Supplementary material for "Modeling and Inferences for Bivariate Signed Integer-valued Autoregressive Models" by Sangyeol Lee and Minyoung Jo

Table 1: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2)$, $\phi = 0$, and no outliers exist.

γ	statistics	estimators								$\mathrm{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.482	0.202	0.195	0.393	1.995	2.993	2.974	2.023	
	variance $\times 10^3$	4.601	5.022	2.729	4.819	45.121	49.988	37.961	53.488	**1.815**
	MSE $\times 10^2$	**0.433**	**0.377**	**0.272**	**0.457**	**4.469**	**4.952**	**3.824**	**4.867**	
0.1	mean	0.509	0.205	0.208	0.404	2.018	3.063	3.041	2.018	
	variance $\times 10^3$	4.490	3.783	3.696	5.241	45.291	54.548	47.308	48.820	2.722
	MSE $\times 10^2$	0.453	0.410	0.373	0.481	4.517	5.809	4.852	4.952	
0.2	mean	0.507	0.205	0.207	0.403	2.013	3.062	3.036	2.018	
	variance $\times 10^3$	4.515	4.116	3.794	5.504	47.420	59.338	48.589	49.694	2.908
	MSE $\times 10^2$	0.453	0.430	0.380	0.520	4.712	6.263	4.944	5.349	
0.3	mean	0.499	0.205	0.201	0.409	1.983	3.058	3.042	2.012	
	variance $\times 10^3$	4.378	4.315	4.037	4.525	53.299	66.968	49.912	53.976	5.177
	MSE $\times 10^2$	0.487	0.466	0.399	0.535	5.302	6.970	5.118	5.358	
0.5	mean	0.506	0.203	0.204	0.401	2.002	3.054	3.022	2.019	
	variance $\times 10^3$	5.150	4.704	4.251	5.410	53.870	61.422	52.455	54.730	6.832
	MSE $\times 10^2$	0.514	0.497	0.422	0.546	5.333	6.381	5.242	5.457	
1	mean	0.505	0.199	0.199	0.399	1.991	3.042	3.006	2.023	
	variance $\times 10^3$	6.476	5.580	5.231	6.133	70.057	72.963	59.274	66.575	12.127
	MSE $\times 10^2$	0.644	0.552	0.517	0.607	6.943	7.404	5.872	6.647	

Table 2: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2), \phi = 0.5$, and no outliers exist.

γ	statistics	estimators								$\text{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.490	0.191	0.197	0.400	2.011	2.953	2.957	2.030	**2.121**
	variance $\times 10^3$	2.804	3.238	2.148	3.350	47.453	51.233	46.419	50.038	
	MSE $\times 10^2$	**0.286**	**0.327**	**0.213**	**0.331**	**4.712**	**4.344**	**4.705**	**5.049**	
0.1	mean	0.532	0.206	0.205	0.416	2.081	3.037	3.037	2.090	3.663
	variance $\times 10^3$	3.142	3.363	2.371	3.891	48.259	49.003	53.801	49.169	
	MSE $\times 10^2$	0.418	0.336	0.237	0.412	5.444	4.564	4.776	5.073	
0.2	mean	0.533	0.205	0.205	0.415	2.083	3.034	3.030	2.096	3.765
	variance $\times 10^3$	3.518	3.881	2.592	4.097	48.959	48.780	51.279	49.574	
	MSE $\times 10^2$	0.438	0.387	0.259	0.429	5.546	4.904	5.058	5.682	
0.3	mean	0.525	0.210	0.215	0.410	2.077	3.039	3.038	2.060	4.515
	variance $\times 10^3$	3.770	3.887	3.550	4.006	40.562	47.977	46.032	47.519	
	MSE $\times 10^2$	0.460	0.395	0.375	0.406	4.619	4.950	5.170	5.845	
0.5	mean	0.532	0.205	0.203	0.412	2.077	3.007	3.013	2.097	6.811
	variance $\times 10^3$	4.434	5.073	3.627	5.064	53.930	43.820	50.915	49.634	
	MSE $\times 10^2$	0.547	0.505	0.360	0.517	5.943	4.994	5.445	5.863	
1	mean	0.531	0.200	0.197	0.410	2.061	2.978	2.993	2.092	10.019
	variance $\times 10^3$	5.719	6.502	4.793	5.813	65.399	45.640	54.952	56.436	
	MSE $\times 10^2$	0.664	0.643	0.474	0.585	6.854	5.286	5.466	6.446	

Table 3: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0$, and no outliers exist.

