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Stationary entangled radiation from micromechanical motion

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THEORETICAL MODEL

A. Hamiltonian of double resonator electromechanics

Our electromechanical system consists of a mechanical resonator (MR) that is capacitively coupled to two superconducting microwave resonators as depicted in Fig. 1 of the main text. These resonators' driving fields are at radian frequencies $\omega_{\mathrm{d},j} = \omega_{c,j} - \Delta_{0,j}$, where the $\Delta_{0,j}$ are the detunings from their resonant frequencies $\omega_{c,j}$, with $j = 1, 2$. We include intrinsic losses for these resonators with rates κ_j^{in}, and use κ_j^{ex} to denote their input-port coupling rates. The Hamiltonian of the coupled system in terms of annihilation and creation operators has been studied in Ref. [1], and is given by

$$\hat{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar \sum_{j=1,2} \Big[\omega_{c,j}\hat{a}_j^\dagger \hat{a}_j + g_{0,j}(\hat{b}^\dagger + \hat{b})\hat{a}_j^\dagger \hat{a}_j + \mathrm{i}E_j(\hat{a}_j^\dagger e^{-\mathrm{i}\omega_{\mathrm{d},j}t} - \hat{a}_j e^{\mathrm{i}\omega_{\mathrm{d},j}t})\Big]. \tag{1}$$

Here, $\hat{b}$ is the annihilation operator of the MR whose resonant frequency is ω_m, $\hat{a}_j$ is the annihilation operator for resonator j whose coupling rate to the MR is $g_{0,j}$. The microwave-driving strength for resonator j is $E_j = \sqrt{P_j \kappa_j^{\mathrm{ex}}/\hbar\omega_{\mathrm{d},j}}$, where P_j is the amplitude of the microwave driving field [1].

In the interaction picture with respect to $\hbar\omega_{\mathrm{d},1}a_1^\dagger a_1 + \hbar\omega_{\mathrm{d},2}a_2^\dagger a_2$, and neglecting terms oscillating at $\pm 2\omega_{\mathrm{d,j}}$, the system Hamiltonian reduces to

$$\hat{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar \sum_{j=1,2} \Big[\Delta_{0,j} + g_{0,j}(\hat{b}^\dagger + \hat{b})\Big]\hat{a}_j^\dagger \hat{a}_j + \hat{H}_{\mathrm{dri}}, \tag{2}$$

where the Hamiltonian associated with the driving fields is $\hat{H}_{\mathrm{dri}} = \mathrm{i}\hbar \sum_{j=1,2} E_j(\hat{a}_j^\dagger - \hat{a}_j)$.

We can linearize Hamiltonian (2) by expanding the resonator modes around their steady-state field amplitudes, $\hat{c}_j = \hat{a}_j - \sqrt{n_j}$, where $n_j = |E_j|^2/(\kappa_j^2 + \Delta_j^2) \gg 1$ is the mean number of intracavity photons induced by the microwave pumps [2], the $\kappa_j = \kappa_j^{\mathrm{in}} + \kappa_j^{\mathrm{ex}}$ are the total resonator decay rates, and the Δ_j are the effective resonator detunings. It is then convenient to move to the interaction picture with respect to the free Hamiltonian, $\hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar \sum_{j=1,2} \omega_{c,j}\hat{a}_j^\dagger \hat{a}_j$, where the linearized Hamiltonian becomes

$$\hat{H} = \hbar \sum_{j=1,2} G_j(\hat{b}\mathrm{e}^{-\mathrm{i}\omega_m t} + \hat{b}^\dagger \mathrm{e}^{\mathrm{i}\omega_m t})(\hat{c}_j^\dagger \mathrm{e}^{\mathrm{i}\Delta_j t} + \hat{c}_j \mathrm{e}^{-\mathrm{i}\Delta_j t}), \tag{3}$$

where $G_j = g_{0,j}\sqrt{n_j}$. By setting the effective resonator detunings so that $\Delta_1 = -\Delta_2 = -\omega_m$ and neglecting the terms rotating at $\pm 2\omega_m$, the above Hamiltonian reduces to

$$\hat{H} = \hbar G_1(\hat{c}_1\hat{b} + \hat{b}^\dagger \hat{c}_1^\dagger) + \hbar G_2(\hat{c}_2\hat{b}^\dagger + \hat{b}\hat{c}_2^\dagger), \tag{4}$$

as specified in the main text.

