Supplementary Information: Electric Dipole Induced Spin Resonance in Disordered Semiconductors

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1 Calculation of the spin vertex correction

Here, we give a more detailed account of the evaluation of the polarization $S^i(\omega)$, Eq. (A11) of the Article, in terms of the spin-spin and the spin-momentum terms, X, Y, respectively, and the spin vertex correction Σ. The simultaneous presence of the internal and external fields, $\alpha(\mathbf{p} \times \mathbf{e}_z)$ and $\mathbf{b}_0$, resp., breaks the symmetry in orbital space such that no closed analytical expression for the integrals in Eq. (A10) and hence for $X^{\mu\nu}$ can be obtained in the general case. However, the most important regime for EDSR is given by the regime where the internal field is much smaller than the (perpendicular) static external magnetic field, as in standard paramagnetic resonance (see also next section). Thus, without any essential restriction we can concentrate on the regime with the SOI being small compared to $\mathbf{b}_0$, i.e. $a = \alpha p_F / 2b_0 = x/2\omega_L\tau \ll 1$. First, upon inspection of Eq. (A11) we note that the contribution of Y to the polarization is due to the momentum-part of the velocity and thus must vanish in the absence of SOI. Thus, the leading order term in Eq. (A11) coming from Y is at least linear in a (assuming analyticity). More precisely, with a calculation similar to the one outlined below for X, we obtain for the spin-momentum diagram

$$Y^{\mu j} = -\epsilon_{\mu j 3} \frac{\alpha}{\lambda(\omega)} + O(\alpha^3), \tag{1}$$

i.e. the same result as before for $b_0 = 0$. Note that only odd powers in α appear due to the symmetry constraints induced by the angular integration occurring in Y. Thus, the expression Eq. (A5) is linear in the SOI α in leading order. In order to expand the polarization Eq. (A5) in leading order in $\alpha \propto a$ it is therefore sufficient to calculate the spin-spin diagram with setting α to zero and retaining only the $\mathbf{b}_0$ dependence. This way, we obtain the spin vertex correction $\Sigma(\omega)$ which is singular at resonance, i.e. when $\omega = \omega_L$ (Larmor frequency), reflecting the presence of Rabi oscillations. This shows, however, that at resonance the next-to-leading order contributions of the spin-spin diagram $X = X_{(0)} + a^2 X_{(2)}$

become relevant[1] for the matrix inversion and must be kept. Indeed, they represent the dominant contribution in $\Sigma(\omega) = (\mathbf{1} - X_{(0)}(\omega) - a^2 X_{(2)}(\omega))^{-1}$ if the determinant of the first term $\mathbf{1} - X_{(0)}$ vanishes. Obviously, at resonance the dominant a-dependence of Σ becomes $1/a^2$. Hence, we concentrate on the evaluation of the spin-spin diagram up to order of a^2, with $X_{(2)}$ characterizing the behavior of the polarization around the resonance (where our analysis is valid).

The spin-spin diagram is given by

$$X^{\mu\nu} = \frac{1}{4E_F\tau} \sum_{s,s'=\pm 1} \int \frac{d^2Q}{(2\pi)^2} \frac{T^{\mu\nu}_{s,s'}(\mathbf{Q})}{(1 + w - Q^2 - sB(\mathbf{Q}) + ir/2)(1 - Q^2 - s'B(\mathbf{Q}) - ir/2)} \tag{2}$$

expressed in terms of the dimensionless quantities[2] $w = \hbar\omega/E_F$, $r = \hbar/E_F\tau$, $Q = q/p_F$, $B_0 = b_0/E_F$ and the effective magnetic field

$$\mathbf{B}(\mathbf{Q}) = \frac{\mathbf{b}_{\text{eff}}(\mathbf{Q}\,p_F)}{E_F} = \mathbf{B}_0 + 2aB_0(\mathbf{Q} \times \mathbf{e}_z) \tag{3}$$

with modulus

$$B(\mathbf{Q}) = B_0\sqrt{1 + 4a\,\hat{\mathbf{B}}_0 \cdot (\mathbf{Q} \times \mathbf{e}_z) + 4a^2Q^2}. \tag{4}$$

Here, $\hat{\mathbf{B}}_0 = \mathbf{B}_0/B_0$ is the unit vector of the external field taken along the y-axis such that the mixed product in Eq.(4) becomes $\hat{\mathbf{B}}_0 \cdot (\mathbf{Q} \times \mathbf{e}_z) = -Q\cos\varphi$.

