nature physics
SUPPLEMENTARY INFORMATION
DOI: 10.1038/NPHYS4003

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Characterizing quantum channels with non-separable states of classical light

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DETERMINING THE TURBULENCE STRENGTH

The strength of a turbulent medium can be characterised by the Strehl ratio [1], which is defined as

$$S_R = \frac{I}{I_0}, \tag{1}$$

where I and I_0 are the on-axis intensities of the aberrated and non-aberrated Gaussian modes, respectively. This is applicable for both the weak and strong turbulence regimes, where 1 represents no turbulence and 0 represents a highly turbulent medium. Figure 1 illustrates the detrimental effects of different turbulence strengths on a vector vortex mode.

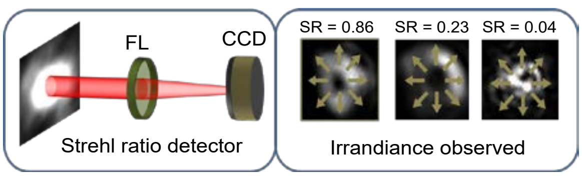

FIG. 1. The turbulence strength, given by the Strehl ratio, is measured by the relative drop in a peak intensity between a perturbed and non-perturbed Gaussian beam.

ENCODING TURBULENCE ON AN SLM

The Kolmogorov power spectrum is given by [1]

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \tag{2}$$

with $1/L_0 \leq \kappa \leq 1/l_0$, where L_0 and l_0 are the inner and outer scales of the turbulence, and define the limits within which the above power spectrum describes an isotropic and homogeneous atmosphere. The turbulence phase screens are generated by Fourier transforming the product of a random function with the power spectrum above. Using a SLM, we digitally generated turbulence phase screens and obtained the calibration curve illustrated in figure 2

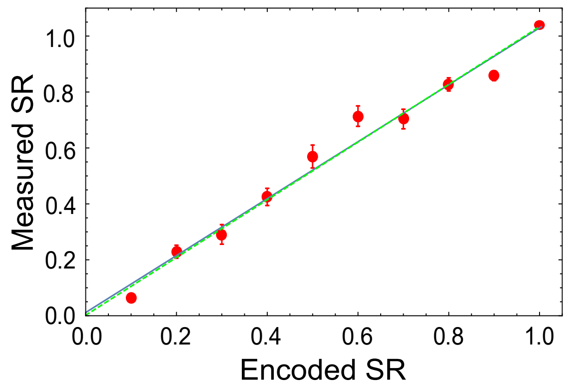

FIG. 2. Calibration of turbulence phase screens encoded on a SLM through a comparison of the encoded and measured SR values. The error bars represent the standard errors in the averages over the multiple turbulence realisations.

ERROR CORRECTION FOR TURBULENCE

A particular realisation of the turbulent atmosphere acting on the spatial modes and resulting in modal crosstalk, together with the detection of a subspace using a filter, can be represented by a single rank-two Kraus operator M, defined here as follows

$$\mathrm{M} = p_0 \left|\ell\right\rangle \left\langle\ell\right| + p_{-2\ell} \left|\ell\right\rangle \left\langle-\ell\right| + p_{2\ell} \left|-\ell\right\rangle \left\langle\ell\right| + p_0 \left|-\ell\right\rangle \left\langle-\ell\right|, \tag{3}$$

The Kraus operator in equation (3) can be expressed in polar decomposition as

$$M = U|M| = U(\lambda_0 \left|0\right\rangle \left\langle 0\right| + \lambda_1 \left|1\right\rangle \left\langle 1\right|), \tag{4}$$

where U is unitary and $\lambda_0 \left|0\right\rangle \left\langle 0\right| + \lambda_1 \left|1\right\rangle \left\langle 1\right|$ is the spectral decomposition of the positive operator $|M| = \sqrt{M^\dagger M}$. The action of the filter M can be compensated by a second 'conjugate' filter $\tilde{M}$ with $\tilde{M}M \propto \mathbb{1}$ given by

$$\tilde{M} = (\lambda_1 \left|0\right\rangle \left\langle 0\right| + \lambda_0 \left|1\right\rangle \left\langle 1\right|)U^\dagger, \tag{5}$$

which can be physically implemented. In the example of error correction discussed in the paper, the action of the channel M was compensated by processing the measurement data with M^{-1}. To visualise the correction process, the amplitudes of the orthogonal modes are shown in a crosstalk matrix in figure 3.

