

Description of Supplementary Files

File Name: Supplementary Information

Description: Supplementary figures

File Name: Supplementary Movie 1

Description: Turbulent active nematic. Fluorescence micrographs of the active nematic in contact with an isotropic oil.

File Name: Supplementary Movie 2

Description: Active nematic swirls in contact with toroidal focal conic domains (TFCDs).

File Name: Supplementary Movie 3

Description: Influence of the activity on the defect trajectories of confined active nematics.

File Name: Supplementary Movie 4

Description: Preservation of the topological charge inside active swirls.

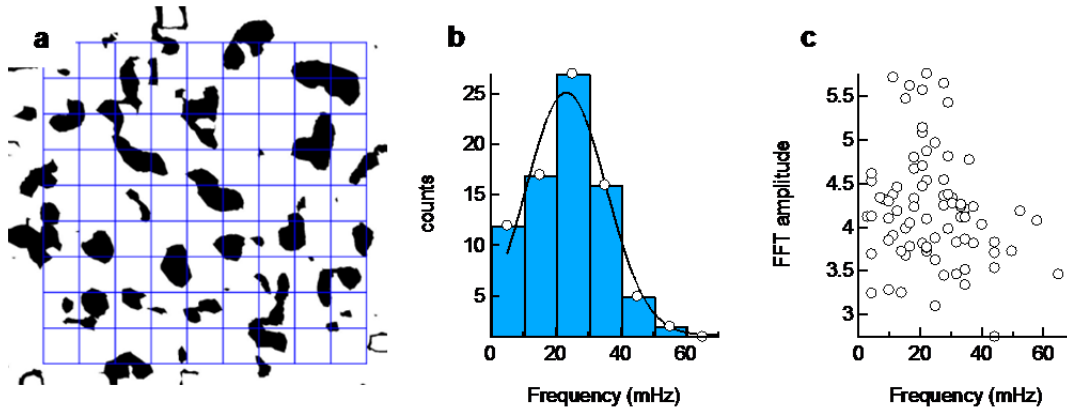
File Name: Supplementary Movie 5

Description: Active nematic swirls are metastable.

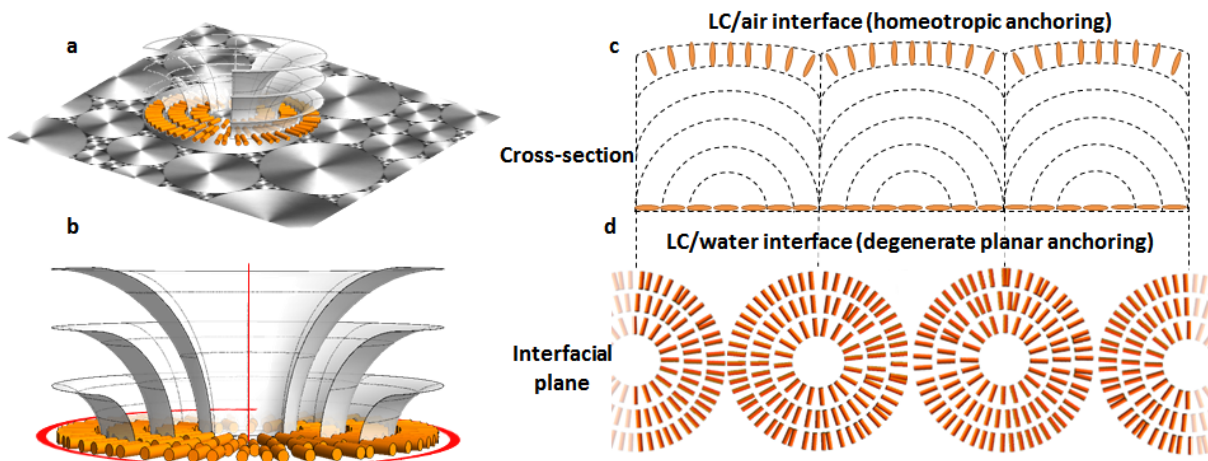
File Name: Supplementary Movie 6

Description: Active nematic swirls induce localized flows.

