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# **Carbon star formation as seen through the non-monotonic initial–final mass relation**

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## **Supplementary Information**

### **Carbon star formation as seen through the non-monotonic initial-final mass relation**

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#### **The WD mass distribution**

The observed field-WD mass distribution (WDMD) in the MW can be employed to further check the impact of the IFMR kink and, more generally, of any assumed IFMR. Ideally, one may simultaneously use the observed IFMR data and the WDMD to constrain the coefficients of a parametric functional form expressing  $M_{\mathsf{f}}$  as a function of  $M_{\mathsf{i}}.$  However, at present the application of this method is made troublesome by the fact that a detailed consideration of the WDMD would introduce additional parameters to be examined, affecting both the population synthesis simulations (initial mass function, star formation history, chemical enrichment law) and the underlying stellar evolutionary models, in particular the metallicity dependence of all relevant processes involved (3DU and mass loss). This approach would introduce further uncertainties in our analysis with the consequence that the significance of our main observable, the IFMR, would be weakened. Therefore, at this stage, our strategy is to adopt the IFMR as the primary calibrator and employ the WDMD as an auxiliary verification tool to test the impact of different fitting forms. The fits examined here are illustrated in Supplementary Fig. [1.](#page-2-0) They adopt different assumptions concerning the IFMR kink and the behaviour of the IFMR through the region (2.0  $\lesssim M_1/M_{\odot} \lesssim 2.8$ ) that shows a substantial lack of data.

Supplementary Fig. [2](#page-3-0) shows the resulting WDMDs for the five considered IFMR fits. All sim-ulations were carried out with the TRILEGAL code,<sup>[1](#page-10-0)</sup> assuming a Kroupa initial mass function,<sup>[2](#page-10-1)</sup> an age-metallicity relation and a star formation history suitable for the MW disk. The stellar evolutionary tracks are taken from the  $\texttt{PARSEC}^3$  $\texttt{PARSEC}^3$  and  $\texttt{COLIBRI}^{4,5}$  $\texttt{COLIBRI}^{4,5}$  $\texttt{COLIBRI}^{4,5}$  $\texttt{COLIBRI}^{4,5}$  $\texttt{COLIBRI}^{4,5}$  databases, tailored to a grid of H-burning Post-AGB tracks and WD cooling sequences.<sup>[6](#page-10-5)</sup> Here we examine the distribution of WDs with masses from 0*.*55 M*<sup>⊙</sup>* to 1*.*2 M*⊙*. We exclude WDs with *M*<sup>f</sup> *<* 0*.*55 M*<sup>⊙</sup>* because their proper consideration would require taking care of various aspects that are still quite uncertain or go beyond the scope of this study, like the metallicity dependence of the IFMR (the metal-poor regime, in particular) and the effects of binary interactions. Within the considered mass range, the SDSS data[7](#page-10-6) has 4597 WDs of type DA with a signal-to-noise ratio *>* 10. The simulated distributions were re-scaled to the same number of WDs as observed.

The simplest case (fit #1) corresponds to a monotonic relation that runs through the data over the range  $0.8 \lesssim M_{\rm i}/\rm M_{\odot} \lesssim 3.1$ . The monotonic representation has been the standard choice of all IFMR studies so far. This fit produces a rough approximation of the observations, but results in a WDMD peak located at too high mass, from  $0.60$  M<sub>⊙</sub> to  $0.65$  M<sub>☉</sub> (Supplementary Fig. [2a](#page-3-0)). The shift is the consequence of the higher intercept we derive when attempting to describe the observed data through a single linear representation. This inconsistency, in addition to its poor representation of the IFMR data around 1*.*7 M*<sup>⊙</sup> −* 2*.*0 M*<sup>⊙</sup>* further argues against a simple monotonic fit.