The full quantum treatment of the system can be given in terms of the quantum Langevin equations in which we add to the Heisenberg equations the quantum noise acting on the mechanical resonator ($\hat{b}_{\mathrm{in}}$ with damping rate γ_m), as well as the resonators' input fluctuations ($\hat{c}_{j,\mathrm{ex}}$, for $j = 1, 2$, with rates κ_j^{ex}), plus the intrinsic losses of the resonator modes ($\hat{c}_{j,\,\mathrm{in}}$, for $j = 1, 2$, with loss rates κ_j^{in}). These noises have the correlation functions

$$\langle \hat{c}_{j,\mathrm{ex}}(t)\hat{c}_{j,\mathrm{ex}}^\dagger(t')\rangle = \langle \hat{c}_{j,\mathrm{ex}}^\dagger(t)\hat{c}_{j,\mathrm{ex}}(t')\rangle + \delta(t-t') = (\bar{n}_j^T + 1)\delta(t-t'), \tag{5a}$$

$$\langle \hat{c}_{j,\,\mathrm{in}}(t)\hat{c}_{j,\,\mathrm{in}}^\dagger(t')\rangle = \langle \hat{c}_{j,\,\mathrm{in}}^\dagger(t)\hat{c}_{j,\,\mathrm{in}}(t')\rangle + \delta(t-t') = (\bar{n}_j^{\mathrm{in}} + 1)\delta(t-t'), \tag{5b}$$

$$\langle \hat{b}_{\,\mathrm{in}}(t)\hat{b}^\dagger_{\,\mathrm{in}}(t')\rangle = \langle \hat{b}^\dagger_{\,\mathrm{in}}(t)\hat{b}_{\,\mathrm{in}}(t')\rangle + \delta(t-t') = (\bar{n}_m + 1)\delta(t-t'), \tag{5c}$$

where $\bar{n}_j{}^{\mathrm{in}}$, $\bar{n}_j$, and $\bar{n}_m$ are the Planck-law thermal occupancies of each bath. The resulting Langevin equations for the resonator modes and MR are

$$\dot{\hat{c}}_1 = -\frac{\kappa_1}{2}\hat{c}_1 - \mathrm{i}G_1\hat{b} + \sqrt{\kappa_1^{\mathrm{ex}}}\hat{c}_{1,\mathrm{ex}} + \sqrt{\kappa_1^{\mathrm{in}}}\hat{c}_{1,\mathrm{in}}, \tag{6a}$$

$$\dot{\hat{c}}_2 = -\frac{\kappa_2}{2}\hat{c}_2 - \mathrm{i}G_2\hat{b}^\dagger + \sqrt{\kappa_2^{\mathrm{ex}}}\hat{c}_{2,\mathrm{ex}} + \sqrt{\kappa_2^{\mathrm{in}}}\hat{c}_{2,\mathrm{in}}, \tag{6b}$$

$$\dot{\hat{b}} = -\frac{\gamma_m}{2}\hat{b} - \mathrm{i}G_1\hat{c}_1^\dagger - \mathrm{i}G_2\hat{c}_2 + \sqrt{\gamma_m}\hat{b}_{\,\mathrm{in}}. \tag{6c}$$

