Supplementary Information for "Coherent optical wavelength conversion via cavity-optomechanics"

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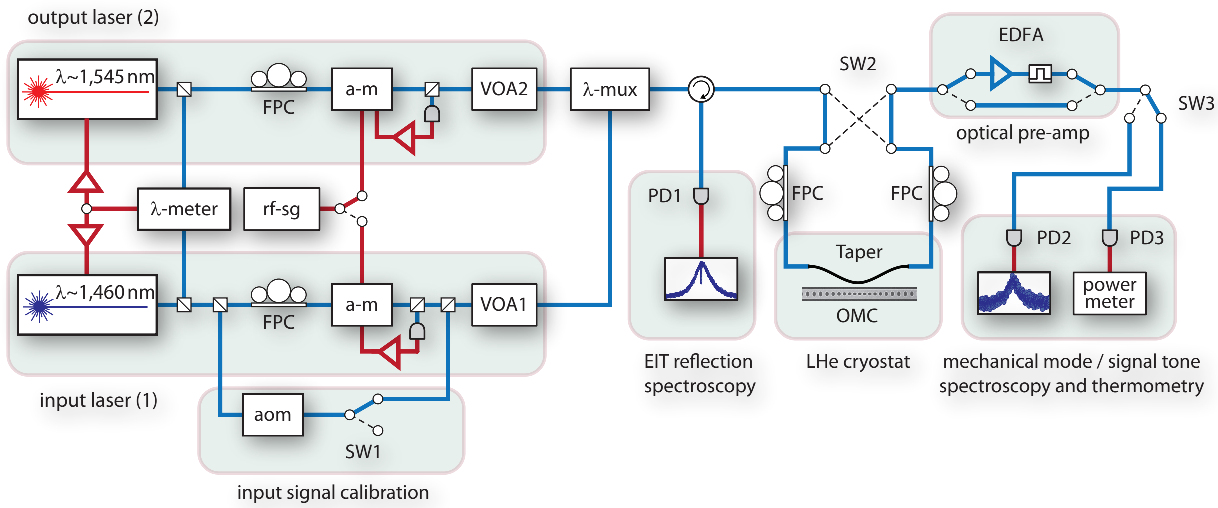
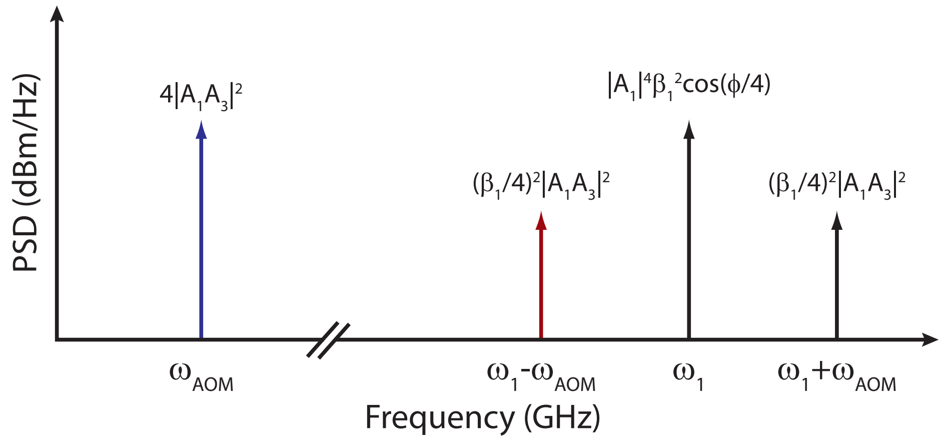
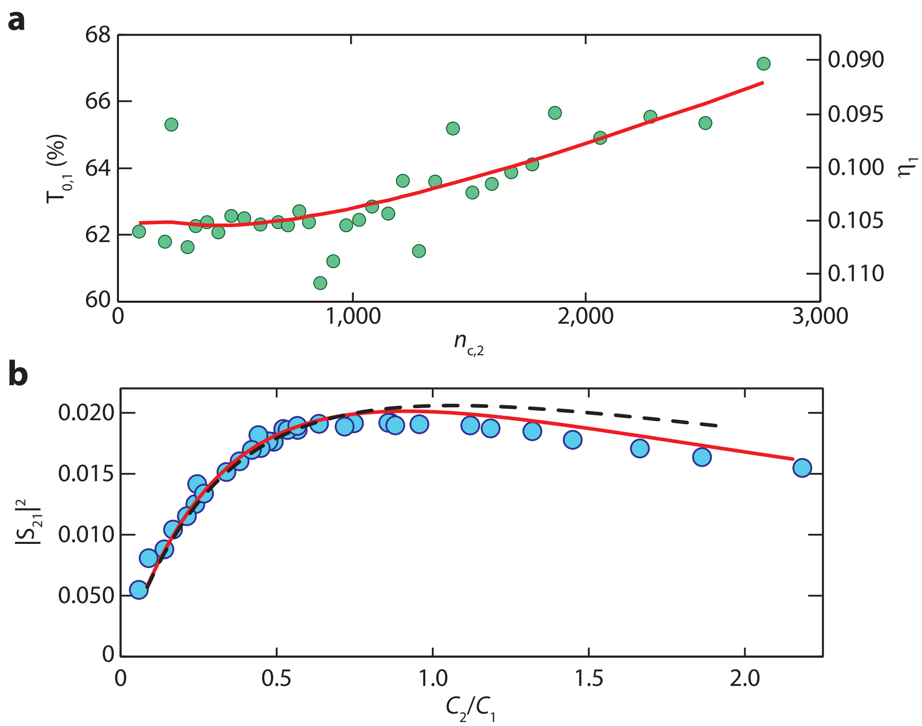
SUPPLEMENTARY FIGURES