As next step we relax the monotonic assumption. We first fit the rising part of the IFMR up to a peak of  $M_{\rm f}\simeq 0.71\,{\rm M}_\odot$  located at  $M_{\rm i}\simeq 1.8\,{\rm M}_\odot$ , and then, beyond it at larger  $M_{\rm i}$ , we test a few trends. We start by adding a plateau (fit #2) of  $M_f \simeq 0.71 M_{\odot}$  to join the increasing branch starting at *M*<sup>i</sup> *≃* 2*.*8 M*⊙*. In this way we exclude the WD belonging to NGC 752 from the fit. Clearly, the case #2 provides a very poor match of the whole observed WDMD from  $M_f \simeq 0.6$  M<sub> $\odot$ </sub> to  $M_f \simeq 0.8$  M<sub> $\odot$ </sub>, with a dramatic excess of WDs of  $M_f \approx 0.70 - 0.75$  M<sub>⊙</sub> (Supplementary Fig. [2b](#page-3-0)).

<span id="page-2-0"></span>

**Supplementary Figure 1:** Sample of five fitting relations compatible with the semi-empirical IFMR, which mainly differ in the description of the interval 1.7  $\lesssim M_{\rm i}/\rm M_{\odot}$   $\lesssim$  2.8. They are meant to explore the effect of introducing a non-monotonic feature in the IFMR, and to analyse its impact on the WDMD when varying the width and the amplitude of the associated kink. The error bars in the semi-empirical IFMR cover the range of  $\pm 1 \sigma$ .

Recognised that the plateau with  $M_f \simeq 0.71 M_{\odot}$  over the range  $1.8 \lesssim M_i/M_{\odot} \lesssim 2.8$  does not work, we reintroduce the NGC 752 data and impose that the fit #3 goes through the weighted averages of the WDs in the clusters over the interval  $1.65 \lesssim M_{\rm i}/\rm M_{\odot} \lesssim 2.1$ . After reaching a peak of  $M_f \simeq 0.71 \text{ M}_{\odot}$  at  $M_i \simeq 1.91 \text{ M}_{\odot}$ , the fit #3 decreases down to recover the mass  $M_f \simeq 0.65 \text{ M}_{\odot}$ of N752-WD01 at  $M_i$   $\simeq 2.1 M_{\odot}$ . From this point a small plateau with  $M_f$   $\simeq 0.65 M_{\odot}$  extends to *M*<sub>i</sub> ≃ 2.4 M<sub>⊙</sub>, where it intersects the extrapolated linear relation that describes the rising branch at larger masses (*M*<sup>i</sup> *>* 2*.*8 M*⊙*). Introducing this kink certainly improves the representation of the IFMR over the relevant  $M_{\rm i}$  range, and at the same time it reduces the discrepancy in the WDMD that affects the previous plateau case (fit #2). We see that also the fit #3 yields an excess of WD with *M*<sup>f</sup> *≃* 0*.*65 *−* 0*.*70 M*⊙*, though at a lower level (Supplementary Fig. [2](#page-3-0)c).

In the previous case (fit #3), we have tested the assumption that the IFMR stays flat over the range between  $M_i \simeq 2.1 \, \text{M}_\odot$  and  $M_i \simeq 2.4 \, \text{M}_\odot$ , just where the IFMR is essentially unconstrained due to the lack of data. Therefore, we are driven to analyse other trends in this region. Now we examine the case (fit #4) in which, after reaching a minimum of  $M_f \simeq 0.65$  M<sub>⊙</sub> in correspondence to N752-WD01, the IFMR regains a positive slope connecting to the branch starting at  $M_{\rm i}\simeq 2.8$  M $_{\odot}$ . While the performance of the fit  $#4$  is the same as the previous  $#3$  – the two only differ in the region void of data–, the impact on the WDMD is sizeable. With the fit #4 the excess of WDs in the range  $M_f$   $\simeq 0.65 - 0.70$  M<sub>☉</sub> appears now amplified (Supplementary Fig. [2](#page-3-0)d).

