Table S1: The stellar sample including atmospheric parameters and Na abundances.

Type	ID	T_{eff}	log (g)	ξ	log (N_{Na})	[Na/Fe]	Error
AGB	FGJ000022	4607	1.414	1.765	4.85	0.06	0.09
AGB	FGJ000025	4371	1.146	1.851	4.71	-0.08	0.09
AGB	FGJ000031	4460	1.285	1.806	4.73	-0.06	0.09
AGB	FGJ000044	4629	1.537	1.725	4.63	-0.16	0.07
AGB	FGJ000052	4787	1.740	1.660	4.74	-0.05	0.09
AGB	FGJ000053	4688	1.636	1.693	4.73	-0.06	0.10
AGB	FGJ000059	4772	1.719	1.666	4.82	0.03	0.06
AGB	FGJ000060	4685	1.654	1.687	4.58	-0.21	0.10
AGB	FGJ000061	4714	1.689	1.676	4.69	-0.10	0.10
AGB	FGJ000065	4677	1.540	1.724	4.91	0.12	0.14
AGB	FGJ000075	4763	1.764	1.652	4.55	-0.24	0.07
AGB	FGJ000076	4881	1.850	1.624	4.74	-0.05	0.05
AGB	FGJ000078	4868	1.855	1.623	4.82	0.03	0.07
AGB	FGJ000080	4829	1.843	1.627	4.67	-0.12	0.08
AGB	FGJ000083	4825	1.849	1.625	4.66	-0.13	0.08
AGB	FGJ000089	4861	1.868	1.618	4.80	0.01	0.04
AGB	FGJ000094	4925	1.937	1.596	4.75	-0.04	0.10
AGB	FGJ000097	4946	1.978	1.583	4.74	-0.05	0.05
AGB	FGJ000104	4874	1.907	1.606	4.56	-0.23	0.08
AGB	FGJ201620	4864	1.938	1.596	4.74	-0.05	0.11
RGB	FGJ000012	4270	1.062	1.878	5.06	0.27	0.12
RGB	FGJ000023	4360	1.181	1.840	5.18	0.39	0.12
RGB	FGJ000027	4425	1.290	1.805	4.75	-0.04	0.11
RGB	FGJ000029	4298	1.102	1.865	4.67	-0.12	0.12
RGB	FGJ000030	4294	1.070	1.876	5.13	0.34	0.10
RGB	FGJ000035	4439	1.353	1.784	5.41	0.62	0.10
RGB	FGJ000043	4443	1.359	1.782	5.49	0.70	0.10
RGB	FGJ000050	4404	1.267	1.812	5.02	0.23	0.13
RGB	FGJ000054	4496	1.487	1.741	4.92	0.13	0.12
RGB	FGJ000064	4436	1.353	1.784	5.42	0.63	0.12
RGB	FGJ000069	4583	1.587	1.709	5.34	0.55	0.12
RGB	FGJ000091	4665	1.776	1.648	5.12	0.33	0.11
RGB	FGJ000092	4612	1.711	1.669	4.73	-0.06	0.10
RGB	FGJ000107	4662	1.822	1.633	5.01	0.22	0.11
RGB	FGJ000129	4717	1.939	1.596	5.03	0.24	0.09
RGB	FGJ000155	4726	1.992	1.579	4.62	-0.17	0.07
RGB	FGJ000161	4775	2.052	1.559	5.17	0.38	0.05
RGB	FGJ000170	4794	2.083	1.549	5.33	0.54	0.12
RGB	FGJ000186	4800	2.117	1.538	4.70	-0.09	0.10
RGB	FGJ000193	4806	2.134	1.533	4.55	-0.24	0.04
RGB	FGJ000217	4813	2.161	1.524	5.22	0.43	0.05
RGB	FGJ000262	4855	2.252	1.495	5.13	0.34	0.09
RGB	FGJ000276	4858	2.260	1.492	5.15	0.36	0.11
RGB	FGJ200619	4760	1.940	1.595	5.43	0.64	0.11

The evolutionary status of each star is indicated in column 1. ID codes are designations of the current study. T_{eff}, log (g), and ξ are the surface temperature, gravity, and microturbulence values used in the abundance determinations. log (N_{Na}) and [Na/Fe] are the final Na abundances. The final column shows the internal errors in [Na/Fe].