γ	statistics	estimators								tr$(As.\hat{var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.479	0.187	0.288	0.481	3.000	2.971	2.002	1.972	**1.985**
	variance $\times 10^3$	2.762	3.849	2.818	2.430	61.601	59.856	42.816	40.158	
	MSE $\times 10^2$	**0.315**	**0.396**	**0.292**	**0.273**	**6.098**	**6.011**	**4.239**	**4.051**	
0.1	mean	0.524	0.216	0.295	0.506	3.067	3.046	2.015	2.046	2.347
	variance $\times 10^3$	4.491	4.818	2.254	4.033	70.016	48.620	39.329	51.071	
	MSE $\times 10^2$	0.428	0.505	0.225	0.373	5.234	5.027	3.918	5.043	
0.2	mean	0.510	0.212	0.318	0.495	3.016	3.046	2.034	1.999	2.822
	variance $\times 10^3$	4.220	6.695	2.874	4.076	52.607	73.292	58.886	50.944	
	MSE $\times 10^2$	0.502	0.634	0.314	0.404	5.265	5.501	4.909	5.273	
0.3	mean	0.512	0.223	0.301	0.506	3.044	3.028	2.035	2.002	3.909
	variance $\times 10^3$	5.140	6.509	3.374	4.421	51.192	54.754	48.318	58.373	
	MSE $\times 10^2$	0.524	0.679	0.316	0.405	6.086	6.005	5.948	5.305	
0.5	mean	0.514	0.206	0.311	0.486	3.032	3.048	1.967	1.983	7.217
	variance $\times 10^3$	5.491	6.366	3.068	3.602	73.180	58.323	70.384	62.879	
	MSE $\times 10^2$	0.562	0.701	0.334	0.442	7.348	6.081	6.946	5.779	
1	mean	0.519	0.202	0.295	0.502	3.001	2.982	2.010	2.010	13.518
	variance $\times 10^3$	6.989	8.708	3.540	4.629	61.477	61.100	70.065	53.480	
	MSE $\times 10^2$	0.731	0.862	0.352	0.458	7.381	7.472	7.073	6.252	

Table 4: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0.5$, and no outliers exist.

γ	statistics	estimators								$\text{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.477	0.194	0.290	0.479	2.950	2.935	2.060	2.006	**3.014**
	variance $\times 10^3$	2.539	3.454	2.044	2.758	43.301	51.233	41.637	52.868	
	MSE $\times 10^2$	**0.301**	**0.345**	**0.211**	**0.316**	**4.535**	**5.483**	**4.489**	**5.238**	
0.1	mean	0.530	0.217	0.313	0.508	3.064	3.025	2.106	2.105	3.812
	variance $\times 10^3$	2.935	3.889	2.898	4.135	44.753	62.499	46.443	49.833	
	MSE $\times 10^2$	0.374	0.416	0.304	0.417	4.849	5.105	4.152	4.184	
0.2	mean	0.524	0.212	0.319	0.503	3.030	3.054	2.102	2.088	4.025
	variance $\times 10^3$	3.156	5.477	2.811	4.665	50.410	60.975	41.998	46.603	
	MSE $\times 10^2$	0.384	0.491	0.312	0.440	5.065	5.962	4.301	4.388	
0.3	mean	0.527	0.215	0.304	0.509	3.028	3.046	2.112	2.096	5.341
	variance $\times 10^3$	4.576	5.431	3.136	4.365	50.365	70.849	55.542	67.989	
	MSE $\times 10^2$	0.495	0.558	0.316	0.463	5.081	6.254	5.213	5.388	
0.5	mean	0.531	0.209	0.324	0.490	3.029	3.019	2.069	2.055	9.091
	variance $\times 10^3$	4.013	5.786	3.657	7.689	64.989	51.172	37.081	39.167	
	MSE $\times 10^2$	0.530	0.561	0.389	0.494	5.905	6.334	5.721	6.036	
1	mean	0.526	0.216	0.312	0.508	2.940	2.935	2.081	2.080	16.218
	variance $\times 10^3$	4.821	4.689	3.767	4.921	56.011	56.011	36.808	37.781	
	MSE $\times 10^2$	0.546	0.582	0.420	0.769	6.518	7.230	6.767	7.655	

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Table 5: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2), \phi = 0$, and outliers exist.

