nature portfolio

Peer Review File

Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The authors present a study on Josephson junction arrays and claim the observation of a "Josephson Diode Effect".

1.) Arrays of junctions in parallel and series configuration have been intensively studied and reported in the literature. There is nothing new here.

2.) I am very skeptical of the "diode effect". Asymmetric junctions are very common in poor quality devices due to Schottky barriers at the contact interfaces.

Reviewer #2 (Remarks to the Author):

The manuscript describes a Josephson junction array obtained by proximitizing a BST film (a topological insulator) by Nb islands. The manuscripts reports the observation of interference diffraction patterns and supercurent nonreciprocity in this array.

I can, in principle, recommend the publication of the manuscript in Communication Physics, provided the authors clarify the following points.

0) It is not clear to me why the authors used a TI as a normal platform for the array. More precisely, the authors write that ``Our experiments demonstrate that topological insulator-superconductor arrays can behave as efficient superconducting rectifiers.''. The BST TI also appears in the title, yet it is not clear which property (if any) of TIs is needed for the reported observations.

1) First paragraph after the section ``Results''.
The authors write that ``The resistance is measured by calculating the slope of the best linear fit to the V-I curve. The V-I curve is measured using a low frequency ac signal, the amplitude of which is indicated on the graph. The V-I is nonlinear, thus explaining why the resistance is generally larger for larger amplitudes of the bias current (Ibias).''

It is still not clear to me how the IVs are measured. Typically AC excitation is used to measure differential resistance (e.g. with a lock-in). IVs are measured typically sweeping a DC current bias and measuring the DC voltage drop. Other solutions are possible, but I would like that the authors describe carefully how the IVs are measured (which quantity is swept, which quantity is correspondingly measured).

2) The word "vortices" is, in the context of Josephson junction arrays, not univocal. It can indicate Abrikosov vortices, Josephson vortices in junctions (the alternating local supercurrent in an extended junction) or the loop of supercurrent that encircles one or more plaquette in an array. It is not clear to me which vortices the authors mean in Fig.4 and its description in the text. From what is written I would think about Abrikosov vortices. But to obtain an Abrikosov vortex with core within the junction, one would need a proximity effect so strong that the order parameter (the gap) is finite also in the BST region in between the islands and not just below the Nb. The authors write, however, that they assume Delta=0 in between the islands (first few lines of the first paragraph of ``Model'') (in this latter case, the supercurrent is carried by Andreev bound states shuttling back and forth from the SC leads). I found this inconsistent. I wonder if, instead, the results could be interpreted in terms of vortices of Josephson supercurrent looping around groups of one or more plaquettes.
In any case, the authors should clarify this point (i.e. the degree of proximitization of the region in

between the Nb islands).
3) After Eq. 13, the authors write ``Thus, in this approximation the array is essentially replaced by a single junction with the width W.''
I think it will help the reader a lot if the authors declared this approximation much earlier in the discussion, e.g. before eq 2.

4) This is my most relevant comment. By symmetry argument, to obtain a diode effect something must break the spatial symmetry of the array. This spatial symmetry breaking is parametrized by the authors using the ``dimensionless asymmetry parameter'' beta.
It is not clear to me where this asymmetry comes from (I mean, physically, since the array is nominally symmetric and so it appears). Also, it is not clear why this asymmetry can be measured by measuring the retrapping current (incidentally, beta here has nothing to do with the McCumber parameter).

5) Before the manuscript is published, please check whether the arxiv preprints have been finally published (Ref 34 and perhaps 35).

Reviewer #3 (Remarks to the Author):

This is an interesting work on interference, diffraction and diode effects in a superconducting array proximitized to an intrinsic topological insulator (TI) Bi0.8Sb1.2Te3. This paper experimentally shows that exotic magnetic-field quantum inference grating of such a proximity-coupled superconducting array allows for dramatic sharpening of a critical current peak and going far beyond the relevant functionality of a standard SQUID. I think that this study brings a practical insight into developing highly sensitive magnetic-field sensors and low-field-operable Josephson supercurrent rectifiers.
I would recommend this paper for publication in Communications Physics if the following questions are properly answered.

