle\langle e_1| - \epsilon_1|h_1\rangle\langle h_1|\big) \\
&\quad + \big(\epsilon_2|e_2\rangle\langle e_2| - \epsilon_2|h_2\rangle\langle h_2|\big), \\
H_1' &= \big(\lambda_1|\Phi_1\rangle\langle e_1| - \lambda_1^*|\Phi_1\rangle\langle h_1|\big) + \text{h.c.}, \\
H_2' &= \big(\lambda_2|\Phi_2\rangle\langle e_2| - \lambda_2^*|\Phi_2\rangle\langle h_2|\big) + \text{h.c.}. \qquad (5)
\end{aligned}
$$

Qualitatively, electron transmission between the QDs via the MZMs can be described by the T matrix in quantum scattering theory, which encodes the key process characterized by the second-order expansion of the tunnel coupling Hamiltonian. For instance, let us consider the effective coupling of the electron block (in terms of the BdG description). We have

$$
\begin{aligned}
\langle e_2|T^{(2)}(\omega)|e_1\rangle &= \langle e_2|H_2' G_0(\omega) H_1'|e_1\rangle \\
&= \lambda_2^*\lambda_1\langle\Phi_2|G_0(\omega)|\Phi_1\rangle, \qquad (6)
\end{aligned}
$$

where the *free* Green's function of the MZMs reads $G_0(\omega) = (\omega - H_M)^{-1}$. One can easily check that, as $\epsilon_M \to 0$, $\langle\Phi_2|G_0(\omega)|\Phi_1\rangle = 0$. This becomes extremely clear in the time domain, say, $\langle\Phi_2|U(\tau)|\Phi_1\rangle = 0$ when $\epsilon_M \to 0$ by noting that the propagator (evolution operator) reads $U(\tau) = e^{-iH_M\tau}$, which is the counterpart of $G_0(\omega)$ in the time domain.

For the convenience of latter discussion, it is also instructive to convert the above description into using the eigenstate basis

$$
\begin{aligned}
|E_0\rangle &= (|\Phi_1\rangle - i|\Phi_2\rangle)/2, \\
|-E_0\rangle &= (|\Phi_1\rangle + i|\Phi_2\rangle)/2. \qquad (7)
\end{aligned}
$$

Here we use $|E_0\rangle$ and $|-E_0\rangle$ to denote the *positive* and *negative* energy states, while keeping in mind that $E_0 = \epsilon_M$. In this eigenstate basis, the Hamiltonian of the MZMs can be expressed as $H_M = E_0|E_0\rangle\langle E_0| + (-E_0)|-E_0\rangle\langle -E_0|$, while the tunnel couplings are described by

$$
\begin{aligned}
H_1' = &\big(\lambda_1|E_0\rangle\langle e_1| + \lambda_1|-E_0\rangle\langle e_1| \\
&- \lambda_1^*|E_0\rangle\langle h_1| - \lambda_1^*|-E_0\rangle\langle h_1|\big) + \text{h.c.}, \\
H_2' = i&\big(\lambda_2|E_0\rangle\langle e_2| - \lambda_2|-E_0\rangle\langle e_2| \\
&- \lambda_2^*|E_0\rangle\langle h_2| + \lambda_2^*|-E_0\rangle\langle h_2|\big) + \text{h.c.}. \qquad (8)
\end{aligned}
$$

We notice that, within this BdG formalism, either the electron or the hole component of the QDs couples simultaneously to the *positive* and *negative* states of the MZM-s, *being always in terms of quantum superposition.* The dramatic consequence of *destructive interference* from this quantum superposition is that the electron-hole excitations in the Majorana wire would localize as the edge state $|\Phi_1\rangle$ or $|\Phi_2\rangle$, resulting thus in vanishing transmission through the MZMs as $\epsilon_M \to 0$.

Again, from the T-matrix-based argument, the state propagation in $\langle e_2|T^{(2)}(\omega)|e_1\rangle$, i.e., $\langle\Phi_2|G_0(\omega)|\Phi_1\rangle$, corresponds to the time evolution as

$$\begin{aligned}&\langle\Phi_2|U(\tau)\big(|E_0\rangle + |-E_0\rangle\big)\\ =\ &\langle\Phi_2|\big(e^{-iE_0\tau}|E_0\rangle + e^{iE_0\tau}|-E_0\rangle\big)\,. \end{aligned} \tag{9}$$

If $E_0 = \epsilon_M \neq 0$, the difference of the phase factors along time would result in the Rabi-type oscillation between $|\Phi_1\rangle$ and $|\Phi_2\rangle$. Otherwise, if $\epsilon_M = 0$, no phase difference can accumulate along time and no transition between $|\Phi_1\rangle$ and $|\Phi_2\rangle$ can take place, leading thus to no transmission of electron between the two quantum dots.

C. POVM Measurement Perspective

In this subsection we further discuss the difference between the two treatments and their different conclusions, from the quantum measurement perspective of positive-operator-value-measure (POVM) [9]. Actually, the negative-energy eigenstate $|-E_0\rangle$ is the dual counterpart of the Bogoliubov quasi-particle state $|E_0\rangle$, with the basic meaning of removing an existing quasi-particle with positive energy E_0. Then, in certain sense, the superposed state $(|E_0\rangle+|-E_0\rangle)/\sqrt{2}$ owing to the *simultaneous coupling* of the QD electron or hole to $|\pm E_0\rangle$ may correspond to $(|1\rangle+|0\rangle)/\sqrt{2}$ if using the occupation-number state representation. However, *this correspondence needs very special pre-occupied condition* of the Majorana wire (to be further clarified in Sec. III of this Supplemental Material).

Now let us consider if it is possible to extract electron from the state $|\Psi_M\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, through the tunnel coupling $H_2' = \lambda_2 d_2^\dagger\gamma_2$ + h.c.. In terms of the POVM formalism of quantum measurement [9], if we detect and find an electron in the right QD, the measurement-result-conditioned backaction implies that the Majorana wire state changes as $\widetilde{\rho} = \mathcal{M}\rho\mathcal{M}^\dagger/||\bullet||$, with $||\bullet||$ the normalization factor. Here the standard notation of $\mathcal{M} = \gamma_2$ and $\rho = |\Psi_M\rangle\langle\Psi_M|$ is assumed. Then, we obtain $|\widetilde{\Psi}_M\rangle = i(|1\rangle - |0\rangle)/\sqrt{2}$ as the state of the Majorana wire after measurement, by noting that $\widetilde{\rho} = |\widetilde{\Psi}_M\rangle\langle\widetilde{\Psi}_M|$.

Actually, registration of an electron in the right QD corresponds to two possible processes which can result in this result. One is the normal tunneling from the Majorana wire excitation through $H_2' \sim f d_2^\dagger$; another is the Andreev process through $H_2' \sim f^\dagger d_2^\dagger$. Here we emphasize that even through the normal tunneling process, it is also possible to extract electron from the state $|\Psi_M\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, by setting the Kraus measurement operator as $\mathcal{M} = f$.

To summarize, from the POVM measurement perspective, we conclude that it is possible to extract an electron into the right QD from the superposed state $|\Psi_M\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. This differs from the prediction of the BdG-type treatment which claims that, through the excitation $|\Psi_M\rangle = (|E_0\rangle + |-E_0\rangle)/\sqrt{2} = |\Phi_1\rangle$, it is impossible to extract electron from the Majorana wire into the right QD.

II. ANALYSIS BASED ON THE SINGLE-ELECTRON WAVEFUNCTION APPROACH OF QUANTUM TRANSPORT

The S matrix scattering approach plays a central role in the Landauer-Büttiker formalism for quantum transport through mesoscopic systems [10]. An alternative method, say, the single-electron wavefunction (SEWF) approach [11, 12], can be regarded as a time-dependent version of the S matrix scattering theory. The basic idea of the SEWF method is keeping track of the quantum evolution of an electron initially in the source lead and computing the various transition rates such as the transmission rate to the drain lead, and/or the Andreev-reflection rate back to the source lead as a hole [2]. In the presence of Andreev process, we have shown that inserting the *conventional BdG treatment* into the SEWF approach (for the *central device* state evolution) can recover, as a stationary limit, the usual steady-state result from either the S matrix scattering theory or the nonequilibrium Green's function technique [2]. Moreover, the wavefunction approach can show very clearly how the *redundancy* of the "negative eigenenergy" states is involved in the treatment and that, without loss of any physics (the essential physics of Andreev process), eliminating this redundancy would result in a different conclusion.

