Frequency-specific flow control in microfluidic circuits with passive elastomer features

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SUPPLEMENTARY INFORMATION

1. Physics of fluidic resistors and deformable capacitors.

<u>*Fluidic resistance:*</u>

For laminar viscous flow, the pressure drop along a channel is proportional to the flow rate. Thus, $\Delta p = Rq$, where Δp is the pressure drop between two points in the channel, q is the volumetric flow rate, and R is the fluidic resistance of the channel. The fluidic resistance in a channel with rectangular cross-section is given by [1]:

$$R = \frac{12\mu L_R}{d^3 w} f\left(\frac{d}{w}\right), \qquad \text{(S1)}$$

Where μ is the fluid viscosity, and L_R, w, and d are the length, width, and depth of the channel, respectively. f is a dimensionless shape factor function that approaches unity in the limit that $d << w$. This theoretical result has been verified extensively with experiments, including the modifications necessary for deviations from purely rectangular cross-sections [1]. Shape factors such as f have a weak effect on the predicted network dynamic response, because this is controlled by ratios of fluidic resistances (see Section 2 below). For Figure 2 in the paper, the above result for a purely rectangular cross-section was used with nominal dimensions.

<u>*Fluidic capacitance (stiffness):*</u>

As the pressure is changed on a capacitor, there is fluid flow into or out of the bulge formed by the deformable film. The relationship between the flow rate of the capacitor and the pressure difference acting on either side of the film is given by $q = C\left(\dot{p}_{ext} - \dot{p}_{in}\right)$, where C is the fluidic capacitance, which is determined by the deformability of the capacitor film.

The relationship between pressure (p) and deflection (δ) of an elliptical plate is given by [2,3]:

$$\delta = \frac{3pa^4b^4}{2\bar{E}t^3\left(3a^4 + 2a^2b^2 + 3b^4\right)} = \frac{p}{k} \qquad \text{(S2)}$$

where a and b are the half-widths of the ellipse, and $\bar{E}$, t are the effective plane-strain modulus and thickness the deformable layer, respectively. The effective modulus and thickness of a bilayer, required in the present experiments because of a reflective gold coating on the PDMS capacitor film, can be calculated following the derivation in [4,5]. (Contrast with the monolithic modulus, *E*, thickness, *h*, and half-widths, *a* and *b*, that are referred to in the text and in FIG. 1.)

In the above, *k* is the "stiffness" of the capacitor (i.e. the pressure required to cause a unit deflection of the centerpoint, $p \equiv k\delta$.) The stiffness is needed to relate network pressures (appearing in the circuit analysis) to the deflection of the capacitors (which is measured).

The volume stored underneath the bulge in the plate can be found by integrating over the deformed profile of the plate [3], the result being:

$$V = \frac{\pi ab}{3}\delta = \frac{\pi ab}{3k}p \tag{S3}$$

The fluidic capacitance is therefore given by:

$$C = \frac{\partial V}{\partial p} = \frac{\pi ab}{3k} \tag{S4}$$

2. Governing equations and frequency-response-function (FRF).

The circuit analysis for the circuit shown in Figure 2 utilizes the pressure-flow relationships described above, i.e. $\Delta p = Rq$ and $q = C\left(\dot{p}_{ext} - \dot{p}_{in}\right)$. These are combined with the concept of mass conservation, which dictates that the total volume of fluid diverted from a channel into a capacitor must be equal to the difference of the input and output flow rates. (This is akin to 'Kirchoff's Law' for electronic circuits, completing the pressure-voltage, flow rate-current analogy.)

In the following, the "input" capacitor refers to the capacitor that is subjected to externally-applied periodic pressure, while the "output" capacitor refers to the passive deformable film. Since the deflections of the input and output capacitors are directly measured, it is desirable to cast the governing equations in terms of film deflections. This is simply achieved by substituting $p_{net} = k\delta$, where k is the film stiffness defined above, and δ is the film deflection. Hence, the flow rate into the capacitor is given by: $q_i = C_i k_i \dot{\delta}_i$, where $\dot{\delta}_i$ is the time rate of change of the deflection of the input capacitor. Using a similar relationship for the passive (output) capacitor, the differential equation relating input and output capacitor deflections is:

$$\dot{\delta}_i = \left(\frac{C_0 k_0}{C_i k_i}\right)\left(\frac{R_1 + R_3}{R_3}\right)\dot{\delta}_0 + \left(\frac{k_0}{C_i k_i}\right)\left(\frac{R_1 + R_2 + R_3}{R_2 R_3}\right)\delta_0 = \alpha\,\dot{\delta}_0 + \beta\,\delta_0 \tag{S5}$$