1) It would be great if the authors could carry out a control experiment by quenching the topological surface states (SSs) of Bi0.8Sb1.2Te3 to investigate how the presence and absence of topological SSs influence the device properties (e.g. critical current peak sharpening, diode efficiency).
2) I think that underlying physical mechanisms behind the observation of low-field supercurrent rectification in this proximity-coupled superconducting array, especially whether intrinsic or extrinsic, remain unspecified. Can the authors perform any further measurements on the existing superconducting array device to narrow down the origin? For instance, field-angle/strength and temperature dependencies of the diode efficiency.
3) Basically, the geometry (d ≈150 nm, w ≈ 1.15 μm, 40-nm-thick Bi0.8Sb1.2Te3, 30-nm-thick Nb) of superconducting array studied here is fixed. I am curious how crucial for the performance of critical current peak sharpening and diode efficiency the device geometry is.

I hope it helps improve the overall quality of the paper to be best suited for the journal of Communications Physics.

RESPONSE TO THE REFEREES

Reviewer #1 (Remarks to the Author):

The authors present a study on Josephson junction arrays and claim the observation of a "Josephson Diode Effect".

1) Arrays of junctions in parallel and series configuration have been intensively studied and reported in the literature. There is nothing new here.

***Our response*:**

Indeed, arrays of Josephson junctions have been intensively studied in the past. However, we focus on a particular property of the array, namely, the diode effect. This important effect is still not well understood.

Superconducting and Josephson diode effects have been demonstrated in various systems recently, including an artificial superlattice without a center of inversion [Nature 584, 373–376 (2020)], a supercurrent interferometer [arXiv:2205.04469v1 (2022)], and a chain of Al islands on top of 2D electron gas [Nat. Nanotechnol. 17, 39–44 (2022)].

Although these papers do report a rectification of the supercurrent, they study qualitatively different systems. Namely, in the first paper the system was an artificial [Nb/V/Ta]$_n$ superlattice which exhibits zero resistance in only one direction. The non-reciprocal critical current is considered to be related to the magnetochiral anisotropy caused by breaking of the spatial-inversion and time-reversal symmetries. The physics of these samples is qualitatively different from our system. We want to emphasize that our efficiency, defined as (Ic^+-Ic^-)/(Ic^++Ic^-) is almost an order of magnitude larger than this previous publication. Note also that this previous publication is quite recent, dated 2020.

The second paper is a theory paper. It is also a recent one (2022). It contains a prediction of a superconducting diode effect. It shows that this topic is a focus of modern research. The efficiency they predict is somewhat higher than our observed efficiency, but still of the same order of magnitude.

In the third paper the system was a highly transparent Josephson junction chains fabricated on InAs quantum wells. Although the efficiency, reported in this recent paper, was somewhat higher than ours, the principle of operation and the sample geometry were qualitatively different from ours. In particular, we employ a topological insulator epitaxial film, and we use a 2D array. Also, our results are related to the asymmetry of the junctions while the cited paper links the results to the asymmetry of the current-phase relationship.

In summary, we agree with the Reviewer that many papers on the physics of Josephson arrays have been published earlier. Many of these papers are recent, which means this field remains very active. More importantly, most of these papers do not report any diode effect,

i.e., the VI curves presented there are symmetrical. Below we list some examples which illustrate our view:

1. Bøttcher, C. G. L., et al. "Dynamical vortex transitions in a gate-tunable Josephson junction array." arXiv preprint arXiv:2212.08651 (2022).

2. Panghotra, R., et al. "Giant fractional Shapiro steps in anisotropic Josephson junction arrays." Communications Physics 3.1 (2020): 53.

3. Kasaei, Leila, et al. "Reduced critical current spread in planar MgB 2 Josephson junction array made by focused Helium ion beam." IEEE Transactions on Applied Superconductivity 29.5 (2019): 1-6.

2) I am very skeptical of the "diode effect". Asymmetric junctions are very common in poor quality devices due to Schottky barriers at the contact interfaces.

***Our response*:**

If the asymmetry comes from Schottky barriers at the contact interfaces, we expect to see it above Tc (at T> Tc), since the Schottky effect is not related to superconductivity [see, e.g., "Towards spin injection from silicon into topological insulators: Schottky barrier between Si and Bi2Se3. Appl. Phys. Lett. 101, 023102 (2012); https://doi.org/10.1063/1.4733388]. Yet, the VI curves we took above Tc are perfectly symmetric and practically linear (see the example below, Fig.R1). We have also explicitly measured the rectification coefficient and found that it disappears very quickly with increasing temperature. Above T=0.52K the VI curves become completely symmetric. It is also important that the rectification occurs only at non-zero magnetic field, while the Schottky barrier should be independent of the field. All these observations imply that the Schottky barrier cannot explain our results.

