Diffusive regimes in a two-dimensional chiral fluid. Supplementary Material

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Supplementary Figure 1. This figure illustrates in more detail the results in Figure 1 (c) of the main file. (a) Displacement-velocity correlation function $(\langle \Delta y(t)v_x(0) \rangle)$ for a series of experiments for varying $\overline{\omega}$ (see color bar at the right), at $\phi = 0.45$. The transition from negative to positive $\overline{\omega}$ yields a transition from positive to negative values of the mean cross correlation, $\langle \Delta y(t)v_x(0) \rangle$. We can see that larger $|\overline{\omega}|$ yields longer curves; i.e., larger diffusivity values. In the inset, where only short times are represented, is very clear that the curvature of $\langle \Delta y(t)v_x(0) \rangle$ alternates signs locally, which suggests the existence of a continuous transition in the odd diffusion dynamics. (b) Dimensionless version of displacement-velocity correlations, here scaled with the maximum absolute value of each series ($\langle \Delta y(t)v_x(0) \rangle / |\langle \Delta y(t_{\max})v_x(0) \rangle|$). This dimensionless representation highlights curve shapes. The cases with large $|\overline{\omega}|$ tend to display a spiral form, whereas the states with smaller $\overline{\omega}$ show very clearly that the curvature sign is not well defined and thus the spirals are actually not present in general as described in previous works for analogous systems. In particular, this effect is very prominent for $\overline{\omega} \simeq 0$.



Supplementary Figure 2. 3D contour-plot representation of the excess kurtosis $\epsilon = \frac{\langle \Delta r \rangle^4}{\langle \Delta r^2 \rangle^2} - 3$ (deviation of the fourth moment from its Gaussian-like value) of the distribution function of displacements $f(\Delta r(t))$. Over the cube faces the contour of ϵ are represented vs. the relevant parameters in our system $(t, \overline{\omega}, \text{ and } \phi \text{ in the vertical axis})$. The distribution tends to be highly platykurtic distribution in most regions of the parameter space (while in others is weakly platykurtic).



Supplementary Figure 3. Time evolution of the distribution function of cross-displacements, $f(\Delta x(t)\Delta y(t))$, for $\overline{\omega} = -0.12 \text{ s}^{-1}, \overline{\omega} = 0.03 \text{ s}^{-1}, \overline{\omega} = 0.26 \text{ s}^{-1}$, in panels (a), (b) and (c) respectively. Each case is representative of each of the different diffusive regimes, found at $\overline{\omega} < 0, \overline{\omega} \simeq 0, \overline{\omega} > 0$. Packing fraction: $\phi = 0.45$. The distribution function is characteristically asymmetric except for $\overline{\omega} \simeq 0$, thus indicating odd diffusion originates out of unbalanced cross displacements only when chiral flow is present.



Supplementary Figure 4. Sample trajectories for three experiments, with $\phi = 0.45$, for $\overline{\omega} < 0$, $\overline{\omega} \simeq 0$, $\overline{\omega} > 0$ respectively (trajectory color indicates time value according to the color bar at the right). For $\overline{\omega} < 0$ and $\overline{\omega} > 0$, odd diffusion at long time scales is observed in the form of as Brownian trajectories along chiral flow particle streamlines (in grey). However, this is not observed for $\overline{\omega} \simeq 0$ we do not observe odd diffusion at any time scale, and trajectories look more normal-diffusion-like Brownian movement. Moreover, we can observe that at shorter time scales all three cases ($\overline{\omega} < 0, \overline{\omega} \simeq 0, \overline{\omega} > 0$) are not distinguishable and present particle trajectories more Brownian-like. These results suggest that odd diffusion (D^{odd}) has in general a longer characteristic time than Brownian diffusion (characterized by the D coefficient).



Supplementary Figure 5. Diffusion coefficients D, D^{odd} vs. reduced particle activity (defined here as the mean spin velocity $\overline{\Omega}$ divided by the square root of the mean kinetic energy, moment of inertia and inverse mass $\sqrt{I\overline{T}/m}$). We see that large variations of the diffusion coefficients for very similar values of the reduced activity. Moreover, diffusion coefficients are not monovalued with respect to reduced particle activity. This is a strong indication that particle activity is not in general the control parameter in diffusion for two-dimensional chiral fluids. Results are represented for three different packing fractions: $\phi = 0.25, 0.45, 0.55$.

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