Supplementary Figure S1: **Detailed schematic of the experimental setup.** Two tunable, external cavity diode lasers (ECDL) are used as control laser beams used to drive the wavelength conversion process. The wavelength of both lasers is monitored and locked to an absolute frequency of better than ± 5 MHz using a wavemeter (λ-meter). Both control beams can be amplitude modulated (a-m) using an RF signal generator (rf-sg), which produces the necessary optical sidebands to perform EIT-like spectroscopy (see main text) and to generate the input signal for the wavelength conversion process. An acousto-optic modulator (aom) is used to calibrate the input signal (switching in the calibration signal with SW1). The light from both lasers are attenuated (VOA1, VOA2) to the desired power, combined using a wavelength multiplexer (λ-mux), and then sent into the dimpled optical fibre taper which is used to optically couple to the optomechanical crystal. The optomechanical crystal sample and fibre taper are both located in a continuous-flow liquid helium cryostat. Before entering the cryostat, the light passes through fibre polarization controllers (FPC) to align the polarization of the light with that of the predominantly in-plane linear polarization of the optomechanical crystal cavity modes. The light also passes through an optical switch (SW2), allowing the light to be switched between either direction of propagation through the fibre taper, and providing an accurate calibration of the losses in the taper before and after the optomechanical crystal cavity (measuring the input-power-dependent optomechanical damping in both transmission directions allows one to separate the total loss of the taper into losses before and after the cavity). The transmitted light from the cavity is either optically pre-amplified using an erbium doped fibre amplifier (EDFA) and sent to a high-speed photodetector (PD2) for measurement of the microwave power spectrum, or using optical switch (SW3), sent to an optical power meter (PD3, power meter) for power calibration.



Supplementary Figure S2: **Calibration tones.** Schematic of the four tones appearing in the electronic power spectrum of the photocurrent generated by the optical input signal and the additional calibration signal. The three additional tones proportional to A_3 are generated to calibrate the input signal modulation index, β_1.