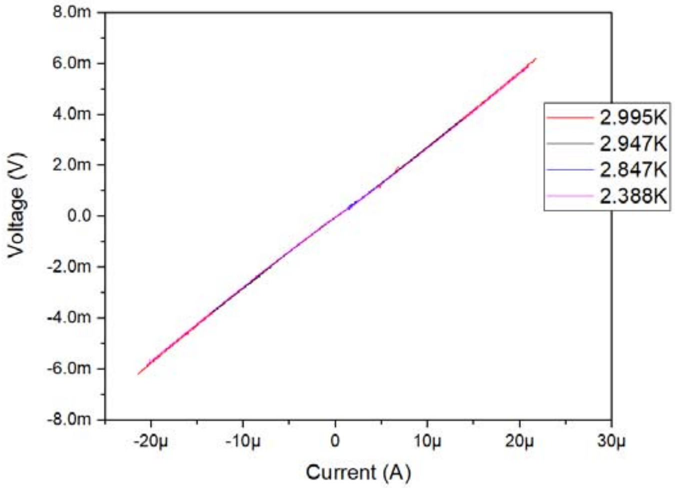

Fig. R1. Voltage-Current dependence of the Josephson junction array device taken above critical temperature Tc.

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***Our response*:**

The BiSbTe topological insulator (TI) material is technically important. There are theoretical predictions suggesting that superconducting qubits made with such intrinsic TI materials can allow non-Abelian Majorana Zero Modes, which then can be used to build topologically protected quantum computers. Yet, previous reports suggested that it is very difficult if not impossible to introduce a significant proximity superconductivity into the intrinsic topological insulator BST. Here we show that, in fact, a strong proximity effect is possible. In our work we demonstrate that it is possible to establish high Ic and good contact, hence stronger proximity.

It is the large sheet resistance $R_\square$(~ 1kOhm) which is the key property of the BST TI we use. As explained by the proposed model, large $R_\square$ value allows pronounced asymmetrical VI curves to occur. It is, of course, possible that there exist other materials which would generate similar effects. It is useful to compare our results to the results obtained on similar arrays made with regular metals, such as Cu or Au [Phys. Rev.B 28, 6578 (1983)][Phys. Rev.B 94, 024510 (2016)]. The diode effect was not reported in those cases. Also, the peak sharpening was much less pronounced.

BST has its advantages in device fabrication when compared with other commonly used materials like graphene or metallic films. For example, graphene is often used in Josephson junction devices, but they cannot survive the ion milling cleaning process without damaging its structure. For BST material, ion milling will only make its top surface cleaner and TI-metal contact better. For metallic films, it is not easy to get normal metal that has comparable $R_\square$ value as BST in the first place.

1) First paragraph after the section "Results".
The authors write that "The resistance is measured by calculating the slope of the best linear fit to the V-I curve. The V-I curve is measured using a low frequency ac signal, the amplitude of

which is indicated on the graph. The V-I is nonlinear, thus explaining why the resistance is generally larger for larger amplitudes of the bias current (Ibias)."

It is still not clear to me how the IVs are measured. Typically AC excitation is used to measure differential resistance (e.g. with a lock-in). IVs are measured typically sweeping a DC current bias and measuring the DC voltage drop. Other solutions are possible, but I would like that the authors describe carefully how the IVs are measured (which quantity is swept, which quantity is correspondingly measured).

***Our response*:**

The current bias of the sample was set by taking an ac voltage, U, from a National Instrument data acquisition card NI-DAQ USB-6216 and applying it to the sample connected in series with a standard resistor of R_{st} = 9.95kOhm [see Fig.R2] and a set of filters. Then, we use the same DAQ to measure the voltage on the sample and on the standard resistor, which is then recalculated into the current in the circuit, using Ohms law ($I=V_{st}/R_{st}$, for each measured point). Then we plot the voltage on the sample versus the bias current in the LabView environment and thus get a V-I curve on the screen. The program makes the best linear fit, and the slope of this fit is the sample resistance provided that the V-I curve is linear. Naturally, we choose the bias current sufficiently low so that the VI curve remains linear, to get the zero-bias resistance R. Note that the voltages from the sample and the standard resistor are amplified with analog low-noise preamplifiers (SR560 Stanford Research Systems) before being supplied to the DAQ card. Other details: The sample resistance was much lower than the standard resistor. The lowest frequency was 0.1Hz, low enough so that further reduction of the frequency would not change the shape of the V-I curve. Yet it was possible to push the frequency up to 10Hz and still obtain reliable reading of the VI curves.