We can solve the above equations in the Fourier domain to obtain the microwave resonators variables. By substituting the solutions of Eqs. (6a)–(6c) into the corresponding input-output formula for the resonators' variables, i.e., $\hat{d}_j \equiv \hat{c}_{j,\text{out}} = \sqrt{\kappa_j^{\text{ex}}}\hat{c}_j - \hat{c}_{j,\text{ex}}$, we obtain

$$\hat{d}_1(\omega) = \alpha_1(\omega)\hat{c}_{1,\text{ex}} + \alpha_{12}(\omega)\hat{c}^\dagger_{2,\text{ex}} + \alpha_{1m}(\omega)\hat{b}^\dagger_{\text{in}} + \alpha_{1in}(\omega)\hat{c}_{1,\text{in}} + \alpha_{12in}(\omega)\hat{c}^\dagger_{2,\text{in}}, \tag{7a}$$

$$\hat{d}_2(\omega) = \alpha_2(\omega)\hat{c}_{2,\text{ex}} + \alpha_{21}(\omega)\hat{c}^\dagger_{1,\text{ex}} + \alpha_{2m}(\omega)\hat{b}_{\text{in}} + \alpha_{2in}(\omega)\hat{c}_{2,\text{in}} + \alpha_{21in}(\omega)\hat{c}^\dagger_{1,\text{in}}, \tag{7b}$$

where

$$\alpha_1(\omega) = -1 + \frac{2\eta_1\left[\tilde{\omega}_2\tilde{\omega}_b + \mathcal{C}_2\right]}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{8a}$$

$$\alpha_{12}(\omega) = \frac{2\sqrt{\eta_1\eta_2\,\mathcal{C}_1\mathcal{C}_2}}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{8b}$$

$$\alpha_{1m}(\omega) = \frac{2\mathrm{i}\sqrt{\eta_1\,\mathcal{C}_1}\tilde{\omega}_2}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{8c}$$

$$\alpha_{1in}(\omega) = \frac{2\sqrt{\eta_1(1-\eta_1)}\,(\tilde{\omega}_2\tilde{\omega}_b + \mathcal{C}_2)}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{8d}$$

$$\alpha_{12in}(\omega) = \frac{2\sqrt{\eta_1(1-\eta_2)\,\mathcal{C}_1\mathcal{C}_2}}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{8e}$$

$$\tag{8f}$$

and

$$\alpha_2(\omega) = -1 + \frac{2\eta_2\left[\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1\right]}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{9a}$$

$$\alpha_{21}(\omega) = -\frac{2\sqrt{\eta_1\eta_2\,\mathcal{C}_1\mathcal{C}_2}}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{9b}$$

$$\alpha_{2m}(\omega) = -\frac{2\mathrm{i}\sqrt{\eta_2\,\mathcal{C}_2}\tilde{\omega}_1}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{9c}$$

$$\alpha_{2in}(\omega) = \frac{2\sqrt{\eta_2(1-\eta_2)}\,(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{9d}$$

$$\alpha_{21in}(\omega) = -\frac{2\sqrt{\eta_2(1-\eta_1)\,\mathcal{C}_1\mathcal{C}_2}}{\tilde{\omega}_1\mathcal{C}_2 + \tilde{\omega}_2(\tilde{\omega}_1\tilde{\omega}_b - \mathcal{C}_1)} \tag{9e}$$

$$\tag{9f}$$

with $\tilde{\omega}_j = 1 - \mathrm{i}\omega/\kappa_j$, $\tilde{\omega}_b = 1 - \mathrm{i}\omega/\gamma_m$, $\eta_i = \kappa_i^{\text{ex}}/\kappa_i$, and $\mathcal{C}_j = 4G_j^2/\kappa_j\gamma_m$. The coefficients (8)–(9) become much simpler at $\omega \simeq 0$, which corresponds to take a narrow frequency band around each resonator resonance, viz.,

$$\alpha_1(\omega) = -1 + \frac{2\gamma_m\eta_1\left[1 + \mathcal{C}_2\right]}{\gamma_{\text{eff}}} \tag{10a}$$