<span id="page-3-0"></span>

**Supplementary Figure 2:** The field white dwarf mass distribution in the MW. The observed distri-bution (purple histogram) is taken from the SDSS DR[7](#page-10-6).<sup>7</sup> Over the range  $0.55 \lesssim M_i/M_{\odot} \lesssim 1.2$  we overplot the results of population synthesis simulations (orange histograms) obtained assuming the same numbers of WDs as observed, and five different IFMR fits, shown in the insets and in Supplementary Fig. [1.](#page-2-0)

This fact leads us to increase the depth of the kink. With the fit  $#5$  we assume that, after matching N752-WD01, the decrease continues down to a slightly deeper minimum ( $M_f \simeq 0.62$  M<sub> $\odot$ </sub> at *M*<sup>i</sup> *≃* 2*.*2 M*⊙*), beyond which a linear positive trend is re-established so as to pass through the data at intermediate  $M_{\rm i}$ . In this way the surplus of WDs with  $M_{\rm f}\simeq 0.7$  M $_{\odot}$  is reduced to a satisfactory level. The resulting theoretical distribution much better matches the observed peak and approximately recovers the intermediate to high-mass wing (Supplementary Fig. [2e](#page-3-0)).

From the tests performed we conclude that a reasonably good reproduction of both the IFMR and the WDMD supports the existence of a kink in the IFMR. The decline that follows the peak at *M*<sup>i</sup> *≃* 1*.*8 *−* 1*.*9 M*<sup>⊙</sup>* is needed to avoid the occurrence of a sizeable excess of WDs with *M*<sup>f</sup> *≃*  $0.7\,\textsf{M}_\odot$ . Clearly, a more precise assessment of the extension (in  $M_\textsf{i}$ ) and the amplitude (in  $M_\textsf{i}$ ) of the kink will require further efforts involving additional observational data across the range 2.1  $\lesssim M_{\rm i}/M_{\odot}$   $\lesssim$  2.8, coupled to a more detailed modelling of the WDMD. Future studies will be devoted to address these aspects.





### **Other supporting evidence: Galactic semi-regular variables**

Observed properties of Galactic AGB stars belonging to the class of semi-regular variables provide hints that support the proposed physical explanation. Let us examine the most relevant facts.

**Progenitors of the IFMR kink.** The picture proposed to explain the existence of the IFMR kink relies on the existence of carbon stars with small carbon enrichment and mild mass loss. In fact, Galactic carbon stars with low C *<sup>−</sup>* O (*<sup>&</sup>lt;* <sup>8</sup>*.*2) and low C/O (*<sup>&</sup>lt;* <sup>1</sup>*.*5) are observed,[14](#page-10-11) and have spectra dominated by dust-free stellar photospheres<sup>[15](#page-10-12)</sup> (see also Fig. 3, main article). They have moderate mass-loss rates[16](#page-11-0) (median value of *<sup>≃</sup>* <sup>2</sup> *<sup>×</sup>* <sup>10</sup>*−*<sup>7</sup> <sup>M</sup>*<sup>⊙</sup>* yr*−*<sup>1</sup> ), not exceeding 10*−*<sup>6</sup> <sup>M</sup>*<sup>⊙</sup>* yr*−*<sup>1</sup> , as well as low expansion velocities,[16](#page-11-0) typically with *v*exp *<* 10 km s*−*<sup>1</sup> (median value *<sup>≃</sup>* <sup>7</sup> *<sup>−</sup>* 8 km s*−*<sup>1</sup> ). They are long-period variables,<sup>[14](#page-10-11), [16](#page-11-0)</sup> with pulsation periods  $P < 400$  days (median value  $\simeq 200$ days), mostly belonging to the classes of Irregulars (Lb) and Semi-regulars (Sra, Srb).

Overall, these facts converge to the conclusion that, as long as C/O remains low enough, carbon stars have weak winds, not sustained by radiation pressure on dust grains, but likely, by low-amplitude stellar pulsation.[17](#page-11-1)–[19](#page-11-2)