Trace. In Eq.(2) we introduced the trace over spin states

$$\begin{aligned} T^{\mu\nu}_{s,s'}(\mathbf{Q}) &= \mathrm{tr}\{\sigma^\mu(1 + s\,\hat{\mathbf{B}}(\mathbf{Q}) \cdot \boldsymbol{\sigma})\sigma^\nu(1 + s'\,\hat{\mathbf{B}}(\mathbf{Q}) \cdot \boldsymbol{\sigma})\} \\ &= 4\,\delta^{\mu\nu}[\delta^{\mu 0}\delta_{s,s'} + \delta^{\mu\neq 0}\delta_{s,-s'}] \\ &+ 4\,[\delta^{\mu\neq 0}\delta^{\nu 0}\hat{B}_\mu(\mathbf{Q}) + \delta^{\mu 0}\delta^{\nu\neq 0}\hat{B}_\nu(\mathbf{Q})]s\delta_{s,s'} \\ &+ 4\,i\epsilon_{\mu\nu k}\hat{B}_k(\mathbf{Q})\delta^{\mu\neq 0}\delta^{\nu\neq 0}s\delta_{s,-s'} \\ &+ 4\,s\,s'\hat{B}_\mu(\mathbf{Q})\hat{B}_\nu(\mathbf{Q})\delta^{\mu\neq 0}\delta^{\nu\neq 0}\ , \end{aligned} \tag{5}$$

where $\delta^{\mu\neq 0} = 1 - \delta^{\mu 0}$ etc., and where summation over repeated indices is implied. There are terms containing none, one, or two normalized magnetic fields $\hat{\mathbf{B}}(\mathbf{Q}) = \mathbf{B}(\mathbf{Q})/B(\mathbf{Q})$, which is relevant for the momentum integration. The trace $T^{\mu\nu}_{s,s'}(\mathbf{Q})$ and the direction of $\mathbf{B}_0$ determine the matrix structure of the spin-spin diagram, i.e. which components $X^{\mu\nu}$ are nonzero.

Momentum integration. The components $X^{\mu\nu}$ are obtained by the mo-

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[1]As a general property of linear SOI the first order in a vanishes due to the symmetry in the angular integration. Indeed, we note that the angular dependence (in the integrals in Eq. (A10)) comes from the magnetic field (Eq. (3)) where φ always occurs in terms of a trigonometric function simultaneously with a factor a. Expanded in a the linear terms thus vanish upon angular integration.

[2]In this Section 1 the capital letters $\mathbf{B}, \mathbf{B}_0$ and B, B_0 denote dimensionless magnetic fields measured in units of the Fermi energy E_F.