γ	statistics	estimators								$\mathrm{tr}(As.\hat{var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.359	0.135	0.146	0.292	2.648	3.550	3.509	2.594	3.822
	variance $\times 10^3$	4.960	4.304	3.833	5.165	102.817	94.428	97.152	106.611	
	MSE $\times 10^2$	2.463	0.841	0.666	1.665	52.297	39.693	35.587	45.890	
0.1	mean	0.397	0.121	0.150	0.354	2.662	3.991	3.865	2.693	3.804
	variance $\times 10^3$	5.415	5.478	4.681	7.580	103.331	129.009	123.604	131.032	
	MSE $\times 10^2$	1.582	1.170	0.711	0.956	54.107	111.021	87.120	61.111	
0.2	mean	0.417	0.154	0.151	0.318	2.242	3.319	3.233	2.250	**2.912**
	variance $\times 10^3$	6.030	3.679	4.031	6.836	59.711	59.869	70.969	60.685	
	MSE $\times 10^2$	1.270	**0.570**	**0.633**	1.337	**11.783**	**16.135**	**12.483**	**12.271**	
0.3	mean	0.430	0.131	0.153	0.374	2.399	3.772	3.668	2.436	4.531
	variance $\times 10^3$	4.126	4.739	4.830	6.376	99.685	114.971	113.431	103.967	
	MSE $\times 10^2$	0.896	0.936	0.690	0.697	25.860	71.007	55.978	29.369	
0.5	mean	0.451	0.145	0.162	0.399	2.297	3.631	3.555	2.325	6.169
	variance $\times 10^3$	4.151	5.527	5.577	6.359	97.248	100.633	116.944	103.023	
	MSE $\times 10^2$	0.642	0.848	0.693	**0.629**	18.456	49.865	42.440	20.788	
1	mean	0.489	0.174	0.178	0.483	2.173	3.451	3.389	2.195	12.217
	variance $\times 10^3$	4.436	8.071	7.469	8.146	90.461	94.461	106.081	133.363	
	MSE $\times 10^2$	**0.451**	0.864	0.786	0.954	11.987	29.714	25.675	17.023	

Table 6: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2), \phi = 0.5$, and outliers exist.

γ	statistics	estimators								$\mathrm{tr}(A\hat{s.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.400	0.137	0.153	0.314	2.540	3.409	3.377	2.549	3.715
	variance $\times 10^3$	4.201	3.371	3.128	3.470	117.109	97.991	93.450	120.109	
	MSE $\times 10^2$	1.404	0.721	0.529	1.071	40.806	26.490	23.478	42.113	
0.1	mean	0.458	0.164	0.175	0.349	2.488	3.388	3.308	2.377	3.231
	variance $\times 10^3$	4.871	4.112	3.121	5.308	97.680	64.430	57.436	70.245	
	MSE $\times 10^2$	0.681	0.631	0.522	0.853	33.513	21.448	15.372	21.196	
0.2	mean	0.464	0.163	0.173	0.365	2.296	3.276	3.268	2.309	4.211
	variance $\times 10^3$	3.908	3.381	4.216	6.084	76.706	72.749	82.480	76.106	
	MSE $\times 10^2$	0.655	0.531	0.489	0.826	16.784	14.828	15.190	17.115	
0.3	mean	0.467	0.172	0.169	0.353	2.302	3.229	3.169	2.220	6.214
	variance $\times 10^3$	5.402	4.120	3.385	6.185	77.053	51.493	48.617	64.319	
	MSE $\times 10^2$	0.637	0.498	0.444	0.814	16.414	10.384	7.691	11.243	
0.5	mean	0.478	0.185	0.172	0.363	2.234	3.157	3.104	2.171	5.112
	variance $\times 10^3$	5.947	4.812	3.719	6.870	69.946	50.782	46.480	65.158	
	MSE $\times 10^2$	0.633	0.482	0.429	0.779	12.401	7.514	5.686	9.381	
1	mean	0.490	0.200	0.177	0.380	2.178	3.096	3.052	2.130	**2.036**
	variance $\times 10^3$	6.786	6.373	4.851	8.219	65.034	47.870	53.786	74.489	
	MSE $\times 10^2$	**0.514**	**0.471**	**0.369**	**0.722**	**9.629**	**5.670**	**5.600**	**9.070**	

Table 7: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0$, and outliers exist.

