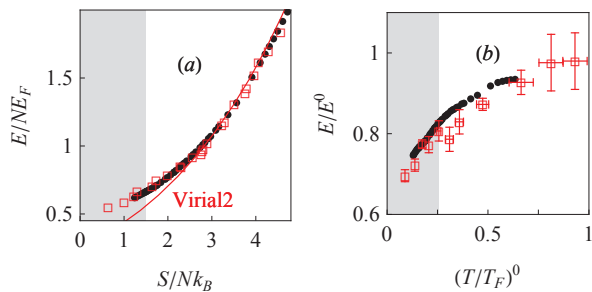## I.  SUPPLEMENTARY FIGURES



**Supplementary Figure 1:** (color online) Equations of state of the trapped unitary gas. (a) Comparison between our EOS $E/NE_F$ as a function of $S/Nk_B$ (black dots) and the EOS measured in [2] (open red squares). The red solid line is the second-order virial equation of state. (b) Comparison between our EOS $E/E^0$ as a function of $(T/T_F)^0$ (black dots) and the EOS measured on $^{40}$K in [1] (open red squares). The grey regions correspond to the superfluid phase.

## II.  SUPPLEMENTARY DISCUSSION

### A.  Equation of State of the Trapped Unitary Gas

In this work, we have measured the equation of state of the homogeneous unitary gas. We can deduce from our data the EOS of the trapped balanced unitary gas, which has been measured in [1, 2].

Using the local density approximation, the total atom number $N = \int n \, dr^3$ is expressed as a function of the temperature $T$ and the chemical potential $\mu^0$ at the center, involving the function $h(1,\zeta)$:

$$N = \frac{-2}{\sqrt{\pi}} \left( \frac{k_B T}{\hbar \omega} \right)^3 \int_{\zeta_0}^{\infty} \frac{d \log^{1/2}(\zeta/\zeta_0)}{d\zeta} f_{5/2}(-\zeta^{-1}) h(\zeta) d\zeta, \tag{1}$$

where $\zeta_0 = \exp(-\mu^0/k_B T)$ and $\omega = (\omega_r^2 \omega_z)^{1/3}$. We use for the calculation a discretized version of (1) taken solely on our experimental values of $h$, *i.e.* without using any interpolating or fitting function. Similar expressions are used to calculate the Fermi temperature $E_F = k_B T_F = \hbar \omega (3N)^{1/3}$, the total entropy $S$ and energy $E$ of the cloud. The equation of state $E/NE_F$ as a function of $S/Nk_B$, displayed in Supplementary Fig.1a, is in very good agreement with [2].

The normal-superfluid phase transition for the trapped gas occurs when at the trap center $\zeta_0 = \zeta_c =$

$\exp(-(k_B T/\mu)_c^{-1})$, with $(k_B T/\mu)_c = 0.32(3)$, as measured on the homogeneous EOS $h(1,\zeta)$. At this point we get $(T/T_F)_c = 0.19(2)$, $(S/Nk_B)_c = 1.5(1)$ and $(E/NE_F)_c = 0.67(5)$.

In order to make the comparison with [1], we also express the equation of state $E/E^0$ as a function of $(T/T_F)^0$, where the superscript $^0$ refers to the quantities evaluated on a non-interacting Fermi gas having the same entropy (Supplementary Fig.1b). The good agreement with the measurement in [1], performed on $^{40}$K clouds, illustrates the universality of the unitary gas.

### B.  Physical interpretation of the pressure in the normal mixed phase

We have shown that the pressure in the normal mixed phase can be described as the sum of the Fermi pressures of ideal gases of majority atoms and of polarons:

$$P = \frac{1}{15\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \left( \mu_1^{5/2} + \left( \frac{m_p^*}{m} \right)^{3/2} (\mu_2 - A\mu_1)^{5/2} \right). \tag{2}$$

Here, we evaluate the corresponding canonical EOS relating the energy $E$ to the densities $n_1, n_2$, in order to compare with the Fixed Node Monte Carlo prediction [3]. Since at unitarity we have $E = 3PV/2$ [4], we just express the chemical potentials in terms of densities by using the thermodynamical identities $n_i = \partial_{\mu_i} P$, which yield respectively:

$$n_2 = \frac{1}{10\pi^2} \left[ \frac{2m^*}{\hbar^2} (\mu_2 - A\mu_1) \right]^{3/2} \tag{3}$$

$$n_1 = \frac{1}{10\pi^2} \left( \frac{2m\mu_1}{\hbar^2} \right)^{3/2} - An_2. \tag{4}$$