All the above-mentioned properties belong to the group of Galactic carbon stars inside the light blue region *B* of Supplementary Fig. [4](#page-6-0) and, according to the proposed scenario, should also characterise the carbon-rich progenitors of the WDs we see now populating the IFMR kink. To better investigate this point we computed the evolution of the terminal wind velocity along a few selected TP-AGB tracks by means of a stationary AGB wind model,<sup>[20](#page-11-3)</sup> which follows the growth of the dust grains as a function of stellar parameters (C/O, *M*, *L*, *T*eff, *M*˙ ). The adopted density profile across the circumstellar envelope was modified to include the effects of shocks in the inner wind.<sup>[21](#page-11-4)</sup> We see that the model with  $M_i = 1.85 M_{\odot}$  spends most of its carbon-star phase inside or close to the region *B*, never exceeding  $\dot{M} \simeq 600$  few  $10^{-6}$  M $_{\odot}$  yr $^{-1}$  and  $v_{\rm exp}~\simeq 10$  km s $^{-1}$ . The predicted low values of the carbon excess are in agreement with the measured chemical abundances of this group of carbon stars.

These results support the proposed picture: the observed mild carbon enrichment and moderate mass loss, which define this sub-sample of SRV stars, set the suitable conditions for a prolongation of the carbon-star phase, hence for the growth of the core mass. We note that stellar models with larger initial masses ( $M_i = 2.0 M_{\odot}$  and  $M_i = 2.8 M_{\odot}$ ) may also cross the region *B*, but they soon leave it and attain larger carbon excess, as well as higher mass-loss rates and outflow velocities. Their properties are consistent with the group of infrared carbon stars, dom-inated by the presence of dust in their observed spectra,<sup>[22](#page-11-5)</sup> such as the well-known IRC+10126<sup>[23](#page-11-6)</sup> which is characterised by  $C/O \approx 2.5$ , as inferred from molecular emission lines.<sup>[24](#page-11-7)</sup>

**Mass loss of Galactic carbon stars.** A closer look at the infrared properties of long-period variables in the Milky Way provides further support to the proposed mass-loss scenario for carbon stars.

To this aim it is useful to consider the colour *K−*[22], where [22] is the 22 *µ*m band of the Wide-field Infrared Survey Explorer (WISE) space observatory.<sup>[25](#page-11-8)</sup> As pointed out in a recent study,<sup>[26](#page-11-9)</sup> the colour *K−*[22] is an indicator of the mass-loss rate, as can be appreciated in Supplementary Fig. [6,](#page-9-0) which combines photometric data with recent mass-loss rate measurements, based on molecular line emission, for a group of MW long-period variables. We see a clear positive correlation between  $K - [22]$  and  $\dot{M}$ . The SRVs mostly populate the branch steeply rising in mass-loss rate (*M*˙ up to *<sup>≈</sup>* <sup>10</sup>*−*<sup>6</sup> *<sup>M</sup><sup>⊙</sup>* yr*−*<sup>1</sup> ) characterised by low values of the infrared colour, up to *K −* [22] *≈* 3. Then, the positive trend extends with the Mira stars which draw an elongated branch at larger mass-loss rates, reaching quite red colours, up to  $K - [22] ≈ 11$ .

We fit the data of Supplementary Fig. [5](#page-7-0) by adopting a functional form  $y = a/(1 + b \times x) + c$ , where  $x = K - [22]$ ,  $y = \log(M_6)$  and  $M_6$  is the mass-loss rate in units of  $10^{-6} M_{\odot}$  yr<sup>−1</sup>. The coefficients of the best-fitting solutions are  $a = -13.068$ ,  $b = 0.814$ , and  $c = 3.205$  for M-type

<span id="page-6-0"></span>

**Supplementary Figure 4:** Mass-loss rates versus terminal wind velocities of Galactic intrinsic carbon stars. The observed data include a sample<sup>[16](#page-11-0)</sup> of optically-bright carbon stars (circles: Semi-regular and Irregular variables; diamonds: Mira variables) and an IR-complete sample<sup>[22](#page-11-5)</sup> (crosses). When the photospheric abundances of carbon and oxygen are available from spectroscopic analyses,[14,](#page-10-11) [27](#page-11-10) the stars are colour-coded according to the carbon excess, C *<sup>−</sup>* O. The light-blue region (labelled with *B*) include Galactic carbon stars with low C/O and C *−* O, moderate *M* and low  $v_{exp}$ , small-amplitude pulsation, mostly dust-free spectra. The median value of the measured carbon excess is *≃* 7*.*5, well below the threshold of dust-driven winds. Carbon stars outside the *B* region, such as the infrared carbon star IRC+10126 (pentagon), exhibit larger values of all these parameters (C/O, C *<sup>−</sup>* O, *<sup>M</sup>*˙ , *v*exp, *P*), and their spectra are dominated by dust. For comparison, we overplot three evolutionary tracks during the carbon-star phase, with initial masses (in  $M_{\odot}$ ) as indicated. The track with  $M_{\rm i} = 1.85\,M_{\odot}$  and initial solar metallicity runs mostly within this region and exemplifies the evolution of the progenitor carbon stars that populate the IFMR kink with their WD remnants.