Supplementary Figure S3: **Effect of large intracavity photon population.** (**a**), Recorded coupling depth of the first order optical cavity as a function of the intracavity photon population of the second order mode (green circles). The black curve represents a fit to the extracted coupling depth. (**b**), Conversion efficiency as a function of C_2/C_1. The dashed black line represents the theoretical values from the nominal system parameters while the solid red line takes into account the effect of the large intracavity photon population of cavity mode 2 ($n_{c,2}$). The blue circles are the extracted data points and show much better correspondence to theory when this effect is accounted for.

SUPPLEMENTARY METHODS

Theory of two-mode optomechanical system

We begin by considering the Hamiltonian describing the optomechanical interaction between two distinct optical modes (indexed $k = 1, 2$) coupled to a shared mechanical mode with annihilation operator $\hat{b}$,

$$\hat{H} \;=\; \sum_k \hbar\delta_k \hat{a}_k^\dagger \hat{a}_k + \hbar\omega_\mathrm{m}\hat{b}^\dagger\hat{b} + (\hat{b} + \hat{b}^\dagger)\sum_k \hbar g_k \hat{a}_k^\dagger \hat{a}_k. \tag{S1}$$

Each optical mode has a frequency ω_k, and is driven by a laser at frequency $\omega_{\mathrm{l},k}$. The Hamiltonian above is written in the interaction picture with $\delta_k = \omega_k - \omega_{\mathrm{l},k}$.

As shown below, strong driving at a mechanical frequency red detuned from each cavity mode, $\delta_k \cong \omega_\mathrm{m}$, causes an effective beam splitter interaction to take place between the mechanical mode and each optical cavity mode at an enhanced coupling rate $G_k = g_k|\alpha_k|$, where α_k is the square root of the photon occupation in optical mode k. As long as this coupling is weak with respect to the optical linewidths κ_k ($G_k \ll \kappa_k$), an adiabatic elimination of the optical cavities results in new effective mechanical loss rates $\gamma_{\mathrm{OM},k}$ into each of the k optical degrees of freedom in the system. This "loss" can provide an effective coupling between the optical cavity modes by allowing the exchange of excitations between the optical resonances through the mechanical motion of the system. We calculate exactly what these conversion rates are by using a scattering matrix formulation to understand the behaviour of the system. Though completely general expressions can be derived, to best understand the processes involved we focus on the parameter regime relevant to this experiment, i.e. the weak-coupling, sideband-resolved case where $\omega_\mathrm{m} \gg \kappa \gg \gamma_\mathrm{OM}$, and follow a scattering matrix derivation of the induced optomechanical coupling between the optical waveguides coupled to each cavity mode.

The linearized Heisenberg-Langevin equations of motion are found using Eq. (S1), the displacement $\hat{a}_k \rightarrow \alpha_k + \hat{a}_k$, and the inclusion of the optical input signal ($\hat{a}_{\mathrm{in},k}(t)$), optical vacuum noise ($\hat{a}_{\mathrm{i},k}(t)$), and mechanical thermal fluctuation fields ($\hat{b}_{\mathrm{in}}(t)$):

$$\begin{aligned}\dot{\hat{b}}(t) &= -\left(i\omega_{\mathrm{m}}+\frac{\gamma_{\mathrm{i}}}{2}\right)\hat{b}-i\sum_k G_k(\hat{a}+\hat{a}^{\dagger})-\sqrt{\gamma_{\mathrm{i}}}\hat{b}_{\mathrm{in}}(t)\\ \dot{\hat{a}}_k(t) &= -\left(i\delta_k+\frac{\kappa_k}{2}\right)\hat{a}_k-iG_k(\hat{b}+\hat{b}^{\dagger})\\ &\quad -\sqrt{\kappa_{\mathrm{e},k}/2}\hat{a}_{\mathrm{in},k}(t)-\sqrt{\kappa'_k}\hat{a}_{\mathrm{i},k}(t).\end{aligned}$$