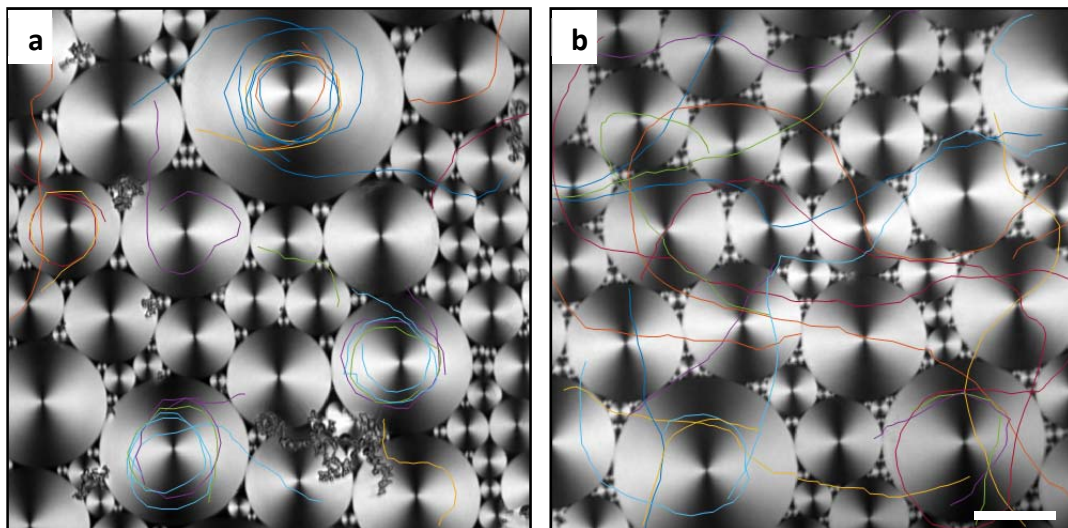
File Name: Peer Review File



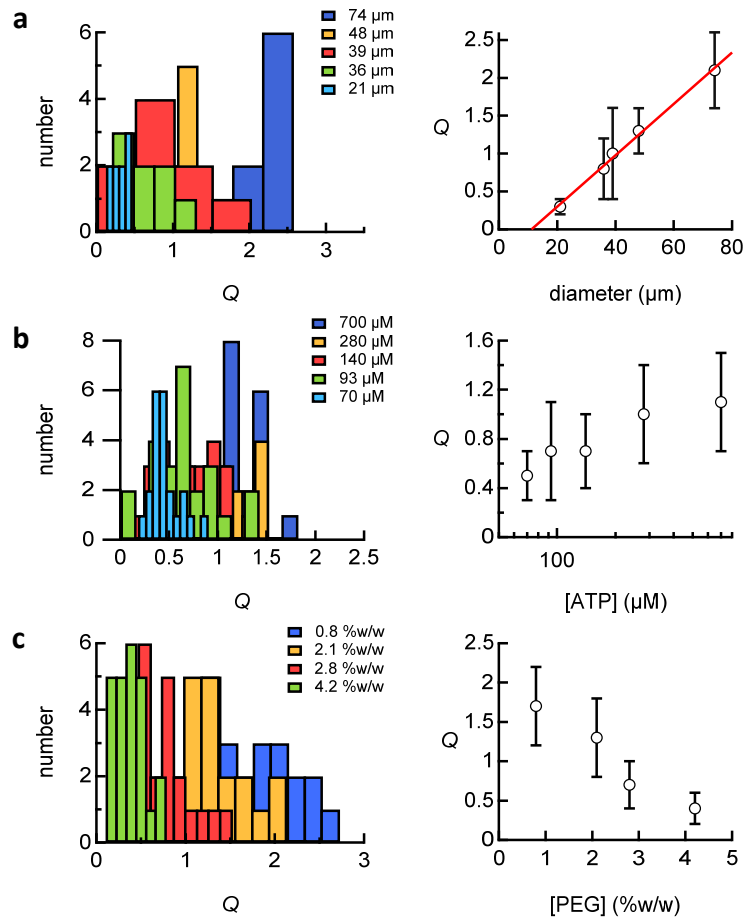
Supplementary Figure 1. Characteristic frequency of the active nematic in the turbulent regime. (a) Snapshot of the AN in the unconstrained turbulent regime for the same conditions described in Fig. 1 of the main text. Here, the fluorescence images have been binarized considering the Okubo-Weiss parameter and the criterion $OW = (\partial_x v_x)^2 + \partial_y v_x \cdot \partial_x v_y < 0$. To obtain a statistical assessment of the vortex lifetime in this regime, we divide the area in $30 \times 30 \mu\text{m}^2$ cells, a size consistent with the characteristic length scale extracted from the distribution of vortex sizes (see main text and Fig. 1). We then measure the area fraction with $OW < 0$ for each of the cells as a function of time (during 1000 s), and we compute the Fast Fourier Transform (FFT) of this signal. (b) Distribution of characteristic frequencies for the ensemble of 81 boxes in a. The solid line is a Gaussian fit. (c) Height of the highest peak in the power spectra of the data in a as a function of its frequency, for the ensemble of 81 boxes.