A. Conventional BdG Treatment

Let us turn to the more experimentally relevant setup of the MZMs directly coupled to transport leads. Applying the low-energy effective description for the MZMs, the conventional BdG formalism allows us to express the single-particle wavefunction as

$$\begin{aligned}|\Psi(t)\rangle &= u_0|E_0\rangle + v_0|-E_0\rangle \\ &+ \sum_{\alpha=1,2}\sum_k \big(u_{\alpha k}|e_{\alpha k}\rangle + v_{\alpha k}|h_{\alpha k}\rangle\big)\,. \end{aligned} \tag{10}$$

This wavefunction is a superposition of all possible electron and hole basis states of the MZMs and the transport leads, as a consequence of quantum evolution with the initial state of a single electron in the source lead, i.e., $u_{1k}(\tau=0)=\delta_{k\bar{l}}$ and the other coefficients being zero initially. For the sake of a simple notation, we omitted here the time variable τ in the superposition coefficients.

Let us split the tunneling Hamiltonian, Eq. (1) in the main text, into two parts $H'=H'_1+H'_2$ and reexpress them as

$$\begin{aligned} H'_\alpha &= \gamma_\alpha \sum_k B_{\alpha k}\,, \\ B_{\alpha k} &= t_{\alpha k} b_{\alpha k} - t^*_{\alpha k} b^\dagger_{\alpha k}\,. \end{aligned} \tag{11}$$

Noting that $\gamma_1=f+f^\dagger$ and $\gamma_2=i(f-f^\dagger)$, again, we convert the tunneling Hamiltonian into the projection operator form as

$$\begin{aligned} H'_1 =& \sum_k \big(t_{1k}|E_0\rangle\langle e_{1k}| - t^*_{1k}|E_0\rangle\langle h_{1k}| \\ &+ t_{1k}|-E_0\rangle\langle e_{1k}| - t^*_{1k}|-E_0\rangle\langle h_{1k}|\big) + \text{h.c.}\,, \\ H'_2 =& \, i\sum_k \big(t_{2k}|E_0\rangle\langle e_{2k}| - t^*_{2k}|E_0\rangle\langle h_{2k}| \\ &- t_{2k}|-E_0\rangle\langle e_{2k}| + t^*_{2k}|-E_0\rangle\langle h_{2k}|\big) + \text{h.c.}\,. \end{aligned} \tag{12}$$

In this conversion, we have applied the following correspondence consideration:

$$\begin{aligned} |E_0\rangle\langle e_{1k}| &\longleftrightarrow f^\dagger b_{1k}\,, \\ |-E_0\rangle\langle e_{1k}| &\longleftrightarrow f b_{1k}\,, \\ |E_0\rangle\langle h_{1k}| &\longleftrightarrow f^\dagger b^\dagger_{1k}\,, \\ |-E_0\rangle\langle h_{1k}| &\longleftrightarrow f b^\dagger_{1k}\,. \end{aligned} \tag{13}$$

Similar correspondence has been applied as well to the right-side tunnel-coupling Hamiltonian H'_2. We notice that in this treatment the inclusion of $|-E_0\rangle$ actually implies a "redundancy" (i.e., a repetition). For instance, $|-E_0\rangle\langle e_{1k}|$ simply describes the inverse process of $|E_0\rangle\langle h_{1k}|$, by noting that the former describes the Andreev reflection process while the latter corresponds to the splitting of a Cooper pair. However, the Hermitian conjugated term of $|E_0\rangle\langle h_{1k}|$ in the tunnel-coupling Hamiltonian has described the same process of the term $|-E_0\rangle\langle e_{1k}|$.

Here, we may point out again, as analyzed in the previous section using the "QD–MZMs-QD" toy model, that it is the *redundant use of* $|-E_0\rangle$ *in the transport* that causes the difference between the (conventional) BdG treatment and the occupation-number-state approach. This issue will become transparent in the following after we carry out the explicit result based on this SEWF approach.

Substituting the wavefunction $|\Psi\rangle$ of Eq. (10) into the time-dependent Schrödinger equation, we obtain

$$\begin{aligned} i\dot{u}_0 &= E_0 u_0 + \sum_k (t_{1k}u_{1k} - t^*_{1k}v_{1k}) \\ &\quad + i\sum_k (t_{2k}u_{2k} - t^*_{2k}v_{2k})\,, \\ i\dot{v}_0 &= -E_0 v_0 + \sum_k (t_{1k}u_{1k} - t^*_{1k}v_{1k}) \\ &\quad - i\sum_k (t_{2k}u_{2k} - t^*_{2k}v_{2k})\,, \\ i\dot{u}_{1k} &= \epsilon_k u_{1k} + t^*_{1k}(u_0+v_0)\,, \\ i\dot{v}_{1k} &= -\epsilon_k v_{1k} - t_{1k}(u_0+v_0)\,, \\ i\dot{u}_{2k} &= \epsilon_k u_{2k} - it^*_{2k}(u_0-v_0)\,, \\ i\dot{v}_{2k} &= -\epsilon_k v_{2k} + it_{2k}(u_0-v_0)\,. \end{aligned} \tag{14}$$

For the convenience of applying a matrix solving algebra, let us introduce the vectors $\vec{u}_k=(u_{1k},v_{1k},u_{2k},v_{2k})^T$ and $\vec{u}=(u_0,v_0)^T$. The last four equations can be expressed in a compact form as $i\dot{\vec{u}}_k = H_k\vec{u}_k + T_{1k}\vec{u}$, with the two matrices given by

$$\begin{aligned} H_k &= \begin{pmatrix} \epsilon_k & 0 & 0 & 0 \\ 0 & -\epsilon_k & 0 & 0 \\ 0 & 0 & \epsilon_k & 0 \\ 0 & 0 & 0 & -\epsilon_k \end{pmatrix}, \\ T_{1k} &= \begin{pmatrix} t^*_{1k} & t^*_{1k} \\ -t_{1k} & -t_{1k} \\ -it^*_{2k} & it^*_{2k} \\ it_{2k} & -it_{2k} \end{pmatrix}. \end{aligned} \tag{15}$$

The first two equations can be expressed as $i\dot{\vec{u}} = H_0\vec{u} + \sum_k T_{2k}\vec{u}_k$, where the two matrices read as

$$\begin{aligned} H_0 &= \begin{pmatrix} E_0 & 0 \\ 0 & -E_0 \end{pmatrix}, \\ T_{2k} &= \begin{pmatrix} t_{1k} & -t^*_{1k} & it_{2k} & -it^*_{2k} \\ t_{1k} & -t^*_{1k} & -it_{2k} & it^*_{2k} \end{pmatrix}. \end{aligned} \tag{16}$$

Then, in the frequency domain (after Laplace transformation), simple matrix-manipulation-algebra yields

$$(\omega - H_0)\vec{u} = \sum_k T_{2k}G_kT_{1k}\vec{u} + \sum_k T_{2k}G_k(i\delta_{k\bar{l}})\vec{e}_i\,, \tag{17}$$

where the lead-electron Green's function reads $G_k=(\omega-H_k)^{-1}$ and the vector $\vec{e}_i=(1,0,0,0)^T$ has been introduced. The last term is originated from the initial condition $u_{1k}(\tau=0)=\delta_{k\bar{l}}$ and has the specific result of

$$\frac{it_{1\bar{l}}}{\omega-\epsilon_{\bar{l}}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Straightforwardly, the self-energy matrix (owing to coupling to the leads) can be evaluated as

$$\begin{aligned}\mathbf{\Sigma} &\equiv \sum_k T_{2k} G_k T_{1k} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \\ &= i \begin{pmatrix} \Gamma_1 + \Gamma_2 & \Gamma_1 - \Gamma_2 \\ \Gamma_1 - \Gamma_2 & \Gamma_1 + \Gamma_2 \end{pmatrix} \\ &\equiv i \begin{pmatrix} \Gamma_+ & \Gamma_- \\ \Gamma_- & \Gamma_+ \end{pmatrix} . \end{aligned} \tag{18}$$

Here we have introduced $\Gamma_1 = (\Gamma_1^e + \Gamma_1^h)/2$ and $\Gamma_2 = (\Gamma_2^e + \Gamma_2^h)/2$, while Γ_α^e and Γ_α^h are the coupling rates with the lead α through the electron and hole components, respectively.