The mechanical loss rate $\gamma_{\mathrm{i}}=\omega_{\mathrm{m}}/Q_{\mathrm{m}}$, determines the coupling of the system to the thermal bath, and provides the most significant contribution in terms of noise processes relevant to the performance of the optomechanical wavelength converter. The other loss rates, κ_k, $\kappa_{e,k}$, and κ'_k are, respectively for optical cavity mode k, the total optical loss rate, the optical loss rate associated with coupling into the waveguide k, and parasitic optical loss rate into all other channels that are undetected representing a loss of information. Due to the local evanescent side-coupling scheme studied here (which bi-directionally couples to the cavity mode), $\kappa_k=\kappa_{\mathrm{e},k}/2+\kappa'_k$, i.e. only a fraction $\eta_k\equiv\kappa_{\mathrm{e},k}/2\kappa_k$ of the photons leaving the cavity get detected and $\eta_k\leq 1/2$.

The equations of motion are linear and thus the system can be analyzed more simply in the frequency domain. Solving for the spectrum of the mechanical mode $\hat{b}(\omega)$ in terms of the input noise operators, we find:

$$\begin{aligned}\hat{b}(\omega) &= \tfrac{-\sqrt{\gamma_{\mathrm{i}}}\hat{b}_{\mathrm{in}}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2}\\ &+\textstyle\sum_k \tfrac{iG_k}{i(\delta_k-\omega)+\kappa_k/2}\tfrac{\sqrt{\kappa_{\mathrm{e},k}/2}\hat{a}_{\mathrm{in},k}(\omega)+\sqrt{\kappa'_k}\hat{a}_{\mathrm{i},k}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2}\\ &+\textstyle\sum_k \tfrac{iG_k}{-i(\delta_k+\omega)+\kappa_k/2}\tfrac{\sqrt{\kappa_{\mathrm{e},k}/2}\hat{a}^{\dagger}_{\mathrm{in},k}(\omega)+\sqrt{\kappa'_k}\hat{a}^{\dagger}_{\mathrm{i},k}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2},\end{aligned}$$

where the mechanical frequency ω_m is now modified by the optical spring, and the mechanical linewidth is given by $\gamma\equiv\gamma_{\mathrm{i}}+\gamma_{\mathrm{OM},1}+\gamma_{\mathrm{OM},2}$. The optomechanical damping terms $\gamma_{\mathrm{OM},k}$ come from coupling of the mechanical system to the optical mode k, and are given by the relation

$$\begin{aligned}\gamma_{\mathrm{OM},k} &= 2|G_k|^2\mathrm{Re}\left[\frac{1}{i(\delta_k-\omega_{\mathrm{m}})+\kappa_k/2}\right.\\ &\quad\left.-\frac{1}{-i(\delta_k+\omega_{\mathrm{m}})+\kappa_k/2}\right]\\ &= \frac{4|G_k|^2}{\kappa_k}\quad \text{for } \delta_k\approx\omega_{\mathrm{m}}.\end{aligned} \tag{S2}$$

The last expression is a simplification that is often made, and is equivalent to looking at spectral properties at detunings from the mechanical frequency much smaller than κ (so the optical lineshape isn't taken into account). Under this approximation, the optical spectrum is

$$\begin{aligned}\frac{\kappa_k}{2}\hat{a}_k(\omega) &= \tfrac{iG_k\sqrt{\gamma_{\mathrm{i}}}\hat{b}_{\mathrm{in}}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2} \\ &\quad +\textstyle\sum_j \tfrac{2G_jG_k}{\kappa_j}\tfrac{\sqrt{\kappa_{\mathrm{e},j}/2}\hat{a}_{\mathrm{in},j}(\omega)+\sqrt{\kappa_j'}\hat{a}_{\mathrm{i},j}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2} \\ &\quad +\textstyle\sum_j \tfrac{iG_jG_k}{2\omega_{\mathrm{m}}}\tfrac{\sqrt{\kappa_{\mathrm{e},j}/2}\hat{a}^\dagger_{\mathrm{in},j}(\omega)+\sqrt{\kappa_j'}\hat{a}^\dagger_{\mathrm{i},j}(\omega)}{i(\omega_{\mathrm{m}}-\omega)+\gamma/2} \\ &\quad -\sqrt{\kappa_{\mathrm{e},k}/2}\hat{a}_{\mathrm{in},k}(\omega)-\sqrt{\kappa_k'}\hat{a}_{\mathrm{i},k}(\omega). \end{aligned} \tag{S3}$$