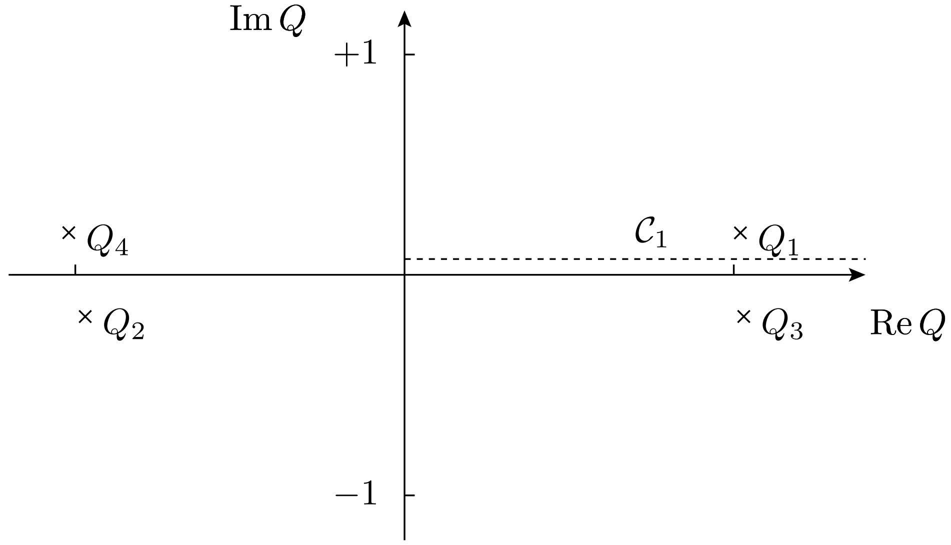

Figure 1: View of the poles Q_i of the retarded and advanced Green functions in the complex plane (cf. Eq. (6)). The contour $\mathcal{C}_1$ of the momentum integral (Eq. (7)) running from 0 to $+\infty$ is shown.

mentum integration in Eq. (2) where the poles Q_i of the denominator yield the main contribution. Assuming that E_F represents the largest energy scale such that w, r, and B_0 are small compared to one, the poles are located essentially at $|Q| \approx 1$ (cf. the denominator in Eq. (2)) with corrections $O(w, B_0, r)$. Making use of $a \ll 1$ we expand the denominator of the integrand in Eq. (2) in a up to second order. From this we obtain the following four poles

$$\begin{aligned} Q_1 &= +1 + k_1 + i\frac{r}{2}\ , \qquad k_1 = \frac{w}{2} - s\frac{B_0}{2} + B_0 as\cos\varphi - a^2 sB_0\sin^2\varphi \\ Q_2 &= -1 + k_2 - i\frac{r}{2}\ , \qquad k_2 = -\frac{w}{2} + s\frac{B_0}{2} + B_0 as\cos\varphi + a^2 sB_0\sin^2\varphi \\ Q_3 &= +1 + k_3 - i\frac{r}{2}\ , \qquad k_3 = -s'\frac{B_0}{2} + B_0 as'\cos\varphi - a^2 s'B_0\sin^2\varphi \\ Q_4 &= +1 + k_4 + i\frac{r}{2}\ , \qquad k_4 = s'\frac{B_0}{2} + B_0 as'\cos\varphi + a^2 s'B_0\sin^2\varphi\ . \end{aligned} \tag{6}$$

These poles are approximately of order one with a small correction k_i and a small imaginary part $r/2 = \hbar/2E_F\tau$, thus showing that the above approximation for small a is self-consistent. Decomposing into linear factors the spin-spin diagram can be recast into the form

$$X^{\mu\nu} = \frac{1}{8\pi E_F\tau} \sum_{s,s'=\pm1} \langle \int_0^{+\infty} \frac{dQ\, Q\, T^{\mu\nu}_{s,s'}(Q)}{(Q-Q_1)\ldots(Q-Q_4)c_\varphi} \rangle_\varphi\ , \tag{7}$$

where $\langle ... \rangle_\varphi$ denotes integration over the polar angle (normalized by 2π), and $c_\varphi = 1 + a^2 B_0(s+s')\sin^2\varphi$.

Note that the Q-dependence of $1/B(Q)$ in $T^{\mu\nu}_{s,s'}$ for $a \neq 0$ generates additional singularities at $Q_{5/6} = \exp(\pm i\varphi)/2a$ when analytically continued into the complex plane away from the real axis. The application of complex contour integration is thus non-trivial. A direct calculation, however, carried out by decomposing the denominator in Eq.(7) into partial fractions shows that these poles do not contribute to X within the accuracy $O(w, B_0, r)$.