γ	statistics	estimators								tr($\hat{As.var}$)
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.392	0.121	0.238	0.301	3.479	3.501	2.791	2.751	5.119
	variance $\times 10^3$	4.571	4.122	5.233	10.545	82.681	73.353	118.079	117.270	
	MSE $\times 10^2$	1.600	1.020	0.901	4.983	31.216	32.394	74.336	68.075	
0.1	mean	0.456	0.155	0.264	0.367	3.381	3.419	2.499	2.490	3.217
	variance $\times 10^3$	4.091	4.509	3.988	7.787	70.391	97.860	89.105	69.540	
	MSE $\times 10^2$	0.595	**0.642**	0.519	2.519	21.449	27.247	33.748	30.951	
0.2	mean	0.474	0.151	0.278	0.403	3.288	3.278	2.267	2.282	4.858
	variance $\times 10^3$	7.167	6.298	3.262	6.562	94.224	80.802	47.435	96.949	
	MSE $\times 10^2$	0.774	0.857	**0.370**	1.583	17.664	15.770	11.859	17.574	
0.3	mean	0.464	0.177	0.269	0.421	3.205	3.178	2.263	2.224	7.215
	variance $\times 10^3$	4.527	6.014	3.683	8.524	73.561	68.152	76.867	72.781	
	MSE $\times 10^2$	**0.572**	0.645	0.459	1.464	11.525	9.916	14.535	12.241	
0.5	mean	0.465	0.198	0.275	0.445	3.156	3.158	2.186	2.189	6.823
	variance $\times 10^3$	5.368	8.288	4.949	9.236	71.952	69.489	103.485	62.488	
	MSE $\times 10^2$	0.649	0.820	0.547	1.212	**9.560**	9.379	13.842	9.794	
1	mean	0.502	0.202	0.292	0.503	3.154	3.117	2.082	2.107	**2.989**
	variance $\times 10^3$	6.639	9.257	4.407	5.915	75.865	80.658	68.810	75.534	
	MSE $\times 10^2$	0.657	0.917	0.441	**0.586**	9.911	**9.375**	**7.485**	**8.628**	

Table 8: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0.5$, and outliers exist.

γ	statistics	estimators								$\text{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.407	0.158	0.253	0.352	3.335	3.396	2.704	2.732	6.101
	variance $\times 10^3$	3.856	4.466	4.914	7.601	85.716	86.890	125.797	143.755	
	MSE $\times 10^2$	1.231	0.615	0.704	2.923	19.755	24.338	62.026	67.863	
0.1	mean	0.365	0.110	0.229	0.290	3.792	3.833	3.059	3.142	5.935
	variance $\times 10^3$	2.743	3.439	6.536	9.276	163.374	135.454	220.784	223.580	
	MSE $\times 10^2$	2.080	1.140	1.144	5.311	78.989	82.809	134.099	152.656	
0.2	mean	0.484	0.183	0.284	0.428	3.212	3.251	2.395	2.385	10.210
	variance $\times 10^3$	4.742	5.792	3.693	8.053	61.820	68.633	79.088	81.756	
	MSE $\times 10^2$	**0.493**	0.601	**0.388**	1.305	10.645	13.112	23.434	22.935	
0.3	mean	0.489	0.189	0.277	0.448	3.160	3.176	2.323	2.323	12.264
	variance $\times 10^3$	5.579	5.298	3.497	7.338	49.805	55.533	59.419	78.161	
	MSE $\times 10^2$	0.564	0.535	0.397	0.989	7.491	**8.615**	16.370	18.207	
0.5	mean	0.505	0.180	0.292	0.463	3.098	3.158	2.231	2.256	**2.853**
	variance $\times 10^3$	5.592	4.708	5.450	6.540	60.666	68.969	57.592	85.551	
	MSE $\times 10^2$	0.557	**0.505**	0.545	**0.778**	**6.970**	9.352	**11.046**	**15.074**	
1	mean	0.438	0.139	0.247	0.426	3.261	3.277	2.353	2.384	5.257
	variance $\times 10^3$	5.078	8.084	5.987	7.337	72.304	88.478	84.953	98.477	
	MSE $\times 10^2$	0.881	1.160	0.872	1.265	13.988	16.461	20.883	24.552	

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Table 9: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2), \phi = 0$, and a big outlier exists.