In this context, rather than solving the time-dependent dynamics associated with injection of an electron or a hole from the leads, we simply consider the state propagation in the frequency domain which is characterized by the full Green's function (propagator)

$$\begin{aligned}\mathbf{G} &= (\omega - H_0 + \mathbf{\Sigma})^{-1} = \begin{pmatrix} \tilde{a} & c \\ d & \tilde{b} \end{pmatrix}^{-1} \\ &= \frac{1}{D} \begin{pmatrix} \tilde{b} & -c \\ -d & \tilde{a} \end{pmatrix} . \end{aligned} \tag{19}$$

In this result, we have introduced

$$\begin{aligned} \tilde{a} &= \omega - E_0 + i\Gamma_+ , \\ \tilde{b} &= \omega + E_0 + i\Gamma_+ , \\ c &= d = i\Gamma_- , \\ D = \tilde{a}\tilde{b} - cd &= (\omega^2 - E_0^2) + 2i\omega(\Gamma_1 + \Gamma_2) - 4\Gamma_1\Gamma_2 . \end{aligned} \tag{20}$$

One can check that this Green's function, which governs the state evolution in the SEWF approach, is precisely the same one either inserted in the S matrix scattering approach [7, 8] or solved from the equation-of-motion method associated with the Heisenberg equation of fermion operators (see the following subsection II (C)), while the latter is equivalent to (and rooted in) the broadly applying nonequilibrium Green's function technique [6, 13].

B. Modified BdG Treatment

In the previous subsection, we see that when converting the tunneling Hamiltonian into the projection operator form, i.e., from Eq. (11) to Eq. (12), redundancy (repetition) has been involved in the BdG treatment. That is, all the tunnel-coupling terms in Eq. (12) via $|-E_0\rangle$ are the same of those via $|E_0\rangle$.

Whether we should include the redundancy and the quantum superposition of $|E_0\rangle$ and $|-E_0\rangle$ may be also justified by a careful examination for the introduction of the "electron" and "hole" states of the transport lead electrons. Starting with an electron initially in the lead state $|\bar{l}\rangle$, let us reexpress Eq. (10) as

$$\begin{aligned} |\Psi(t)\rangle &= u_{\bar{l}}|e_{\bar{l}}\rangle + \sum_{\alpha=1,2} \sum_{k_1(\neq \bar{l})} u_{\alpha k_1}|e_{\alpha k_1}\rangle \\ &+ \sum_{\alpha=1,2} \sum_{k_2} v_{\alpha k_2}|h_{\alpha k_2}\rangle + u_0|E_0\rangle + v_0|-E_0\rangle \end{aligned}$$

Here, the sum of the k_1 terms is owing to the usual normal tunneling process, while the sum of the k_2 terms is owing to the Andreev process. The important point is that the electron state $|e_{\alpha k_1}\rangle$ and the hole state $|h_{\alpha k_2}\rangle$ correspond to, respectively, two different electrons, but not the "electron" (occupied) and "hole" (unoccupied) states of the same wavefunction state (e.g., $|k\rangle$). Therefore, the superposition $u_0|E_0\rangle + v_0|-E_0\rangle$ (associated with a single subgap quasiparticle state) is different from the electron/hole description for the transport-lead electrons.

Actually, using only the state $|E_0\rangle$ for the Majorana quasiparticle and introducing "electron" and "hole" states for the lead electrons, we can reexpress the tunneling Hamiltonian as

$$\begin{aligned} H_1' &= \sum_k \left(t_{1k}|E_0\rangle\langle e_{1k}| - t_{1k}^*|E_0\rangle\langle h_{1k}| \right) + \text{h.c.} , \\ H_2' &= i \sum_k \left(t_{2k}|E_0\rangle\langle e_{2k}| - t_{2k}^*|E_0\rangle\langle h_{2k}| \right) + \text{h.c.} . \end{aligned} \tag{21}$$

We emphasize that this expression in terms of the projection operators is complete, which contains all the normal tunneling process and the Andreev process (say, the Cooper-pair creation and splitting process). For example, the two displayed terms in H_1' correspond to, respectively, the normal tunneling term $t_{1k} f^\dagger b_{1k}$ and the inverse Andreev reflection term $t_{1k}^* b_{1k}^\dagger f^\dagger$.

Following the same procedures outlined in the previous subsection, we can construct the single particle wavefunction as

$$|\Psi\rangle = u_0|E_0\rangle + \sum_{\alpha=1,2} \sum_k \left(u_{\alpha k}|e_{\alpha k}\rangle + v_{\alpha k}|h_{\alpha k}\rangle \right) . \tag{22}$$

Substituting this wavefunction and the above tunnel-coupling Hamiltonian into the Schrödinger equation, we obtain

$$\begin{aligned} i\dot{u}_0 &= E_0 u_0 + \sum_k (t_{1k} u_{1k} - t_{1k}^* v_{1k}) \\ &\quad + i \sum_k (t_{2k} u_{2k} - t_{2k}^* v_{2k}) , \\ i\dot{u}_{1k} &= \epsilon_k u_{1k} + t_{1k}^* u_0 , \\ i\dot{v}_{1k} &= -\epsilon_k v_{1k} - t_{1k} u_0 , \\ i\dot{u}_{2k} &= \epsilon_k u_{2k} - i t_{2k}^* u_0 , \\ i\dot{v}_{2k} &= -\epsilon_k v_{2k} + i t_{2k} u_0 . \end{aligned} \tag{23}$$

Again (as in the above subsection), simple matrix-manipulation-algebra in the frequency domain after

Laplace transformation yields

$$(\omega - E_0)u_0 = \sum_k T_{2k} G_k T_{1k} u_0 + \frac{it_{1\bar{l}}}{\omega - \epsilon_{\bar{l}}}\,, \tag{24}$$

where the two coupling matrices read as

$$\begin{aligned} T_{1k} &= (t_{1k}^*, -t_{1k}, -it_{2k}^*, it_{2k})^T\,, \\ T_{2k} &= (t_{1k}, -t_{1k}^*, it_{2k}, -it_{2k}^*)\,. \end{aligned} \tag{25}$$

With this identification, the coupling self-energy is explicitly obtained as

$$\begin{aligned} \Sigma &\equiv \sum_k T_{2k} G_k T_{1k} \\ &= i\,(\Gamma_1^e + \Gamma_1^h + \Gamma_2^e + \Gamma_2^h)/2\,. \end{aligned} \tag{26}$$

Therefore, in frequency domain, the state propagation is characterized by the full Green's function $G = (\omega - E_0 + i\Gamma)^{-1}$. Here we introduced $\Gamma = (\Gamma_1^e + \Gamma_1^h + \Gamma_2^e + \Gamma_2^h)/2$, i.e., the sum of the various coupling rates of the quasi-particle state $|E_0\rangle$ to the left and right leads via the electron and hole components.

Inserting this Green's function into the S matrix scattering approach would lead to predictions different from the conventional BdG treatment, especially when $E_0 = \epsilon_M \to 0$, as highlighted and demonstrated in detail in the main text, e.g., the *survival of the non-vanishing teleportation channel* which supports electron transmission and cross-Andreev-process even when $\epsilon_M \to 0$.

C. Further Remarks

In this subsection we make further remarks based on a comparison between the conventional BdG SEWF treatment in Sec. II (A) and the Heisenberg equations of electron operators, while the latter are essentially equivalent to the equation-of-motion of the Green's functions [13]. Let us formally consider the quantum average $\bar{\mathcal{O}}(t) = \langle \Psi_i | U^\dagger(t) \mathcal{O} U(t) | \Psi_i \rangle$. Corresponding to the SEWF approach, the initial state can be specified as $|\Psi_i\rangle = |e_{\bar{l}}\rangle \otimes |G\rangle$, which assumes a "starting electron" in the lead state $|e_{\bar{l}}\rangle$ and the superconductor in ground state $|G\rangle$. The SEWF approach is actually considering the state evolution $|\Psi(t)\rangle = U(t)|\Psi_i\rangle$, which should properly take into account the ground state property $f|G\rangle = 0$, i.e., $|G\rangle$ is the annihilating state of quasiparticle operators. Importantly, it is this constraint that leads to the modified BdG scheme in Sec. II (B). Further discussions are referred to Sec. III, the next section.