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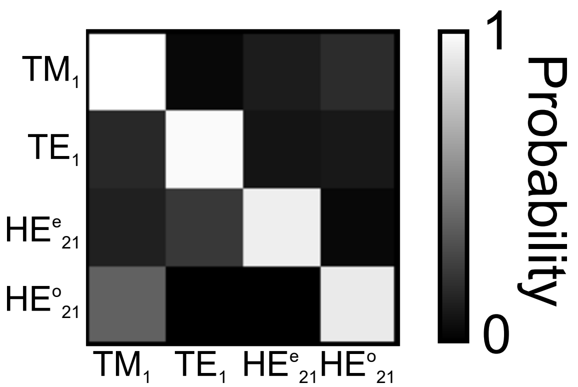

FIG. 3. Measured cross-talk of the orthogonal vector modes through a turbulent channel of strength SR = 0.3.

EFFECTS OF OAM CROSSTALK ON THE CONCURRENCE

For a hybrid OAM-polarisation qubit state $|\Psi\rangle_\ell = \alpha|\ell\rangle|R\rangle + \gamma|\ell\rangle|L\rangle + \beta|-\ell\rangle|L\rangle + \tau|-\ell\rangle|R\rangle$, the concurrence is computed as follows [2]

$$\mathcal{C}(|\Psi\rangle_\ell) = 2|\alpha\beta - \gamma\tau|. \quad (6)$$

For a vector vortex mode defined as $|\Psi_\ell\rangle = \alpha|\ell\rangle|R\rangle + \beta|-\ell\rangle|L\rangle$, the concurrence reduces to $\mathcal{C}(|\Psi\rangle_\ell) = 2|\alpha\beta|$.

Recall the expression derived for the concurrence of the input and output states:

$$\mathcal{C}(|\Psi_\ell\rangle_{\text{out}}) = |p_0^2 - p_{2\ell}p_{-2\ell}|\ \mathcal{C}(|\Psi_\ell\rangle_{\text{in}}), \quad (7)$$

where we omitted a normalization constant for the sake of simplicity. We can extend our analysis by imposing conditions on p_ℓ. We want a symmetric distribution with its maximum $p_\ell = 1$ centered at $\ell = 0$. For the sake of the argument, we will assume a Gaussian-like discrete function for p_ℓ

$$p_\ell = \exp(-\ell^2/2\Delta^2), \quad (8)$$

where ℓ is the OAM index and Δ is the width of the distribution, which depends on turbulence. We can rewrite equation 7 as:

$$\mathcal{C}(|\Psi_{\text{out}}\rangle) = |1 - \exp(-\ell^2/\Delta^2)|\ \mathcal{C}(|\Psi_{\text{in}}\rangle). \quad (9)$$

If $\ell = 0$, then the concurrence $C(|\Psi_\ell\rangle_{\text{out}}) = 0$, which is explained by the fact that the input beam is not a vector beam. In the case of $\ell \to \infty$, the concurrence of the output state will be equal to that of the input state: $C(|\Psi_\ell\rangle_{\text{out}}) = C(|\Psi_\ell\rangle_{\text{in}})$. This is because the OAM modes are so far apart that the crosstalk resulting from the turbulence will not affect the measured OAM modes, as illustrated in figure 4. If $\Delta = 0$, then $C(|\Psi_\ell\rangle_{\text{out}}) = C(|\Psi_\ell\rangle_{\text{in}})$, as this implies that the initial state is not perturbed (no turbulence).

[1] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation Through Random Media*, SPIE Press, Bellingham, Washington, (1998).

[2] W. Wootters, Entanglement of formation and concurrence, Quantum Inf. Comput. **1**, 27–44 (2001)
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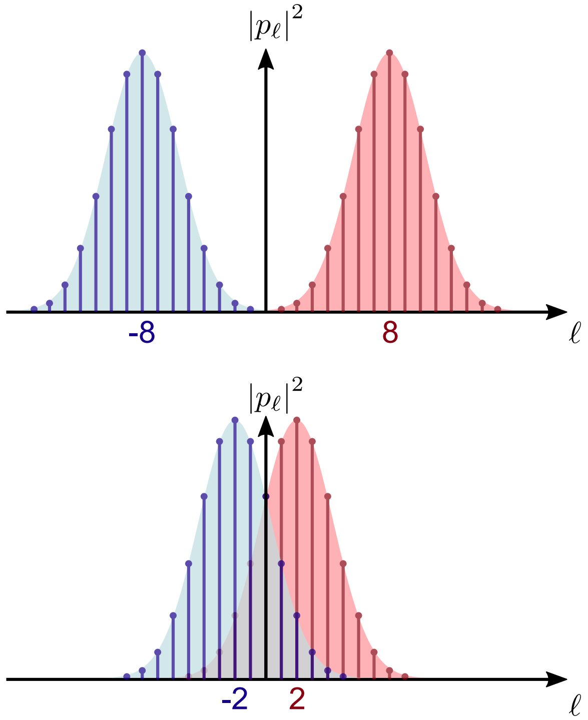

FIG. 4. Schematic representation of OAM crosstalk. Increasing the separation of the modes or decreasing the width of each Gaussian curve (decreasing turbulence) reduces the crosstalk.