From this expression, we see that there are several noise operators incident on optical mode k. The thermal fluctuations from the environment, $\hat{b}_{\mathrm{in}}(\omega)$, are converted into noise photons over the mechanical bandwidth γ. There is also an induced coupling to the optical mode $j \neq k$, and photons originally incident only on mode j, i.e. $\hat{a}_{\mathrm{in},j}(\omega)$ are now also coupled into $\hat{a}_k$. Finally we note that vacuum noise creation operators are also present in this expression. These give rise to quantum noise through spontaneous emission of phonons, though we show below that as ω_{m} is made very large with respect to κ_k, these terms diminish in importance.

From outside the optomechanical system, we only have access to photons sent into the system, $\hat{a}_{\mathrm{in},j}$, and those leaving the system, $\hat{a}_{\mathrm{out},k}$, on transmission. The relation between these operators is best understood through scattering parameters, and can be derived using the equation for $\hat{a}_k(\omega)$ (Eq. S3) and the input-output boundary conditions $\hat{a}_{\mathrm{out},k}(\omega) = \hat{a}_{\mathrm{in},k}(\omega) + \sqrt{\kappa_{\mathrm{e},k}/2}\hat{a}_k(\omega)$. After some algebra, the output operator is expressed as

$$\begin{aligned}\hat{a}_{\mathrm{out},k}(\omega) &= s_{\mathrm{th},k}(\omega)\hat{b}_{\mathrm{in}}(\omega) \\ &\quad +t_k(\omega)\hat{a}_{\mathrm{in},k}(\omega)+s_{kj}(\omega)\hat{a}_{\mathrm{in},j}(\omega) \\ &\quad +\sum_m n_{\mathrm{opt},m}(\omega)\hat{a}_{\mathrm{i},m}(\omega) \\ &\quad +\sum_m s_{\mathrm{adj,in},m}(\omega)\hat{a}^\dagger_{\mathrm{in},m}(\omega) \\ &\quad +\sum_m s_{\mathrm{adj,i},m}(\omega)\hat{a}^\dagger_{\mathrm{i},m}(\omega) \end{aligned} \tag{S4}$$

The scattering coefficient $s_{\mathrm{th},k}(\omega)$ is the conversion efficiency of mechanical thermal noise to photons, and is given by

$$s_{\mathrm{th},k}(\omega) = i\sqrt{\eta_k}\frac{\sqrt{\gamma_\mathrm{i}\gamma_{\mathrm{OM},k}}}{i(\omega_\mathrm{m}-\omega)+\gamma/2}. \tag{S5}$$

The coefficient $t_k(\omega)$ in our system is transmission amplitude, given by

$$t_k(\omega) = (1-2\eta_k) + \eta_k\frac{\gamma_{\mathrm{OM},k}}{i(\omega_\mathrm{m}-\omega)+\gamma/2}. \tag{S6}$$

In principle, this coefficient is the electromagnetically induced transparency transmission coefficient, though here it is written about the mechanical frequency for detunings on the order of γ and the κ's are too large ($\kappa \gg \gamma$) for the optical lineshape to be considered in the expression. This is the expected result of the weak coupling approximation. Finally, the most important coefficient is the wavelength conversion coefficient $s_{kj}(\omega)$ which is given by

$$s_{kj}(\omega) = \sqrt{\eta_k\eta_j}\frac{\sqrt{\gamma_{\mathrm{OM},k}\gamma_{\mathrm{OM},j}}}{i(\omega_\mathrm{m}-\omega)+\gamma/2}. \tag{S7}$$