On the other hand, in the Heisenberg picture, let us consider the evolution of operator, $U^\dagger(t)\mathcal{O}U(t)$. Obviously, this evolution is just governed by the Hamiltonian and is *free from the annihilating property* of the ground state $|G\rangle$. This might resemble somehow the situation of *symmetry breaking* between the Hamiltonian and the ground state. For the transport setup under consideration, we straightforwardly carry out the Heisenberg equations for the individual electron operators:

$$\begin{aligned} \dot{f} &= -iE_0 f - i\sum_k (t_{1k} b_{1k} - t_{1k}^* b_{1k}^\dagger) \\ &\quad - \sum_k (t_{2k} b_{2k} - t_{2k}^* b_{2k}^\dagger)\,, \\ \dot{f}^\dagger &= iE_0 f^\dagger - i\sum_k (t_{1k} b_{1k} - t_{1k}^* b_{1k}^\dagger) \\ &\quad + \sum_k (t_{2k} b_{2k} - t_{2k}^* b_{2k}^\dagger)\,, \\ \dot{b}_{1k} &= -i\epsilon_k b_{1k} - it_{1k}^*(f + f^\dagger)\,, \\ \dot{b}_{1k}^\dagger &= i\epsilon_k b_{1k}^\dagger + it_{1k}(f + f^\dagger)\,, \\ \dot{b}_{2k} &= -i\epsilon_k b_{2k} + t_{2k}^*(f - f^\dagger)\,, \\ \dot{b}_{2k}^\dagger &= i\epsilon_k b_{2k}^\dagger - t_{2k}(f - f^\dagger)\,. \end{aligned} \tag{27}$$

The structure of this set of equations is the same as Eq. (14). Following the procedures of solving Eq. (14), one can also introduce the operator vectors $\vec{f} = (f, f^\dagger)^T$ and $\vec{B}_k = (b_{1k}, b_{1k}^\dagger, b_{2k}, b_{2k}^\dagger)^T$, and reexpress the equations-of-motion in terms of the operator vectors. Performing the Fourier transformation and manipulating the simple matrix algebra, in frequency domain, one can obtain the same solution of Eqs. (19) and (20).

It is well known that the operator equation-of-motion in Heisenberg picture is the central procedure to derive the equation-of-motion of many-body Green's functions [13]. For transport through noninteracting systems, the transmission coefficient from channel α to β can be well expressed as $\mathcal{T}_{\alpha\beta} = \mathrm{Tr}[\Gamma_\alpha G^r \Gamma_\beta G^a]$, where $\Gamma_{\alpha(\beta)}$ is the coupling rate, and $G^{r(a)}$ is the retarded (advanced) Green's function which carries the spectral information and usually contains no information of occupations. Actually, this result derived within the framework of the nonequilibrium Green's functions [13] is connected with the S matrix approach through the simple formula $\mathcal{T}_{\alpha\beta} = |s_{\alpha\beta}|^2$, with $s_{\alpha\beta}$ the scattering matrix element between the channels α and β.

Therefore, the intrinsic connections pointed out above allow us to understand why the SEWF approach in Sec. II (A) based on the conventional BdG treatment by introducing both $|E_0\rangle$ and $|-E_0\rangle$ leads to the same result from the S matrix approach [7, 8]. We are also allowed to understand that the treatment arriving to such result is free from the annihilating property of the ground state of superconductors. After accounting for the annihilating property, we actually arrived to the modified BdG treatment in Sec. II (B). Further elaborations are referred to next section.

III. ON THE BdG SCHEME: CONCEPTUALLY REVISITED

In this section we carefully examine the meaning and condition of the "negative-energy-eigenstate" excitations in the BdG formalism. For a superconductor, for instance, described by a lattice Hamiltonian such as the Kitaev model [14], it is well known that the electron and hole basis states on the lattice sites $\{|e_j\rangle, |h_j\rangle; j = 1, 2, \cdots, N\}$ and the eigenstates of quasiparticle excitations (after Hamiltonian diagonalization) $\{| \pm E_n\rangle; n = 0, 1, \cdots, N-1\}$ constitute mutually a unitary transformation relationship. Indeed, the mutual transformation obeys the general rule of quantum mechanics, being associated with the BdG matrix Hamiltonian. We may point out that in this BdG-type formalism, either the lattice hole state $|h_j\rangle$ or the negative eigenenergy state $|-E_n\rangle$ simply means *the removal of an existing particle* on the conjugated state, say, on $|e_j\rangle$ or $|E_n\rangle$.

However, if we consider the injection of an electron or a hole from outside, can the electron-hole excitations in the superconductor really be the superposition of the positive and negative energy eigenstates, simply following the principle of quantum state transformation? Our analysis in this section will show that, if we want to maintain the validity of this type of excitation picture caused by an external injection, quite special –but unrealistic– condition must be assumed.

A. Based on the Kitaev Lattice Model

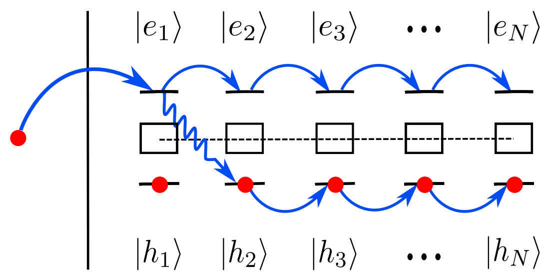

FIG. 1: Dynamical picture of the BdG treatment by considering injection of an external electron into a ground-state superconductor. On each site of the lattice model (such as the Kitaev model [14]), electron and hole states are introduced. The nearest-neighbor hopping of electron and hole is explicitly illustrated, while the wavy-arrow-line indicates the transition from an electron to a hole state which is accompanied by formation of a Cooper pair. This picture is simply based on the quantum mechanics associated with the BdG matrix of Hamiltonian for diagonalization. However, as explained in detail in the main text, *this dynamics is not true* when an external electron is injected into the ground-state superconductor.

Let us start the analysis with the well-known Kitaev model [14]

$$H_W = \sum_{j=1}^{N} \left[-\mu c_j^\dagger c_j - t(c_j^\dagger c_{j+1} + \text{h.c.}) \right] + \Delta \sum_{j=1}^{N} (c_j c_{j+1} + \text{h.c.}) \,. \tag{28}$$

In this spinless p-wave superconductor model, μ is the chemical potential, Δ is the superconducting order parameter, and t is the hopping energy between the nearest neighbor sites with $c_j^\dagger$ (c_j) the associated electron creation (annihilation) operators. Using the lattice electron and hole state basis $\{|e_1\rangle, \cdots, |e_N\rangle;\ |h_1\rangle, \cdots, |h_N\rangle\}$, the BdG Hamiltonian matrix reads as

$$H_{\text{BdG}} = \begin{pmatrix} T & \Omega \\ -\Omega & -T \end{pmatrix} , \tag{29}$$

while the block matrices are given by

$$T = \begin{pmatrix} -\mu & -t & 0 & \cdots & \cdots \\ -t & -\mu & -t & 0 & \cdots \\ 0 & -t & -\mu & -t & \cdots \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix} , \tag{30}$$

and

$$\Omega = \begin{pmatrix} 0 & \Delta & 0 & \cdots & \cdots \\ -\Delta & 0 & \Delta & 0 & \cdots \\ 0 & -\Delta & 0 & \Delta & \cdots \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix} . \tag{31}$$

The BdG matrix of Hamiltonian can be diagonalized straightforwardly. Let us denote the eigenstates as

$$|E_n\rangle = \sum_{j=1}^{N} (u_{nj}|e_j\rangle + v_{nj}|h_j\rangle) \,, \quad |-E_n\rangle = \sum_{j=1}^{N} (u_{nj}^*|h_j\rangle + v_{nj}^*|e_j\rangle) \,, \tag{32}$$

which correspond to the positive eigenenergy E_n and negative eigenenergy $-E_n$, respectively. For the convenience of labelling the subgap zero-energy state (using $n = 0$), we may arrange n from 0 to $N-1$. For each pair of the n-labeled states, the associated creation operator of Bogoliubov quasiparticle is given by

$$\gamma_n^\dagger = \sum_{j=1}^{N} (u_{nj} c_j^\dagger + v_{nj} c_j) \,. \tag{33}$$

The inverse transformation can be carried out either in state or in operator form, for example, as

$$|e_j\rangle = \sum_{n=0}^{N-1} (u_{nj}^*|E_n\rangle + v_{nj}|-E_n\rangle) \,, \quad c_j^\dagger = \sum_{n=0}^{N-1} (u_{nj}^* \gamma_n^\dagger + v_{nj}\gamma_n) \,. \tag{34}$$

Here we see that, for an isolated superconductor, the unitary transformation between the lattice-site electron and hole basis and the quasiparticle eigenstates, or in terms of the operator form, does follow the standard rule of quantum mechanics.