$$\alpha_{12}(\omega) = \frac{2\gamma_m\sqrt{\eta_1\eta_2\,\mathcal{C}_1\mathcal{C}_2}}{\gamma_{\text{eff}}} \tag{10b}$$

$$\alpha_{1m}(\omega) = \frac{2\mathrm{i}\gamma_m\sqrt{\eta_1\,\mathcal{C}_1}}{\gamma_{\text{eff}}} \tag{10c}$$

$$\alpha_{1in}(\omega) = \frac{2\gamma_m\sqrt{\eta_1(1-\eta_1)}\,(1 + \mathcal{C}_2)}{\gamma_{\text{eff}}} \tag{10d}$$

$$\alpha_{12in}(\omega) = \frac{2\gamma_m\sqrt{\eta_1(1-\eta_2)\,\mathcal{C}_1\mathcal{C}_2}}{\gamma_{\text{eff}}} \tag{10e}$$

$$\tag{10f}$$

and

$$\alpha_2(\omega) = -1 + \frac{2\gamma_m \eta_2 \big[1 - \mathcal{C}_1\big]}{\gamma_{\rm eff}} \tag{11a}$$

$$\alpha_{21}(\omega) = -\frac{2\gamma_m \sqrt{\eta_1\eta_2\,\mathcal{C}_1\mathcal{C}_2}}{\gamma_{\rm eff}} \tag{11b}$$

$$\alpha_{2m}(\omega) = -\frac{2{\rm i}\gamma_m \sqrt{\eta_2\,\mathcal{C}_2}}{\gamma_{\rm eff}} \tag{11c}$$

$$\alpha_{2in}(\omega) = \frac{2\gamma_m \sqrt{\eta_2(1-\eta_2)}\,(1-\mathcal{C}_1)}{\gamma_{\rm eff}} \tag{11d}$$

$$\alpha_{21in}(\omega) = -\frac{2\gamma_m \sqrt{\eta_2(1-\eta_1)\,\mathcal{C}_1\mathcal{C}_2}}{\gamma_{\rm eff}} \tag{11e}$$

$$\tag{11f}$$

with $\gamma_{\rm eff} = \gamma_m(1 + \mathcal{C}_2 - \mathcal{C}_1)$ is the effective damping rate of the MR. Furthermore, when the internal losses are negligible, i.e., $\eta_j = 1$, then we get $\alpha_{1in} = \alpha_{2in} = \alpha_{12in} = \alpha_{21in} = 0$, and Eqs. (7a)–(7b) reduce to the simple forms

$$\hat{d}_1 = \alpha_1 \hat{c}_{1,\rm ex} + \alpha_{12}\hat{c}^{\dagger}_{2,\rm ex} + \alpha_{1m}\hat{b}^{\dagger}_{\rm in} \tag{12a}$$

$$\hat{d}_2 = \alpha_2 \hat{c}_{2,\rm ex} + \alpha_{21}\hat{c}^{\dagger}_{1,\rm ex} + \alpha_{2m}\hat{b}_{\rm in}, \tag{12b}$$

with coefficients given by

$$\alpha_1 = -1 + \frac{2\gamma_m\big[1 + \mathcal{C}_2\big]}{\gamma_{\rm eff}} \tag{13a}$$

$$\alpha_2 = -1 + \frac{2\gamma_m\big[1 - \mathcal{C}_1\big]}{\gamma_{\rm eff}} \tag{13b}$$

$$\alpha_{12} = -\alpha_{21} = \frac{2\gamma_m\sqrt{\mathcal{C}_1\mathcal{C}_2}}{\gamma_{\rm eff}} \tag{13c}$$

$$\alpha_{1m} = \frac{2{\rm i}\gamma_m\sqrt{\mathcal{C}_1}}{\gamma_{\rm eff}} \tag{13d}$$

$$\alpha_{2m} = -\frac{2{\rm i}\gamma_m\sqrt{\mathcal{C}_2}}{\gamma_{\rm eff}}, \tag{13e}$$