The subsequent angular integrations become simple when expanding in the small parameter a. In this way, we obtain the components $X^{\mu\nu}$ and via the matrix inversion $\Sigma = (\mathbf{1} - X)^{-1}$ the spin vertex correction in Eq. (A12) of the paper. In particular, the vertex components

$$\begin{aligned}\Sigma^{11} &= \frac{(\omega_L\tau)^2 + (\lambda-1)\lambda + x^2\gamma_{11}}{(\omega_L\tau)^2 - (\omega\tau)^2 + x^2\gamma} \\ \Sigma^{13} = -\Sigma^{31} &= \frac{\omega_L\tau + x^2\gamma_{13}}{(\omega_L\tau)^2 - (\omega\tau)^2 + x^2\gamma}\end{aligned} \tag{8}$$

with the complex damping function

$$\gamma(\omega) = \frac{3(\lambda-1)\lambda^3 - (\omega_L\tau)^2\lambda(6\lambda-1) - (\omega_L\tau)^4}{2\lambda(\lambda^2 + (\omega_L\tau)^2)^2} \tag{9}$$

characterizing the linewidth and the functions

$$\begin{aligned}\gamma_{11}(\omega,\omega_L) &= -\frac{\lambda[(\omega_L\tau)^2 - \lambda^2]}{[(\omega_L\tau)^2 + \lambda^2]^2} \\ \gamma_{13}(\omega,\omega_L) &= -\frac{2\omega_L\tau\lambda^2}{[(\omega_L\tau)^2 + \lambda^2]^2}\end{aligned} \tag{10}$$

are relevant for the subsequent calculation of the spin polarization and spin Hall current. The frequency dependence and resonance behaviour of the spin polarization and current are discussed in the main text.

2 Regime of validity

We now give a summary of the parameters controlling the regime of validity of the present theory.

A first constraint ensures the validity of the linear response approach. For this, we give a heuristic argument based on the analogy to conventional ESR[1] expressed by Eq. (A2) of the paper. For this case, we consider the Bloch equations

$$\dot{S}^1 = \gamma \left[\mathbf{S} \times (\mathbf{B}_0 + \mathbf{B}_1(t))\right]_1 - \frac{S^1}{T_2}$$
$$\dot{S}^3 = \gamma \left[\mathbf{S} \times (\mathbf{B}_0 + \mathbf{B}_1(t))\right]_3 - \frac{S^3}{T_2}$$
$$\dot{S}^2 = \gamma \left[\mathbf{S} \times (\mathbf{B}_0 + \mathbf{B}_1(t))\right]_2 - \frac{S^2 - S_{eq}}{T_1} \qquad (11)$$

describing magnetic moments subject to a constant magnetic field $\mathbf{B}_0 || \mathbf{e}_2$ and a circularly polarized field $\mathbf{B}_1(t)$ perpendicular to it, which oscillates with frequency ω. The familiar steady state solutions in the rotating frame for the longitudinal component S^2 and the transverse components S^u and S^v, respectively are

$$S^2 = S_{\text{eq}} \frac{1 + \Delta\omega^2 T_2^2}{1 + \Delta\omega^2 T_2^2 + T_1 T_2 \omega_1^2}$$
$$S^u = S_{\text{eq}} \frac{\Delta\omega\omega_1 T_2^2}{1 + \Delta\omega^2 T_2^2 + T_1 T_2 \omega_1^2}$$
$$S^v = S_{\text{eq}} \frac{\omega_1 T_2}{1 + \Delta\omega^2 T_2^2 + T_1 T_2 \omega_1^2}, \qquad (12)$$

where $\omega_1 = \gamma B_1$, $\omega_L = \gamma B_0$, and $\Delta\omega = \omega_L - \omega$.

The resulting transverse polarization close to resonance is, thus, proportional to $\omega_1/[(\omega - \omega_L)^2 + \omega_1^2 (T_1/T_2) + 1/T_2^2]$ with the phenomenological relaxation rate $1/T_2$. Thus, two relaxation terms are present, viz. the 'external' damping given by $1/T_2^2$ and an intrinsic term $\omega_1^2(T_1/T_2)$ given by the driving rf field itself.