However, *unusual insight is gained* if we analyze carefully the electron-hole excitation and its propagation in the superconductor by considering injection of an external electron or hole. To be specific, let us consider injecting an electron via the tunneling Hamiltonian $H' = \sum_k (t_k b_k c_1^\dagger + \text{h.c.})$ to generate the excitation $|e_1\rangle$. The subsequent dynamics of this excitation driven by the BdG Hamiltonian includes (i) electron hopping on the *empty* lattice sites, (ii) transition between electron and hole ($|e_j\rangle \leftrightarrow |h_{j+1}\rangle$), and (iii) hole hopping on the *occupied* lattice sites, as schematically shown in Fig. 1. However, in order to make this type of electron-hole dynamics be possible, we need a *starting state* as

$$|\widetilde{G}\rangle = \big(|0\rangle_1|0\rangle_2\cdots|0\rangle_N\big)_e \otimes \big(|1\rangle_1|1\rangle_2\cdots|1\rangle_N\big)_h \,, \tag{35}$$

where $|0\rangle_j$ and $|1\rangle_j$ are the occupation-number states on the lattice site j. The former sector $(\cdots)_e$ of this product state is for "electron" excitation, while the latter sector $(\cdots)_h$ is for "hole" excitation. We may notice the very unusual feature of the state $|\widetilde{G}\rangle$: *on one aspect, all the lattice sites are empty; while on the other aspect, they all are occupied.* Based on $|\widetilde{G}\rangle$, the "electron" and "hole" excitations can be generated through acting on the individual sectors as $c_j^\dagger|\widetilde{G}\rangle = |e_j\rangle$ and $c_j|\widetilde{G}\rangle = |h_j\rangle$. It is also this background state $|\widetilde{G}\rangle$ that can support the propagation of the electron-hole excitations in the lattice chain, which is actually the dynamic counterpart of diagonalization of the BdG Hamiltonian matrix.

Alternatively, based on the above unitary transformation between the electron-hole basis and the eigenstates, the initial excitation can be expressed also as $|e_1\rangle = \sum_{n=0}^{N-1}(u_{n1}^*|E_n\rangle + v_{n1}|-E_n\rangle)$, while the corresponding operator form reads as $c_1^\dagger = \sum_{n=0}^{N-1}(u_{n1}^*\gamma_n^\dagger + v_{n1}\gamma_n)$. Again, this correspondence needs the background state $|\widetilde{G}\rangle$ expressed in the representation of the Bogoliubov quasiparticle eigenstates as

$$|\widetilde{G}\rangle = \big(|0\rangle_{E_0}|0\rangle_{E_1}\cdots|0\rangle_{E_{N-1}}\big)_e \otimes \big(|1\rangle_{E_0}|1\rangle_{E_1}\cdots|1\rangle_{E_{N-1}}\big)_h \,. \tag{36}$$

Here $|0\rangle_{E_n}$ and $|1\rangle_{E_n}$ are the occupation-number states associated with the eigen-wavefunction $|E_n\rangle$. Also, the very unusual feature is that on one aspect all the quasiparticle eigenstates are empty, while on the other aspect they all are occupied.

In particular, for the case of topological phase in the presence of the MZMs, the low-energy effective description may make the above $|e_1\rangle$ excitation corresponding to $|\Phi_1\rangle = (|E_0\rangle + |-E_0\rangle)/\sqrt{2}$, owing to $c_1^\dagger|\widetilde{G}\rangle \sim \gamma_1|\widetilde{G}\rangle = |\Phi_1\rangle$. We know that the MZM $|\Phi_1\rangle$ localizes at the left side as electron-hole excitation.

However, for the *true* ground state $|G\rangle$, rather than the artificially constructed state $|\widetilde{G}\rangle$, we have $\gamma_1|G\rangle \sim (f+f^\dagger)|G\rangle = |E_0\rangle = (|\Phi_1\rangle + i|\Phi_2\rangle)/\sqrt{2}$. In sharp contrast to the above conclusion, this subgap Bogoliubov quasiparticle accommodates the electron-hole excitations at the two sides of the lattice chain.

B. Based on the BCS Model

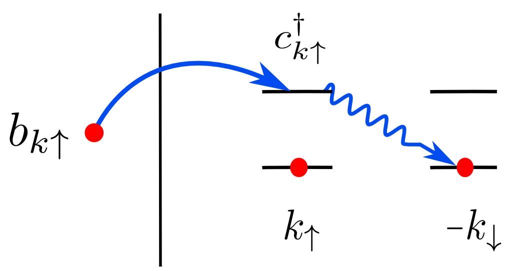

FIG. 2: Dynamical picture of the BdG treatment by considering injection of an external electron into the superconductor described by the BCS Hamiltonian. The wavy-arrow-line indicates the transition (Rabi-type oscillation) between a pair of electron and hole states (accompanied by formation and splitting of a Cooper pair). However, again, as illustrated by the Kitaev lattice model in Fig. 1, *this dynamics is not true* when an external electron is injected into the ground-state superconductor.

We further apply similar analysis to the even better known BCS Hamiltonian which reads

$$H_{\rm BCS} = \sum_{k,\sigma=\uparrow,\downarrow}(\epsilon_k-\mu)c_{k\sigma}^\dagger c_{k\sigma} + \sum_k[\Delta_k^* c_{k\uparrow}c_{-k\downarrow} + \text{h.c.}] \,. \tag{37}$$

The first term describes the free noninteracting electrons in the momentum representation, with also an explicit inclusion of the spin components. The second term describes the superconducting pairing under the mean-field approximation. For an s-wave superconductor, the superconducting order parameter is of k-independence. We thus denote it as $\Delta_k \equiv \Delta$. Using the free electron and hole state basis $\{|e_{k\uparrow}\rangle, |h_{-k\downarrow}\rangle\}$, the BdG matrix of the BCS Hamiltonian simply reads

$$H_{\rm BCS} = \begin{pmatrix} \epsilon_k-\mu & \Delta \\ \Delta^* & -(\epsilon_k-\mu) \end{pmatrix} . \tag{38}$$

Diagonalizing this Hamiltonian matrix gives the eigenenergies $E_{k\uparrow}$ and $-E_{-k\downarrow}$. We also have $E_{k\uparrow} = E_{-k\downarrow} = \sqrt{(\epsilon_k-\mu)^2+|\Delta|^2} \equiv E_k$. The corresponding eigenstates are given by

$$|E_{k\uparrow}\rangle = u_k|e_{k\uparrow}\rangle - v_k|h_{-k\downarrow}\rangle \,, \quad |-E_{-k\downarrow}\rangle = u_k|h_{-k\downarrow}\rangle + v_k|e_{k\uparrow}\rangle \,, \tag{39}$$

with the coefficients

$$u_k^2 \;=\; \frac{1}{2}\big(1+\frac{\epsilon_k-\mu}{E_k}\big)\,,$$
$$v_k^2 \;=\; \frac{1}{2}\big(1-\frac{\epsilon_k-\mu}{E_k}\big)\,. \qquad (40)$$

From this diagonalized solution, we know also the Bogoliubov quasiparticle operators

$$\gamma^\dagger_{k\uparrow} \;=\; u_k c^\dagger_{k\uparrow} - v_k c_{-k\downarrow}\,,$$
$$\gamma_{-k\downarrow} \;=\; u_k c_{-k\downarrow} + v_k c^\dagger_{k\uparrow}\,. \qquad (41)$$

Following similar analysis of previous subsection, let us consider, as illustrated by Fig. 2, injecting a spin-up electron from outside (e.g., transport lead) via the tunneling Hamiltonian $H'=\sum_{k,\sigma}(t_k c^\dagger_{k\sigma} b_{k\sigma}+\text{h.c.})$, where $b_{k\sigma}$ is the annihilation operator of the lead electron. The BdG treatment allows the creation operator $c^\dagger_{k\uparrow}$ to create an excitation $|e_{k\uparrow}\rangle$ in the superconductor, which can be also decomposed into a superposition of the eigenstates as $|e_{k\uparrow}\rangle = u_k|E_{k\uparrow}\rangle + v_k|-E_{-k\downarrow}\rangle$. In terms of the original electron and hole basis, the excitation would manifest itself as a Rabi-type oscillation between $|e_{k\uparrow}\rangle$ and $|h_{-k\downarrow}\rangle$, before the excitation is destroyed owing to tunneling into the lead as either an electron or a hole.