γ	statistics	estimators								$\mathrm{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.337	0.105	0.133	0.272	2.514	3.106	3.389	2.105	7.213
	variance $\times 10^3$	2.916	4.459	4.442	4.532	86.098	82.197	88.097	84.173	
	MSE $\times 10^2$	2.822	1.145	0.685	1.865	31.058	26.308	40.887	15.495	
0.1	mean	0.491	0.171	0.198	0.421	2.249	3.605	3.617	2.334	6.553
	variance $\times 10^3$	2.277	4.697	4.035	2.931	72.194	88.122	116.673	83.520	
	MSE $\times 10^2$	0.232	0.548	**0.400**	**0.333**	13.379	45.374	49.630	19.478	
0.2	mean	0.495	0.194	0.207	0.456	2.235	3.545	3.497	2.309	4.251
	variance $\times 10^3$	2.350	4.583	4.204	3.574	56.056	74.960	86.060	67.281	
	MSE $\times 10^2$	0.234	**0.456**	0.421	0.678	11.118	37.221	33.295	16.234	
0.3	mean	0.514	0.186	0.201	0.457	2.257	3.475	3.520	2.248	4.369
	variance $\times 10^3$	1.969	4.631	4.741	3.946	84.815	82.753	91.604	84.782	
	MSE $\times 10^2$	**0.216**	0.475	0.469	0.722	15.043	30.849	36.131	14.552	
0.5	mean	0.519	0.186	0.200	0.464	2.208	3.415	3.460	2.207	6.220
	variance $\times 10^3$	2.267	5.065	5.497	4.425	86.134	83.850	94.374	90.839	
	MSE $\times 10^2$	0.263	0.521	0.544	0.859	12.886	25.556	30.547	13.282	
1	mean	0.525	0.181	0.190	0.462	2.116	3.189	3.363	2.123	**1.894**
	variance $\times 10^3$	3.511	6.390	6.349	6.656	90.613	94.321	126.191	111.817	
	MSE $\times 10^2$	0.413	0.667	0.638	1.046	**10.334**	**17.741**	**25.726**	**12.586**	

Table 10: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.2, 0.4, 2, 3, 3, 2), \phi = 0.5$, and a big outlier exists.

γ	statistics	estimators								$\mathrm{tr}(A\hat{s}.var)$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.328	0.107	0.124	0.275	2.528	3.182	3.379	2.417	8.003
	variance $\times 10^3$	2.102	3.083	2.183	2.437	72.628	67.215	55.323	76.792	
	MSE $\times 10^2$	3.150	1.153	0.782	1.791	35.091	37.329	33.296	28.973	
0.1	mean	0.449	0.142	0.165	0.395	2.524	3.605	3.556	2.488	6.267
	variance $\times 10^3$	2.092	4.175	3.588	3.591	112.222	89.754	82.526	107.946	
	MSE $\times 10^2$	0.460	0.740	0.471	**0.357**	38.643	45.575	39.150	34.557	
0.2	mean	0.462	0.153	0.173	0.418	2.472	3.525	3.495	2.436	5.154
	variance $\times 10^3$	2.649	4.511	3.600	3.344	100.803	80.910	76.806	92.398	
	MSE $\times 10^2$	0.400	0.660	**0.426**	0.364	32.267	35.581	32.133	28.184	
0.3	mean	0.471	0.158	0.180	0.423	2.435	3.467	3.455	2.401	7.903
	variance $\times 10^3$	2.350	4.340	4.053	3.535	82.913	70.145	72.099	87.600	
	MSE $\times 10^2$	0.316	**0.601**	0.438	0.403	27.180	28.838	27.907	24.831	
0.5	mean	0.478	0.156	0.184	0.430	2.396	3.408	3.409	2.363	4.010
	variance $\times 10^3$	2.961	4.715	4.335	3.798	79.034	68.091	64.388	85.059	
	MSE $\times 10^2$	0.337	0.654	0.452	0.468	23.507	23.410	23.109	21.623	
1	mean	0.496	0.152	0.181	0.439	2.305	3.262	3.284	2.257	**2.782**
	variance $\times 10^3$	3.224	5.175	5.486	5.050	74.625	62.800	60.695	93.587	
	MSE $\times 10^2$	**0.320**	0.739	0.577	0.655	**16.710**	**13.119**	**14.092**	**15.897**	

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Table 11: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0$, and a big outlier exists.