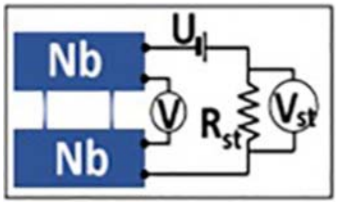


Fig.R2. Schematic of the measurement setup. A voltage, U, is swept from a DAQ card (or a precision function generator). The voltage is applied to the standard resistor, connected in series with the sample. Thus, the current bias is set up. The voltage on the standard resistor and the voltage on the sample are measured using preamplifier (not shown) and the DAQ card (not shown). The V-I curves are plotted in the LabVIEW program and then analyzed.

2) The word "vortices" is, in the context of Josephson junction arrays, not univocal. It can indicate Abrikosov vortices, Josephson vortices in junctions (the alternating local supercurrent in an extended junction) or the loop of supercurrent that encircles one or more plaquette in an array. It is not clear to me which vortices the authors mean in Fig.4 and its description in the text. From

what is written I would think about Abrikosov vortices. But to obtain an Abrikosov vortex with core within the junction, one would need a proximity effect so strong that the order parameter (the gap) is finite also in the BST region in between the islands and not just below the Nb. The authors write, however, that they assume Delta=0 in between the islands (first few lines of the first paragraph of "Model") (in this latter case, the supercurrent is carried by Andreev bound states shuttling back and forth from the SC leads). I found this inconsistent. I wonder if, instead, the results could be interpreted in terms of vortices of Josephson supercurrent looping around groups of one or more plaquettes.
In any case, the authors should clarify this point (i.e. the degree of proximitization of the region in between the Nb islands).

***Our response*:**

We discuss the Josephson vortices, which may, indeed, extend over several islands. These vortices can be described solely by variation of the Josephson phase difference in the direction perpendicular to the current. They are not related to Abrikosov vortices and do not require suppression of Δ in the leads or in the gaps between the Nb islands. Such vortices have been studied a lot in wide Josephson junctions and they are often called "fluxons". In the revised version of the paper, we explain this point more clearly.

3) After Eq. 13, the authors write "Thus, in this approximation the array is essentially replaced by a single junction with the width W."
I think it will help the reader a lot if the authors declared this approximation much earlier in the discussion, e.g. before eq 2.

***Our response*:**

Yes, we agree. Now we add a new explanation above Eq.2.

4) This is my most relevant comment. By symmetry argument, to obtain a diode effect something must break the spatial symmetry of the array. This spatial symmetry breaking is parametrized by the authors using the "dimensionless asymmetry parameter" beta.
It is not clear to me where this asymmetry comes from (Imean, physically, since the array is nominally symmetric and so it appears). Also, it is not clear why this asymmetry can be measured by measuring the retrapping current (incidentally, beta here has nothing to do with the McCumber parameter).

***Our response*:**

In simple terms, the "dimensionless asymmetry parameter" β characterizes the difference between the critical currents of the Josephson junctions located at the opposite edges of the array. If $\beta > 0$, the superconducting current flowing through the array tends to create a big vortex because the current density at one edge of the array is different from that at the other edge. Under ideal fabrication conditions all junctions are identical and $\beta = 0$. However, fabrication errors result in the spread in the critical currents and lead to $\beta > 0$.

Unfortunately, we cannot precisely determine the parameter β. Therefore, we rely on an indirect estimate, which we explain below. First, we find the spread of the critical currents from the jumps visible in the re-trapping current. It is possible because these jumps occur at critical currents of individual junctions, when they switch back to the superconducting state one by one. As we reduce the bias current from large values to zero, the junction with the largest I_C switches first and the junctions with the smallest critical current switches the last. In this way we estimate the difference between the maximum and the minimum critical currents in the array. Next, we make an assumption that the junctions with the maximum and the minimum currents are located at the opposite edges of the array. With this assumption, we obtain $\beta \approx (I^{max}_C - I^{min}_C)/(I^{max}_C + I^{min}_C)$, where I^{max}_C is the maximum value of the retrapping current, i.e., the current at which the first retrapping step occurs of the VI curve. And I^{min}_C is the minimum retrapping current, i.e. current at which the last step occurs and the voltage drops to zero. Both of these currents related to the branch for the VI curve at which the current is reduced from its maximum to zero.