However, the above picture of excitation and the subsequent dynamics is possible only by assuming the following artificial state

$$|\widetilde{G}\rangle \;=\; \prod_k \big(|0\rangle_{k\uparrow}|0\rangle_{-k\downarrow}\big)_e \otimes \big(|1\rangle_{k\uparrow}|1\rangle_{-k\downarrow}\big)_h$$
$$\;=\; \prod_k \big(|0\rangle_{E_{k\uparrow}}|0\rangle_{E_{-k\downarrow}}\big)_e \otimes \big(|1\rangle_{E_{k\uparrow}}|1\rangle_{E_{-k\downarrow}}\ \big)_h\,. (42)$$

Again, we observe the unusual feature that *each basis state is simultaneously unoccupied and occupied*, forming thus a Hilbert space expanded by the direct product of electron and hole subspaces. The creation operator creates a particle through acting onto the electron subspace (the former sector of the product space), while the annihilation operator removes a particle –creates a hole– in the hole subspace (the second one of the product space). One can check that acting $c^\dagger_{k\uparrow} = u_k\gamma^\dagger_{k\uparrow} + v_k\gamma_{-k\downarrow}$ onto $|\widetilde{G}\rangle$ would generate the electronic excitation $|e_{k\uparrow}\rangle$, while acting $c_{-k\downarrow} = u_k\gamma_{-k\downarrow} - v_k\gamma^\dagger_{k\uparrow}$ onto $|\widetilde{G}\rangle$ would generate the hole-type excitation $|h_{-k\downarrow}\rangle$.

Unfortunately, $|\widetilde{G}\rangle$ is not the true ground state $|G\rangle$ of the superconductor, since we know that $c^\dagger_{k\uparrow}|G\rangle \to u_k|E_{k\uparrow}\rangle$, which is not at all the superposition of $|E_{k\uparrow}\rangle$ and $|-E_{-k\downarrow}\rangle$. Obviously, the true ground state of the superconductor does dot support the picture of excitation and the subsequent dynamics imagined above.

C. Summarizing Remarks

The above analysis shows that the *dynamical interpretation* of the BdG treatment requires a background state $|\widetilde{G}\rangle$, which assumes that all the basis states, on one aspect, are fully empty for the sake of supporting electronic excitations, yet on the other aspect, are fully occupied at the same time in order to support hole-type excitations. Or, being equivalent, in the representation of Bogoliubov quasiparticle states, they are empty for the positive-energy-state excitation; but at the same time, they are fully occupied for the excitation of the negative-energy-states.

Obviously, the background state $|\widetilde{G}\rangle$ is not the true ground state $|G\rangle$ of the superconductor. It is well known that the true ground state is the annihilating state of the Bogoliubov quasiparticle annihilation operators. From $|G\rangle$, it is impossible to create the negative-eigenenergy-state $|-E_n\rangle$. Actually, in standard literature, the BdG treatment is largely a technique of diagonalization, which replaces the Bogoliubov transformation in terms of operators. Formally, the BdG technique introduces the Nambu representation of field operators, then diagonalize the BdG matrix of Hamiltonian to obtain the eigenenergy spectrum of quasiparticles and the associated creation and annihilation operators. In this formulation, the negative eigenenergy states are the redundant counterparts of the positive ones, which simply mean removal of the already existing quasiparticles.

However, when applying the BdG matrix of Hamiltonian to the dynamical evolution of the electron-hole excitations, one should take particular care of the nature of the superconductor ground state. That is, the "creation operator" of a negative state, which is actually the *annihilation operator* of a quasiparticle, cannot create such a negative state from the ground state, unlike the formal indication based on the transformation between the electron/hole basis and the positive/negative eigenenergy states. Actually, starting with the superconductor ground state, injecting an electron (or a hole) from outside can cause only the positive energy quasiparticle excitation. Of course, it can also annihilate an existing quasiparticle through the Andreev process, which has yet been properly accounted for in our modified BdG treatment, without loss of any physics.

IV. ADDITIONAL TECHNICAL INFORMATION

A. Solving the MME

In this part we present some details of solving the Majorana master equation (MME), which was constructed in the main text as

$$\dot\rho \;=\; -i[H_M,\rho] + \sum_{\alpha=1,2}\Big(\Gamma^{(+)}_\alpha\mathcal{D}[f^\dagger]\rho + \Gamma^{(-)}_\alpha\mathcal{D}[f]\rho\Big)$$
$$+ \sum_{\alpha=1,2}\Big(\widetilde{\Gamma}^{(+)}_\alpha\mathcal{D}[f]\rho + \widetilde{\Gamma}^{(-)}_\alpha\mathcal{D}[f^\dagger]\rho\Big)\,,$$

where the Majorana Hamiltonian reads as $H_M = i\epsilon_M\gamma_1\gamma_2 = \epsilon_M(f^\dagger f - \frac{1}{2})$, and the Lindblad superoperator is defined through $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$. As mentioned in the main text, this MME contains *anomalous terms* which describe the Andreev processes most clearly, compared with constructing the master equation by coupling the Majorana operators $\gamma_{1,2}$ to the transport leads, as to be further remarked below near the end of this subsection. To simplify the notation, let us introduce two joint rates

$$\begin{aligned} r_1 &= \Gamma_1^{(+)} + \Gamma_2^{(+)} + \widetilde{\Gamma}_1^{(-)} + \widetilde{\Gamma}_2^{(-)}, \\ r_2 &= \Gamma_1^{(-)} + \Gamma_2^{(-)} + \widetilde{\Gamma}_1^{(+)} + \widetilde{\Gamma}_2^{(+)}. \end{aligned} \tag{43}$$

Here we may notice also that $r_1 + r_2 = 2(\Gamma_1 + \Gamma_2) = 2\Gamma$, owing to $\Gamma_\alpha^{(+)} + \Gamma_\alpha^{(-)} = \widetilde{\Gamma}_\alpha^{(+)} + \widetilde{\Gamma}_\alpha^{(-)} = \Gamma_\alpha$. Then, we arrive at a very simple Lindblad-type master equation as follows:

$$\dot{\rho} = -i[H_M, \rho] + r_1\mathcal{D}[f^\dagger]\rho + r_2\mathcal{D}[f]\rho, \tag{44}$$

which shows clearly a quasi-particle excitation term with the total rate r_1, and an annihilation term with the total rate r_2.

In general, let us express the density matrix as

$$\rho = \rho_{00}|0\rangle\langle 0| + \rho_{11}|1\rangle\langle 1| + \rho_{01}|0\rangle\langle 1| + \rho_{10}|1\rangle\langle 0|. \tag{45}$$

Taking use of $f^\dagger = |1\rangle\langle 0|$ and $f = |0\rangle\langle 1|$, we obtain

$$\begin{aligned} f^\dagger\rho f &= \rho_{00}|1\rangle\langle 1|, \\ f\rho f^\dagger &= \rho_{11}|0\rangle\langle 0|, \\ \{ff^\dagger, \rho\} &= 2\rho_{00}|0\rangle\langle 0| + \rho_{01}|0\rangle\langle 1| + \rho_{10}|1\rangle\langle 0|, \\ \{f^\dagger f, \rho\} &= 2\rho_{11}|1\rangle\langle 1| + \rho_{10}|1\rangle\langle 0| + \rho_{01}|0\rangle\langle 1|. \end{aligned} \tag{46}$$

In the f particle number-state basis $\{|0\rangle, |1\rangle\}$, we thus reexpress the MME as

$$\begin{aligned} \dot{\rho}_{00} &= -r_1\rho_{00} + r_2\rho_{11}, \\ \dot{\rho}_{11} &= -r_2\rho_{11} + r_1\rho_{00}, \\ \dot{\rho}_{10} &= -i\epsilon_M\rho_{10} - \frac{r_1 + r_2}{2}\rho_{10}, \\ \dot{\rho}_{01} &= i\epsilon_M\rho_{01} - \frac{r_1 + r_2}{2}\rho_{01}. \end{aligned} \tag{47}$$

Importantly, this result shows that the diagonal elements of the density matrix do not couple to the off-diagonal elements, implying that no quantum coherence between $|0\rangle$ and $|1\rangle$ will be induced via tunnel-coupling to the incoherent electron reservoirs (the transport leads), if initially they have no quantum superposition. Alternatively, if we construct the master equation based on the tunneling Hamiltonian through the Majorana operators $\gamma_{1,2}$, we may encounter Lindblad terms such as $\mathcal{D}[f + \tilde{c}f^\dagger]\rho$ (with $\tilde{c}$ a combination coefficient). We notice that the "anti-rotating-wave" terms, $f\rho f = \rho_{10}|0\rangle\langle 1|$ and $f^\dagger\rho f^\dagger = \rho_{01}|1\rangle\langle 0|$, only modify the last two equations of (47) by adding a ρ_{10} term in $\dot{\rho}_{01}$ and a ρ_{01} term in $\dot{\rho}_{10}$. Therefore, starting with a mixed state (especially with the ground state $|0\rangle$), it is impossible to create a quantum superposition of $|0\rangle$ and $|1\rangle$ by the tunneling process with the transport leads, then the "anti-rotating-wave" terms will have no effect owing to the absence of off-diagonal elements of ρ. This observation justifies the validity of the MME constructed in the main text, in the presence of Andreev process.