<span id="page-7-0"></span>

**Supplementary Figure 5:** Relation between the gas mass-loss rate and the infrared colour K-[22] for a sample of MW semi-regular (triangles) and Mira variables (squares). Mass-loss measurements for Miras are taken from an extensive compilation of data, $26$  but restricted to the most recent analyses.<sup>[28–](#page-11-11)[32](#page-12-0)</sup> Data for SRVs are obtained from a new study<sup>[33](#page-12-1)</sup> that employs DR2 Gaia distances.[34](#page-12-2) Stars with spectral types M and C are coloured in blue and red, respectively. The solid lines are two fit relations with the same colour coding as for the data, the dashed line represents a linear fit (over the approximate range  $1.5 \leq K - [22] \leq 7.5$ ) proposed in a recent work.<sup>[26](#page-11-9)</sup>

stars and  $a = -4.036$ ,  $b = 0.273$ , and  $c = 1.554$  for C-type stars. We use the two relations to colour a few evolutionary tracks and compare the results with observations of AGB variable stars in the diagram that relates the pulsation period with the infrared colour *K−*[22] (Supplementary Fig. [6\)](#page-9-0). Predicted periods for different radial models are obtained on the basis of an extensive theoretical work.<sup>[35](#page-12-3)</sup> We apply the predicted fundamental period,  $P_0$ , an empirical correction (log $(P_0^{\sf cor})=$  $log(P_0) + 2.341 \times log(T_{\text{eff}}) - 8.33$ ), which has shown to yield a satisfactory reproduction of the sequence C, mostly populated by Miras, in the period-luminosity diagram of the Small Magellanic Cloud.<sup>[36](#page-12-4)</sup> In this respect it is worth underlining that the main focus here are the not the Mira variables but rather the SRVs, as is clear from the previous analyses on dust condensation (Fig. 3, main article), terminal wind velocities (Supplementary Fig. [4\)](#page-6-0), as well as from the discussion that follows.

The diagram in Supplementary Fig. [6](#page-9-0) is helpful to test the plausibility of the proposed massloss scenario, including the drop in mass loss during the very early stages of the C-star phase. The choice of the two evolutionary models with initial masses, 1*.*95 M*<sup>⊙</sup>* and 2*.*40 M*⊙*, is motivated by the fact that the former should represent a stellar progenitor of the IFMR kink, while the latter exemplifies the case of a star which gets more efficiently enriched in carbon and populates the IFMR beyond the kink. We also note that the density of points along the tracks is controlled by the sampling of the pulse-cycle phase and it is not indicative of the observational probability, for which dedicated population synthesis simulations are necessary. In addition to our reference CDYN mass-loss prescription (Supplementary Figs. [6](#page-9-0)a-[6](#page-9-0)b), we also consider TP-AGB models computed with the VW93 and B95 formulas, which do not depend explicitly on the carbon abundance (Supplementary Figs. [6c](#page-9-0)-[6](#page-9-0)f).