These input-output relations preserve the bosonic commutation relations, i.e., when the operators on the right in Eqs. (12a) and (12b) satisfy those commutation relations, we get $[\hat{d}_i, \hat{d}^{\dagger}_j] = \delta_{i,j}$ and $[\hat{d}_i, \hat{d}_j] = [\hat{d}^{\dagger}_i, \hat{d}^{\dagger}_j] = 0$, for $i, j \in 1, 2$.

The system is stable if the Routh-Hurwitz criterion is satisfied. For $\mathcal{C}_i \gg 0$, this criterion reduces to the following necessary and sufficient condition [3]:

$$\kappa_2\,\mathcal{C}_2 - \kappa_1\mathcal{C}_1 > \tilde{\mathcal{C}}\max\left\{\kappa_2 - \kappa_1, \frac{\kappa_1^2 - \kappa_2^2}{2\gamma_m + \kappa_1 + \kappa_2}\right\},$$

where $\tilde{\mathcal{C}} = \frac{\mathcal{C}_2}{1+\kappa_1/\kappa_2} + \frac{\mathcal{C}_1}{1+\kappa_2/\kappa_1}$.

B. Covariance matrix of a two-mode Gaussian state

In order to quantify entanglement, we first determine the covariance matrix (CM) of our system in the frequency domain, which can be expressed as

$$V_{ij} = \frac{1}{2}\langle u_i u_j + u_j u_i\rangle, \tag{14}$$

where

$$\mathbf{u} = [X_1, Y_1, X_2, Y_2]^T, \tag{15}$$

and $X_j = (D_j + D_j^\dagger)/\sqrt{2}$, $Y_j = (D_j - D_j^\dagger)/\mathrm{i}\sqrt{2}$ with $j = 1, 2$. Note that the vacuum noise has variance $1/2$ in these quadratures. Here we have defined the filtered output operators

$$D_j(B) = \int_{-\infty}^{\infty} d\omega' f_j(\omega', B) d_j(\omega') \tag{16}$$

where a filter function $f_j(\omega, B)$ with bandwidth B is applied on the output of the each resonator. Now, by using Eqs. (7a), (7b) and (14), we obtain the CM for the quadratures of the resonators outputs, which is given by the normal form

$$\mathbf{V}(\omega) = \begin{pmatrix} V_{11} & 0 & V_{13} & 0 \\ 0 & V_{11} & 0 & -V_{13} \\ V_{13} & 0 & V_{33} & 0 \\ 0 & -V_{13} & 0 & V_{33} \end{pmatrix}, \tag{17}$$

Note that Eq. (17) is the typical CM of a two-mode squeezed thermal state [4, 5] where the elements of the CM can be written in terms of photon numbers n_i, squeezing angle ϕ and squeezing parameter r, reads

$$V_{11} = V_{22} = \frac{(1 + n_1 + n_2)\cosh(2r) + (n_1 - n_2)}{2}, \tag{18a}$$

$$V_{33} = V_{44} = \frac{(1 + n_1 + n_2)\cosh(2r) - (n_1 - n_2)}{2}, \tag{18b}$$

$$V_{13} = -V_{24} = \frac{(1 + n_1 + n_2)\sinh(2r)\cos\phi}{2}, \tag{18c}$$

when $n_i = 0$ the Gaussian state is called two-mode squeezed vacuum. Squeezing in the two-mode squeezed thermal state can be determined by following expression

$$S(\phi) = V_{11} + V_{33} - 2V_{13} = \frac{1}{2}(1 + n_1 + n_2)\Big(\cosh(2r) - \sinh(2r)\cos\phi\Big), \tag{19}$$

For $\phi = 0$ and $n_i = 0$ we get $S(0) = e^{-2r}/2$.