Based on Eq. (47), we can easily solve the steady-state density matrix, which is denoted as $\bar{\rho} = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1|$, yielding

$$p_0 = \frac{r_2}{r_1 + r_2}, \quad p_1 = \frac{r_1}{r_1 + r_2}. \tag{48}$$

They are, respectively, the empty and occupied probabilities of the f quasiparticle state. Using this solution, a simple rate process counting yields the steady-state current components shown by Eq. (5) in the main text.

B. ϵ_M-Dependence of the Nonlocal Teleportation Conductance

In this subsection we present the detailed derivation of Eqs. (16)-(18) in the main text, and add supplemental information for the ϵ_M-dependence of the nonlocal conductance, which is much relevant to the Majorana teleportation issue.

1. BCS Charge Illustrated by the Kitaev Model

Taking use of the Kitaev model [14], Eq. (28), we diagonalize the BdG matrix of the Hamiltonian to obtain the eigenstates Eq. (32). In particular, we are interested in the subgap states $|\pm E_0\rangle$. If $E_0 = \epsilon_M = 0$, we know that for all lattice sites, $|u_{0j}|^2 = |v_{0j}|^2$, i.e., the particle-hole symmetry in the ideal Majorana case. However, if $\epsilon_M \neq 0$, strictly speaking, for the subgap state $|E_0\rangle$, we find $|u_{0j}|^2 \neq |v_{0j}|^2$ and introduce the so-called BCS charge as $q_j = |u_{0j}|^2 - |v_{0j}|^2$. Since the MZMs are locating at the two sides of the lattice chain, we may denote the local BCS charges at the two edge sites as q_l and q_r, with l and r corresponding to the lattice sites $j = 1$ and N, respectively. In Fig. 3, we plot the ϵ_M-dependence of the reduced BCS charge $\tilde{q}_r = q_r/|u_r|^2$ (here $u_r \equiv u_{0N}$) and find an almost linear scaling behavior. As will see below, this behavior will help the conventional BdG treatment to conclude an approximate scaling behavior of $\propto \epsilon_M^2$ for the teleportation conductance.

2. Derivation of the Nonlocal Conductance and Its ϵ_M-Dependence

Keeping the insight gained from the above numerical result based on the Kitaev model, let us return to the low-energy effective description for the MZMs. We would like

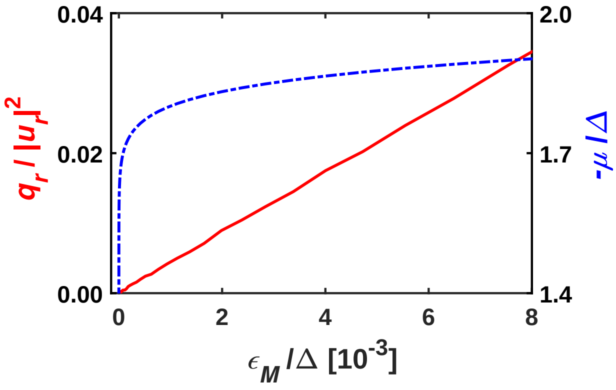

FIG. 3: Linear ϵ_M (Majorana coupling energy) dependence of the local BCS charge at the edge site of the Majorana wire, simulated using the Kitaev lattice model. We fix $t = \Delta = 1.0$ and change the chemical potential μ to slightly violate the ideal Majorana condition, causing thus $\epsilon_M \neq 0$.

to extend the treatment in Ref. [7] to the more detailed one in Ref. [8], i.e., taking into account the effect of the BCS charge when $\epsilon_M \neq 0$ to derive the results of Eqs. (16)-(18) in the main manuscript.

We consider a homogeneous Majorana wire, which renders the electron/hole components of the wavefunctions of the left and right MZMs to be symmetric/antisymmetric. Based on the numerical result of the Kitaev model, we assume $u_l = u_r = u$ and $v_l = -v_r = v$, with u and v real values. Without loss of physics, we also assume symmetric couplings to the left and right leads, say, $t_l = t_r = \widetilde{t}$. We further introduce the coupling rates as

$$\begin{aligned}\Gamma_\alpha^e &= 2\pi\nu|t_\alpha|^2|u_\alpha|^2\,,\\ \Gamma_\alpha^h &= 2\pi\nu|t_\alpha|^2|v_\alpha|^2\,,\end{aligned} \tag{49}$$

where ν is the density-of-states (assumed flat) of the leads. For the convenience of following use, we may introduce the total coupling rate

$$\Gamma = (\Gamma_1^e + \Gamma_1^h + \Gamma_2^e + \Gamma_2^h)/2\,. \tag{50}$$

Here we also labeled the coupling rates using $\alpha = 1$ and 2, simply corresponding to coupling to the leads trough the left (l) and right (r) edge sites of the Majorana wire. For symmetric coupling, we have $\Gamma_1^e = \Gamma_2^e \equiv \Gamma^e$ and $\Gamma_1^h = \Gamma_2^h \equiv \Gamma^h$. When $\epsilon_M \neq 0$, owing to $u \neq v$, we also introduce $\xi = \Gamma^e - \Gamma^h$.

Following Refs. [7, 8], we apply the S matrix scattering approach to carry out the result based on the conventional BdG treatment, and show its prediction for the ϵ_M scaling behavior, which differs from the one predicted by the modified BdG treatment (and self-consistently, by the MME approach). The conventional BdG treatment includes coupling of both $|E_0\rangle$ and $|-E_0\rangle$ to the electron and hole components of the left (L) and right (R) leads, $\{|e_L\rangle, |e_R\rangle, |h_L\rangle, |h_R\rangle\}$, described by the coupling matrix

$$\begin{aligned}W &= \begin{pmatrix} t_l u_l & t_r u_r & -t_l v_l^* & -t_r v_r^* \\ t_l v_l & t_r v_r & -t_l u_l^* & -t_r u_r^* \end{pmatrix}\\ &= \widetilde{t}\begin{pmatrix} u & u & -v & v \\ v & -v & -u & -u \end{pmatrix}.\end{aligned} \tag{51}$$

Using it, the scattering S matrix is given by

$$S(\omega) = 1 - 2\pi i\nu W^\dagger(\omega - H_M + i\pi\nu WW^\dagger)^{-1}W\,. \tag{52}$$

Simple algebra yields

$$\begin{aligned}&\omega - H_M + i\pi\nu WW^\dagger\\ &= \begin{pmatrix} \omega - \epsilon_M + i\Gamma & 0 \\ 0 & \omega + \epsilon_M + i\Gamma \end{pmatrix},\end{aligned} \tag{53}$$

and its inverse matrix

$$\begin{aligned}&(\omega - H_M + i\nu WW^\dagger)^{-1}\\ &= \frac{1}{z}\begin{pmatrix} \omega + \epsilon_M + i\Gamma & 0 \\ 0 & \omega - \epsilon_M + i\Gamma \end{pmatrix}.\end{aligned}$$

Here we introduced $z = \omega^2 - \epsilon_M^2 - \Gamma^2 + 2i\omega\Gamma$. For short, let us introduce $a = \omega + \epsilon_M + i\Gamma$ and $b = \omega - \epsilon_M + i\Gamma$ and obtain the S matrix as

$$\begin{aligned}&S(\omega) = 1 - 2\pi i\nu\widetilde{t}^2 z^{-1}\\ &\times\begin{pmatrix} u^2a + v^2b & u^2a - v^2b & -uva - uvb & uva - uvb \\ u^2a - v^2b & u^2a + v^2b & -uva + uvb & uva + uvb \\ -uva - uvb & -uva + uvb & v^2a + u^2b & -v^2a + u^2b \\ uva - uvb & uva + uvb & -v^2a + u^2b & v^2a + u^2b \end{pmatrix}\end{aligned} \tag{54}$$

Based on this result, we straightforwardly obtain the transmission coefficients from the left to right leads, for electron $\mathcal{T}_{12}^{ee}$ and hole $\mathcal{T}_{21}^{hh}$, respectively, as

$$\begin{aligned}\mathcal{T}_{12}^{ee}(\omega) &= |s_{12}|^2 = \frac{4\pi^2\nu^2\widetilde{t}^4}{|z|^2}\,|u^2a - v^2b|^2\\ &= \frac{|\Gamma_e(\omega + \epsilon_M + i\Gamma) - \Gamma_h(\omega - \epsilon_M + i\Gamma)|^2}{|z|^2}\\ &= \frac{|(\omega + i\Gamma)\xi + \epsilon_M\Gamma|^2}{|z|^2}\,,\\ \mathcal{T}_{21}^{hh}(\omega) &= |s_{43}|^2 = \frac{4\pi^2\nu^2\widetilde{t}^4}{|z|^2}\,|-v^2a + u^2b|^2\\ &= \frac{|-\Gamma_h(\omega + \epsilon_M + i\Gamma) + \Gamma_e(\omega - \epsilon_M + i\Gamma)|^2}{|z|^2}\\ &= \frac{|(\omega + i\Gamma)\xi - \epsilon_M\Gamma|^2}{|z|^2}\,.\end{aligned} \tag{55}$$