Similarly, the same intrinsic mechanism should be expected if the driving field B_1 is generated by a SOI-mediated bias like in the case considered in the paper. We thus anticipate a total spin relaxation rate of the form[3] $\sqrt{\omega_R^2 + \Gamma^2}$ with Rabi frequency $\omega_R = eE_0\alpha/\hbar\omega_L$ derived at resonance from Eq. (A2). Here, E_0 denotes the amplitude of the electric field $\mathbf{E}(t) = E_0 \mathbf{e}_y \cos(\omega t)$ and Γ is given by Eq. (A7). However, the Rabi frequency occurring in the rate $\sqrt{\omega_R^2 + \Gamma^2}$, being E-field dependent, must be negligible for a polarization S^i which is calculated in linear response with respect to $\mathbf{E}(t)$. This imposes the self-consistent condition

$$\omega_R \ll \Gamma \quad \Leftrightarrow \quad \frac{\hbar e E_0}{p_F \omega_L \tau} \ll 2\alpha p_F \left(1 + \frac{1}{2[1 + (\omega_L\tau)^2]}\right) \qquad (13)$$

for the validity of the linear response approach. A more systematic approach for estimating the validity of the linear response regime requires an explicit evaluation of the non-linear response, which, however, is beyond the scope of the present work.

[3] assuming $T_1 = T_2$ for simplicity

Secondly, in order to carry out the momentum integrals in Eq.(2) we introduced a condition limiting the SOI strength

$$a = \frac{\alpha p_F}{\hbar \omega_L} \ll 1. \tag{14}$$

This constraint not only simplified our analysis but also defines the most interesting regime for EDSR. Indeed, in order to have a pronounced resonance, the width of the resonance peak needs to be smaller than the resonance frequency, i.e. $\Gamma \ll \omega_L$, which is equivalent to $\alpha p_F x \ll \hbar \omega_L$ (see Eq. (A7)). For self-consistency we need to assume $x \leq 1$ (see the text before Eq. (A6)), and thus we see that $a \ll 1$ ensures $\Gamma \ll \omega_L$.

In this context we note the somewhat counterintuitive fact that the height of the resonance decreases with increasing SOI, see Eq. (A6). Indeed, on one hand the polarization is proportional to α via the driving rf field, and thus increases with increasing SOI. On the other hand, at resonance the polarization becomes proportional to $1/\Gamma$ (due to disorder) which gives then rise to a suppression factor $1/\alpha^2$. Thus, in total the polarization decrease as $1/\alpha$ with increasing SOI at resonance.

Our last constraints

$$\frac{b_0}{E_F}, \frac{\hbar \omega}{E_F}, \frac{\hbar}{E_F \tau} \ll 1 \tag{15}$$

correspond to the physically relevant situation where the Fermi energy E_F is the largest energy in the system. Further, the condition $x = 2\alpha p_F \tau/\hbar \ll 1$ does not restrict the validity of Eqs. (A12) and (A13) but permits us to represent Eq. (A6) in terms of two Lorentzians. In the case $\omega\tau \approx 1$, however, it becomes equivalent to the inequality (14).

3 Numerical estimates

To illustrate the predicted effects we now evaluate the polarization explicitly using typical GaAs parameters (cf. table 1), thereby making sure that we stay within the range of validity of our approximations. With a typical sheet density $n_2 = p_F^2/2\pi\hbar^2 = 4 \times 10^{15}$ m^{-2}, effective mass $m^* = 0.067\, m_e$ and a high mobility scattering time $\tau = 2 \times 10^{-11}$ s taken from [2] we can estimate the maximum polarization P as the ratio of the peak polarization per unit area and the sheet density

$$P = \frac{S^3_{max}}{n_2} = \frac{eEm^*}{(2\pi\hbar n_2)^2 2\alpha\tau} \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}} \left(1 + \frac{1}{2(1+\omega^2\tau^2)}\right)^{-1} . \tag{16}$$

In order to stay within the condition (14) we choose a small Rashba - parameter $\alpha = 10^{-14}$ eVm and find $x = 0.1$. Assuming a realistic microwave frequency $\omega = 50 \times 10^9$ s^{-1} corresponding to $\omega\tau = 1$ and a voltage amplitude of $V = 0.1$ V over a sample length of $l = 600$ μm we find an electric field $E_0 = 166$ Vm^{-1} and a polarization of

