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SUPPLEMENTARY INFORMATION
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Demonstration of an ultracold micro-optomechanical oscillator in a cryogenic cavity

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SUPPLEMENTARY INFORMATION

Effective Mass

We have estimated the effective mass of the fundamental mode of our micromechanical structure using both analytic models and FEM analysis. The experimentally observed value of 43 ± 2 ng agrees to within 10% with the estimated value of 53 ± 5 ng.

The total mass of the dielectric Bragg mirror (radius $R \approx 24.5 \pm 0.5$ μm) made of 36 alternating layers of Ta_2O_5 ($\rho \approx 8200$ kg/m^3, $t = 126.4$ nm) and SiO_2 ($\rho = 2200$ kg/m^3, $t = 179.6$ nm) is 45 ± 5 ng, not taking into account the lateral etch and tapering of the mirror pad. The large error stems from the uncertainty in the exact value of the Ta_2O_5 density, which can vary between 6800 and 8300 kg/m^3. The mass of the Si_3N_4 resonator ($\rho = 3000$ kg/m^3, approximate dimensions of $100 \times 50 \times 1$ μm^3) is approx. 11 ng, resulting in a maximum total mass of 56 ± 5 ng for the full optomechanical device.

The mode mass, i.e. the actual mass contributing to the motion of the Si_3N_4 resonator fundamental mode, is approx. 74% of the total mass of the Si_3N_4 resonator (see any standard literature on elasticity theory, for example [S1]). This would result in a total mode mass of the optomechanical resonator (Si_3N_4 beam plus micromirror) of approx. 53 ± 5 ng. However, because of the flat-top mode shape of our actual device (see the FEM simulation shown in

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Figure S1), this value is only a conservative lower bound. A more realistic value that takes into account the actual mode shape can be obtained directly from FEM simulation and is approx. 56 ± 5 ng (see below).

Finally, to calculate the effective mass one has to take into account the mode overlap between the mechanical resonator mode and the mode of the optical probe beam (for a detailed analysis on the calculation of the effective mass see for example [S2]). Based on the experimentally obtained optical finesse, which is limited by intensity losses due to a finite mirror size, we can provide an upper bound on the cavity beam waist at the micromirror position of 8 ± 2 μm. If we assume a mechanical mode shape of an ideal doubly-clamped beam of dimensions $100 \times 50 \times 1\ \mu m^3$ we would calculate an effective mass (see e.g. [S2,S3]) of 50 ± 5 ng, Again, the actual flat-top mode shape of our device results in a decreased mean square displacement (by approx. 6%) compared to the ideal doubly-clamped beam. Taking this into account yields a final effective mass of 53 ± 5 ng, which agrees to within 10% with the experimentally observed value of 43 ± 2 ng.

The abovementioned FEM simulations make use of the exact geometry and material data for our resonator. The main idea is to impose a force on the structure and have the FEM simulation calculate the deflection. Using Hooke's law one can then extract the spring constant k of the device. The mode mass can be extracted by using $\omega_m = \sqrt{k/m_{mode}}$. For our specific device the FEM solver provides us with a spring constant of 2196 N/m and a fundamental mode at $\omega_m = 2\pi \times 945$ kHz, which results in $m_{mode} = 57 \pm 5$ ng.

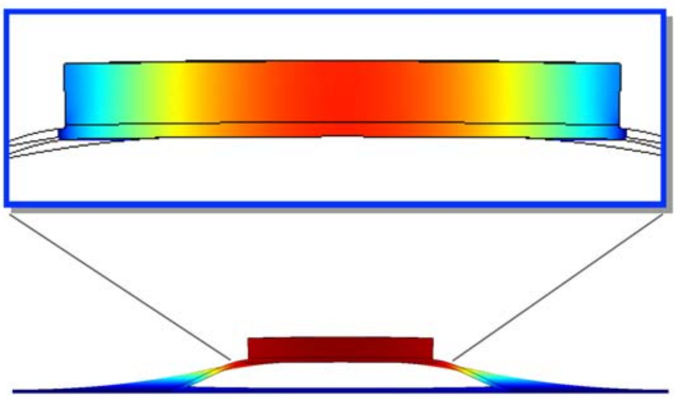

Figure S1: *FEM simulation of our optomechanical device. Shown is the side-view of the fundamental resonance mode at its maximum displacement (below). The cylindrical mirror pad on top of the Si_3N_4 beam induces a flat-top mode shape (inset).*