The last term in equation (4) clearly indicates the increased majority density due to the presence of the minority component. Expressing the pressure as a function of $n_i$ in (2) yields the energy:

$$E = E_{\text{FP}} \left[ (1 + Ax)^{5/3} + \frac{m}{m^*} x^{5/3} \right],$$

where $E_{FP}$ is the energy of the fully polarized gas and $x = n_2/n_1$. Expanding $E$ to order $x^2$ finally leads to an expression similar to that obtained in [3]:

$$E(x) = E_{\text{FP}} \left( 1 + \frac{5}{3} Ax + \frac{m}{m^*} x^{5/3} + Bx^2 + .... \right),$$

with $B = 5A^2/9 = 0.2$. Our value of $B$ is close to the calculated value $B \simeq 0.14$ from [3].

## C.  Trap Anharmonicity

First, in the axial direction $z$, the confinement is produced magnetically and the corresponding anharmonicity is negligible. In the radial direction, we develop the gaussian potential to fourth order around $\rho = 0$:

$$V_r(\rho) = V_0 \left( 1 - \exp \frac{-\rho^2}{\sigma^2} \right) \simeq \frac{1}{2} m \omega_r^2 \rho^2 + \epsilon \rho^4,$$

where $m\omega_r^2 = 2V_0/\sigma^2$ and $\epsilon = -V_0/2\sigma^4$. In the balanced case, we have

$$\overline{n}(z) = \int d^2\rho \, n \left( \mu^0 - \frac{1}{2}m\omega_z^2 z^2 - \frac{1}{2}m\omega_r^2\rho^2 - \epsilon\rho^4 \right).$$

Introducing $n = \partial P/\partial \mu$ and defining $u = m\omega_r^2\rho^2/2 + \epsilon\rho^4$ we obtain, to lowest order,

$$\frac{m\omega_r^2}{2\pi}\overline{n}(z) = P(\mu_z) + \int_0^\infty P(\mu_z - u)\frac{du}{V_0}.$$

The error on the measurement of $h$ is then

$$\frac{m\omega_r^2 \, \overline{n}(z)}{2\pi P_1(\mu_z, T)} - h(1,\zeta) = \frac{k_B T}{V_0} \int_\zeta^\infty \frac{f_{5/2}(-\zeta'^{-1})}{f_{5/2}(-\zeta^{-1})} \frac{h(1,\zeta')}{\zeta'} d\zeta'. \tag{5}$$

We evaluate the integral in (5) using the experimental values of $h(1,\zeta)$. In our shallowest trap, the worst case anharmonicity effect is 5%.

## D.  An exact inequality on the equation of state of an attractive Fermi gas

Writing the hamiltonian as $\widehat{H} = \widehat{H}_0 + \widehat{U}$, where $\widehat{H}_0$ is the single-particle part of the hamiltonian and $\widehat{U}$ is the inter-particle interaction, one has the general inequality $\Omega \le \Omega_0 + \langle V \rangle_0$, where $\Omega_0$ is the grand potential associated with $\widehat{H}_0$ and $\langle \cdot \rangle_0$ is the thermal average related to $\widehat{H}_0$ [5]. Taking for $U$ a short range square potential of depth $U_0 < 0$ recovering the true scattering length, one has trivially $\langle \widehat{V} \rangle_0 < 0$, hence $\Omega \le \Omega_0$. Using the thermodynamic identity $\Omega = -PV$, and recalling that $\Omega_0 = -2P_1 V$ and $h = P/P_1$, we finally get the inequality

$$h(1,\zeta) \ge 2.$$

## E.  Extension to a Multi-Component System

We extend the equation (2) to a mixture of species $i$, of mass $m_i$, trapped in a harmonic trap of transverse frequencies $\omega_{ri}$, following the calculations in [6]. Using Gibbs-Duhem relation at a constant temperature $T$, $dP = \sum_i n_i d\mu_i$, then

$$\sum_i \frac{m_i\omega_{ri}^2}{2\pi}\overline{n}_i = \int \sum_i \frac{m_i\omega_{ri}^2}{2\pi} dx dy \frac{\partial P}{\partial \mu_i} = \int \sum_i d\mu_i \frac{\partial P}{\partial \mu_i},$$

where we have used local density approximation ($\mu_i(\mathbf{r}) = \mu_i^0 - V(\mathbf{r})$) to convert the integral over space to an integral on the chemical potentials. The integral is straightforward and yields to

$$P(\mu_{iz}, T) = \frac{1}{2\pi}\sum_i m_i\omega_{ri}^2\overline{n}_i(z).$$

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