$$P = 10^{-4} . \tag{17}$$

Description	Parameter	Value
sheet density	n_2	4×10^{11} cm^{-2}
effective mass	m^*	$0.067\, m_e$
scattering time	τ	2×10^{-11} s
frequency	$f = \omega/2\pi$	8 GHz
Larmor frequency	$f = f_L = \omega_L/2\pi$	8 GHz
Rashba Parameter	α	10^{-12} eV cm
electric field	E	1.66 V cm^{-1}
polarization	P	10^{-4}
SOI vs. scattering	x	0.1
spin relaxation rate	$\Gamma/2\pi$	0.05 GHz
resonance shift	$\delta\omega/2\pi$	0.01 GHz
Rabi frequency	$\omega_R/2\pi$	0.012 GHz
validity conditions		
linear response	ω_R/Γ	0.27
relative SOI strength	$a = \alpha p_F/\hbar\omega_L$	0.05

Table 1: Numerical estimates

Note that the size of the chosen $E-$field satisfies the linear response condition (13) (and poses no severe limitation for a real experiment).

The corresponding number of excess spins $N_\uparrow - N_\downarrow$ in a laser spot of size $5\mu m \times 5\mu m$ is 200. This number is measurable with state-of-the-art optical detection techniques such as Faraday rotation[3].

We can further quantify the peak width Γ and the frequency shift $\delta\omega$. Making use of Eqs. (A7) and (A8) of the main text we find

$$\Gamma = 0.3 \times 10^9 \text{ s}^{-1}$$
$$\delta\omega = 0.06 \times 10^9 \text{ s}^{-1}. \quad (18)$$

As a further characterization of the resonance we estimate the Rabi frequency ω_R, given by the amplitude of $b_1(t)$ in Eq. (A2). Assuming a bias $\mathbf{E}(t) = E_0\mathbf{e}_y \cos(\omega t)$ we find

$$\omega_R = \frac{eE_0\alpha}{\hbar\omega} = 0.08 \times 10^9 \text{ s}^{-1}, \quad (19)$$

evaluated at resonance[4] $\omega_L = \omega$ with the parameters given above. A summary

[4]corresponding to a magnetic field $B \approx 1$ T for $|g| = 0.44$

of the above calculation and a check of the constraints Eqs. (14,13) is given in table 1.

4 Spin Hall current and polarization

We show now that the obtained polarization (magnetization/μ_B) S^i can be related to the spin current (defined below) via an exact relation. More generally, we consider the spin density operator

$$\rho^i(\mathbf{x}) = \frac{1}{2}\{\sigma^i, \delta(\mathbf{x} - \hat{\mathbf{x}})\}, \tag{20}$$

defined as the (symmetrized) product of the spin with the particle density operator $\delta(\mathbf{x} - \hat{\mathbf{x}})$ where $\hat{\mathbf{x}}$ is the position operator. Integrating over space (homogeneous limit) and taking expectation values we get the spin polarization $S^i = \int d^2x < \rho^i(\mathbf{x}) >$. The spin current density associated with ρ^i is defined in the usual way[4, 5, 6, 7, 8, 9, 10]

$$j^i_k(\mathbf{x}, t) = \frac{1}{2}\{\sigma^i, j_k(\mathbf{x})\} \tag{21}$$

in terms of the current operator $j_k(\mathbf{x}, t) = \frac{1}{2}\{\delta(x - \hat{\mathbf{x}}), v_k\}$ where, in contrast to the linear response treatment of the paper, the velocity operator $v_k = i/\hbar[H, x_k] = (p_k - (e/c)A_k)/m + \alpha(\boldsymbol{\sigma} \times \mathbf{e}_z)_k$ contains the kinetic momentum including the (homogenous) vector potential $\mathbf{A}$.

The two operators ρ^i and j^i_k are related via the Heisenberg equation of motion

$$\frac{d}{dt}\rho^\eta(\mathbf{x}, t) = \frac{i}{\hbar}[H, \rho^\eta] \tag{22}$$

given by the Hamiltonian Eq. (A1). Analogous to [9] where the Rashba- and Dresselhaus SOI has been considered it forms an exact operator identity