γ	statistics	estimators								$\mathrm{tr}(\hat{As.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.331	0.095	0.190	0.312	3.431	3.085	2.675	2.200	4.122
	variance $\times 10^3$	1.729	2.339	2.625	3.517	62.311	66.613	72.772	89.273	
	MSE $\times 10^2$	3.009	1.315	1.462	3.880	24.711	17.327	52.781	12.854	
0.1	mean	0.459	0.145	0.284	0.436	3.459	3.497	2.429	2.443	5.115
	variance $\times 10^3$	2.698	5.622	4.170	4.685	97.206	71.053	116.627	108.364	
	MSE $\times 10^2$	0.428	0.848	0.438	0.871	30.757	31.793	30.030	30.375	
0.2	mean	0.476	0.144	0.285	0.455	3.433	3.442	2.357	2.333	5.948
	variance $\times 10^3$	3.276	5.606	3.015	4.155	65.272	78.245	82.906	120.361	
	MSE $\times 10^2$	0.380	0.857	0.320	0.605	25.234	27.301	20.996	23.010	
0.3	mean	0.473	0.170	0.280	0.471	3.449	3.437	2.356	2.355	6.112
	variance $\times 10^3$	2.629	6.014	2.656	3.456	71.451	71.426	72.689	100.594	
	MSE $\times 10^2$	0.329	**0.683**	**0.299**	**0.422**	27.307	26.210	19.917	22.585	
0.5	mean	0.481	0.157	0.278	0.477	3.371	3.346	2.211	2.268	5.078
	variance $\times 10^3$	2.840	7.310	4.044	4.863	116.421	109.163	93.647	107.599	
	MSE $\times 10^2$	**0.314**	0.908	0.448	0.532	25.291	22.836	13.763	17.853	
1	mean	0.497	0.166	0.286	0.477	3.220	3.209	2.130	2.143	**3.246**
	variance $\times 10^3$	5.404	10.186	4.692	5.228	100.859	60.149	84.740	81.243	
	MSE $\times 10^2$	0.535	1.120	0.482	0.570	**14.827**	**10.329**	**10.079**	**10.096**	

Table 12: Sample mean, variance, and MSE of estimators for the BSINAR(1) model when $(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \psi_{11}, \psi_{12}, \psi_{21}, \psi_{22}) = (0.5, 0.2, 0.3, 0.5, 3, 3, 2, 2), \phi = 0.5$, and a big outlier exists.

γ	statistics	estimators								$\mathrm{tr}(A\hat{s.var})$
		$\hat{\mu}_{11,n}$	$\hat{\mu}_{12,n}$	$\hat{\mu}_{21,n}$	$\hat{\mu}_{22,n}$	$\hat{\psi}_{11,n}$	$\hat{\psi}_{12,n}$	$\hat{\psi}_{21,n}$	$\hat{\psi}_{22,n}$	
0(QMLE)	mean	0.325	0.097	0.178	0.275	3.325	3.390	2.644	2.330	8.226
	variance $\times 10^3$	1.574	1.734	3.150	4.366	112.475	82.293	149.495	163.423	
	MSE $\times 10^2$	3.197	1.229	1.778	5.473	21.725	8.970	56.347	27.090	
0.1	mean	0.414	0.132	0.252	0.374	3.468	3.528	2.559	2.617	9.211
	variance $\times 10^3$	3.050	4.614	4.253	6.138	118.359	106.733	171.623	172.299	
	MSE $\times 10^2$	1.032	0.915	0.646	2.193	33.652	38.452	48.265	55.197	
0.2	mean	0.440	0.129	0.264	0.401	3.427	3.411	2.542	2.546	8.834
	variance $\times 10^3$	2.793	3.435	4.833	6.899	88.413	92.857	130.642	134.567	
	MSE $\times 10^2$	0.627	0.833	0.602	1.659	26.992	26.126	42.344	43.155	
0.3	mean	0.442	0.150	0.267	0.408	3.389	3.429	2.424	2.445	7.011
	variance $\times 10^3$	3.111	4.727	5.981	6.089	99.756	107.494	140.457	102.989	
	MSE $\times 10^2$	0.644	**0.710**	0.697	1.444	25.037	29.051	31.906	30.058	
0.5	mean	0.436	0.150	0.262	0.416	3.321	3.333	2.371	2.395	3.964
	variance $\times 10^3$	2.841	6.140	4.038	5.374	69.652	80.009	86.088	75.130	
	MSE $\times 10^2$	0.690	0.848	**0.539**	1.236	17.205	19.026	22.358	23.070	
1	mean	0.474	0.130	0.257	0.445	3.185	3.169	2.301	2.230	**3.826**
	variance $\times 10^3$	3.019	5.653	4.821	6.623	80.451	57.844	104.112	79.826	
	MSE $\times 10^2$	**0.362**	1.039	0.658	**0.954**	**11.422**	**8.608**	**19.420**	**13.229**	