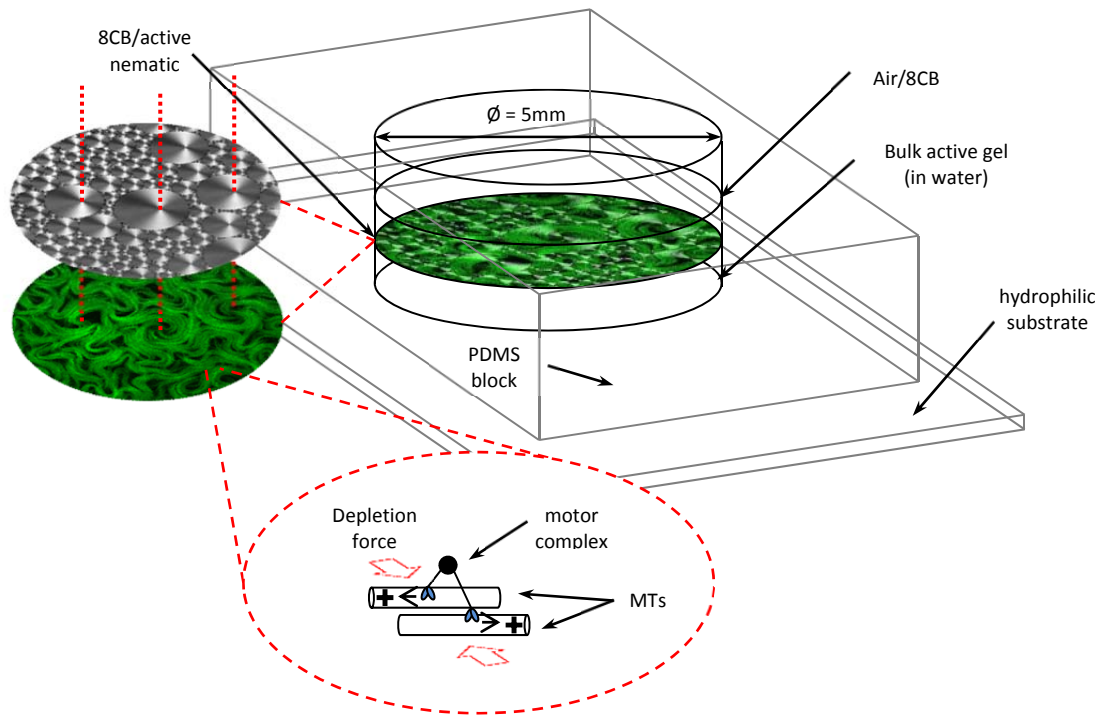
Supplementary Figure 2: Structure of the TFCDs. **a** Confocal reflection micrograph (field of view $300 \times 300 \mu\text{m}^2$) showing the toroidal focal conic domains at the 8CB/water interface. A sketch of the 3D SmA planes and the orientation of the mesogen molecules on top of a domain is included. **b** SmA lamellae bend around two singular lines (depicted in red in the sketch) namely, a circle at the LC/water interface, and a straight line running from the circle center into the bulk of the LC. **c** Cross section of the bulk structure of the lamellar planes in the LC across the sample thickness. **d** Organization of the LC molecules at the interface.



Supplementary Figure 3. Trapping of defects by Toroidal Focal Conic Domains. **a** Confocal reflection micrograph of different TFCD and overlaid trajectories of $+1/2$ defects. Each color corresponds to a different defect. Experimental conditions of the AN are $[\text{ATP}] = 700 \mu\text{M}$, $[\text{PEG}] = 0.8\% \text{w/w}$. **b** Similar study but with experimental conditions of the AN are $[\text{ATP}] = 70 \mu\text{M}$, $[\text{PEG}] = 0.8\% \text{w/w}$. The scale bar is $25 \mu\text{m}$.



Supplementary Figure 4. Distribution of winding number as a function of different control parameters. **a** Dependence of Q with TFCD diameter, for experiments performed with $[ATP] = 700 \mu\text{M}$ and $[PEG]=0.8 \text{ \%w/w}$. The line is a linear fit to the data. **b** Dependence of Q with $[ATP]$ for circulation around TFCD with diameter $60 \mu\text{m}$, and experiments performed with $[PEG] = 1.6 \text{ \%w/w}$. **c** Dependence of Q with $[PEG]$ for circulation around TFCD with diameter $60 \mu\text{m}$, and experiments performed with $[ATP] = 1.4 \text{ mM}$. Error bars are the standard deviation for the measurements under each condition.



Supplementary Figure 5: Sketch of the experimental setup. The active nematic forms at the LC/water interface contained in the open cavity of a custom PDMS block, where all the experiments are performed.