$$\begin{aligned}
\frac{d}{dt}\rho^1(\mathbf{x}, t) + \nabla \cdot \mathbf{j}^1(\mathbf{x}, t) &= -\frac{2\alpha m}{\hbar} j^3_x(\mathbf{x}, t) - \frac{2}{\hbar}\left[\rho^2(\mathbf{x}, t)b_{0,z} - \rho^3(\mathbf{x}, t)b_{0,y}\right] \quad (23)\\
\frac{d}{dt}\rho^2(\mathbf{x}, t) + \nabla \cdot \mathbf{j}^2(\mathbf{x}, t) &= -\frac{2\alpha m}{\hbar} j^3_y(\mathbf{x}, t) - \frac{2}{\hbar}\left[\rho^3(\mathbf{x}, t)b_{0,x} - \rho^1(\mathbf{x}, t)b_{0,z}\right]\\
\frac{d}{dt}\rho^3(\mathbf{x}, t) + \nabla \cdot \mathbf{j}^3(\mathbf{x}, t) &= +\frac{2\alpha m}{\hbar}\left[j^1_x(\mathbf{x}, t) + j^2_y(\mathbf{x}, t)\right]\\
&\quad - \frac{2}{\hbar}\left[\rho^1(\mathbf{x}, t)b_{0,y} - \rho^2(\mathbf{x}, t)b_{0,x}\right],
\end{aligned}$$

for the case of an additional static magnetic field with components $b_{0,i}$, $i = x, y, z$, which holds independently of the impurity potential as ρ^i commutes with the position operator.

In deriving Eq. (23) the definition of j^i_k arises naturally as a divergence term of a current associated with the spin density. Together with the time derivative $\dot{\rho}^i$ it forms the left-hand side of a continuity equation. The right hand side, however, is nonzero and describes the dynamics of the spin due to the external magnetic field $\mathbf{b}_0$ and the internal SOI field. The definition of Eq.(21) as a 'spin current' is thus ambiguous[11, 12] and it is not clear to what extent the quantity Eq. (21) can be identified with actual spin transport, i.e. with spin polarized currents which are experimentally accessible[13].

In spite of the above concerns we note that in the homogeneous limit the spin Hall current can be expressed entirely in terms of the polarization. Namely, going over to the spin Hall current $I_k^i = \int d^2x < j_k^i(x) >$ such that the gradient in Eq.(23) vanishes we find the expectation value of the spin Hall current given by

$$\begin{aligned} I_x^3(\omega) &= \frac{\hbar}{2\alpha m}\left[i\omega S^1(\omega) + \omega_L S^3(\omega)\right] \\ &= \frac{e}{2\pi\hbar}E_2(\omega)\left[i\omega\tau\left(1 - \Sigma^{11}\left(1 - \frac{1}{\lambda}\right)\right) - \omega_L\tau\Sigma^{31}\left(1 - \frac{1}{\lambda}\right)\right]. \end{aligned} \tag{24}$$

[This relation can be obtained directly from the Heisenberg equation of motion $d\sigma^1/dt = i[H, \sigma^1]$, and by noting that $I_x^3 = \sigma^3 p_x$.] Since S^3 vanishes for $\omega = 0$ it is obvious from Eq. (24) that there is no spin Hall current in the dc limit $\omega \to 0$ for a homogenous infinite sample[6, 9, 10]. This means a generalization of the argument given in [9, 10] to the case of a finite magnetic field. For finite frequencies, however, Eq. (24) predicts a non-vanishing oscillating spin current expressed in terms of the polarization components perpendicular to the applied electric rf field. With the results for S^i inserted we find the ac spin Hall conductivity evaluated at resonance ($\omega = \omega_L$) as

$$\sigma_{xy}^{3,res} \equiv \frac{I_x^3\hbar/2}{E_y} = \frac{e}{4\pi}\frac{i\omega_L\tau}{1 + 2\lambda(\omega_L)}. \tag{25}$$

We emphasize that this relation provides a direct link between the experimentally accessible polarization and the spin current. For $\omega_L\tau \gg 1$ this becomes

$$\sigma_{xy}^{3,res} = -\frac{e}{8\pi} \tag{26}$$

giving a universal value for the spin current at resonance. It is quite remarkable that the same result, Eq. (26), can be obtained when inserting the solutions S^i obtained from the Bloch equations Eq. (11) close at resonance into Eq. (24).

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