# SUPPLEMENTARY INFORMATION

**Supplementary Note 1**:

The distinction between the BEC and BCS limits is theoretically unambiguous. However, there is no unique definition of the "BCS to BEC crossover regime." Here, we have adopted a thermodynamic definition in terms of the change in the chemical potential. Others, notably in Ref. [26], have proposed a broader definition to include a range of conditions in which there is a significant fluctuational regime above $T_c$.

**Supplementary Note 2**:

Whether superconducting coherence is destroyed thermally at $T > T_c$ or by vortex proliferation for $H > H_{c2}$, in the BEC limit the "normal" state proximate to the superconducting state is an uncondensed phase of "preformed" Cooper pairs, with vanishing density of electron quasi-particles.

**Supplementary Note 3**:

For a recent review. see Ref. [79].

**Supplementary Note 4**:

There are photoemission experiments reporting gapped nodes in highly underdoped cuprates below p = 0.08 - 0.10 [74, 80–82]. However, these gaps preserve the nodal Fermi momenta, lack coherent quasi-particles, and are all two orders of magnitude smaller compared to the Fermi energy.

**Supplementary Note 5**:

One should note that there exists fine-tuned circumstances - both in terms of electron density (i.e. 'charge-neutrality') and band-structure considerations - under which an exact particle-hole symmetry pins the chemical potential to a specific value, independent of $T$ or whether one is in the normal or superconducting state. Obviously in this case, the chemical potential cannot be used as a metric of the BCS to BEC crossover. Such a symmetry is manifestly absent in the cuprates.

**Supplementary Note 6**:

Such tight-binding fits to the low energy electronic structure capture the Fermi surface shape accurately, but substantially underestimate the binding energy of the band bottom at $\Gamma$ [43].

Hence, our simulation is already biased towards a crossover regime with an underestimated $E_F$.

**Supplementary Note 7**:

$E_F$ can only be directly related to the electron density in cases in which the band-structure is well approximated by a free electron dispersion, which is not the case in the cuprates. The values of $E_F$ that we quote are obtained directly from photoemission so the issue of whether the electron density is $x$ or $(1 + x)$ need not be resolved for our purposes.

**Supplementary Note 8**:

A different approach to analyzing the magnetization data has been applied in Ref. [83] which leads to a much narrower inferred regime of superconducting fluctuations. The delineation of the fluctuation regime is an important open issue, but not of immediate relevance for present purposes.

**Supplementary Note 9**:

Both samples whose ARPES spectra are shown in Fig. 2 are moderately overdoped, which makes it is easier to track dispersing features. The deviations of the low energy spectra from the local density approximation (LDA) are even more pronounced in underdoped samples; they exhibit a pseudo-gap and extremely broad features in their energy distribution curves (EDCs). However, the high energy portions of the spectra - which are of primary importance for present considerations - are not highly doping dependent in the relevant range of hole-doping [42].