We completely agree that β does not have any direct relation to the topic of the McCumber parameter.

5) Before the manuscript is published, please check whether the arxiv preprints have been finally published (Ref. 34 and perhaps 35).

***Our response*:**
Thank you for the reminder. All the citations are up to date.

Reviewer #3 (Remarks to the Author):

This is an interesting work on interference, diffraction and diode effects in a superconducting array proximitized to an intrinsic topological insulator (TI) Bi0.8Sb1.2Te3. This paper experimentally shows that exotic magnetic-field quantum inference grating of such a proximity-coupled superconducting array allows for dramatic sharpening of a critical current peak and going far beyond the relevant functionality of a standard SQUID. I think that this study brings a practical insight into developing highly sensitive magnetic-field sensors and low-field-operable Josephson supercurrent rectifiers.
I would recommend this paper for publication in Communications Physics if the following questions are properly answered.

1) It would be great if the authors could carry out a control experiment by quenching the topological surface states (SSs) of Bi0.8Sb1.2Te3 to investigate how the presence and absence of topological SSs influence the device properties (e.g. *critical current peak sharpening*, *diode efficiency*).

***Our response*:**

At temperature T_0 = 520 mK the diode effect disappears, i.e. $\eta = 1$ for $T > T_0$ (Fig.R3). Interestingly, the dependence $I_C(T)$ has a kink at T_0. In a recent experiment by P. Schuffelgen et al, Nat. Nanotechnol. 14, 825 (2019), similar $I_C(T)$ dependence has been observed. It has been interpreted in terms of the two parallel conducting channels for the Cooper pairs: ballistic surface states in BiSbTe and diffusive states in the bulk of the material. The diffusive transport can be characterized by the normal state sheet resistance $R_\square$ and can be associated with the diode effect as we explained in the paper. According to this interpretation, at temperature T_0 the diffusive channel (which provides a relatively small contribution in our type of topological insulator) switches off because the induced superconducting gap in this channel vanishes. Therefore, at $T > T_0$ only the ballistic surface states, which do not induce the diode effect, contribute to the Josephson current. Thus, these observations support our model.

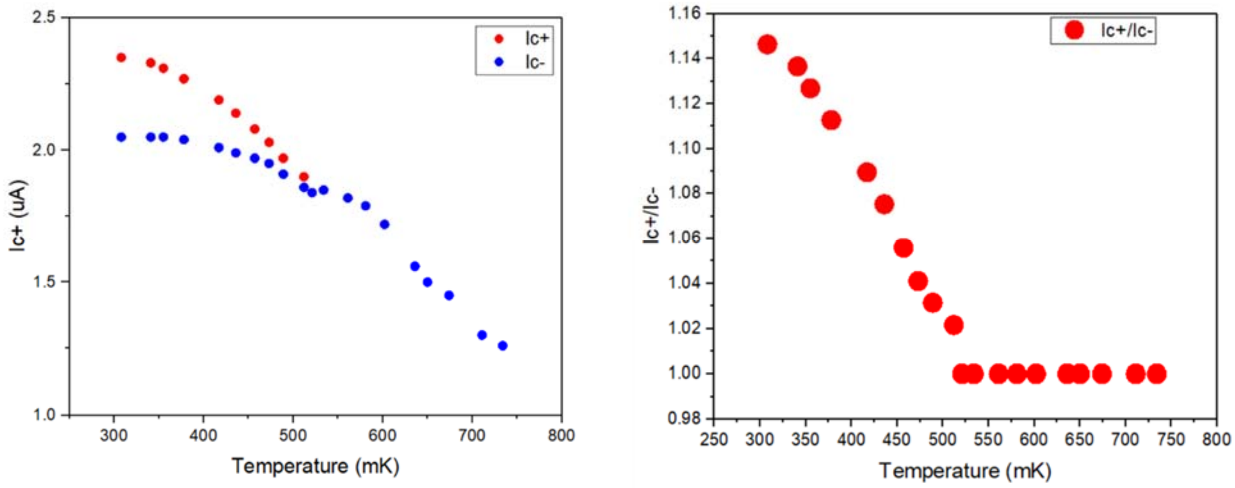


Fig. R3. The temperature dependence of the critical current and of the diode asymmetry parameter η.

