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Supplementary Information on Theoretical Modeling of Inertia-Driven Spin Switching in Antiferromagnets

The dynamics of an antiferromagnet with two sublattices can be describedon the basis of a system of two Landau–Lifshitz equations for the sublattice magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$, $|\mathbf{M}_1|=|\mathbf{M}_2|=M_s/2$, where M_s is the maximum possible value of the magnetization of the antiferromagnet when the sublattices are parallel. It is more convenient to use the equations for the irreducible vectors, the net magnetization $\mathbf{M}=\mathbf{M}_1+\mathbf{M}_2$ and the antiferromagnetic vector $\mathbf{L}=\mathbf{M}_1-\mathbf{M}_2$, subject to the constraints $(\mathbf{M}\cdot\mathbf{L})=0$, $\mathbf{M}^2+\mathbf{L}^2=M_s^2/2$. The equations for $\mathbf{M}$ and $\mathbf{L}$ can be written as $d\mathbf{L}/dt=\gamma(\mathbf{L}\times\mathbf{H}_M)+\gamma(\mathbf{M}\times\mathbf{H}_L)$, $d\mathbf{M}/dt=\gamma(\mathbf{L}\times\mathbf{H}_L)+\gamma(\mathbf{M}\times\mathbf{H}_M)$, where $\mathbf{H}_L=-\partial W/\partial\mathbf{L}$ and $\mathbf{H}_M=-\partial W/\partial\mathbf{M}$, W is the energy of the antiferromagnet, written in terms of the vectors $\mathbf{M}$ and $\mathbf{L}$. Using the conditions $|\mathbf{M}|<<|\mathbf{L}|$ which is natural for antiferromagnets (we will verify the condition below), and the inequality for the material parameters $H_e >> |\mathbf{H}|, |\mathbf{H}_D| >> H_a$, where H_e and H_a are exchange field and anisotropy field, respectively, and the vectors $\mathbf{H}$ and $\mathbf{H}_D$ determine the external field and Dzyaloshinskii field, respectively, we can neglect the dependence of the anisotropy energy W_a on $\mathbf{M}$ and the energy takes the form $W=(H_e/2M_s)\mathbf{M}^2-\mathbf{H}_{tot}\mathbf{M}+W_a(\mathbf{L})$, where $\mathbf{H}_{tot}=\mathbf{H}+\mathbf{H}_D$. The second term in the right hand side of the equation for $d\mathbf{L}/dt$ is of the order of MH_a, and is small compared to the first term, that is of order H_eM^2 or MH_D. This leads to an essential simplification: the vector $\mathbf{M}$ appears to be only a slave variable and is determined by the vector $\mathbf{L}$ and its time derivative, as follows, $\mathbf{M}=(M_s/H_e)[\mathbf{H}_{tot}-\mathbf{l}(\mathbf{H}_{tot}\mathbf{l})+(d\mathbf{l}/dt\times\mathbf{l})/\gamma]$, where the antiferromagnetic unit vector $\mathbf{l}=\mathbf{L}/|\mathbf{L}|$ is

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introduced. Then, excluding **M** from the equation for d**M** /dt, one can obtain the second order dynamical equation for the unit vector **l** only. The equation for **l** is too long to be written here, but can be obtained through the variation of the Langrangian (see equation 1 in the manuscript).

It is worth discussing here the condition $|\mathbf{M}| \ll |\mathbf{L}|$ which is a key point in deriving the sigma-model equation. To comply with it, we need to satisfy not only the natural condition $|\mathbf{H}_{tot}| \ll H_e$, but the inequality $|d\mathbf{l}/dt| \ll \gamma H_e$ as well. For the description of free spin oscillations, these two conditions are in fact equivalent, but the last one could be a much more serious limitation for the usage of the sigma-model for the problem of ultrafast pulse pumping, where the characteristic time scale Δt is much shorter than the spin wave period $2\pi/\omega$. For typical orthoferrites, $H_e = 1.3 \cdot 10^3$ T, the value of γH_e exceed 30 THz, and the sigma-model is adequate even for pulse durations like 30 fs.

In equation (3), the anisotropy energy W_a is modelled by a standard expansion over components of the vector **l** till fourth order. For a magnetic field pulse of short duration $\Delta t \ll 1/\omega$, equation 3 was solved numerically and analytically, with replacing the real pulse shape by a Dirac delta-function, $H(t) \rightarrow \delta(t) H_p \Delta t = \delta(t) \int H(t) dt$. Then before ($t<0$) and after ($t>0$) the action of the pulse, the spin dynamics is described by equation 2 with $H(t)=0$. Assuming $\varphi=0$ at $t<0$, it is easy to find the initial conditions of the form, $\varphi=0$, $d\varphi/dt = \gamma^2 H_p H_D \Delta t / \sin\theta_0$, which gives the solution of equation 4 in the same form of a damped nonlinear oscillation, as in Fig. 3.