Independently of the assumed mass loss, both TP-AGB tracks with  $M_i = 1.95 M_{\odot}$  and  $M_i =$ 2*.*4 M*<sup>⊙</sup>* show a similar evolution of their pulsational properties. During the oxygen-rich stages (with  $C/O < 1$ ) pulsation takes place in the 2<sup>nd</sup> and 1<sup>st</sup> overtone modes with periods  $\lesssim 200$  days. Soon after the transition to the C-star domain (with  $C/O \gtrsim 1$ ), the evolution is initially still dominated by the 1<sup>st</sup> overtone modes, until when the fundamental mode becomes excited, remaining dominant up to the end of the quiescent TP-AGB evolution. Longer periods (typically  $P_0 > 250$ days) are attained, the colour *K −*[22] becomes redder and the models cross the sequence populated by the Miras. During these stage transitions back to higher overtone modes may take place along the same pulse cycle, typically when a star is in the post-flash low-luminosity dip.<sup>[35](#page-12-3)</sup>

While this picture holds, in general terms, for all the models shown in Supplementary Fig. [6,](#page-9-0) important differences emerge in the infrared colours *K −* [22]. During the C/O *<* 1 stages the models (purple) go through the region occupied by the M-type SRVs (blue ellipse). Their colour *K* − [22] remains rather low, indication that the mass-loss rates are mostly moderate. Relatively red colours ( $K - [22] \simeq 3$ ) are attained only by the  $M_i = 1.95 M_{\odot}$  at the tip of its 1<sup>st</sup> overtone sequence, just before the transition to the carbon-star regime.

When the stars become carbon rich, the behaviour of the models radically differ depending on the assumed mass-loss scheme. Models that adopt the CDYN prescription (Supplementary Figs. [6](#page-9-0)a-[6b](#page-9-0)) experience a drop in mass loss due to the low carbon excess and experience a temporary pulsation-driven moderate wind phase. The mass-loss drop translates into a colour jump: the model with  $M_i = 1.95 M_{\odot}$  suddenly falls from  $K - [22] \approx 3$  at the tip of the 1<sup>st</sup> overtone Orich sequence down to *K −* [22] *≈* 1-2, values that characterise the group of C-type SRVs (red ellipse). A similar behaviour, but with a less extended colour jump, is exhibited by the model with  $M_i = 2.40 \text{ M}_{\odot}$ .

In summary, the CDYN models are able to reach the region of C-type SRVs as a consequence of the drop in mass loss predicted after the transition to the carbon-star phase. Later, as more TPs take place, the fundamental mode becomes dominant and the models attain larger mass-loss rates evolving through the branch mostly occupied by Mira stars at increasing period and redder *K* − [22]. We also note that the bottom of the fundamental-mode sequence is partly populated by SRVs.

Conversely, the VW93 and B95 models (Supplementary Figs. [6](#page-9-0)c-[6f](#page-9-0)) do not recover the observed bluer *K −* [22] colours characteristic of the C-type SRVs and remain outside the red ellipse. This implies that both prescriptions tend to overestimate the mass-loss rates during the early stages of the carbon-star evolution.

<span id="page-9-0"></span>

**Supplementary Figure 6:** Infrared colour *K −* [22] vs pulsation period of AGB stars in the MW. Symbols denote different classes of variables (SRV: filled triangles; Mira: empty squares) while colours correspond to different spectral types (blue: M-type, cyan: S-type, red: C-type). The location of of the bulk of M-type and C-type SRVs is highlighted with two elliptical shaded areas to help the comparison with the models. Here we include the results of TP-AGB evolutionary calculations for models with  $M_i = 1.95 M_{\odot}$ ,  $2.40 M_{\odot}$ . Three descriptions of mass loss are adopted, namely the reference CDYN (**a**-**b**), B95 (**c**-**d**) and VW93 (**e**-**f**). The model results are shown for discrete values of the pulse-cycle phase  $\phi$  (for each TP  $\phi$  goes from 0.1 to 1 in steps of 0.1). Predicted periods as a function of stellar parameters are obtained from analytic relations<sup>[35](#page-12-3)</sup> for different pulsation modes (cross: second overtone; filled circle: first overtone; filled circle with a yellow dot: fundamental mode), using a colour code that depends on the surface C/O (purple:  $C/O < 1$ , magenta:  $C/O > 1$ ). The labels  $P_2$ ,  $P_1$ ,  $P_0$  are placed close to the model sequences that share the same pulsation mode (second overtone, first overtone, and fundamental, respectively).

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