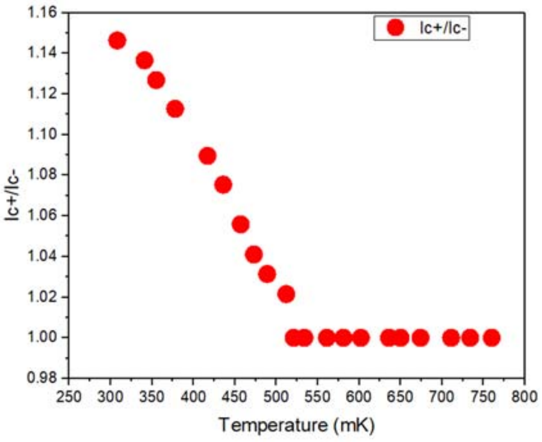
2) I think that underlying physical mechanisms behind the observation of low-field supercurrent rectification in this proximity-coupled superconducting array, especially whether intrinsic or extrinsic, remain unspecified. Can the authors perform any further measurements on the existing superconducting array device to narrow down the origin? For instance, field-angle/strength and temperature dependencies of the diode efficiency.

***Our response*:**

Field-angle dependence: We have performed a simple test in which a parallel magnetic field was applied in addition to the perpendicular one. This constant horizontal field was 3mT. This is 600 times larger than the perpendicular field we apply to the sample to maximize the rectification effect. The behavior of the sample did not change because of the application of the parallel field. Unfortunately, our setup is not equipped for the application of stronger parallel magnetic fields.

Field strength dependence: We presented results in our paper. It is demonstrated that at zero magnetic field the diode efficiency is basically zero. As we increase the perpendicular magnetic field the diode efficiency increases and reaches its maximum at -5uT.

Temperature dependence: We conducted several experiments to trace the asymmetry coefficient Ic+/Ic- vs. temperature (Fig.R4 left). The two V-I curves (Fig.R4 right) taken at a lower temperature and at a higher temperature show that the higher temperature makes the V-I curve essentially symmetric. Two of the curves are attached below. Ic+/Ic- decreases as the temperature increases, from Ic+/Ic- =1.15 (at 308mK) to Ic+/Ic- =1 (at 520mK). At higher temperatures the probability of a phase diffusion increases and because of this the critical current loses its sharpness. Such smearing leads to the suppression of the asymmetry also.



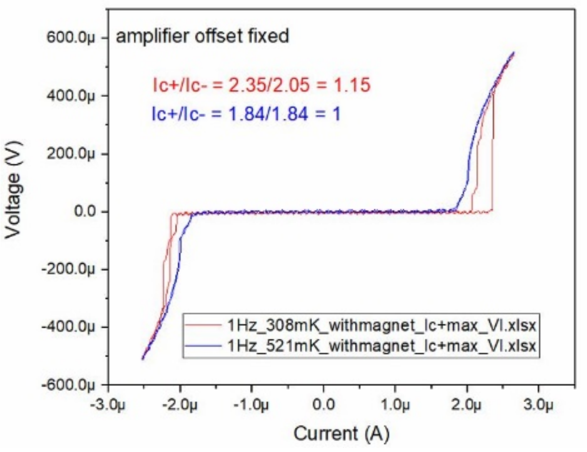


Fig. R4. Left: asymmetry coefficient Ic+/Ic- vs. temperature. Right: two V-I curves taken at a lower temperature and at a higher temperature. Lower temperature curve shows a clear symmetry while the higher temperature curve is symmetrical yet critical current is still strong.

3) Basically, the geometry (d ≈150 nm, w ≈ 1.15 µm, 40-nm-thick Bi0.8Sb1.2Te3, 30-nm-thick Nb) of superconducting array studied here is fixed. I am curious how crucial for the performance of critical current peak sharpening and diode efficiency the device geometry is.

Our response:

Due to various technical reasons, we could not produce a systematic experimental study samples with different geometries. This topic will be a subject of some future research. However, according to our model the geometry of the sample can have a strong impact on the device properties. Our model (see Eq.14 in the manuscript for example) predicts that the diode efficiency will become more pronounced with a larger critical current. To achieve that we can, for example, reduce *d* (the gap between the islands). The critical current should increase sharply if the gap between the Nb islands is reduced. But this approach will require more advanced e-beam lithography. Another approach, which is more straightforward, is to introduce artificial asymmetry into the array, which will make the parameter β larger.

The BST film thickness is not essential. The key property of this film is that it provides large sheet resistance $R_\square$. But this property is independent of the thickness since the current is predominantly carried by the surface states.

The Nb island thickness is not critical if it is thick enough to allow a strong superconducting proximity effect.