Discussion

S2. Relationship between horizontal branch morphology and stellar composition

It has long been speculated that the composition differences between the first generation and second generation populations could have an effect on the colour-magnitude diagram structure of GCs[9]. Recent work has begun to provide some evidence to support this. For example, a new study on M4, which has both a red horizontal branch and a blue horizontal branch, has shown that all red horizontal branch stars in their sample are Na-poor, whilst all their blue horizontal branch stars are Na-rich[29]. They infer that the He content must be different between the two Na populations since it is not expected that Na (or N) could affect the position of stars in the horizontal branch, while He can[30] (see Supplementary Discussion S3 below). As mentioned in the main text, a sample of horizontal branch stars from the redder end of the horizontal branch of NGC 6752 (it only has a blue horizontal branch) was shown to exclusively contain Na-poor stars[17] (that study also reports one star with elevated Na abundance, but, as noted by the authors, the star has evolved off the horizontal branch and probably started from a much bluer position). The same stars have a uniform He abundance that is consistent with Big Bang theory predictions ($\mathrm{Y} = 0.245$), as expected for a first generation population. Thus it appears that the bluer (presumably Na-rich) horizontal branch stars must avoid AGB ascent – leaving only the redder, Na-poor horizontal branch stars to populate the AGB of NGC 6752.

As also mentioned in the main text, if we combine this information with our estimate of the proportion of stars that do not ascend the AGB, we can estimate what horizontal branch colour delineates the border between the two groups. Since our $uvby$ colour-magnitude diagram dataset is not complete at the bluest end of the horizontal branch, we obtained a very high quality UBV photometric dataset[19] for this purpose, courtesy of Dr. Yazan Momany at the European Southern Observatory (Chile). We counted horizontal branch stars in the $U, U - V$ plane, starting at the red edge of the horizontal branch at $U - V = 0.25$. The total number of horizontal branch stars was found to be 320. Thus we expect the reddest 96 stars (30%) to eventually ascend the AGB. We find that this number of stars corresponds to an 'ascension cut-off' in $U - V$ of -0.30. Interestingly this is exactly the colour for the Grundahl jump, a discontinuity in horizontal branch morphology which is seen in all GCs studied to date whose horizontal branch extends beyond $\mathrm{T}_{eff} \approx 11,500$ K[18,19]. Explanations of this discontinuity include radiative levitation of elements heavier than carbon and nitrogen in the high-temperature atmospheres of these stars[18], or the combination of post-zero-age horizontal branch evolution and diffusion effects[19]. At face value, it appears that all stars bluer than the Grundahl jump do not ascend the AGB, at least in NGC 6752. This may represent further evidence that there is some fundamental change in the stellar atmosphere structure and/or mass-loss physics occurring at the Grundahl jump temperature[18]. We note that it is these extremely blue horizontal branch stars that are considered to be the source of excess UV flux in the spectra of elliptical galaxies[31] as well as Galactic and extra-galactic GCs[32].

S3. Stellar model experiments

Theoretical models show that for a given metallicity and core mass at the tip of the red giant branch the position of a star along the horizontal branch is determined by the mass of the hydrogen-rich envelope. The lower the envelope mass, the bluer the star will be. If the envelope mass is extremely low ($\lesssim 10^{-2}$ $\mathrm{M}_{\odot}$)[33], a horizontal branch star will not ascend the AGB. These stars instead evolve directly to the WD cooling track. A horizontal branch star can have such a low envelope mass if it suffered extra mass-loss during the preceding red giant branch phase, or if it formed with an elevated helium abundance. In the latter case the higher He affects the evolution of the star such that it arrives on the horizontal branch with a lower total mass (for a given GC age). In the former case the mechanisms that might affect the mass loss rates are unknown, although rotation is a possibility. In both cases the stars populate the blue end of the horizontal branch[30].

Our stellar model experiments (see Figure 3 in main text) are based on standard assumptions for mass-loss rates, cluster age, and a possible helium variation between the two generations of δY $= 0.04$ (although we note a very recent study has just reported stronger constraints on Y)[34]. Different assumptions of mass-loss rate or cluster age could produce bluer (or redder) horizontal branch morphologies. Our simulations show that standard models cannot reproduce the observations and thus non-standard/improved models are needed. We are currently working on improved horizontal branch models and expect that other groups will also investigate the very high AGB failure rate phenomenon reported in this Letter. Finally we note that our rough test of enhanced mass-loss on the horizontal branch (Figure 3 in main text) resulted in a quite different evolution in the CMD – the star with a 20-fold increase in mass-loss evolved ‘downwards’ (towards lower luminosities), practically along the zero-age horizontal branch line, then spent a significant amount of time at higher luminosities after leaving the zero-age line. This different evolution should be taken into account in future theoretical and observational investigations. We thank a thoughtful referee for inspiring this final paragraph of discussion.

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