Efficiency, Bandwidth, and Noise

We calculate the spectral density of the output field on port k ($S_{\mathrm{out},k}(\omega)$) assuming an input field spectral density on the opposing optical port j ($S_{\mathrm{in},j}(\omega)$), vacuum inputs on all other optical channels, and thermal noise from a phonon bath with thermal occupation n_b. The spectral densities here have units of photons/Hz$\cdot$s and can be interpreted as photon flux per unit bandwidth. At first we ignore the field creation operators in the scattering relation Eq. (S4) (which give rise to quantum noise and can be neglected when the mechanical resonator occupation number is greater than one) and arrive at the following expression:

$$\begin{aligned}
S_{\mathrm{out},k}(\omega) &= \int_{-\infty}^{\infty} \mathrm{d}\omega' \ \langle \hat{a}^\dagger_{\mathrm{out},k}(\omega)\hat{a}_{\mathrm{out},k}(\omega')\rangle \\
&= \int_{-\infty}^{\infty} \ s^*_{\mathrm{th},k}(-\omega)s_{\mathrm{th},k}(\omega')\langle \hat{b}^\dagger_\mathrm{in}(\omega)\hat{b}_\mathrm{in}(\omega')\rangle \\
&\quad + t^*_k(-\omega)t_k(\omega')\langle \hat{a}^\dagger_{\mathrm{in},k}(\omega)\hat{a}_{\mathrm{in},k}(\omega')\rangle \\
&\quad + s^*_{kj}(-\omega)s_{kj}(\omega')\langle \hat{a}^\dagger_{\mathrm{in},j}(\omega)\hat{a}_{\mathrm{in},j}(\omega')\rangle \\
&\quad + n^*_{\mathrm{opt},k}(-\omega)n_{\mathrm{opt},k}(\omega')\langle \hat{a}^\dagger_{\mathrm{i},k}(\omega)\hat{a}_{\mathrm{i},k}(\omega')\rangle \\
&\quad + n^*_{\mathrm{opt},j}(-\omega)n_{\mathrm{opt},j}(\omega')\langle \hat{a}^\dagger_{\mathrm{i},j}(\omega)\hat{a}_{\mathrm{i},j}(\omega')\rangle \mathrm{d}\omega'.
\end{aligned}$$

This reduces to,

$$\begin{aligned} S_{\mathrm{out},k}(\omega) &= \eta_k \frac{\gamma_{\mathrm{i}}\gamma_{\mathrm{OM},k}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)} n_b \\ &\quad + \eta_k\eta_j \frac{\gamma_{\mathrm{OM},k}\gamma_{\mathrm{OM},j}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)^2} S_{\mathrm{in},j}(\omega), \end{aligned}$$

accounting for the noise autocorrelation functions. From here, we see that the maximum photon wavelength conversion efficiency is given by

$$\eta_{\max} = \eta_1\eta_2 \frac{4\gamma_{\mathrm{OM},1}\gamma_{\mathrm{OM},2}}{(\gamma_i+\gamma_{\mathrm{OM},1}+\gamma_{\mathrm{OM},2})^2}. \tag{S8}$$

The conversion process has a bandwidth (BW) equal to the mechanical linewidth

$$\mathrm{BW} \equiv \gamma_{\mathrm{i}} + \gamma_{\mathrm{OM},1} + \gamma_{\mathrm{OM},2}. \tag{S9}$$

From the same expression, we calculate a value for the optical signal to noise ratio (OSNR), to find

$$\mathrm{OSNR}_{kj}^{\mathrm{classical}} = \eta_j \frac{\gamma_{\mathrm{OM},j}}{\gamma_{\mathrm{i}} n_{\mathrm{b}}} S_{\mathrm{in},j}(\omega). \tag{S10}$$