The peak sharpening is an interference effect between multiple parallel islands. In the general sense, it is analogous to an optical diffraction grating. In that case it is known that the diffraction peaks become sharper when the number of scattering groves is larger, provided that the spacing between the groves is constant. The implication for our case is that the peak sharpening (Fig. 7 in the manuscript) can be made stronger if the number of islands is increased, provided that the islands and the spacing between them is constant with sufficiently high precision. Sharper current peaks will lead to better sensitivity of the device to the magnetic field. In order to maximize the diode effect, the optimal geometry is a wide sample with strong asymmetry factor β, generated artificially by, for example, changing the spacing between the islands in a systematic manner.

Reviewers' comments:

Reviewer #2 (Remarks to the Author):

The authors answered my questions in an almost satisfactory fashion. I still have some doubts concerning my question 1 (see below), and I would like that the explanations for questions 2 and 4 are made explicit in the text, since I believe the reader will have my same doubts.

More in detail:

1) The explanation provided by the authors clarify how zero-bias resistance is measured (the authors write in their answer: "provided the VI curve is linear" and "... to get the zero-bias resistance". Still, I do not understand how full VI curves as the one in Fig.2b are measured (where the regime is clearly nonlinear).
Please, all the explanations must be implemented in the text (e.g. in the Methods section).

2) Concerning the answer to Question 2, the authors write that "In the revised version of the paper, we explain this point more clearly." Where? I do not see it.

3) OK.

4) Concerning the answer to Question 4, \beta is an ad hoc parameter introduced by the authors. Without an explanation the reader will not get what it is. Still, in the new version \beta is not defined (after Eq 11 it is just written "where \beta is a dimensionless asymmetry parameter,..."). Please define carefully what beta is, in the main text possibly.

Reviewer #3 (Remarks to the Author):

The new data and discussions have clearly improved the overall quality of this manuscript, and most of my concerns from the previous review have been properly addressed. Thus, I would recommend the revised manuscript for publication in Communications Physics with final/minor suggestions as follows.

1) I think that data of T-dependent Ic+/ Ic- signifying the relative contribution of ballistic surface-state versus diffusive bulk-state transport on the superconducting diode effect across T0 (defined as T where Ic+ and Ic- become identical) need to be presented in the main text.

2) I would recommend the authors add the following statement after the first paragraph on page 4. To identify the exact physical mechanisms for the observation of low-field supercurrent rectification in this proximity-coupled superconducting array, in particular, whether intrinsic or extrinsic, a further study is necessary.

3) It would be great if the authors could add a brief discussion about how to further improve the performance of critical current peak sharpening and diode efficiency via array geometry engineering for non-expert readers.

RESPONSE TO THE REFEREES

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Please, all the explanations must be implemented in the text (e.g. in the Methods section).

***Our response*:**

The current bias for the sample was set by taking an ac voltage, *U*, from a National Instrument data acquisition card NI-DAQ USB-6216 output and applying it to the sample connected in series with a known standard resistor R_{st}. Typically, R_{st} = 10 kOhm. The same NI- DAQ card was used to measure the voltage on the sample, *V*, and on the standard resistor V_{st}, at a high data acquisition rate of the order of 100000 points per second (Fig. R1). Then we plot the voltage on the sample, *V*, versus the bias current, calculate using Ohm's law, $I = V_{st}/R_{st}$, in the LabVIEW environment and thus get a complete V-I curve on the screen of the computer. Thus, we can take V-I curves with different current-bias amplitudes by adjusting the voltage U amplitude supplied by the NI-DAQ card. To get a complete V-I curve where the critical current is visible, the amplitude of the voltage *U* should be sufficiently large so the maximum bias current in the circuit exceeds the critical current of the sample.

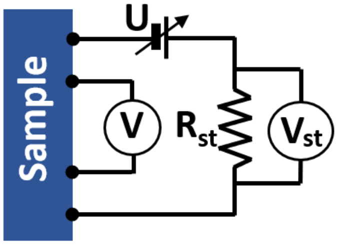

***Figure R1*. Schematic of the electrical circuit. An ac voltage *U* is supplied for a NI-DAQ data acquisition card. The bias current in the sample is set by a high-value standard resistor connected in series with the sample. The same NI-DAQ card is equipped with an analogue-digital converter. So, it is used to measure the voltage on the sample, *V*, and the voltage, V_{st}, on the standard resistor, R_{st}. The measurement is done in real time, as the bias current sweeps up and down. The resulting array of data points is plotted on the screen of the computer in the *V* versus *I* format, where the bias current is calculated as $I=V_{st}/R_{st}$.**

We have added this explanation about the V-I curve measurements in the Methods section of the manuscript.