For periodic response, the ratio of the amplitudes of the input and output deflections is then:

$$\frac{\langle \delta_o \rangle}{\langle \delta_{in} \rangle} = \frac{\omega}{\sqrt{\beta^2 + (\alpha\omega)^2}}, \tag{S6}$$

where α dictates the constant asymptotic limit at large frequencies, and β reflects a characteristic transition frequency. Noting from eqn. S4 that fluidic capacitance and film stiffness are inversely proportional, it is clear that the large-frequency limit α is independent of the capacitor film thickness (see Table 3 below). This is illustrated in the experiments (FIG. 2C in text) and the theoretical values listed in the next section. Note that α depends on capacitor stiffness only through the planar capacitor dimensions, which were not changed. Conversely, changing the output capacitor thickness changes its stiffness, *k*, and hence will affect the transition frequency, β – again, as illustrated by the experiments and theory. Finally, changing the channel dimensions (and hence resistances) changes both the asymptotic limit at high frequencies (α) and the transitional frequency (β).

3. Material/geometry/circuit parameters for devices in Figure 2.

Supplementary Table S1: Nominal dimensions and calculated fluidic resistances for channels.

	CHIP 1		CHIP 2	
	Length, L (mm)	Resistance, R ($kPa \cdot s / mm^3$)	Length, L (mm)	Resistance, R ($kPa \cdot s / mm^3$)
Segment 1	37.5	14.87	50.	19.82
Segment 2	8.5	3.37	3	1.19
Segment 3	21	8.3	4.25	1.68
All channels are $w = 217\ \mu m$ wide by $d = 59\ \mu m$ deep.				

Supplementary Table S2: Nominal dimensions, effective properties and calculated stiffness of capacitors.

OUTPUT CAPACITOR, Chip 1a and Chip 2		
	Reflective gold coating	PDMS layer
Thickness, *h*	50 nm	50 um
Modulus (plane strain), *E*	95 GPa	1.35 Mpa
Effective thickness, *t*	11.8 um	
Effective modulus, $\bar{E}$	409 Mpa	
Half sizes, (*a*,*b*)	0.559 mm x 1.059 mm	
Stiffness, *k*	17.39 kPa/mm	
Fluidic capacitance, *C*	0.0359 mm^3/kPa	

OUTPUT CAPACITOR, Chip 1b		
	Reflective gold coating	PDMS layer
Thickness, *h*	50 nm	25 um
Modulus (plane strain), *E*	95 Gpa	1.35 Mpa
Effective thickness	4.2 um	
Effective modulus, $\bar{E}$	1142 Mpa	
Half sizes, (*a*,*b*)	0.559 mm x 1.059 mm	
Stiffness, *k*	2.19 kPa/mm	
Fluidic capacitance, *C*	0.286 mm^3/kPa	

INPUT CAPACITOR (all chips)		
	Reflective gold coating	PDMS layer
Thickness, *h*	50 nm	340 um
Modulus (plane strain), *E*	95 Gpa	1.35 Mpa
Effective thickness, *t*	195 um	
Effective modulus, $\bar{E}$	26.7 Mpa	
Half sizes, (*a*,*b*)	0.559 mm x 1.559 mm	
Stiffness, *k*	4470 kPa/mm	
Fluidic capacitance, *C*	3.86×10^{-4} mm^3/kPa	

Supplementary Table S3: Calculated FRF parameters for devices in Figure 2.

FRF: $\dfrac{\omega}{\sqrt{\beta^2+(\alpha\omega)^2}}$	High frequency limit parameter, α	Frequency slope parameter, β
Chip 1a (thick output capacitor)	1.89	18.32
Chip 1b (thin output capacitor)	1.89	2.33
Chip 2 (thick output capacitor with different resistances)	8.69	215.15

4. Supplementary video file.

Supplementary Video S1: This video, designed to accompany FIG. 4 in the text, provides an illustration of the frequency-dependent control of the output flow ratio transferred from two microfluidic reservoirs. As the single input pressure source is varied from 0.2 Hz to 7.0 Hz, it is shown that the output ratio of red dye (left) changes from low to high, in comparison to the blue dye. This frequency response was passively programmed into the device shown in FIG. 3 and 4. The timing of the video was adjusted to show approximately the same number of cycles at each frequency. Represented frequency is labeled in the upper right corner of each frame. (AVI file, 4.26 MB in size)

References

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