C. Logarithmic Negativity

Here we quantify the amount of entanglement generated by our microwave entanglement source using standard measures in quantum information theory. In particular, we consider the log-negativity [6, 7], which is an upper bound to the number of distillable entanglement bits (ebits) generated by the source.

The log-negativity E_N is given by [6, 7]

$$E_N = \max[0, -\log(2\zeta^-)], \tag{20}$$

where ζ^- is the smallest partially-transposed symplectic eigenvalue of $\mathbf{V}(\omega)$, given by [8]

$$\zeta^- = 2^{-1/2}\left(V_{11}^2 + V_{33}^2 + 2V_{13}^2 - \sqrt{(V_{11}^2 - V_{33}^2)^2 + 4V_{13}^2(V_{11} + V_{33})^2}\right)^{1/2}. \tag{21}$$

D. Quantum correlations beyond entanglement: Quantum discord

Our microwave source generates a Gaussian state which is mixed, as one can easily check from the numerical values of its von Neumann entropy. It is therefore important to describe its quality in terms of general quantum correlations beyond quantum entanglement. Thus we compute here the quantum discord [9, 10] of the source $D(2|1)$, capturing the basic quantum correlations which are carried by the microwave modes.

Since our source emits a mixed Gaussian state which is a two-mode squeezed thermal state, we can compute its (unrestricted) quantum discord using the formulas of Ref. [5]. In particular, the CM in Eq. (17) can be expressed as

$$\mathbf{V}(\omega) = \begin{pmatrix} (\tau b + \eta)\mathbf{I} & \sqrt{\tau(b^2 - 1)}\mathbf{Z} \\ \sqrt{\tau(b^2 - 1)}\mathbf{Z} & b\mathbf{I} \end{pmatrix}, \quad \begin{array}{l} \mathbf{I} \equiv \mathrm{diag}(1, 1), \\ \mathbf{Z} \equiv \mathrm{diag}(1, -1), \end{array} \tag{22}$$

where

$$b = V_{33}, \quad \tau = \frac{V_{13}^2}{V_{33}^2 - 1}, \quad \eta = V_{11} - \frac{V_{33}V_{13}^2}{V_{33}^2 - 1}. \tag{23}$$

Thus, we may write [5]

$$D(2|1) = h(b) - h(\nu_-) - h(\nu_+) + h(\tau + \eta) \tag{24}$$

$$= h(V_{33}) - h(\nu_-) - h(\nu_+) + h\left[V_{11} + \frac{V_{13}^2(1 - V_{33})}{V_{33}^2 - 1}\right], \tag{25}$$

where ν_- and ν_+ are the symplectic eigenvalues of $\mathbf{V}(\omega)$ and they are given by [8]

$$\nu_\pm = 2^{-1/2}\left(V_{11}^2 + V_{33}^2 - 2V_{13}^2 \pm \sqrt{(V_{11}^2 - V_{33}^2)^2 - 4V_{13}^2(V_{11} - V_{33})^2}\right)^{1/2}. \tag{26}$$

where

$$h(x) \equiv \left(x + \frac{1}{2}\right)\log\left(x + \frac{1}{2}\right) - \left(x - \frac{1}{2}\right)\log\left(x - \frac{1}{2}\right). \tag{27}$$

Note that the expression of the entropic function $h(x)$ is that for vacuum noise equal to $1/2$. Our notation is different from that of Ref. [8], where the vacuum noise is equal to 1.

E. Entropy of formation

The effective number of ebits at the detectors input known is entropy of formation can be expressed in terms of the log-negativity defined in Eq. 20 [11–13]

$$E_f = \sigma_+ \log_2 \sigma_+ - \sigma_- \log_2 \sigma_-, \tag{28}$$

where $\sigma_\pm = (\frac{1}{\sqrt{\theta}} \pm \sqrt{\theta})^2/4$ with $\theta = 2^{-E_N}$.

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