2) Concerning the answer to Question 2, the authors write that "In the revised version of the paper, we explain this point more clearly." Where? I do not see it.

***Our response*:**

We have modified the caption of Fig.4 in the current version of the manuscript to provide this explanation: "Schematic of the sample, which illustrates possible distribution of fluxoids at different values of the magnetic field. The Nb squares are shown as blue. A unit cell is shown by a dashed line. Due to the proximity effect, the Nb squares induce superconductivity into the underlying topological insulator film (yellow). Since the entire array is globally superconducting, we expect that Josephson supercurrents are looping between neighbor Nb squares if a magnetic field is applied. a) The centers of supercurrent loops are shown, schematically, as swirls, for the case $f = 1$. Note that the edges of the Nb squares are colored green to illustrate the regions when the Meissner effect is incomplete. b) Josephson vortex centers are shown as swirls, for the case $f = 2$. In this case each junction has one quantum of the magnetic flux, so the Josephson vortex is located in the center of each junction between each pair of neighbor islands.".

3) OK.

4) Concerning the answer to Question 4, \beta is an ad hoc parameter introduced by the authors. Without an explanation the reader will not get what it is. Still, in the new version \beta is not defined (after Eq 11 it is just written "where \beta is a dimensionless asymmetry parameter,..."). Please define carefully what beta is, in the main text possibly.

***Our response*:**

The meaning of the parameter beta follows from Equation 11. In this sense Equation 11 in the manuscript defines the parameter beta. We explain this in detail in the main text, just below the Equation 11: "This formula provides the definition for the dimensionless asymmetry parameter, β. The meaning of this formula is that the critical current density increases linearly, from the value on the left side of the sample, where it was $j_C(0) = I_C/W\,[1 - \beta/2]$ to the maximum value on the right side of the sample, where it is $j_C(W) = I_C/W\,[1 + \beta/2]$. The parameter β is an unknown and an adjustable parameter, which can be extracted, indirectly, from the shape of the V-I curves, which is explained below, where we compare the model to the experiment."

Reviewer #3 (Remarks to the Author):

The new data and discussions have clearly improved the overall quality of this manuscript, and most of my concerns from the previous review have been properly addressed. Thus, I would recommend the revised manuscript for publication in Communications Physics with final/minor suggestions as follows.

1) I think that data of T-dependent Ic+/ Ic- signifying the relative contribution of ballistic surface-state versus diffusive bulk-state transport on the superconducting diode effect across T0 (defined as T where Ic+ and Ic- become identical) need to be presented in the main text.

***Our response*:**

We added T-dependent Ic+/Ic- as Figure 7 in the manuscript. The results and the discussion are presented in the manuscript as well.

2) I would recommend the authors add the following statement after the first paragraph on page 4. To identify the exact physical mechanisms for the observation of low-field supercurrent rectification in this proximity-coupled superconducting array, in particular, whether intrinsic or extrinsic, a further study is necessary.

***Our response*:**

Thank you. We added the statement to the end of the second paragraph on Page 5, just before the section "Model", after the diode effect has been properly introduced.

3) It would be great if the authors could add a brief discussion about how to further improve the performance of critical current peak sharpening and diode efficiency via array geometry engineering for non-expert readers.

***Our response*:**

We added a discussion at the end of the Conclusion section: "The critical current peak sharpening and the diode efficiency can be improved via array geometry engineering. The peak sharpening is an interference effect between multiple parallel islands. In the general sense, it is analogous to an optical diffraction grating. In that case, it is known that the diffraction peaks become sharper when the number of scattering groves is larger, provided that the spacing between the groves is constant. The implication for our case is that the peak sharpening can be made stronger if the number of islands is increased, provided that the islands and the spacing between them are constant with sufficiently high precision. Sharper current peaks will lead to better sensitivity of the device to the magnetic field. In order to maximize the diode effect, the optimal geometry is a wide sample with strong asymmetry factor generated artificially by, for example, changing the spacing between the

islands in a systematic manner, in order to produce a stronger gradient of the critical current.”.

REVIEWERS' COMMENTS:

Reviewer #2 (Remarks to the Author):

I have no further comments. In my opinion the manuscript is ready and it can be published.