Substituting these two results into the teleportation conductance, Eq. (15) in the main text, we obtain

$$\Delta G_{LL} = \frac{d(\Delta I_L)}{dV_L} = \frac{e^2}{h}\left[\mathcal{T}^{ee}_{12}(\mu_L) + \mathcal{T}^{hh}_{21}(-\mu_L)\right]$$
$$= \frac{e^2}{h}\frac{|\mu_L\xi + \epsilon_M\Gamma + i\Gamma\xi|^2 + |-\mu_L\xi - \epsilon_M\Gamma + i\Gamma\xi|^2}{|z|^2}$$
$$= (\frac{2e^2}{h})\left[(\xi\mu_L + \epsilon_M\Gamma)^2 + \xi^2\Gamma^2\right]/|z|^2 . \quad (56)$$

For the zero-bias (small voltage) limit $\mu_L \to 0$ (under the choice of $\mu_R = 0$) and in the weak interaction regime $\epsilon_M \ll \Gamma$, we have $|z|^2 \simeq \Gamma^4$ and simplify the above expression as

$$\Delta G_{LL} = (\frac{2e^2}{h})\,(K^2 + 1/\Gamma^2)\,\epsilon_M^2 . \quad (57)$$

Here we have introduced $\xi/\Gamma = K\epsilon_M$, with K a proportional coefficient which is weakly dependent on ϵ_M, as shown in the supplementary Fig. 4.

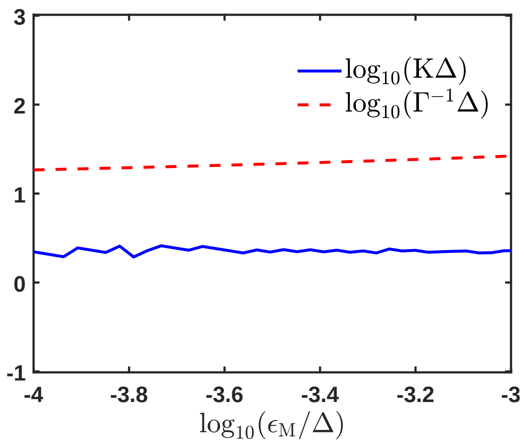

FIG. 4: Weak ϵ_M-dependence of the inverse total coupling rate Γ^{-1} and the coefficient K in the teleportation conductance ΔG_{LL}, Eq. (57), under the condition $\epsilon_M \ll \Gamma$. Model and parameters used here are the same as in Fig. 3.

So far, we have completed the derivation of Eqs. (16)-(18) displayed in the main text. The final expression of ΔG_{LL} indicates an ϵ_M^2-scaling behavior if K and $1/\Gamma$ are weakly dependent on ϵ_M. Indeed, in the supplementary Fig. 4, we illustrate the weak dependence feature, which supports an approximate ϵ_M^2 scaling of ΔG_{LL}.

The important point is that the conventional BdG treatment shown above concludes the teleportation channel vanishing when $\epsilon_M \to 0$. This feature has already been reflected in the results of $\mathcal{T}^{ee}_{12}$ and $\mathcal{T}^{hh}_{21}$ in Eq. (55), by noting that ξ is proportional to the BCS charge while the BCS charge is linearly dependent on ϵ_M, as shown by the supplementary Fig. 3. We may emphasize that, in sharp contrast, the modified BdG treatment predicts $\mathcal{T}^{ee}_{12} \sim \Gamma^e_1\Gamma^e_2/\Gamma^2$ and $\mathcal{T}^{hh}_{21} \sim \Gamma^h_1\Gamma^h_2/\Gamma^2$, which obviously *do not* vanish when $\epsilon_M \to 0$.

3. *Asymmetric Coupling to the Leads*

In this subsection we briefly consider the asymmetric coupling to the leads, i.e., keeping $t_l \neq t_r$ in Eqs. (49) and (51), and show that the approximate ϵ_M^2-scaling behavior remains unchanged from the conventional BdG treatment. Following precisely the same procedures arriving to Eq. (55), we obtain

$$\mathcal{T}^{ee}_{12}(\omega) = |s_{12}|^2 = \frac{|(\omega + i\Gamma)\tilde{\xi} + \epsilon_M\tilde{\Gamma}|^2}{|z|^2},$$
$$\mathcal{T}^{hh}_{21}(\omega) = |s_{43}|^2 = \frac{|(\omega + i\Gamma)\tilde{\xi} - \epsilon_M\tilde{\Gamma}|^2}{|z|^2}. \quad (58)$$

Here, $z = \omega^2 - \epsilon_M^2 - \Gamma^2 + 2i\omega\Gamma$ is formally the same as before but two more combination parameters were introduced

$$\tilde{\xi} = \sqrt{\Gamma^e_1\Gamma^e_2} - \sqrt{\Gamma^h_1\Gamma^h_2} = 2\pi\nu|t_lt_r|(u^2 - v^2),$$
$$\tilde{\Gamma} = \sqrt{\Gamma^e_1\Gamma^e_2} + \sqrt{\Gamma^h_1\Gamma^h_2} = 2\pi\nu|t_lt_r|(u^2 + v^2). \quad (59)$$

Based on this result, we further obtain the teleportation conductance as

$$\Delta G_{LL} = \frac{d(\Delta I_L)}{dV_L} = \frac{e^2}{h}\left[\mathcal{T}^{ee}_{12}(\mu_L) + \mathcal{T}^{hh}_{21}(-\mu_L)\right]$$
$$= \frac{e^2}{h}\frac{|\mu_L\tilde{\xi} + \epsilon_M\tilde{\Gamma} + i\Gamma\tilde{\xi}|^2 + |-\mu_L\tilde{\xi} - \epsilon_M\tilde{\Gamma} + i\Gamma\tilde{\xi}|^2}{|z|^2}$$
$$= (\frac{2e^2}{h})\left[(\tilde{\xi}\mu_L + \epsilon_M\tilde{\Gamma})^2 + \tilde{\xi}^2\Gamma^2\right]/|z|^2 . \quad (60)$$

Notice that now the BCS charge is encoded in $\tilde{\xi}$, which is linearly dependent on ϵ_M as shown in Fig. 3, whereas Γ and $\tilde{\Gamma}$ weakly depend on ϵ_M as shown by Fig. 4. We thus conclude the same ϵ_M^2-scaling behavior for ΔG_{LL}, as in the symmetric coupling case.

[1] S. Tewari, C. Zhang, S. Das Sarma, C. Nayak, and D. H. Lee, Phys. Rev. Lett. **100**, 027001 (2008).
[2] X. Q. Li and L. Xu, Phys. Rev. B **101**, 205401 (2020).
[3] G. W. Semenoff and P. Sodano, *Teleportation by a Ma-*

jorana Medium, arXiv:cond-mat/0601261; *Stretching the electron as far as it will go*, arXiv:cond-mat/0605147.
[4] L. Fu, Phys. Rev. Lett. **104**, 056402 (2010).
[5] B. Zocher and B. Rosenow, Phys. Rev. Lett. **111**, 036802 (2013).
[6] C. J. Bolech and E. Demler, Phys. Rev. Lett. **98**, 237002 (2007).
[7] J. Nilsson, A. R. Akhmerov, and C. W. J. Beenakker, Phys. Rev. Lett. **101**, 120403 (2008).
[8] J. Danon, A. B. Hellenes, E. B. Hansen, L. Casparis, A. P. Higginbotham, and K. Flensberg, Phys. Rev. Lett. **124**, 036801 (2020).
[9] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, UK, 2009).
[10] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, New York, 1995).
[11] S. A. Gurvitz, Phys. Rev. B **44**, 11924 (1991).
[12] F. Li, X. Q. Li, W. M. Zhang, and S. A. Gurvitz, Europhys. Lett. **88**, 37001 (2009).
[13] H. Haug and A. P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer, Berlin, 1996).
[14] A.Y. Kitaev, Phys. Usp. **44**, 131 (2001).