Equation (S10) only takes into account the thermal noise in the system and therefore holds only when $\gamma_{\mathrm{i}} n_{\mathrm{b}}/\gamma_{\mathrm{OM},j} \gg 1$. At higher powers, the quantum noise processes which give rise to the back-action limit in cooling also give rise to excess noise for wavelength conversion. To consider these terms, the creation operators in Eq. (S4) must not be neglected and the full relation is then found to be

$$\begin{aligned} S_{\mathrm{out},k}(\omega) &= \eta_k \frac{\gamma_{\mathrm{OM},k}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)^2} \gamma_{\mathrm{i}} n_{\mathrm{b}} \\ &\quad + \eta_k \frac{\gamma_{\mathrm{OM},k}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)^2} \gamma_{\mathrm{OM},k} \left(\frac{\kappa_k}{4\omega_{\mathrm{m}}}\right)^2 \\ &\quad + \eta_k \frac{\gamma_{\mathrm{OM},k}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)^2} \gamma_{\mathrm{OM},j} \left(\frac{\kappa_j}{4\omega_{\mathrm{m}}}\right)^2 \\ &\quad + \eta_k\eta_j \frac{\gamma_{\mathrm{OM},k}\gamma_{\mathrm{OM},j}}{(\omega+\omega_{\mathrm{m}})^2+(\gamma/2)^2} S_{\mathrm{in},j}(\omega). \end{aligned} \tag{S11}$$

The quantum limited OSNR is then found to be

$$\mathrm{OSNR}_{kj} = \frac{\eta_j\gamma_{\mathrm{OM},j} S_{\mathrm{in},j}(\omega)}{\gamma_{\mathrm{i}} n_{\mathrm{b}} + \gamma_{\mathrm{OM},k}\left(\frac{\kappa_k}{4\omega_{\mathrm{m}}}\right)^2 + \gamma_{\mathrm{OM},j}\left(\frac{\kappa_j}{4\omega_{\mathrm{m}}}\right)^2}. \tag{S12}$$

For the quantum back-action terms to become significant as compared to the thermal noise on the mechanical system, one needs $\gamma_{\mathrm{i}} n_{\mathrm{b}}/\gamma \sim \left(\frac{\kappa}{4\omega_{\mathrm{m}}}\right)^2$. This is a regime that is yet to be reached in experiments and as such the quantum back-action noise terms can be neglected for experiments to date. Perhaps more importantly, we ask “what is the amount of noise added to a signal of a single photon?”. We can express the signal to noise ratios as $\mathrm{OSNR}_{kj} = S_{\mathrm{in},j}(\omega)/n_{\mathrm{added}}$, with

$$n_{\mathrm{added}} = \frac{\gamma_{\mathrm{i}} n_{\mathrm{b}} + \gamma_{\mathrm{OM},k}\left(\frac{\kappa_k}{4\omega_{\mathrm{m}}}\right)^2 + \gamma_{\mathrm{OM},j}\left(\frac{\kappa_j}{4\omega_{\mathrm{m}}}\right)^2}{\eta_j \gamma_{\mathrm{OM},j}}. \tag{S13}$$

Assuming $\kappa_j = \kappa_k = \kappa$, $\gamma_{\mathrm{i}} \ll \gamma$, and $\gamma_{\mathrm{OM},j} = \gamma_{\mathrm{OM},k} = \gamma_{\mathrm{OM}}$, this expression becomes

$$n_{\mathrm{added}} \approx 2\eta_j^{-1}\left(\frac{\gamma_{\mathrm{i}} n_{\mathrm{b}}}{\gamma} + \left(\frac{\kappa}{4\omega_{\mathrm{m}}}\right)^2\right). \tag{S14}$$

To achieve the threshold where a single photon incoming on port j is converted, and is more likely to be detected on port k than an upconverted thermal excitation, we require $n_{\mathrm{added}} < 1$. This requirement is equivalent to stating that in the absence of signals on port j and k, a cooled phonon occupation of $\bar{n} = \eta_j/2$ must be achieved. We note that η_k does not make an appearance in this equation, since reduced output coupling efficiency attenuates the thermal noise and signal equivalently. For the same reason, $1/\eta_j$ is present in the noise quanta expression. The factor of two can be understood to come from the fact that for good conversion, we require the photon to enter, and exit the system before a phonon is up converted. The former two processes happen at γ_{OM}, while the latter is given by the thermalization rate $\gamma_{\mathrm{i}} n_{\mathrm{b}}$.