Error Analysis

The error associated with the noise power spectra peak areas, which provide the mechanical mean square displacement, can be estimated as follows: Assuming that the NPS comprises a sequence of N independent data points (x_i, y_i) (with $i = 1 \ldots N$) with measurement uncertainty $(\delta x_i, \delta y_i)$ one can calculate the area underneath the NPS by Riemann integration as $A = \sum_{i=1}^{N-1} (x_{i+1} - x_i) y_i$ with an uncertainty $\delta A = \sqrt{\sum_{i=1}^{N-1} (x_{i+1} - x_i)^2 (\delta y_i)^2}$, which is obtained by Gaussian error propagation and neglecting the uncertainty in x. The strongly cooled NPS shown in Figure 3a is given by a data set of $N = 5000$ points with $x_{i+1} - x_i = 100$ Hz and with $\delta y_i \approx 1 \times 10^{-34}$ m^2 Hz^{-1} for all *i*. We obtain $A = 3.780 \times 10^{-28}$ m^2 (by numerically integrating the data set), $\delta A \approx \sqrt{N} \times 100$ Hz $\times 1 \times 10^{-34}$ m^2 Hz^{-1} $= 7.1 \times 10^{-31}$ m^2 and an integrated noise floor of $N \times 100$ Hz $\times 7.3 \times 10^{-34}$ m^2 Hz$^{-1} = 3.65 \times 10^{-28}$ m^2. This results in an integrated "real thermal noise" of $(3.78 - 3.65) \times 10^{-28}$ m^2 $= 1.3 \times 10^{-29}$ m^2 with an overall error of approx. $\sqrt{2} \times 7.3 \times 10^{-31}$ m^2 $\approx 1 \times 10^{-30}$ m^2, i.e. with an error of approx. 8%. The SNR of our measurement is therefore sufficient to support our result of $\langle n \rangle = 32$ and accounts for an uncertainty of $\delta \langle n \rangle = \pm 1.5$.

Other possible sources of experimental uncertainty are: an uncertainty related to the absolute displacement amplitude calibration (amounting to approx. 12% relative uncertainty), an uncertainty related to determining the mechanical resonance frequency (known up to an error of approx. 5%) and an uncertainty related to the absolute power calibration of the intracavity optical pump field (known up to an error of approx. 10%). These additional experimental uncertainties add up to an overall error of approx. 25%. All errors are conservatively estimated and finally result in $\langle n \rangle = 32 \pm 4$.

Shot-Noise

The noise floor of our measurement is limited by optical shot-noise. The corresponding displacement noise can be calculated according to [S4] as

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$$\delta x_{Shot} = \frac{\lambda}{16F\sqrt{\frac{P\lambda}{hc}}} \cdot \sqrt{1+\left(\frac{\omega_m}{\kappa}\right)^2} \cdot \sqrt{\frac{T+l}{T}} \cdot \sqrt{\frac{P}{P_{MM}}} .$$

Our experimental parameters (finesse $F = 3900$, input power $P = 14$ μW, $\lambda = 1064$ nm, $\omega_m = 2\pi \times 945$ kHz, $\kappa = 2\pi \times 770$ kHz, input coupler transmission $T = 900$ ppm, overall intra-cavity losses $l = 620$ ppm, optical input power (corrected for imperfect mode-matching) $P_{MM} = 7$ μW) result in a minimal noise-floor of $\delta x_{Shot} = 6 \times 10^{-18}$ m Hz$^{-0.5}$.

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